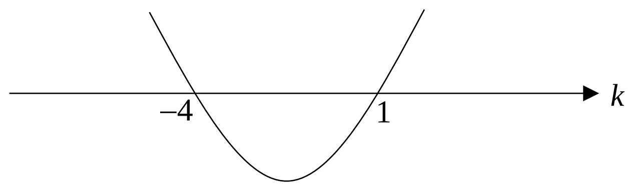
3E A Math WA2 2024 Marking Scheme

1	$(k+3)x^2 - 4x + k$	
	Coefficient of x^2 must be >0, $k+3>0$ $k>-3$	**M1**
	and Discriminant must be <0, $(-4)^2 - 4(k+3)(k) < 0$	**M1**
	$-4k^2 - 12k + 16 < 0$ $4k^2 + 12k - 16 > 0$ $k^2 + 3k - 4 > 0$ $(k+4)(k-1) > 0$	**M1**
	 $k < -4$ (rej $\because$ $k > -3$) or $k > 1$	**A1** – marks will not be awarded without rejection

2(a)	$\mathrm{f}(x)=x^3-9x-10$	
	$\mathrm{f}(-2)=(-2)^3-9(-2)-10=0$	**M1**
	By factor theorem, $(x+2)$ is a factor.	
	$\mathrm{f}(x)=(x+2)(x^2+Ax-5)$ $\quad\quad=x^3+(A+2)x^2+(2A-5)x-10$ By comparing coefficients of x^2 (or alternatively coefficients of x), $A+2=0$ $A=-2$ Or $2A-5=-9$ $A=-2$ --- Alternatively, $\begin{array}{r} x^2-2x-5 \\ x+2\overline{)\,x^3-9x-10} \\ -(x^3+2x^2) \\ \hline -2x^2-9x \\ -(-2x^2-4x) \\ \hline -5x-10 \\ -(-5x-10) \\ \hline 0 \end{array}$	**M1** – if student expand $(x+2)(Ax^2+Bx+C)$ the method mark will only be given for getting linear equation solving for unknown *B* in one of the forms shown in the marking scheme. For long division method, the method mark will be awarded if the student does the first 2 rounds of division correctly.
	$\mathrm{f}(x)=(x+2)(x^2-2x-5)$	**A1**

2(b)	$\mathrm{f}(x)=0$	
	$(x+2)(x^2-2x-5)=0$ $x+2=0$ or $x^2-2x-5=0$	
	$x=\dfrac{-(-2)\pm\sqrt{(-2)^2-4(1)(-5)}}{2(1)}$	**M1**
	$x=\dfrac{2\pm\sqrt{24}}{2}$	
	$x=-2$ or $x=-1.45$ (3s.f.) or $x=3.45$ (3s.f.)	**A1** (if student leaves in the simplest surd form $1\pm\sqrt{6}$, these are accepted too)

3	$\text{Base}=\dfrac{3+\sqrt{75}}{2\sqrt{108}-4}\times 2$	**M1** – correct formula applied using area of triangle
	$=\dfrac{3+\sqrt{75}}{\sqrt{108}-2}$	
	$=\dfrac{3+5\sqrt{3}}{6\sqrt{3}-2}$	**M1** – simplifying surds to simplest form
	$=\dfrac{3+5\sqrt{3}}{6\sqrt{3}-2}\times\dfrac{6\sqrt{3}+2}{6\sqrt{3}+2}$	**M1** - rationalising
	$=\dfrac{28\sqrt{3}+96}{104}$	
	$=\dfrac{24+7\sqrt{3}}{26}$ cm	**A1**

4(a)	$\sqrt{3-11x}=2x$	
	$3-11x=4x^2$	**M1**
	$4x^2+11x-3=0$	
	$(4x-1)(x+3)=0$	**M1** – alternatively, quadratic formula will be accepted too
	$x=\dfrac{1}{4}$ or $x=-3$ (rej $\because$ $\sqrt{3-11x}\geq 0$, thus $2x\geq 0$)	**A1** – marks will not be awarded if rejection with the correct reasons is not provided

4(b)	$\sqrt{15+12\sqrt{5}} = a+b\sqrt{5}$	
	$15+12\sqrt{5} = \left(a+b\sqrt{5}\right)\left(a+b\sqrt{5}\right)$	**M1** – squaring both sides of equation
	$15+12\sqrt{5} = a^2+2ab\sqrt{5}+5b^2$	
	$15+12\sqrt{3} = a^2+5b^2+2ab\sqrt{5}$	
	$2ab=12 \Rightarrow a=\dfrac{6}{b}$ --- (1) $15=a^2+5b^2$ --- (2)	**M1** – correctly identified simultaneous equations by comparing LHS and RHS
	Subs (1) into (2), $15=\left(\dfrac{6}{b}\right)^2+5b^2$ $15=\dfrac{36}{b^2}+5b^2$	**M1** – substitution **A1**

5	$y=x-k$ --- (1) $x^2+3y^2=9$ --- (2) Subs (1) into (2), $x^2+3(x-k)^2=9$	**M1**
	$x^2+3x^2-6kx+3k^2-9=0$ $4x^2-6kx+3k^2-9=0$ For the line to be tangent to the curve, Discriminant $=0$, $(-6k)^2-4(4)(3k^2-9)=0$	**M1**
	$36k^2-48k^2+144=0$ $12k^2-144=0$ $k^2-12=0$ $\left(k+\sqrt{12}\right)\left(k-\sqrt{12}\right)=0$	**M1** – simplify quadratic equation
	$k=\pm\sqrt{12}$ $=\pm 2\sqrt{3}$	**A1** – ± 3.46 (3s.f.) will be accepted too but surds not in the simplest form will not be accepted to be consistent with what we apply for topic on surds

6(a)	By Factor theorem, $\mathrm{f}\left(\frac{2}{3}\right)=0$ $3\left(\frac{2}{3}\right)^3+a\left(\frac{2}{3}\right)^2-5\left(\frac{2}{3}\right)+b=0$ $\frac{4}{9}a+b=\frac{22}{9}$ --- (1)	**M1** – applying factor theorem
	By Remainder theorem, $\mathrm{f}(-2)=-40$ $3(-2)^3+a(-2)^2-5(-2)+b=-40$ $4a+b=-26$ --- (2)	**M1** – applying remainder theorem
	$(2)-(1): \frac{32}{9}a=-\frac{256}{9}$	**M1** – elimination or substitution method to solve simultaneous equation
	$a=-8$ $b=6$	**A1** **A1**

6(b)	$\mathrm{f}(x)=3x^3-8x^2-5x+6$	
	By Remainder Theorem, $\mathrm{f}(-4)=3(-4)^3-8(-4)^2-5(-4)+6$	
	$=-294$	**B1** – if student uses long division, it will be accepted too.