1 A group of students was playing soccer on the school field. One student kicked a soccer ball. The path of the ball is modelled by the equation $y = 2 + x - \frac{49x^2}{800}$, where x metres is the horizontal distance of the ball from the student and y metres is the vertical height of the ball above the ground.

(a) Find the initial height of the ball above the ground. [1]

(b) Find the horizontal distance from the student when the ball next reached this same height. [2]

(c) The ball just passed over the top of a vertical pole located 8 metres from the student. Find the height of the pole. [2]

(d) Find the horizontal distance travelled by the ball when it hit the ground. [3]

2 A tech company, NexSg Labs, produces two types of tablets: the Pro and the Lite. In the month of January, its factory at Kaki Bukit produces 100 Pro tablets and 150 Lite tablets while another factory in Jurong produces 80 Pro tablets and 200 Lite tablets.

This information can be represented by matrix $\mathbf{A} = \begin{pmatrix} 100 & 150 \\ 80 & 200 \end{pmatrix}$, where matrix **A** represents the units produced at the two different factories in January.

The following month, February, its factory at Kaki Bukit produces 120 Pro tablets and 130 Lite tablets while the factory in Jurong produces 90 Pro tablets and 210 Lite tablets.

This information can be represented by matrix $\mathbf{B} = \begin{pmatrix} 120 & 130 \\ 90 & 210 \end{pmatrix}$, where matrix **B** represents the units produced at the two different factories in February.

(a) Find matrix **S**, where **S** = **A** + **B**. State what matrix **S** represents. [2]

(b) Given that **T** = 0.8**S**, where matrix **T** denotes the number of units shipped out in January and February, find the remaining inventory matrix **R**, where $\mathbf{R} = \mathbf{S} - \mathbf{T}$. [3]

The cost price of producing the Pro tablet is \$200 and the Lite tablet is \$150 while the selling price of the Pro tablet is \$450 and the Lite tablet is \$300.

(c) Represent this information in a 2×2 matrix **C**. [1]

(d) Evaluate the matrix **P** = **SC**. Hence, determine which factory generates the highest total revenue. [4]

Turn Over

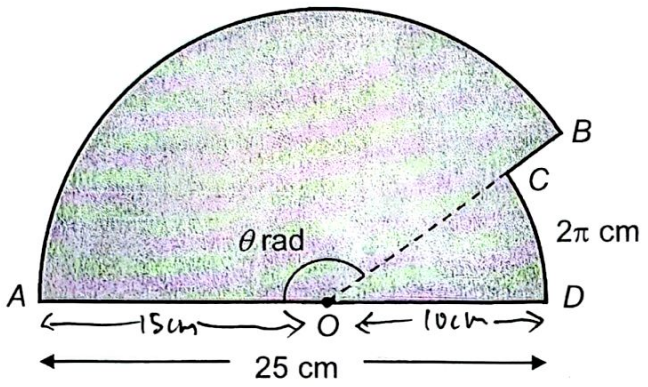
3 The diagram below shows the design of a company logo.

OCD and OAB are sectors of two circles with a common centre O.
AOD and OCB are straight lines.

Arc $CD = 2\pi$ cm, $AD = 25$ cm and angle $AOB = \theta$ radians.
The ratio of angle AOB : angle $COD = 4 : 1$.

θr = peri

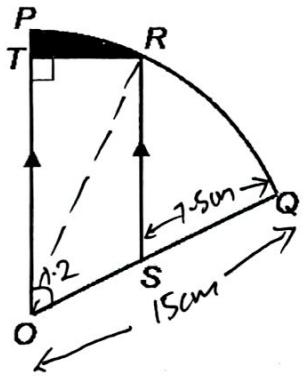
$\frac{1}{2}\theta r^2$ = area



(a) Show that length $OD = 10$ cm. [2]

(b) Find the perimeter of the design, express your answer in the form $(a\pi + b)$ cm where a and b are constants. [3]

4 In the diagram, OPQ is a sector of a circle, centre O and radius 15 cm. OP is parallel to SR. The area of sector OPQ is 135 cm^2 and S is the midpoint of OQ.
(The diagram is not drawn to scale.)



(a) Calculate the length of RT. [2]

(b) Calculate $\angle ROT$ in radians. [2]

(c) Calculate the area of the shaded region. [3]

End of Paper

Name ..... Subject .....

Q
M

C.T Group/C

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20/30

1(a). $y = 2 + x - \frac{49x^2}{800}$

$y = 1600 + 800x - 49x^2$

$y = -49x^2 + 800x + 1600$ ✗ 0

0m

Set $x = 0$

$y = 2$

∴ Initial height = 2m #

$y = 2 + x - \frac{49x^2}{800}$

Initial vertical distance of the ball

Sub $x = 0$

$y = 2$

1(b). $0 = -49x^2 + 800x + 1600$

$x = \frac{-(800) \pm \sqrt{800^2 - 4(-49)(1600)}}{2(-49)}$

$x = -1.801$ or $x = 18.13$

$= -1.80$ (to 3s.f.) (rejected) $= 18.1$ (to 3s.f.)

The distance is 18.1 m.

$2 + x - \frac{49x^2}{800} = 2$

$x - \frac{49x^2}{800} = 0$

$x\left(1 - \frac{49x}{800}\right) = 0$

$\frac{49x}{800} = 1$

$x = 0$ or $1 - \frac{49x}{800} = 0$

$\frac{49x}{800} = 1$

$x = \frac{800}{49}$ or $x = 16\frac{16}{49}$

1(c). $y = 2 + 8 - \frac{49(8^2)}{800}$

$y = 6.08$

The height is 6.08m

2

1(d). $y = -49x^2 + 800x + 1600$

$x = \frac{-800 \pm \sqrt{800^2 - 4(-49)(1600)}}{2(-49)}$

Set $y = 0$

1

4 $x = -1.801$ or $x = 18.13$

The distance is 18.1 m

1

2(a). $S = A + B$

$$= \begin{pmatrix} 100 & 150 \\ 80 & 200 \end{pmatrix} + \begin{pmatrix} 120 & 130 \\ 90 & 210 \end{pmatrix}$$

$$= \begin{pmatrix} 220 & 280 \\ 170 & 410 \end{pmatrix}$$ 2

Matrix S represents the total number of each tablets produced for January and February respectively.

2(b). $T = \begin{pmatrix} 0.8(220) & 0.8(280) \\ 0.8(170) & 0.8(410) \end{pmatrix}$

$$= \begin{pmatrix} 176 & 224 \\ 136 & 328 \end{pmatrix}$$ 1

$R = S - T$

$$= \begin{pmatrix} 220 & 280 \\ 170 & 410 \end{pmatrix} - \begin{pmatrix} 176 & 224 \\ 136 & 328 \end{pmatrix}$$

$$= \begin{pmatrix} 44 & 56 \\ 34 & 82 \end{pmatrix}$$ 2

2(c). $C = \begin{pmatrix} 200 & 450 \\ 150 & 300 \end{pmatrix}$ 1

2(d). $P = SC$

$$= \begin{pmatrix} 220 & 280 \\ 170 & 410 \end{pmatrix} \begin{pmatrix} 200 & 450 \\ 150 & 300 \end{pmatrix}$$

$$= \begin{pmatrix} 220(200) + 280(150) & 220(450) + 280(300) \\ 170(200) + 410(150) & 170(450) + 410(300) \end{pmatrix}$$

$$= \begin{pmatrix} 86000 & 183000 \\ 95500 & 149500 \end{pmatrix}$$ 3

9

183000 − 86000 = 97000

149500 − 95500 = 104000

∴ Jurong factory generates the highest total revenue ($149500)

3(a). ~~$\theta \times AO + \frac{\theta}{4} \times 2\pi = 25$~~

$\theta + \frac{\theta}{4} = \pi$

$\frac{5\theta}{4} = \pi$

$\theta = \frac{4\pi}{5}$

$\therefore COD = \frac{\pi}{5}$

$s = r\theta$

$2\pi = r\left(\frac{\pi}{5}\right)$

$r = 10$

$\therefore OD = 10\text{cm}$ (shown)

3(b). $2\pi \div 10 = 0.6283 = \frac{\pi}{5}$

$= 0.628$ (to 3 s.f.)

$0.628 \times 4 = 2.512$

$2.512 \times (25 - 10) = 37.68\text{ cm}$

$37.68 + 25 + 2\pi + (15 - 10)$

$= 73.96$

$= 74.0\text{ cm}$ (to 3 s.f.) $= (14\pi + 30)\text{ cm}$

4(a). Area of sector OPQ

$= 135\text{cm}^2$

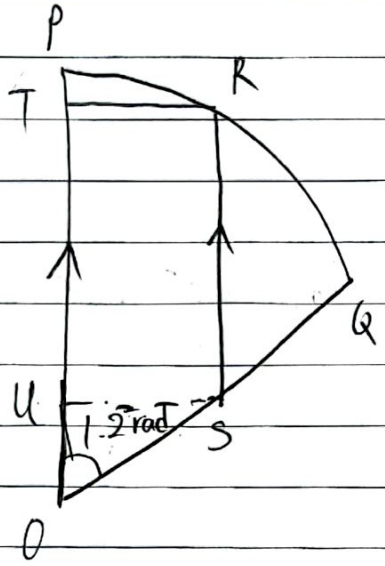
$= \frac{1}{2}\theta(15^2)$

$= 112.5\theta$

$\theta = 135 \div 112.5$

$= 1.2$

3+1 = 4



$US = TR$

$OS = 15 \div 2$

$= 7.5$

$\sin 1.2 = \frac{SO}{US}$

$\sin 1.2 = \frac{7.5}{US}$

$US = 7.5 \div \sin 1.2$

$RT = 8.047$ Check . . .

$RT = 8.05$ cm (to 3s.f.)

$\sin(1.2) = \frac{RT}{7.5}$

$RT = 6.99029$

$RT = 6.99$ cm (3s.f.)

∴ The length of RT is 8.05 cm.

4(b). In ΔTRO,

$\sin \angle RUT = \frac{8.05}{15}$

$\angle RUT = \sin^{-1}\left(\frac{8.05}{15}\right)$

$= 0.5664$

$= 0.566$ rad

$\sin \angle RUT = \frac{RT}{15} = \frac{6.99029}{15}$

AFT

$\angle RUT = \sin^{-1}\left(\frac{6.99029}{15}\right)$

$= 0.48479$

$= 0.485$ rad

4(c). Area of sector PRO

$= \frac{1}{2}(0.566)(15^2)$

$= 63.675$ cm^2

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In $\triangle TRO$,

$RO^2 = TR^2 + TO^2$ (Pythagoras' Theorem)

$15^2 = 8.05^2 + TO^2$

$TO^2 = \sqrt{15^2 - 8.05^2}$

$TO = 12.65$

$= 12.7$ cm (to 3 s.f.) 1

Area of $\triangle TRO$

$= 12.7 \times 8.05 \times \frac{1}{2}$

$= 51.1175\ cm^2$

The area of the shaded region

$= 63675 - 51.1175$

$= 12.55$

$= 12.6\ cm^2$ 1FT

(2)