



RAFFLES INSTITUTION
H2 Mathematics 9758
2025 Year 6 Term 3 Revision 4 (Zeta)
Topic(s): Calculus II
(Integration, Applications, DE) (Solutions)

Source of Question: TJC Prelim 9758/2022/01/Q1

1 Find

(a) $\int \sin^{-1} 2x \, dx,$ [3]

(b) $\int \frac{1}{x^2 - nx + n^2} \, dx,$ where n is a non-zero constant. [2]

Solution:

(a)	$\int \sin^{-1} 2x \, dx = \int 1 \cdot \sin^{-1} 2x \, dx$ $= x \sin^{-1} 2x - \int x \cdot \frac{2}{\sqrt{1-4x^2}} \, dx$ $= x \sin^{-1} 2x + \frac{1}{4} \int (-8x)(1-4x^2)^{-\frac{1}{2}} \, dx$ $= x \sin^{-1} 2x + \frac{1}{4} \cdot \frac{(1-4x^2)^{\frac{1}{2}}}{-\frac{1}{2}} + c$ $= x \sin^{-1} 2x + \frac{1}{2} \sqrt{1-4x^2} + c$ $u = \sin^{-1} 2x$ $\frac{du}{dx} = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2$ $v = x$	$\frac{dv}{dx} = 1$
(b)	$\int \frac{1}{x^2 - nx + n^2} \, dx = \int \frac{1}{\left(x - \frac{1}{2}n\right)^2 - \frac{1}{4}n^2 + n^2} \, dx$ $= \int \frac{1}{\left(x - \frac{1}{2}n\right)^2 + \frac{3}{4}n^2} \, dx$ $= \frac{2}{\sqrt{3}n} \tan^{-1} \left(\frac{x - \frac{1}{2}n}{\frac{\sqrt{3}}{2}n} \right) + c$ $= \frac{2}{\sqrt{3}n} \tan^{-1} \left(\frac{2x - n}{\sqrt{3}n} \right) + c$	

Source of Question: TMJC Prelim 9758/2022/01/Q2(modified)

2 (a) Find $\int \frac{1}{\sqrt{(1-x^2)} \sin^{-1} x} dx$. [2]

(b) Find $\int \frac{x-3}{x^2+2x-3} dx$. [3]

Solution:

(a)	$\int \frac{1}{\sqrt{(1-x^2)} \sin^{-1} x} dx = \int \frac{1}{\sqrt{(1-x^2)}} (\sin^{-1} x)^{-\frac{1}{2}} dx$ $= \frac{(\sin^{-1} x)^{\frac{1}{2}}}{\frac{1}{2}} + C$ $= 2\sqrt{\sin^{-1} x} + C$ The integrand is of the form $f'(x)(f(x))^n$, where $f(x) = \sin^{-1} x$ and $n = -\frac{1}{2}$.
(b)	$\int \frac{x-3}{x^2+2x-3} dx = \int \frac{x-3}{(x-1)(x+3)} dx$ $= -\frac{1}{2} \int \frac{1}{x-1} dx + \frac{3}{2} \int \frac{1}{x+3} dx$ $= -\frac{1}{2} \ln x-1 + \frac{3}{2} \ln x+3 + C$ Alternatively, $\int \frac{x-3}{x^2+2x-3} dx = \frac{1}{2} \int \frac{2x+2}{x^2+2x-3} dx - \int \frac{4}{x^2+2x-3} dx$ $= \frac{1}{2} \ln x^2+2x-3 - 4 \int \frac{1}{(x+1)^2-4} dx$ $= \frac{1}{2} \ln (x+3)(x-1) - \frac{4}{4} \ln \left \frac{x+1-2}{x+1+2} \right + C$ $= \frac{1}{2} \ln x+3 + \frac{1}{2} \ln x-1 - (\ln x-1 - \ln x+3) + C$ $= -\frac{1}{2} \ln x-1 + \frac{3}{2} \ln x+3 + C$

Source of Question: NYJC Prelim 9758/2022/02/Q5 (modified)

3 (a) (i) Differentiate $\sqrt{x^2 + 4}$ with respect to x . [1]

(ii) Hence find $\int \frac{x^3}{\sqrt{x^2 + 4}} dx$. [3]

(b) Find (i) $\int \sin^3 mx dx$, where m is a non-zero constant, [2]

(ii) $\int \cos^4 3x dx$. [4]

Solution:

(a)(i)	$\frac{d}{dx}(\sqrt{x^2 + 4}) = \frac{2x}{2\sqrt{x^2 + 4}} = \frac{x}{\sqrt{x^2 + 4}}$
(a)(ii)	$u = x^2 \quad \frac{dv}{dx} = \frac{x}{\sqrt{x^2 + 4}}$ $\frac{du}{dx} = 2x \quad v = \sqrt{x^2 + 4}$ $\int \frac{x^3}{\sqrt{x^2 + 4}} dx = x^2 \sqrt{x^2 + 4} - \int 2x \sqrt{x^2 + 4} dx$ $= x^2 \sqrt{x^2 + 4} - \frac{2(x^2 + 4)^{\frac{3}{2}}}{3} + C$
(b)(i)	$\int \sin^3 mx dx = \int \sin mx (\sin^2 mx) dx$ $= \int \sin mx (1 - \cos^2 mx) dx$ $= \int \sin mx - \sin mx \cos^2 mx dx$ $= -\frac{\cos mx}{m} + \frac{\cos^3 mx}{3m} + C$
(b)(ii)	$\int \cos^4 3x dx = \frac{1}{4} \int (1 + \cos 6x)^2 dx$ $= \int \frac{1}{4} + \frac{\cos 6x}{2} + \frac{1}{4} \cos^2 6x dx$ $= \int \frac{1}{4} + \frac{\cos 6x}{2} + \frac{1}{4} \left(\frac{1 + \cos 12x}{2} \right) dx$ $= \frac{1}{4}x + \frac{\sin 6x}{12} + \frac{1}{8}x + \frac{\sin 12x}{96} + C$ $= \frac{3}{8}x + \frac{\sin 6x}{12} + \frac{\sin 12x}{96} + C$

Source of Question: NJC Prelim 9758/2022/01/Q1

- 4 (i) Given that $I_n = \int_1^e x(\ln x)^n dx$ for $n \in \mathbb{Z}$, $n \geq 0$, show that

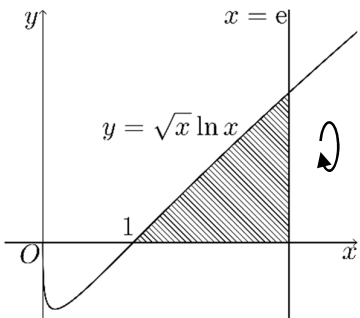
$$I_n = \frac{e^2}{2} - \frac{n}{2} I_{n-1}$$

for all $n \in \mathbb{Z}^+$.

[2]

- (ii) Find the exact volume of the solid generated when the region bounded by the curve $y = \sqrt{x} \ln x$, the x -axis and the line $x = e$ is rotated completely about the x -axis. [3]

Solution:

(i)	$ \begin{aligned} I_n &= \int_1^e x(\ln x)^n dx \\ &= \left[\frac{x^2}{2} (\ln x)^n \right]_1^e - \int_1^e \frac{x^2}{2} \cdot n(\ln x)^{n-1} \frac{1}{x} dx \\ &= \frac{e^2}{2} - \frac{n}{2} \int_1^e x(\ln x)^{n-1} dx \\ &= \frac{e^2}{2} - \frac{n}{2} I_{n-1} \text{ (shown)} \end{aligned} $
(ii)	Volume $ \begin{aligned} &= \pi \int_1^e x(\ln x)^2 dx \\ &= \pi I_2 \\ &= \pi \left(\frac{e^2}{2} - \frac{2}{2} I_1 \right) \\ &= \pi \left[\frac{e^2}{2} - \left(\frac{e^2}{2} - \frac{1}{2} I_0 \right) \right] \\ &= \frac{\pi}{2} I_0 = \frac{\pi}{2} \left[\frac{x^2}{2} \right]_1^e \\ &= \frac{\pi}{4} (e^2 - 1) \text{ units}^3 \end{aligned} $ 

Alternatively:

Volume

$$= \pi \int_1^e x (\ln x)^2 dx$$

$$= \pi \left\{ \left[\frac{x^2}{2} (\ln x)^2 \right]_1^e - \int_1^e \frac{x^2}{2} \cdot 2(\ln x) \frac{1}{x} dx \right\}$$

$$= \pi \left[\frac{e^2}{2} - \int_1^e x (\ln x) dx \right]$$

$$= \pi \left\{ \frac{e^2}{2} - \left[\frac{x^2}{2} (\ln x) \right]_1^e + \int_1^e \frac{x^2}{2} \cdot \frac{1}{x} dx \right\}$$

$$= \pi \left\{ \frac{e^2}{2} - \frac{e^2}{2} + \left[\frac{x^2}{4} \right]_1^e \right\}$$

$$= \frac{\pi}{4} (e^2 - 1) \text{ units}^3$$

$$u = (\ln x)^2$$

$$\frac{dv}{dx} = x$$

$$\frac{du}{dx} = 2(\ln x) \frac{1}{x}$$

$$v = \frac{x^2}{2}$$

$$u = \ln x$$

$$\frac{dv}{dx} = x$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$v = \frac{x^2}{2}$$

Source of Question: ACJC Prelim 9758/2022/01/Q4 (modified)

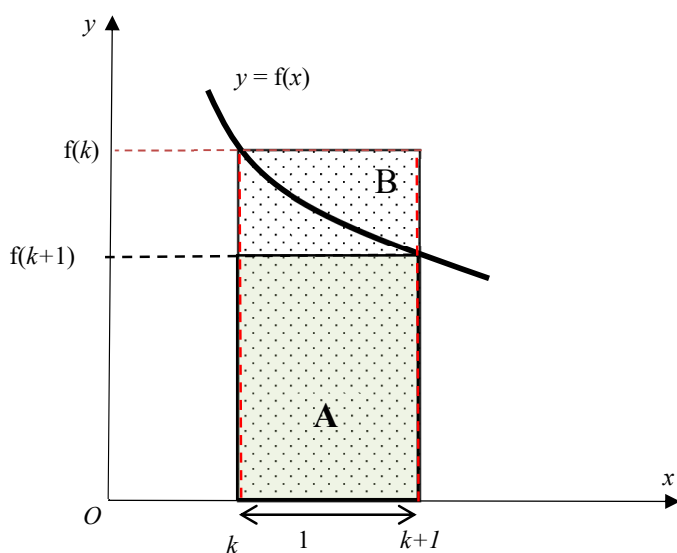
- 5 (a)** The continuous function $f(x)$, where $f(x) > 0$, is strictly decreasing for $x \geq 1$.
 Sketch the curve $y = f(x)$ for $k < x < k+1$ where k is an integer and $k \geq 1$.
 By comparing the areas of appropriate rectangles and the area under the curve $y = f(x)$,
 show that for any integer $k \geq 1$,

$$f(k+1) < \int_k^{k+1} f(x) dx < f(k). \quad [2]$$

- (b)** Hence show that

(i) $\ln 10 < 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{9} < 1 + \ln 9.$ [3]

(ii) $\ln((n-1)!) < n \ln n - n + 1 < \ln(n!)$ for $n > 1$. [4]

Solution:**(a)**

$$\text{Area under curve} = \int_k^{k+1} f(x) dx$$

$$\text{Area of rectangle A} = f(k+1) \times 1$$

$$\text{Area of rectangle B} = f(k) \times 1$$

As seen from diagram:

$$f(k+1) < \int_k^{k+1} f(x) dx < f(k)$$

(b)(i)	Let $f(x) = \frac{1}{x}$. $\int_1^{10} \frac{1}{x} dx = \int_1^2 \frac{1}{x} dx + \int_2^3 \frac{1}{x} dx + \int_3^4 \frac{1}{x} dx + \dots + \int_9^{10} \frac{1}{x} dx$ $< f(1) + f(2) + f(3) + \dots + f(9)$ $= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{9}$ $\int_1^9 \frac{1}{x} dx = \int_1^2 \frac{1}{x} dx + \int_2^3 \frac{1}{x} dx + \int_3^4 \frac{1}{x} dx + \dots + \int_8^9 \frac{1}{x} dx$ $> f(2) + f(3) + f(4) + \dots + f(9)$ $= \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{9} = \sum_{k=1}^9 \frac{1}{k} - 1$ $\Rightarrow 1 + \int_1^9 \frac{1}{x} dx > 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{9}$ Combining both inequalities we have $\int_1^{10} \frac{1}{x} dx < \sum_{k=1}^9 \frac{1}{k} < 1 + \int_1^9 \frac{1}{x} dx$ $[\ln x]_1^{10} < 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{9} < 1 + [\ln x]_1^9$ $\ln 10 < 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{9} < 1 + \ln 9$
(b)(ii)	We consider the function $f(x) = \ln x$. This function is strictly increasing, so the inequality in (a) changes to $f(k) < \int_k^{k+1} f(x) dx < f(k+1)$. We thus have $\sum_{k=1}^{n-1} f(k) < \sum_{k=1}^{n-1} \int_k^{k+1} f(x) dx < \sum_{k=1}^{n-1} f(k+1)$ $\Rightarrow \ln 1 + \ln 2 + \dots + \ln(n-1) < \int_1^2 f(x) dx + \dots + \int_{n-1}^n f(x) dx < \ln 2 + \dots + \ln n$ $\Rightarrow \ln((n-1)!) < \int_1^n \ln x dx < \ln(n!)$ $\Rightarrow \ln((n-1)!) < [x \ln x - x]_1^n < \ln(n!)$ $\Rightarrow \ln((n-1)!) < n \ln n - n + 1 < \ln(n!)$

Source of Question: VJC Prelim 9758/2022/01/Q9

6 The curve C has equation $\frac{1}{3}x^2 + y^2 - 2y = 0$.

(i) Sketch C . [2]

(ii) Use the substitution $x = p \sin \theta$ to show that

$$\int_0^p \sqrt{p^2 - x^2} \, dx = \frac{p^2 \pi}{4},$$

where p is a positive constant. [4]

(iii) The region R is bounded by C , the line $x = \sqrt{3}$ and the x -axis.

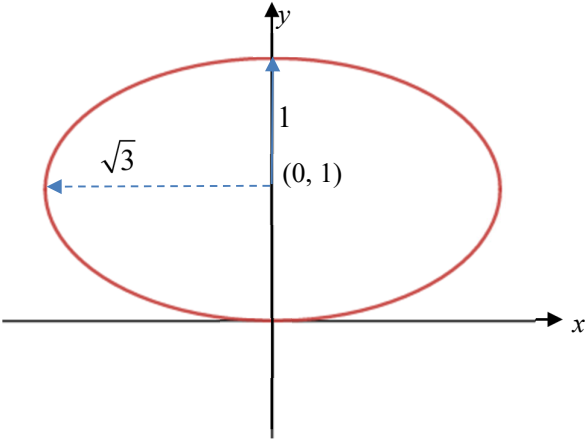
Find the exact area of R . [3]

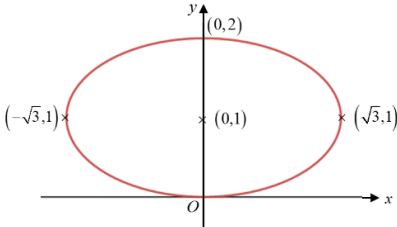
(iv) R is rotated completely about the y -axis.

Find the exact volume of the solid obtained. [3]

(v) Describe a pair of transformations which transforms the graph of C onto the graph of $x^2 + y^2 = 1$. [2]

Solution:

(i) $\frac{1}{3}x^2 + y^2 - 2y = 0$ $\frac{1}{3}x^2 + (y-1)^2 - 1 = 0$ $\frac{1}{3}x^2 + (y-1)^2 = 1$ This is an ellipse centered at $(0, 1)$ with semi-minor axis 1 and semi-major axis $\sqrt{3}$.	
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(ii)	$\int_0^p \sqrt{p^2 - x^2} \, dx$ $= \int_0^{\frac{\pi}{2}} \sqrt{p^2 - p^2 \sin^2 \theta} (p \cos \theta) \, d\theta$ $= p^2 \int_0^{\frac{\pi}{2}} \sqrt{\cos^2 \theta} (\cos \theta) \, d\theta$ $= p^2 \int_0^{\frac{\pi}{2}} \cos^2 \theta \, d\theta$ $= p^2 \int_0^{\frac{\pi}{2}} \left(\frac{\cos 2\theta + 1}{2} \right) d\theta$ $= p^2 \left[\frac{\sin 2\theta}{4} + \frac{\theta}{2} \right]_0^{\frac{\pi}{2}}$ $= p^2 \left(\frac{\sin 2\left(\frac{\pi}{2}\right)}{4} + \frac{\frac{\pi}{2}}{2} \right) = \frac{p^2 \pi}{4}$	$x = p \sin \theta$ $dx = p \cos \theta \, d\theta$ When $x = 0$, $\theta = 0$ When $x = p$, $\theta = \frac{\pi}{2}$
(iii)	$\frac{1}{3}x^2 + (y-1)^2 = 1$ $y = 1 - \sqrt{1 - \frac{1}{3}x^2} \quad (\because y < 1)$ $\text{Area} = \int_0^{\sqrt{3}} \left(1 - \sqrt{1 - \frac{1}{3}x^2} \right) dx$ $= \int_0^{\sqrt{3}} 1 \, dx - \frac{1}{\sqrt{3}} \int_0^{\sqrt{3}} \sqrt{3 - x^2} \, dx$ $= \sqrt{3} - \frac{1}{\sqrt{3}} \left(\frac{3\pi}{4} \right) \quad \text{using (ii)}$ $= \sqrt{3} - \frac{\sqrt{3}\pi}{4}$	
(iv)	$\text{Volume} = \pi (\sqrt{3})^2 (1) - \pi \int_0^1 x^2 \, dy$ $= 3\pi - \pi \int_0^1 (6y - 3y^2) \, dy$ $= 3\pi - \pi [3y^2 - y^3]_0^1$ $= \pi$	
(v)	Translate 1 units in the negative y -direction. Stretch the resultant curve by a factor of $\frac{1}{\sqrt{3}}$ parallel to the x -axis.	

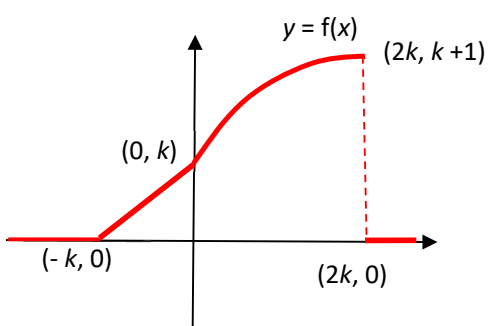
Source of Question: CJC Prelim 9758/2022/02/Q5

7 A curve C_1 has equation $y = f(x)$, where $k > 1$ and

$$f(x) = \begin{cases} x + k & \text{for } -k < x \leq 0, \\ (k+1) - \frac{(2k-x)^2}{4k^2} & \text{for } 0 < x \leq 2k, \\ 0 & \text{otherwise.} \end{cases}$$

- (i) Sketch the curve C_1 . [2]
 (ii) The region R is bounded by the curve C_1 , the line $x = 1$ and the x -axis. Find the exact area of R , leaving your answer in terms of k . [6]
 (iii) On the same diagram on part (i), sketch the curve C_2 with equation $y = x^2 \tan^{-1} x$. [1]
 (iv) The region S is bounded by the curves C_1 and C_2 , and the x -axis. Given that $k = 3$, determine the volume of solid of revolution formed when S is rotated 2π radians about the x -axis, leaving your answer to 2 decimal places. [3]

Solution:

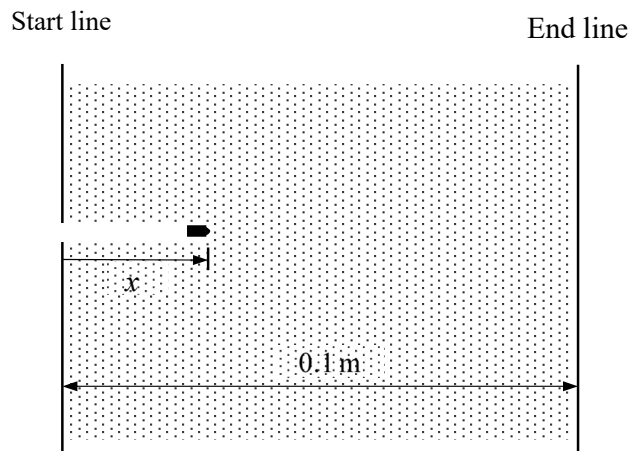
(i)	
(ii)	Required Area $ \begin{aligned} &= \int_{-k}^0 (x+k) dx + \int_0^1 \left((k+1) - \frac{(2k-x)^2}{4k^2} \right) dx \\ &= \left[\frac{x^2}{2} + kx \right]_{-k}^0 + \left[(k+1)x - \frac{(2k-x)^3}{4k^2(3)(-1)} \right]_0^1 \\ &= 0 - \left[\frac{k^2}{2} - k^2 \right] + \left[(k+1) - \frac{(2k-1)^3}{4k^2(3)(-1)} \right] - \left[-\frac{(2k)^3}{4k^2(3)(-1)} \right] \\ &= \frac{k^2}{2} + \frac{k}{3} + 1 + \frac{(2k-1)^3}{12k^2} \end{aligned} $ OR Required Area $ \begin{aligned} &= \frac{1}{2}(k)(k) + \int_0^1 \left((k+1) - \frac{(2k-x)^2}{4k^2} \right) dx \\ &= \frac{k^2}{2} + \left[(k+1)x - \frac{(2k-x)^3}{4k^2(3)(-1)} \right]_0^1 \end{aligned} $

	$= \frac{k^2}{2} + \left[(k+1) - \frac{(2k-1)^3}{4k^2(3)(-1)} \right] - \left[-\frac{(2k)^3}{4k^2(3)(-1)} \right]$ $= \frac{k^2}{2} + \frac{k}{3} + 1 + \frac{(2k-1)^3}{12k^2}$
(iii)	
(iv)	Point of intersection between C_1 and C_2 is at $x = 1.81453$ Volume of S $= \pi \left\{ \int_{-3}^0 (x+3)^2 dx + \int_0^{1.81453} \left[(3+1) - \frac{(2(3)-x)^2}{4(3)^2} \right]^2 dx - \int_0^{1.81453} [x^2 \tan^{-1}(x)]^2 dx \right\}$ $= 77.54 \text{ (2 d.p.) unit}^3$ OR Volume of S $= \frac{1}{3} \pi (3)^2 (3) + \pi \left\{ \int_0^{1.81453} \left[(3+1) - \frac{(2(3)-x)^2}{4(3)^2} \right]^2 dx - \int_0^{1.81453} [x^2 \tan^{-1}(x)]^2 dx \right\}$ $= 77.54 \text{ (2 d.p.) unit}^3$

Source of Question: DHS Prelim 9758/2021/01/Q11

- 8 A company conducts experiments to investigate the resistive power of armoured plates of different materials against a bullet fired from a rifle. The plates used for the experiment have a consistent thickness of 0.1 m. Once the plates are positioned, a bullet is fired at the start line of the plate with a fixed velocity of 100 ms^{-1} .

For a duration of $t \text{ s}$ after it is fired, the bullet is $x \text{ m}$ away from the start line and travels at a velocity $v \text{ ms}^{-1}$. The setup is shown below.



- (i) When a plate of material P is used, v and x are modelled by the following differential equations:

$$(A) \quad \frac{dx}{dt} = v,$$

$$(B) \quad \frac{d^2x}{dt^2} + \beta \left(\frac{dx}{dt} \right)^2 = 0,$$

where β is a positive constant.

- (a) By substituting equation (A), show that equation (B) can be written as $\frac{dv}{dt} = -\beta v^2$. [1]

- (b) Find v in terms of β and t and hence find x in terms of β and t . [5]

- (c) Given that it is found that the bullet exits the end line of the plate with a velocity of 80 ms^{-1} , find the time taken for the bullet to pass through the plate of material P , leaving your answer to 5 decimal places. [3]

- (ii) The company develops another plate made of a new material Q . When the experiment is conducted with a plate of material Q , it is found that the bullet is slowed down in such a way that $\frac{d^2x}{dt^2}$ is inversely proportional to e^{10000t} . It is given that $\frac{dx}{dt} = 100$ and $\frac{d^2x}{dt^2} = -10^6$ when $t = 0$. Write down a differential equation to model the bullet as it

travels through the plate of material Q . Solve this differentiation equation to find x in terms of t for this model. [4]

- (iii) The company claims that material Q provides better protection against bullets than material P . Explain, with justification, whether the company's claim is valid. [1]

Solution:

(i)(a)	$\frac{d^2x}{dt^2} + \beta \left(\frac{dx}{dt} \right)^2 = 0$ Let $v = \frac{dx}{dt}$, $\frac{dv}{dt} = \frac{d^2x}{dt^2}$, we have $\frac{dv}{dt} + \beta v^2 = 0$ $\frac{dv}{dt} = -\beta v^2$
(i)(b)	$v^{-2} \frac{dv}{dt} = -\beta$ $\int v^{-2} dv = -\beta \int 1 dt$ $-\frac{1}{v} = -\beta t + C$ When $t = 0$, $v = 100$, $C = -\frac{1}{100}$. $-\frac{1}{v} = -\beta t - \frac{1}{100}$ $\frac{1}{v} = \beta t + \frac{1}{100}$ $v = \frac{1}{\beta t + 0.01} \quad \text{--- (1)}$ $\frac{dx}{dt} = \frac{1}{\beta t + 0.01}$ $x = \int \frac{1}{\beta t + 0.01} dt$ $x = \frac{1}{\beta} \ln(\beta t + 0.01) + D \quad (\text{since } \beta > 0)$ When $t = 0$, $x = 0$, $0 = \frac{1}{\beta} \ln(0.01) + D$ $D = -\frac{1}{\beta} \ln(0.01)$

	$x = \frac{1}{\beta} \ln(\beta t + 0.01) - \frac{1}{\beta} \ln(0.01)$ $x = \frac{1}{\beta} \left[\ln\left(\frac{\beta t + 0.01}{0.01}\right) \right]$ $x = \frac{1}{\beta} \ln(100\beta t + 1) \quad \text{--- (2)}$
(i)(c)	Let $t = T$ when the bullet leaves plate of material X. When $t = T$, $v = 80$, from (1), $80 = \frac{1}{\beta T + 0.01}$ $\beta T + 0.01 = \frac{1}{80}$ $\beta T = \frac{1}{400} = 0.0025$ $\beta = \frac{1}{400T}$ When $t = T$, $x = 0.1$ and $\beta = \frac{1}{400T}$, from (2), $0.1 = \frac{1}{\left(\frac{1}{400T}\right)} \ln\left(100\left(\frac{1}{400T}\right)T + 1\right)$ $0.1 = 400T \ln\left(\frac{5}{4}\right)$ $T = \frac{0.1}{400 \ln(1.25)}$ $= 0.00112035502$ $= 0.00112 \text{ seconds}$
(ii)	$\frac{d^2x}{dt^2} = \frac{k}{e^{10000t}} = k e^{-10^4 t}$ When $t = 0$, $\frac{d^2x}{dt^2} = -10^6$, $-10^6 = k e^{-10^4(0)}$ $k = -10^6$ Hence, $\frac{d^2x}{dt^2} = -10^6 e^{-10^4 t}$ $\frac{dx}{dt} = \frac{-10^6}{-10^4} e^{-10^4 t} + E$ $= 100 e^{-10^4 t} + E$

	When $t = 0$, $\frac{dx}{dt} = 100$, $100 = 100e^{-10^4(0)} + E$ $E = 0$ $\frac{dx}{dt} = 100e^{-10^4 t} \quad \text{--- (3)}$ $x = -\frac{100}{10^4} e^{-10^4 t} + F$ $x = -0.01e^{-10^4 t} + F$ When $t = 0$, $x = 0$, $0 = -0.01e^{-10^4(0)} + F$ $F = 0.01$ $x = 0.01 - 0.01e^{-10^4 t}$ $x = 0.01(1 - e^{-10^4 t}) \quad \text{--- (4)}$
(iii)	Method 1 When $t = 0.0011204$, from (4), $x = 0.01(1 - e^{-10^4(0.0011204)})$ $\approx 0.001 < 0.1$ In the same amount of time it takes for the bullet to penetrate the entire plate of material P of 0.1m, it only travels 0.01m through plate of material Q. Hence, the company's claim is valid. Method 2 From (4), as $t \rightarrow \infty$, $x \rightarrow 0.01$. This means the bullet will not be able to penetrate the plate of material Q which is 0.1m thick, unlike what it has done to material P. Hence, the company's claim is valid. Method 3 When $t = 0.0011204$, from (3), $\frac{dx}{dt} = 100e^{-10^4(0.0011204)} = 0.00136 \approx 0.$ In the same amount of time it takes for the bullet to penetrate the entire plate of material P to exit with a velocity of 80 ms^{-1}, the bullet that travels through the plate of material Q has its velocity reduced to almost 0 ms^{-1}. Hence, the company's claim is valid.