Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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MA303.1.1 – Gradient-Point Form

At the end of this unit, you should be able to

- **obtain equation of a straight-line using the gradient-point form**

Prior to this unit you were already exposed to some concepts of coordinate geometry, such as midpoint, distance and gradient. You would also remember the gradient-intercept form of an equation of a straight line.

P1. Find the equation of the line joining the points $(4,-2)$ and $(-1,5)$, expressing it in the form $ax+by=c$ where a, b and c are integers and are co-prime.

Gradient $= \frac{5-(-2)}{-1-4}$

$= -\frac{7}{5}$

Equation of line : $y = -\frac{7}{5}x + c$ — ①

Sub $(-1,5)$ into ①

$5 = -\frac{7}{5}(-1) + c$

$c = \frac{18}{5}$

$\therefore y = -\frac{7}{5}x + \frac{18}{5}$

$5y = -7x + 18$

$\therefore 7x + 5y = 18$

E1. A line l which is parallel to the y-axis passes through the intersection of the straight lines $y = 2x-1$ and $x-3y=5$. Determine the equation of the line l.

Let $(y = 2x-1) = (x-3y=5)$

$y = 2x-1$ — ①

$3y = x-5$ — ②

Multiply ① by 3.

$3y = 6x-3$ — ③

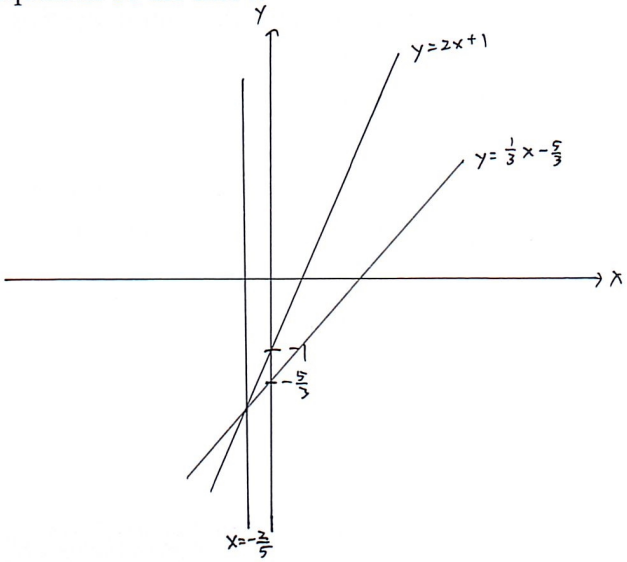
② = ③

$\therefore x-5 = 6x-3$

$5x = -2$

$x = -\frac{2}{5}$

Equation of line l: $x = -\frac{2}{5}$



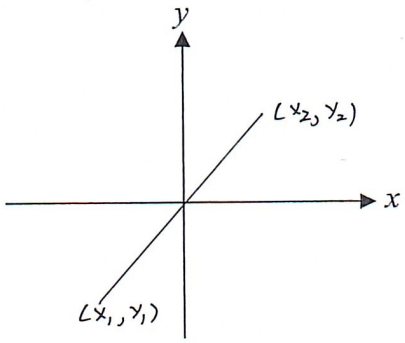
Alternative form of equation of line

Previously we had studied the form of an equation of a line when the gradient and the y-intercept are both known. We now look at another form of the equation, when the coordinates of two points which lie on the line are given.

E2. Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$.

(a) Determine the gradient of the line passing through AB.

Gradient : $\dfrac{y_2 - y_1}{x_2 - x_1}$



(b) If C is a point on the line AB such that the coordinates of C is (x, y), find the gradient of the line in terms of x and y, using the coordinates of A and C.

Gradient = $\dfrac{y - y_1}{x - x_1}$

(c) Is the gradient of AB the same as that of AC? Hence or otherwise show that $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Yes the gradient is the same as they are on the same line.

$(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1)$

(d) If the gradient of AB is denoted by m, show further that

$$y - y_1 = m(x - x_1).$$

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$

$\therefore m = \frac{y - y_1}{x - x_1}$

$m(x - x_1) = y - y_1$ (shown)

Suppose A and B are two points on a line, such that their coordinates are (x_1, y_1) and (x_2, y_2) respectively. Then the equation of the line is given by

$$y - y_1 = m(x - x_1)$$

where $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

P2. Determine the equation of the line passing through

(a) $A(2, 5)$ and $B(-3, -2)$;

Gradient of $AB = \frac{-2-5}{-3-2}$

$= \frac{-7}{-5}$

$= \frac{7}{5}$

$\therefore AB: y = \frac{7}{5}x + k$ —— ①

Sub (2, 5) into ①

$\therefore 5 = \frac{7}{5} \times 2 + k$

$k = \frac{11}{5}$

Equation of line: $y = \frac{7}{5}x + \frac{11}{5}$

(b) $A(a, 0)$ and $B(2a, 3a)$.

Gradient of $AB = \frac{3a - 0}{2a - a}$

$= \frac{3a}{a}$

$= 3$

$\therefore AB: y = 3x + k$ —— ①

Sub $(a, 0)$ into ①

$\therefore 0 = 3a + k$

$k = -3a$

Equation of line: $y = 3x - 3a$

E3. Find the equation of the line passing through $(3, 3)$ and parallel to another line passing through $(1, 3)$ and $(4, 12)$.

Let the equation of other line be $y=mx+c$.

$m = \frac{12-3}{4-1}$

$= \frac{9}{3}$

$= 3$

$\therefore y = 3x+c$ —— ①

Sub $(1, 3)$ into ①

$\therefore 3 = 3+c$

$c = 0$

$\therefore y = 3x$

As the line passing through $(3, 3)$ is parallel to the other line, their gradient is the same.

$\therefore$ Equation of line: $y = 3x+k$ —— ②

Sub $(3, 3)$ into ②

$3 = 3\times 3 + k$

$k = -6$

$\therefore y = 3x-6$

P3. The line $y = mx + c$ passes through the point $(1, -3)$ and is parallel to another line which passes through $(-2, 0)$ and $(3, -4)$. Find the values of m and c.

Let the equation of other line be $y=mx+c$

$m = \frac{-4-0}{3-(-2)}$

$= \frac{-4}{5}$

$= -\frac{4}{5}$

$\therefore y = -\frac{4}{5}x+c$ —— ①

Sub $(3, -4)$ into ①

$-4 = -\frac{4}{5}(3)+c$

$c = -\frac{8}{5}$

$\therefore y = -\frac{4}{5}x - \frac{8}{5}$

Since the two lines are parallel, their gradient is the same.

$\therefore$ Equation of line $= y = -\frac{4}{5}x+k$ —— ②

Sub $(1, -3)$ into ②

$-3 = -\frac{4}{5}+k$

$k = -\frac{11}{5}$

$\therefore y = -\frac{4}{5}x - \frac{11}{5}$

$m = -\frac{4}{5}$

$c = -\frac{11}{5}$

P4. Find the coordinates of the vertices formed by the following three lines.

$$y=2x-3,\ y+2x+3=0,\ x+2y=1$$

$y=2x-3$ —— ①

$y=-2x-3$ —— ②

$y=-\frac{1}{2}x+\frac{1}{2}$ —— ③

Case 1: when ① meets ②

① = ②

$2x-3=-2x-3$

$4x=0$

$x=0$

$y=-3$

Vertice : $(0,-3)$

Case 2 : when ① meets ③

① = ③

$2x-3=-\frac{1}{2}x+\frac{1}{2}$

$\frac{5}{2}x=\frac{7}{2}$

$x=\frac{7}{5}$

$y=-\frac{1}{5}$

Vertice: $(\frac{7}{5}, -\frac{1}{5})$

Case 3 : when ② meets ③

② = ③

$-2x-3=-\frac{1}{2}x+\frac{1}{2}$

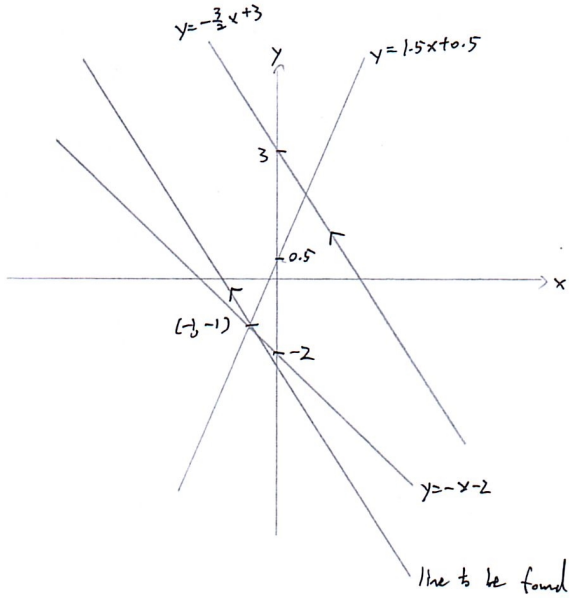
$\frac{3}{2}x=-\frac{7}{2}$

$x=-\frac{7}{3}$

$y=\frac{5}{3}$

Vertice : $(-\frac{7}{3}, \frac{5}{3})$

P5. Find the equation of the line parallel to the line $3x+2y-6=0$ and passing through the point where $x+y+2=0$ intersects $3x-2y+1=0$. Interpret the question using a sketch, including the line required.



$x+y+2=0$

$y=-x-2$ —— ①

$3x-2y+1=0$

$2y=3x+1$

$y=1.5x+0.5$ —— ②

$3x+2y-6=0$

$2y=-3x+6$

$y=-\frac{3}{2}x+3$ —— ③

When ① and ② meet, ① = ②

$-x-2=1.5x+0.5$

$2.5x=-2.5$

$x=-1$

$y=-1$

They meet at $(-1,-1)$

Gradient of the line $=-\frac{3}{2}$ as it is parallel to ③

$\therefore y=-\frac{3}{2}x+c$ —— ④

Sub $(-1,-1)$ into ④

$-1=-\frac{3}{2}(-1)+c$

$c=-\frac{5}{2}$

Equation of line: $y=-\frac{3}{2}x-\frac{5}{2}$