

Class	Register Number	Name
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南洋女子中學校
NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2020
Secondary 3

INTEGRATED MATHEMATICS 2

2 hours

Monday

05 October 2020

08 45 – 10 45

READ THESE INSTRUCTIONS FIRST

1. Write your name, register number and class on all the work you hand in.
2. Answer **all** questions.
3. Write your answers and working on the separate writing paper provided, unless otherwise stated.
4. Write in dark blue or black ink on both sides of the paper.
5. You may use a HB pencil for any diagrams or graphs.
6. Do not use staples, paper clips, glue or correction tape/fluid.
7. Omission of essential working will result in loss of marks.
8. The use of an electronic calculator is expected, where appropriate.
9. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place. For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .
10. At the end of the examination, fasten all your work securely together.
11. The number of marks is given in brackets [] at the end of each question or part question.
12. The total number of marks for this paper is 80.

Setter: E.Liow	This document consists of 5 printed pages, including this cover page. NANYANG GIRLS' HIGH SCHOOL	[Turn over
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Mathematical Formulae

1. ALGEBRA

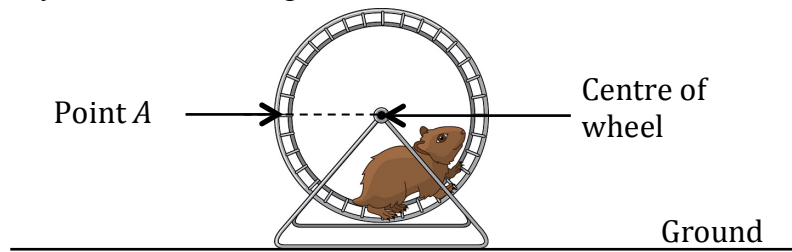
Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- 1 (a) Express $\frac{2\sqrt{3}-5}{5-\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are rational numbers. [3]
- (b) Given that $\lg a + \lg x = b$. Express x in terms of a and b . [2]
- (c) Solve each of the following equations.
- (i) $\log_2(x+1) - 2\log_4 x = 3$ [4]
- (ii) $6(5^{2x}) - 11(5^x) - 2 = 0$ [3]
- 2 Express in partial fractions $\frac{4x}{(x^2+4)(x-2)}$. [4]
- 3 Due to the Covid-19 situation, the number of customers, N , of a food catering company starts to increase exponentially. The number of customers can be modelled by $N = \frac{300}{1+2^{1-1.2t}}$, where t is the number of months after the start of April 2020.
- (i) Write down the initial number of customers at the start of April 2020. [1]
- (ii) How many days after the start of April 2020 do they have 150 customers?
Interpret one month as 30 days. [3]
- (iii) The current resources of the company can cater up to 350 customers. This trend on the number of customers is expected to continue. Explain with a reason, why there is no need for the company to expand their resources as a result of this trend. [2]
- 4 The graph of $y = f(x)$ passes through the point $A(4, 5)$. Find the image of A under each of the following transformations.
- (i) $y = f(x) + 2$ [1]
- (ii) $y = f(x - 1)$ [1]
- (iii) $y = f(-x)$ [1]
- (iv) $y = \frac{1}{4}f(x)$ [1]
- (v) $y = f\left(\frac{x}{2}\right)$ [1]
- (vi) $y = f^{-1}(x)$ [1]

- 5 A hamster is observed to run at constant speed on a circular hamster wheel which is supported by an isosceles triangular base.



(Source: <http://clipart-library.com/clipart/1597815.htm>)

At a particular instant, a fixed point A , on the wheel is observed to be at the same height above the ground as the centre of the wheel, as shown in the diagram. t seconds after that instant, point A is y cm above the ground, where $y = 11 + 9 \sin 4\pi t$.

- (i) State the amplitude and period of $y = 11 + 9 \sin 4\pi t$. [2]
 - (ii) Hence, state the diameter of the wheel, and find the running speed of the hamster. [3]
 - (iii) Sketch the graph of $y = 11 + 9 \sin 4\pi t$ for $0 \leq t \leq 0.5$. [2]
- 6 The expression $f(x) = 6x^3 + px^2 - 19x + q$ is exactly divisible by $x + 3$ but leaves a remainder of 4 when divided by $x - 1$.
- (i) Show that $p = 11$ and $q = 6$. [4]
 - (ii) Factorise $f(x)$ completely. [2]
 - (iii) Solve the equation $f(x) = 0$. [2]
 - (iv) Hence, by using a suitable substitution, solve the equation

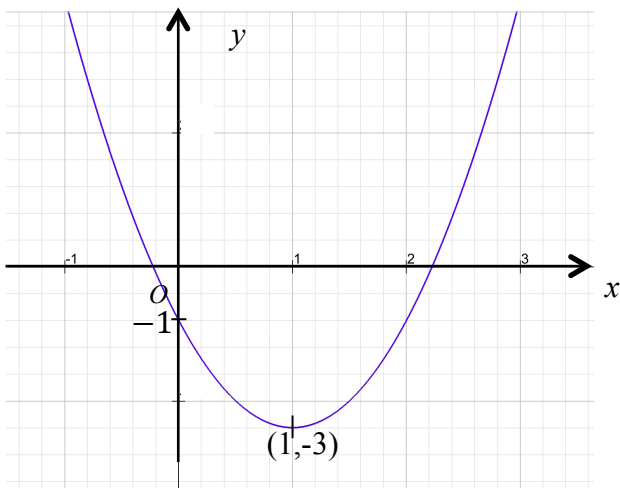
$$48y^3 + 44y^2 - 38y + 6 = 0. \quad [2]$$
- 7
- (a) Find the range of values of x for which $x - \frac{4}{3} \leq 5x + 2 < -3x^2$. [4]
 - (b) Solve the equation $|4x^2 - 3| = 4x$. [5]

- 8** The angles A and B are in the same quadrant such that $\sin A > 0$ and $\cos B < 0$.
- (a) Explain why $\sin(A + B)$ is less than zero. [1]
- (b) Given further that $\sin A = \frac{2}{3}$ and $\cos B = -\frac{4}{5}$, without using a calculator, find the value of
- (i) $\cos(-A)$, [1]
- (ii) $\operatorname{cosec} B$, [1]
- (iii) $\tan(180^\circ + A)$, [1]
- rationalising the denominator, where necessary.
- 9** Functions f and g are defined for real values of x by $f : x \mapsto x - 1$ and $g : x \mapsto 1 - x^2$.
- (i) State the range of $g(x)$. [1]
- (ii) Express in similar form, the functions f^2 and gf . [3]
- (iii) Given that the domain of gf is restricted to $x \geq k$, state the smallest value of k for which $(gf)^{-1}$ exists. Hence, find the expression for $(gf)^{-1}$, stating its domain clearly. [4]
- 10** (a) (i) By completing the square, find the minimum value of $2x^2 - 4x - 1$. [3]
- (ii) Sketch the graph of $y = 2x^2 - 4x - 1$, indicating clearly the y -intercept and the coordinates of the turning point. [2]
- (iii) k is an integer such that $2x^2 - 4x - 1 + k > 0$ for all real values of x . Given that x and k can vary, state the integer value of k that corresponds to the maximum value of $\frac{1}{2x^2 - 4x - 1 + k}$. [1]
- (b) Find the range of values of c , such that the graph of $y = 2cx^2 - 4x - 1 + c$ is completely below the x -axis for all real values of x . [5]
- 11** In order to solve the equation $5^{(2 \log_5 x) - 1} + \log_2 20x = 0$, a suitable straight line has to be drawn on the same set of axes as the graph of $y = x^2 + 5 \log_2 5x$. Find the equation of the straight line. [3]

End of Paper

2020 Sec 3 IM2 EOY Answer Key

No	Answer Key
1a	$-\frac{19}{22} + \frac{5}{22}\sqrt{3}$
1b	$x = \frac{1}{a}10^b$
1ci	$x = \frac{1}{7}$
1cii	$x \approx 0.431$
2	$\frac{4x}{(x^2+4)(x-2)}$ $= \frac{2-x}{x^2+4} + \frac{1}{x-2}$
3i	100
3ii	25 days
3iii	$1 + \frac{2^{1-1.2t}}{300} > 1$ $\frac{2^{1-1.2t}}{300} < 300$ No need to expand as the number of customers will not exceed 300.
4i	(4, 7)
4ii	(5, 5)
4iii	(-4, 5)
4iv	$\left(4, \frac{5}{4}\right)$
4v	(8, 5)
4vi	(5, 4)
5i	Amplitude = 9; Period = 0.5.
5ii	Diameter = 18 cm Running speed ≈ 113 cm/s
5iii	
6ii	$f(x) = (x+3)(2x-1)(3x-2)$
6iii	$x = -3, \frac{1}{2}$ or $\frac{2}{3}$
6iv	$y = -\frac{3}{2}, \frac{1}{4}$ or $\frac{1}{3}$
7a	$-\frac{5}{6} \leq x < -\frac{2}{3}$
7b	$x = \frac{1}{2}$ or $\frac{3}{2}$

8a	Angles A and B are in 2 nd quadrant. So, angle $(A + B)$ is in the 3 rd or 4 th quadrant. Hence, $\sin(A + B) < 0$.
8bi	$-\frac{\sqrt{5}}{3}$
8bii	$\frac{5}{3}$
8biii	$-\frac{2\sqrt{5}}{5}$
9i	$g(x) \leq 1$
9ii	$f^2 : x \mapsto x - 2$ $gf : x \mapsto -x^2 + 2x$
9iii	$(gf)^{-1}(x) = 1 + \sqrt{1 - x}, x \leq 1$
10 ai	$2x^2 - 4x - 1$ $= 2(x - 1)^2 - 3$ Hence, minimum value is -3 .
10 aii	
10 aiii	4
10 b	$c < -1$.
11	Draw the line $y = -10$