Qn			
1	$k^2+2k+3=0$ AE: $k=\dfrac{-4\pm\sqrt{4-4(1)(3)}}{2}$ $=-2\pm\sqrt{2}\mathrm{i}$ CF: $y=\mathrm{e}^{-2x}\left(A\cos\sqrt{2}x+B\sin\sqrt{2}x\right)$ PI: Let $y=a\sin x+b\cos x$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=a\cos x-b\sin x$ $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-a\sin x-b\cos x$ Sub into DE, $-a\sin x-b\cos x+2(a\cos x-b\sin x)+3(a\sin x+b\cos x)=\cos x$ $2a-2b=0$ $2a+2b=1$ Solving, $a=b=\dfrac{1}{4}$ $\therefore$ General solution is $y=\mathrm{e}^{-2x}\left(A\cos\sqrt{2}x+B\sin\sqrt{2}x\right)+\dfrac{1}{4}(\sin x+\cos x)$		

2	**2023/TJC/JC2 MYE/I/Q1** (i) $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}r\cos\alpha\\ r\sin\alpha\end{pmatrix}=\begin{pmatrix}r\cos\theta\cos\alpha-r\sin\theta\sin\alpha\\ r\sin\theta\cos\alpha+r\cos\theta\sin\alpha\end{pmatrix}$ $=\begin{pmatrix}r\cos(\theta+\alpha)\\ r\sin(\theta+\alpha)\end{pmatrix}$ The transformation T rotates every vector in $\mathbb{R}^2$ **anti-clockwise about the origin through the angle θ.** (ii) $\det(\mathbf{A}-\lambda\mathbf{I})=0$ $\begin{vmatrix}\cos\theta-\lambda & -\sin\theta\\ \sin\theta & \cos\theta-\lambda\end{vmatrix}=0$ $(\cos\theta-\lambda)^2+\sin^2\theta=0$ $\cos^2\theta-2\lambda\cos\theta+\lambda^2+\sin^2\theta=0$ $\lambda^2-2\lambda\cos\theta+1=0$ For real eigenvalues, discriminant ≥ 0 $4\cos^2\theta-4\geq 0$ $(\cos\theta-1)(\cos\theta+1)\geq 0$ $\cos\theta\leq -1$ or $\cos\theta\geq 1$ Since $-1\leq\cos\theta\leq 1$, $\cos\theta=-1$ or $\cos\theta=1$ $\theta=0$ or π When $\theta=0$, $\mathbf{A}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ which is the identity matrix. Eigenvalue = 1 When $\theta=\pi$, $\mathbf{A}=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$. Eigenvalue = -1		

3(i)	Note that $T(e_1+e_2)=2(e_1+e_2)$. Thus $v=e_1+e_2$. e_1+e_2 is an eigenvector of the matrix representation of T with corresponding eigenvalue 2. (Geometrically, the line $\mathbf{r}=\alpha\left(e_1+e_2\right),\alpha\in\mathbb{R}$, is invariant under the transformation, and each point is transformed onto a different point on the line)		
3(ii)	$T(e_3)=e_3-T(e_1+e_2)=-2e_1-2e_2+e_3$ Thus $\mathbf{A}=\begin{pmatrix}0&2&-2\\1&1&-2\\0&0&1\end{pmatrix}$		
3(iii)	Since 2 is an eigenvalue of **A**, we have $\ker(\mathbf{A})=\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}\right\}$. Since $\mathbf{A}-2\mathbf{I}=\begin{pmatrix}-2&2&-2\\1&-1&-2\\0&0&-1\end{pmatrix}$, a basis for R_1 is $\ker(\mathbf{A})=\left\{\begin{pmatrix}2\\-1\\0\end{pmatrix},\begin{pmatrix}2\\2\\1\end{pmatrix}\right\}$.	Note that we are solving $\left(\mathbf{A}-2\mathbf{I}\right)\mathbf{v}=\mathbf{0}$. Since dim of nullspace is 1, dim of range space(i.e. column space) is 2. We can just take the 2 linearly independent columns (since 1st 2 columns are multiples of each other)	
4(i)	Let $\mathbf{x}\in\ker(\mathbf{A})$. Thus $\mathbf{Ax}=\mathbf{0}$. Accordingly, $\mathbf{BA}\left(\mathbf{x}\right)=\mathbf{B}\left(\mathbf{Ax}\right)=\mathbf{0}$. Thus $\mathbf{x}\in\ker(\mathbf{BA})$ Thus $\ker(\mathbf{A})\subseteq\ker(\mathbf{BA})$		
4(ii)	Since **BA** is invertible, thus $\lvert\mathbf{BA}\rvert\neq 0\Rightarrow\lvert\mathbf{B}\rvert\lvert\mathbf{A}\rvert\neq 0$, both $\lvert\mathbf{B}\rvert\neq 0$ and $\lvert\mathbf{A}\rvert\neq 0$, thus **A** and **B** are both invertible.		
4(iii)	$\mathbf{A}\left(\mathbf{A}+\mathbf{B}\right)^{-1}\mathbf{B}=\mathbf{A}\left(\mathbf{A}+\mathbf{B}\right)^{-1}(\mathbf{B}+\mathbf{A}-\mathbf{A})$ $=\mathbf{A}-\mathbf{A}\left(\mathbf{A}+\mathbf{B}\right)^{-1}\mathbf{A}$ $=\mathbf{A}-\left(-\mathbf{B}+\mathbf{A}+\mathbf{B}\right)\left(\mathbf{A}+\mathbf{B}\right)^{-1}\mathbf{A}$ $=\mathbf{A}+\mathbf{B}\left(\mathbf{A}+\mathbf{B}\right)^{-1}\mathbf{A}-\mathbf{A}$ $=\mathbf{B}\left(\mathbf{A}+\mathbf{B}\right)^{-1}\mathbf{A}$		

<table>
<tr><td>5</td><td>2023/TJC/JC2 MYE/I/Q7
(i) $\begin{aligned} 6b_r + 8c_r &= a_{r+1} \\ 0.5a_r &= b_{r+1} \\ 0.5b_r &= c_{r+1} \end{aligned} \Rightarrow \begin{pmatrix} 0 & 6 & 8 \\ 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix}\begin{pmatrix} a_r \\ b_r \\ c_r \end{pmatrix} = \begin{pmatrix} a_{r+1} \\ b_{r+1} \\ c_{r+1} \end{pmatrix}$
$\mathbf{M} = \begin{pmatrix} 0 & 6 & 8 \\ 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix}$
Given $\mathbf{x_0} = \begin{pmatrix} 120 \\ 120 \\ 120 \end{pmatrix}$
$\mathbf{x_1} = \begin{pmatrix} 0 & 6 & 8 \\ 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix}\begin{pmatrix} 120 \\ 120 \\ 120 \end{pmatrix} = \begin{pmatrix} 1680 \\ 60 \\ 60 \end{pmatrix}$
$\mathbf{x}_2 = \begin{pmatrix} 0 & 6 & 8 \\ 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix}\begin{pmatrix} 1680 \\ 60 \\ 60 \end{pmatrix} = \begin{pmatrix} 840 \\ 840 \\ 30 \end{pmatrix}$
The number of rabbits in first age class, second age class and third age class after 2 years are 840, 30 and 30 respectively.</td><td></td><td></td></tr>
<tr><td>5</td><td>(ii) To find $\mathbf{Mx} = \lambda\mathbf{x}$
$(\mathbf{M} - \lambda\mathbf{I})\mathbf{x} = \mathbf{0}$
The characteristic equation is det $(\mathbf{M} - \lambda\mathbf{I}) = 0$
$\det\begin{pmatrix} -\lambda & 6 & 8 \\ 0.5 & -\lambda & 0 \\ 0 & 0.5 & -\lambda \end{pmatrix} = 0 \Rightarrow -\lambda(\lambda^2) - 0.5(-6\lambda - 4) = -\lambda^3 + 3\lambda + 2 = 0$
$\lambda = -1$ (repeated) or $\lambda = 2$
Taking $\lambda = 2$, (rejected the negative eigenvalue as population cannot be negative)
Augmented matrix:</td><td></td><td></td></tr>
</table>

	$\begin{pmatrix} -2 & 6 & 8 & 0 \\ 0.5 & -2 & 0 & 0 \\ 0 & 0.5 & -2 & 0 \end{pmatrix} \xrightarrow{\text{RREF using GC}} \begin{pmatrix} 1 & 0 & -16 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ eigenvector, $\mathbf{x} = \begin{pmatrix} 16t \\ 4t \\ t \end{pmatrix}$ Smallest initial population $= \begin{pmatrix} 16 \\ 4 \\ 1 \end{pmatrix}$ when $t = 1$		
6(i)	If n is odd, then $x_{n+2} = 2x_{n+1} + 1$ and $x_{n+1} = 2x_n$, Combining, we have $x_{n+2} = x_{n+1} + x_{n+1} + 1 = x_{n+1} + 2x_n + 1$ If n is even, then $x_{n+2} = 2x_{n+1}$ and $x_{n+1} = 2x_n + 1$ Combining, we have $x_{n+2} = x_{n+1} + x_{n+1} = x_{n+1} + 2x_n + 1$		
6(ii)	From GC, $x_{11} = 1365 < 2021$ $x_{12} = 2730 > 2021$ Thus max number of rings = 11.		
6(iii)	Since $x_1 = 1,\ x_2 = 2x_1 = 2,\ x_3 = 2x_2 + 1 = 5$ $x_n = A(-1)^n + B(2)^n + C$ $x_1 = 1,\ 1 = A(-1)^1 + B(2)^1 + C \Rightarrow -A + 2B + C = 1$ $x_2 = 2,\ 2 = A(-1)^2 + B(2)^2 + C \Rightarrow A + 4B + C = 2$ $x_2 = 5,\ 5 = A(-1)^3 + B(2)^3 + C \Rightarrow -A + 8B + C = 5$ Using GC, $x_n = \frac{2}{3}(2^n) - \frac{1}{6}(-1)^n - \frac{1}{2}$ Alternatively, $m^2 - m - 2 = 0$ $m = 2$ or $m = -1$ Complementary solution: $x_n = A(-1)^n + B(2)^n$ Particular solution: $x_n = C$		

	$C=C+2C+1$ $C=-\frac{1}{2}$ Since $x_1=1,\ x_2=2x_1=2$ Also, $x_n=A(-1)^n+B(2)^n-\frac{1}{2}$ $x_1=1,\ 1=A(-1)^1+B(2)^1-\frac{1}{2}\Rightarrow -A+2B=\frac{3}{2}$ $x_2=2,\ 2=A(-1)^2+B(2)^2-\frac{1}{2}\Rightarrow A+4B=\frac{5}{2}$ $A=-\frac{1}{6},\ B=\frac{2}{3}$ $x_n=\frac{2}{3}(2^n)-\frac{1}{6}(-1)^n-\frac{1}{2}$		
7	Source: NJC Promo 2025 Qn 3 $z=\mathrm{e}^{-(y-1)^2}\cos\left(x-\frac{\pi}{3}\right),\ \left\{(x,y)\mid -\pi\le x\le\pi,\ y\in\mathbb{R}\right\}.$ $\frac{\partial z}{\partial x}=-\mathrm{e}^{-(y-1)^2}\sin\left(x-\frac{\pi}{3}\right),\ \frac{\partial z}{\partial y}=-2(y-1)\mathrm{e}^{-(y-1)^2}\cos\left(x-\frac{\pi}{3}\right).$ At stationary points, $\frac{\partial z}{\partial x}=0$ and $\frac{\partial z}{\partial y}=0$. $-\mathrm{e}^{-(y-1)^2}\sin\left(x-\frac{\pi}{3}\right)=0$ $\sin\left(x-\frac{\pi}{3}\right)=0\ (\because -\mathrm{e}^{-(y-1)^2}<0\ \forall y\in\mathbb{R})$ $x=-\frac{2\pi}{3}$ or $\frac{\pi}{3}$ and		

$-2(y-1)e^{-(y-1)^2}\cos\left(x-\frac{\pi}{3}\right)=0$

$(y-1)\cos\left(x-\frac{\pi}{3}\right)=0$ ($\because -e^{-(y-1)^2}<0\ \forall y\in\mathbb{R}$)

$y=1$ or $\cos\left(x-\frac{\pi}{3}\right)=0$.

When $x=-\frac{2\pi}{3}$, $\cos\left(-\frac{2\pi}{3}-\frac{\pi}{3}\right)=-1\neq 0$.

When $x=\frac{\pi}{3}$, $\cos\left(\frac{\pi}{3}-\frac{\pi}{3}\right)=1\neq 0$.

Therefore, the stationary points in D are $\left(-\frac{2\pi}{3},1,-1\right)$ and $\left(\frac{\pi}{3},1,1\right)$.

$$\frac{\partial^2 z}{\partial x^2}=-e^{-(y-1)^2}\cos\left(x-\frac{\pi}{3}\right)=-z,$$

$$\frac{\partial^2 z}{\partial y\partial x}=2(y-1)e^{-(y-1)^2}\sin\left(x-\frac{\pi}{3}\right)=-2(y-1)\frac{\partial z}{\partial x},$$

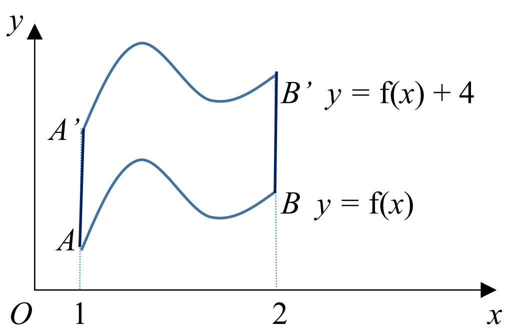
$$\begin{aligned}\frac{\partial^2 z}{\partial y^2}&=\left(4(y-1)^2e^{-(y-1)^2}-2e^{-(y-1)^2}\right)\cos\left(x-\frac{\pi}{3}\right)\\&=\left(4(y-1)^2-2\right)e^{-(y-1)^2}\cos\left(x-\frac{\pi}{3}\right)\\&=\left(4(y-1)^2-2\right)z.\end{aligned}$$

At $\left(-\frac{2\pi}{3},1,-1\right)$, $D=(-1\cdot-1)(-2\cdot-1)-(0)^2=2>0$.

Also, $\frac{\partial^2 z}{\partial x^2}=1>0$. $\therefore\left(-\frac{2\pi}{3},1,-1\right)$ is a local minimum point.

At $\left(\frac{\pi}{3},1,1\right)$, $D=(-1\cdot 1)(-2\cdot 1)-(0)^2=2>0$.

Also, $\frac{\partial^2 z}{\partial x^2}=-1<0$. $\therefore\left(\frac{\pi}{3},1,1\right)$ is a local maximum point.

8(i)	$\overset{\frown}{AB}=\int_1^2\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x=\int_1^2\sqrt{\frac{\left(x^4+1\right)^2}{4x^4}}\,\mathrm{d}x=\int_1^2\sqrt{\frac{\left(x^4+1\right)^2}{4x^4}}\,\mathrm{d}x$ $=\frac{1}{2}\int_1^2\frac{x^4+1}{x^2}\,\mathrm{d}x$ $=\frac{1}{2}\left\{\left[\frac{1}{3}x^3-\frac{1}{x}\right]_1^2\right\}$ $=\frac{17}{12}$		
8(ii)	Surface area $=2\pi\int_1^2 y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x=2\pi\int_1^2\left(\frac{x^4+3}{6x}\right)\left(\frac{x^4+1}{2x^2}\right)\mathrm{d}x$ $=\frac{\pi}{6}\int_1^2\left(x^5+4x+\frac{3}{x^3}\right)\mathrm{d}x$ $=\frac{\pi}{6}\left[\frac{x^6}{6}+2x^2-\frac{3}{2x^2}\right]_1^2$ $=\frac{47}{16}\pi$ $x^4-6xy+3=0\xrightarrow{\mathrm{T}_y\text{ by }4}x^4-6x\left(y-4\right)+3=0\Rightarrow x^4-6xy+24x+3=0$. So Γ_2 is a translation of Γ_1 by 4 units in the positive direction of the y-axis. 		

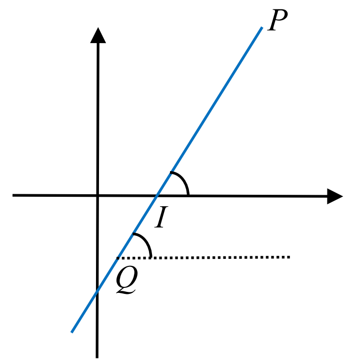
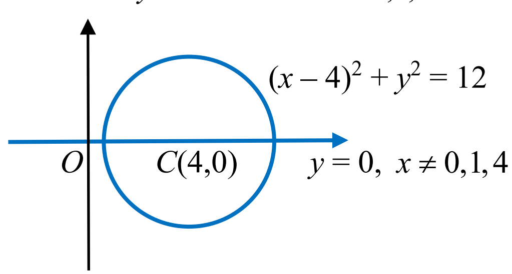
8(iii)	Perimeter $= 2(\overset{\frown}{AB} + AA')$ $= 2(\frac{17}{12} + 4)$ $= \frac{65}{6}$.		
8(iv)	$y = \frac{x^4+3}{6x} = \mathrm{f}\left(x\right)$. By the 'shell' method, volume $2\pi\int_1^2 x\left[\mathrm{f}\left(x\right)+4-\mathrm{f}\left(x\right)\right]\,\mathrm{d}x = 8\pi\int_1^2 x\,\mathrm{d}x$ $= 4\pi\left[x^2\right]_1^2$ $= 12\pi$		
9(i)	$p(x_1) = a_0 + a_1x_1 + a_2{x_1}^2 + \cdots + a_{n-1}{x_1}^{n-1} = y_1$ $p(x_2) = a_0 + a_1x_2 + a_2{x_2}^2 + \cdots + a_{n-1}{x_2}^{n-1} = y_2$ $\vdots$ $p(x_n) = a_0 + a_1x_n + a_2{x_n}^2 + \cdots + a_{n-1}{x_n}^{n-1} = y_n$ We can rewrite the above n equations in matrix form as follows: $\begin{pmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ 1 & x_3 & x_3^2 & \cdots & x_3^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{pmatrix}$		
9(ii)	For the case of $n = 3$, (refer to Section 1 Linear Algebra Notes last page on properties of determinant in relation to row operations)		

	$\begin{vmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{vmatrix} = \begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & x_2 - x_1 & x_2^2 - x_1^2 \\ 0 & x_3 - x_1 & x_3^2 - x_1^2 \end{vmatrix}$ (multiple of 1 row added to another row does not change determinant) $= (x_2 - x_1)\begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & 1 & x_2 + x_1 \\ 0 & x_3 - x_1 & x_3^2 - x_1^2 \end{vmatrix}$ (we can factor out a multiple of one row out of determinant) $= (x_2 - x_1)(x_3 - x_1)\begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & 1 & x_2 + x_1 \\ 0 & 1 & x_3 + x_1 \end{vmatrix}$ $= (x_2 - x_1)(x_3 - x_1)\begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & 1 & x_2 + x_1 \\ 0 & 0 & x_3 - x_2 \end{vmatrix}$ $= (x_2 - x_1)(x_3 - x_1)(x_3 - x_2)\begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & 1 & x_2 + x_1 \\ 0 & 0 & 1 \end{vmatrix}$ $= (x_2 - x_1)(x_3 - x_1)(x_3 - x_2)$ Remark: It can be proven that the determinant of $\mathbf{Q}$ is $\prod_{1 \le i < j \le n} (x_j - x_i)$.		
9(iii)	If the x_i 's are distinct, then $\prod_{1 \le i < j \le n} (x_j - x_i) \neq 0$. Thus $\mathbf{Q}$ is invertible. Hence the system has unique solution and the values of the values of a_0, a_1, …, a_{n-1} are unique. That is, the polynomial the passes through these points is uniquely determined by the distinct points.		
9(iv)	The system is $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 2 \\ 5 \end{pmatrix}$. Thus		

	$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{pmatrix}^{-1} \begin{pmatrix} -1 \\ 0 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 \\ \frac{3}{2} \\ -\frac{2}{3} \\ \frac{1}{6} \end{pmatrix}$ Thus the polynomial is $p(x) = -1 + \frac{3}{2}x - \frac{2}{3}x^2 + \frac{1}{6}x^3$.		
10(i)	$\frac{dx}{dt} = 3y - 2x$ $y = \frac{1}{3}\left(\frac{dx}{dt} + 2x\right)$ Differentiating with respect to t, $\frac{dy}{dt} = \frac{1}{3}\left(\frac{d^2x}{dt^2} + 2\frac{dx}{dt}\right)$ Sub into second equation, $\frac{1}{3}\left(\frac{d^2x}{dt^2} + 2\frac{dx}{dt}\right) = 4x - 3\left(\frac{1}{3}\left(\frac{dx}{dt} + 2x\right)\right)$ $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} = 12x - 3\left(\frac{dx}{dt} + 2x\right)$ $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} - 6x = 0$ Auxiliary equation: $k^2 + 5k - 6 = 0$ $(k-1)(k+6) = 0$ $k = 1$ or $k = -6$ $\therefore x = Ae^{t} + Be^{-6t}$ $\Rightarrow \frac{dx}{dt} = Ae^{t} - 6Be^{-6t}$		

	$y=\frac{1}{3}\left(\frac{\mathrm{d}x}{\mathrm{d}t}+2x\right)$ $=\frac{1}{3}\left(Ae^{t}-6Be^{-6t}+2Ae^{t}+2Be^{-6t}\right)$ $=Ae^{t}-\frac{4}{3}Be^{-6t}$		
10(ii)	$1=A+B$ $1=A-\frac{4}{3}B$ Using GC, $A=1, B=0$ $\therefore x=e^{t}$ $y=e^{t}$		
10(iii)	As $t\to\infty$, both x and $y\to\infty$. Therefore, it will escalate towards conflict/war.		
10(iv)	Using (i), $x=Ae^{t}+Be^{-6t}+C$ $\frac{\mathrm{d}x}{\mathrm{d}t}=Ae^{t}-6Be^{-6t}$ $y=\frac{1}{3}\left(\frac{\mathrm{d}x}{\mathrm{d}t}+2x+10\right)$ $=\frac{1}{3}\left(Ae^{t}-6Be^{-6t}+2Ae^{t}+2Be^{-6t}+2C+10\right)$ $=Ae^{t}-\frac{4}{3}Be^{-6t}+\frac{2}{3}C+\frac{10}{3}$		

10(v)	$1=A+B+C$ $-\frac{7}{3}=A-\frac{4}{3}B+\frac{2}{3}C$ Using GC, $\begin{pmatrix}A\\B\\C\end{pmatrix}=\begin{pmatrix}-3/7\\10/7\\0\end{pmatrix}+\lambda\begin{pmatrix}-6/7\\-1/7\\1\end{pmatrix}$ A possible solution is $A=-9, B=0, C=10$ $x=10-9\mathrm{e}^{t}, y=10-9\mathrm{e}^{t}$ In the long run, $x\to 0, y\to 0$.		
11(i)	$z-1=x-1+y\mathrm{i}$. <u>Case 1</u>: $(x-1,y)$ is in the 1st or 4th quadrant $\arg(z-1)=\tan^{-1}\left(\frac{y}{x-1}\right)$. $\tan\left[\arg(z-1)\right]=\tan\left[\tan^{-1}\left(\frac{y}{x-1}\right)\right]=\frac{y}{x-1}$. <u>Case 2</u>: $(x-1,y)$ is in the 2nd or 3rd quadrant $\arg(z-1)=\tan^{-1}\left(\frac{y}{x-1}\right)\pm\pi$. where '+' is used when $(x-1,y)$ is in the 2nd quadrant and '–' is used when $(x-1,y)$ is in the 3rd quadrant. $\tan\left[\arg(z-1)\right]=\tan\left[\tan^{-1}\left(\frac{y}{x-1}\right)\pm\pi\right]=\dfrac{\tan\left[\tan^{-1}\left(\frac{y}{x-1}\right)\right]\pm\tan\pi}{1\mp\tan\left[\tan^{-1}\left(\frac{y}{x-1}\right)\right]\tan\pi}$ $=\tan\left[\tan^{-1}\left(\frac{y}{x-1}\right)\right]=\frac{y}{x-1}$		

11(ii)	$1-\frac{4}{z}=1-\frac{4}{x+y\mathrm{i}}=1-\frac{4\left(x-y\mathrm{i}\right)}{x^2+y^2}=\frac{x^2+y^2-4x}{x^2+y^2}+\frac{4y}{x^2+y^2}\mathrm{i}$. In a similar manner as (i), we infer that $\tan\left[\arg\left(1-\frac{4}{z}\right)\right]=\frac{4y}{x^2+y^2-4x}$.		
11(iii)	Let $I\equiv\left(1,0\right)$. Since PIQ is a straight line, PI and IQ make the same angle with the positive direction of the real axis. This implies  $\arg\left(z-1\right)=\arg\left(1-\frac{4}{z}\right)$ $\Rightarrow \tan\left[\arg\left(z-1\right)\right]=\tan\left[\arg\left(1-\frac{4}{z}\right)\right]$ $\Rightarrow \frac{y}{x-1}=\frac{4y}{x^2+y^2-4x}$ $\Rightarrow y\left(\frac{4}{x^2+y^2-4x}-\frac{1}{x-1}\right)=0$ $\Rightarrow y\left[\frac{4\left(x-1\right)-\left(x^2+y^2-4x\right)}{\left(x^2+y^2-4x\right)\left(x-1\right)}\right]=0$ $\Rightarrow y=0$ or $x^2+y^2-8x+4=0 \Rightarrow \left(x-4\right)^2+y^2=12$		
11(iv)	$x-1\neq 0$ and $x^2+y^2-4x\neq 0 \Rightarrow x\neq 0,1,4$  remove (0,0), (1,0), (4,0) and the points on circle where $x = 1$		

11(v)	Given that P lies on $(x-4)^2+y^2=12$, $y\neq 0$, $4-\sqrt{12}<x<4+\sqrt{12}$. $(x-4)^2+y^2=12 \Rightarrow x^2+y^2=8x-4$ ------ (*) $\left\|1-\frac{1}{z}\right\|^2$ $=\left\|\frac{z-1}{z}\right\|^2$ $=\frac{\|(x-1)+y\mathrm{i}\|^2}{\|x+y\mathrm{i}\|^2}$ $=\frac{(x-1)^2+y^2}{x^2+y^2}$ $=\frac{x^2+y^2-2x+1}{x^2+y^2}$ $=\frac{8x-4-2x+1}{8x-4}$ by (*) $=\frac{3(2x-1)}{4(2x-1)}$ $=\frac{3}{4}$ Thus $\left\|1-\frac{1}{z}\right\|=\frac{1}{2}\sqrt{3}$.		