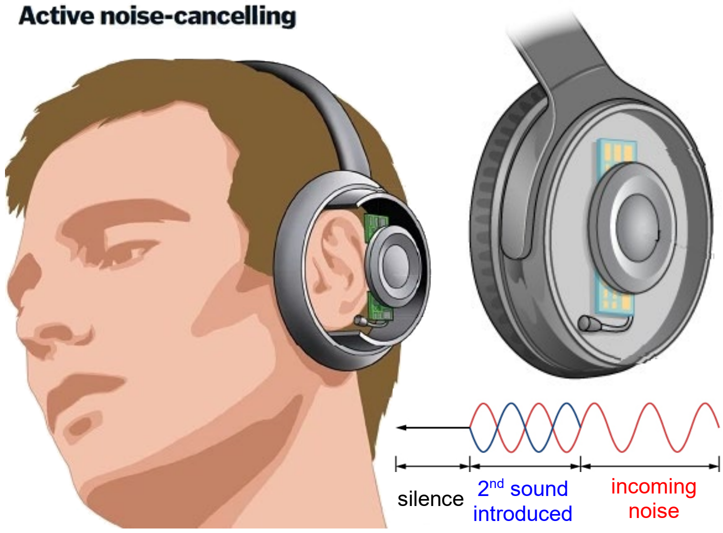
H2 Topic 12a – Superposition



Anti-reflection (AR) coating commonly found on lenses, and Active Noise Cancellation (ANC) commonly found on higher-end audio devices are just two amongst many applications of interference, which is a phenomenon based on the principle of superposition.

Content

- Principle of superposition
- Stationary waves
- Diffraction
- Two-source interference
- Single slit and multiple slit diffraction

Learning Outcomes

Candidates should be able to:

(a) explain and use the principle of superposition in simple applications
(b) show an understanding of the terms interference, coherence, phase difference and path difference
(c) show an understanding of experiments which demonstrate stationary waves using microwaves, stretched strings and air columns
(d) explain the formation of a stationary wave using a graphical method, and identify nodes and antinodes
(e) explain the meaning of the term diffraction
(f) show an understanding of experiments which demonstrate diffraction including the diffraction of water waves in a ripple tank with both a wide gap and a narrow gap
(g) show an understanding of experiments which demonstrate two-source interference using water waves, sound waves, light and microwaves
(h) show an understanding of the conditions required for two-source interference fringes to be observed
(i) recall and solve problems using the equation $\lambda = ax / D$ for double-slit interference using light
(j) recall and use the equation $\sin\theta = \lambda / b$ to locate the position of the first minima for single slit diffraction
(k) recall and use the Rayleigh criterion $\theta \approx \lambda / b$ for the resolving power of a single aperture
(l) recall and use the equation $d \sin\theta = n\lambda$ to locate the positions of the principal maxima produced by a diffraction grating
(m) describe the use of a diffraction grating to determine the wavelength of light (the structure and use of a spectrometer are not required)

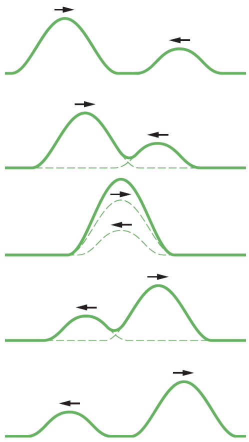
12.0 Introduction

Unlike particles, two (or more) waves can exist at the same point in space, at the same time. In this topic, we will look at the interaction between two or more waves when they *meet* and *overlap*.

12.1 Principle of Superposition

The **Principle of Superposition** states that

when two or more waves meet and overlap, the resultant displacement is the vector sum of the displacement of each individual wave



A series of snapshots that show two pulses travelling in opposite directions along a stretched string.

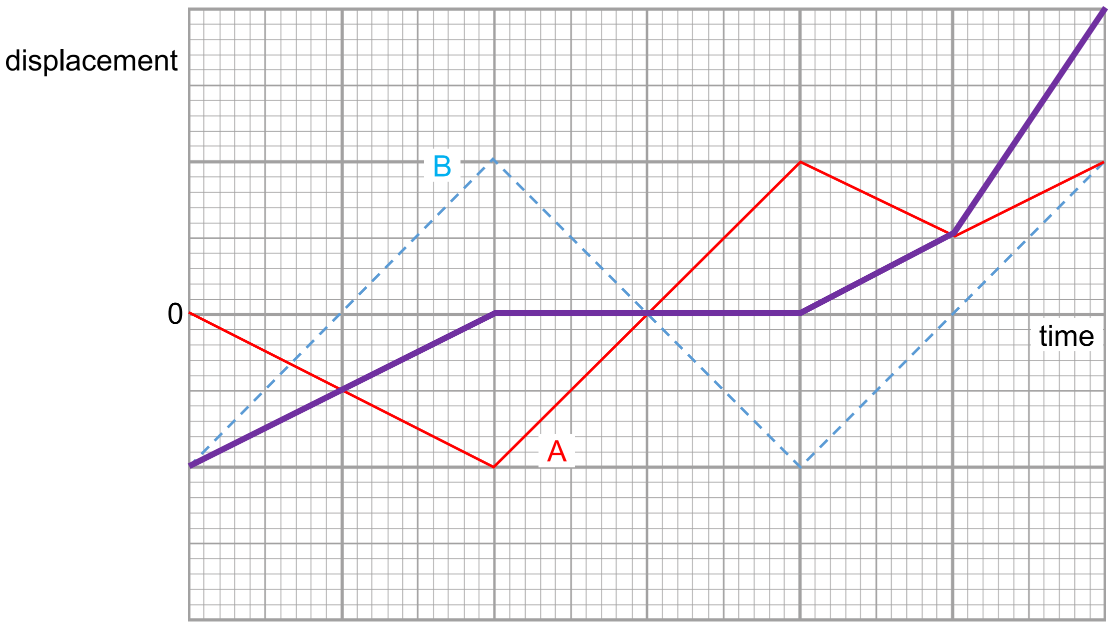
Where the two pulses meet and overlap, we see the resultant wave, not the individual waves.

*The waves' travel are **not** altered after they overlap.*

Be careful to use "*displacement*" rather than "*amplitude*" when defining superposition. In a wave, the displacement of each particle or segment of the wave varies from zero to the maximum possible value, and therefore the resultant displacement when multiple waves meet will change with time

Example 1

On the same axes, use the principle of superposition to sketch the resultant variation of displacement with time when two waves A and B meet and overlap at a single particle.



Note: the principle of superposition applies at each point along the wave, at the same point in time, to give the resultant waveform.

12.2 Interference

An interference pattern is
a pattern of maxima and minima

which results from 2 or more waves meeting and overlapping such that the resultant displacement is the vector sum of the displacement of each individual wave.

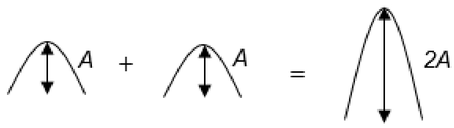
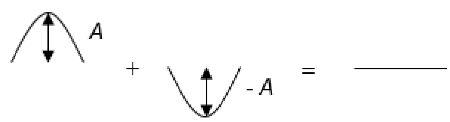
**depending on the context, will need to bring in additional details provided in question to explain the phase difference of the 2 waves meeting -> type of interference -> maxima / minima formed*

It is inevitable to invoke the definition of superposition when describing *interference*.

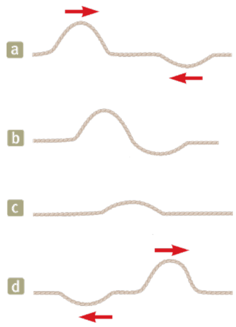
The added consideration is that interference describes a spatial *pattern* where there are large displacements and minimal displacements. Since intensity is directly proportional to the square of amplitude, correspondingly there are regions of high intensity (*maxima*) and regions of minimal intensity (*minima*).

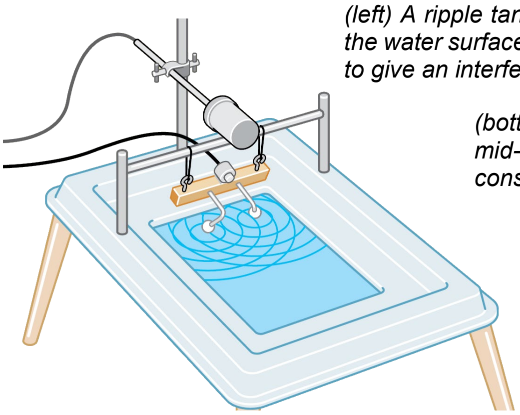
We term these regions of *constructive interference* and *destructive interference.*

By referring to 2 waves meeting and overlapping:

phase difference of waves at that point	type of interference	resultant displacement is …
$0,\ 2\pi,\ 4\pi, ...,\ n(2\pi)$ even integers of π / meet in-phase	constructive	**larger** than the largest *individual* displacement 
$\pi,\ 3\pi, ...,\ (2n+1)\pi$ odd integers of π / meet anti-phase	destructive	**smaller** than the largest *individual* displacement  *not necessarily *zero* resultant

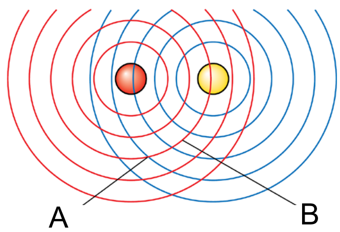
Two pulses can travel on a stretched string in opposite directions and undergo destructive interference when they meet and overlap. Thereafter the pulses continue without having been permanently affected.



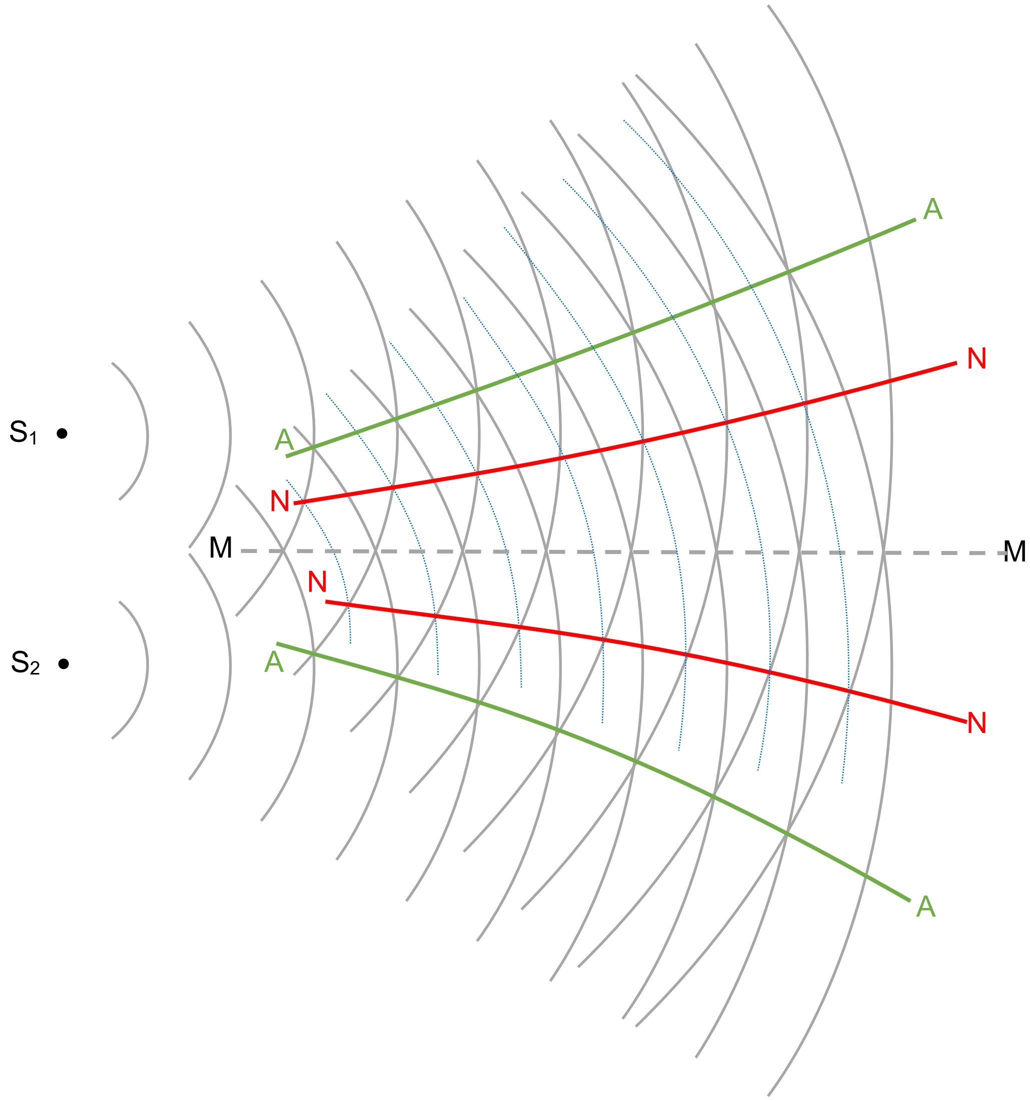


(left) A ripple tank. 2 dippers are connected to the same oscillating bar and just touch the water surface when at rest. Each dipper is a source of circular ripples, which overlap to give an interference pattern.

(bottom) The circular lines are the wavefronts of crests, which means the mid-point between 2 concentric wavefronts are troughs. A is a position of constructive interference; there is destructive interference at B.

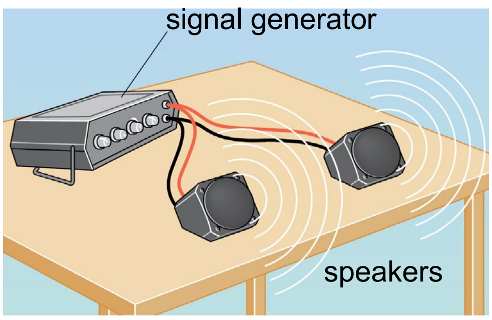


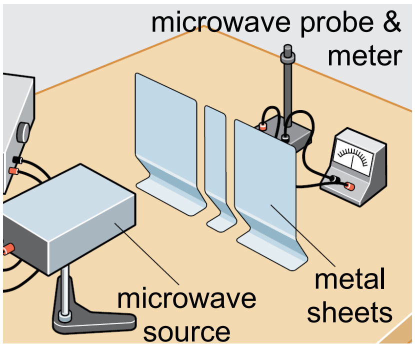
Example 2
Two wave sources S_1 and S_2, which are synchronised to each other, provide wavefronts as shown. The amplitude of the resultant of the waves is observed to be a maximum along the direction MM. On the same diagram, draw a line to show
(i) a second direction along which the amplitude is a maximum (label this line AA),
(ii) a direction in which the amplitude is a minimum (label this line NN).

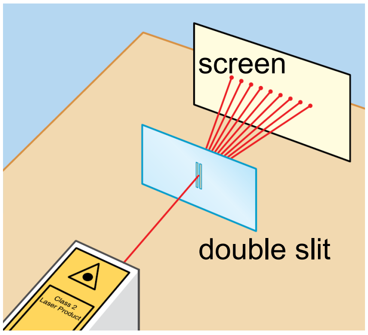


Note: the interference *pattern* comprises the MM, AA, and NN lines; not the wavefronts. The minimum amplitudes are when crests meet troughs.

The *two-source* set up in Example 2 can be created using sound waves, microwaves and light:

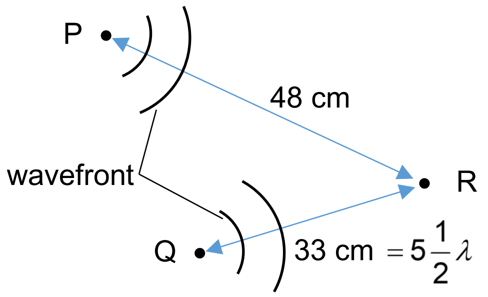






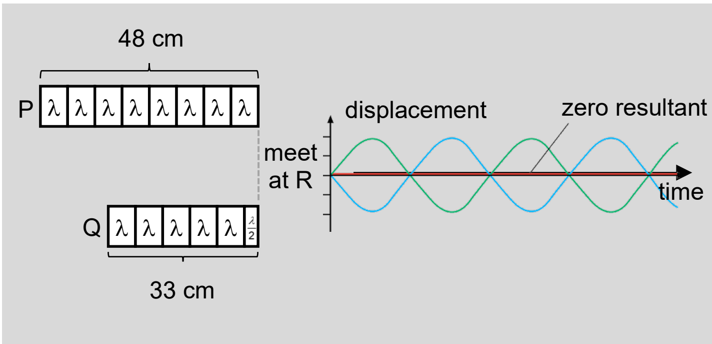
Example 3
Circular water waves may be produced by oscillating dippers at points P and Q as shown. The waves from P alone have the same amplitude at point R as the waves from Q alone. The dippers oscillate in phase with a period of 1.5 s to produce waves of speed 4.0 cm s^{-1}. Explain why the water particles are at rest at point R.

Solution:



$$\text{wavelength } \lambda = \frac{v}{f} = vT = (4)(1.5) = 6.00 \text{ cm}$$

$$\text{path difference } \Delta s = 48 - 33 = 15 \text{ cm} = 2\frac{1}{2}\lambda$$



$$\frac{\Delta s}{\lambda} = \frac{\Delta\phi}{2\pi}$$

$$\text{phase difference } \Delta\phi = \left(\frac{\Delta s}{\lambda}\right)(2\pi) = \left(\frac{15}{6}\right)(2\pi) = 5\pi$$

Waves from P and Q, meet and overlap at point R in anti-phase,
and undergo destructive interference.
Since the waves individually have the same amplitude at point R, the vector sum of displacements from waves from P and Q is zero.

To explain the resultant intensity at a particular position in an interference pattern:

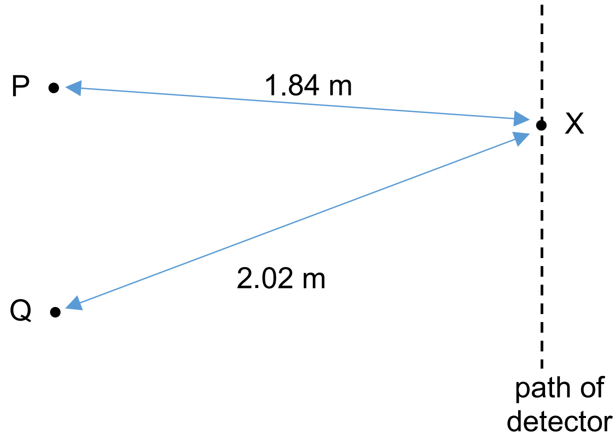
(i) identify the wave sources and the position where the waves *meet and overlap*
(ii) make reference to either *path difference* Δs or *time lag* Δt
(iii) work out the *phase difference* $\Delta\phi$ that the waves meet and overlap via $\frac{\Delta s}{\lambda} = \frac{\Delta\phi}{2\pi} = \frac{\Delta t}{T}$
(iv) determine if there is constructive or destructive interference

For point (iii), it matters if the sources are in-phase or not. See next example as an illustration.

Example 4

Microwaves of the same amplitude and of frequency $f = 5.0 \times 10^9$ Hz are emitted from two sources P and Q as shown, **in anti-phase**. A microwave detector is moved along a path that is parallel to the line joining P and Q. A series of intensity maxima and intensity minima are detected. Describe and explain the intensity of the microwaves detected at X:

Solution:



$$\text{wavelength } \lambda = \frac{c}{f} = \frac{3 \times 10^8}{5.0 \times 10^9} = 0.0600 \text{ m}$$

$$\text{path difference } \Delta s = 2.02 - 1.84 = 0.18 \text{ m}$$

$$\frac{\Delta s}{\lambda} = \frac{\Delta \phi}{2\pi}$$

(if sources are in phase)

$$\text{phase difference } \Delta\phi \text{ of waves meeting at X} = \left(\frac{0.18}{0.06}\right)(2\pi) + \pi = 6\pi + \pi = 7\pi$$

added because sources in anti-phase

Waves from P and Q, meet and overlap at point X in anti-phase,
and undergo destructive interference.
Since the waves individually have the same amplitude at point X, the vector sum of displacements from waves from P and Q is zero, and so is a region of zero intensity.

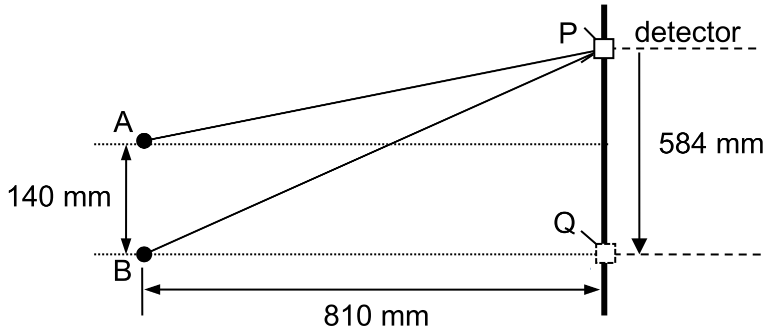
Note: To find the final phase difference at a region of interference we need to account for the phase difference at source. We can simply add the phase difference at source (e.g. for sources in anti-phase) to get the resultant phase difference.

The above are *general* cases involving *two sources*. We will return to such interference set-ups involving light later as there are some neat approximations that can be applied. As a summary:

		path difference Δs	
		$n\lambda$	$\left(n + \frac{1}{2}\right)\lambda$
sources	in phase	constructive	destructive
	in antiphase	destructive	constructive

Example 5

Two microwave sources A and B are in phase with one another. They emit waves of equal amplitude and of wavelength 30 mm. They are placed 140 mm apart and a distance 810 mm from a line along which a detector is moved.

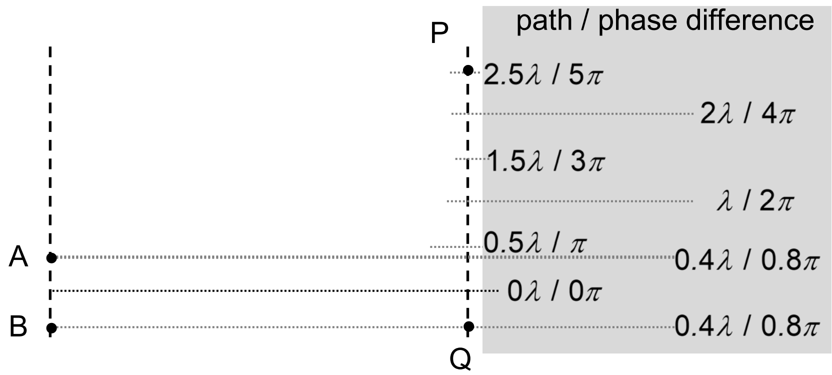


Find the number of regions of high intensities detected as the detector moves from P to Q, a point directly opposite source B.

Solution:

$$\text{length AP} = \sqrt{(584-140)^2 + 810^2}$$
$$\text{length BP} = \sqrt{(584)^2 + 810^2}$$
$$\text{path difference BP} - \text{AP} = 75 \text{ mm} = 2.5\lambda$$
$$\text{phase difference } \Delta\phi = 5\pi$$

$$\text{length AQ} = \sqrt{140^2 + 810^2}$$
$$\text{length BQ} = 810 \text{ mm}$$
$$\text{path difference AQ} - \text{BQ} = 12 \text{ mm} = 0.4\lambda$$
$$\text{phase difference } \Delta\phi = 0.8\pi$$



There are 3 regions of high intensity detected
(at regions where the waves from A and B meet and overlap in-phase: $0\pi,\ 2\pi,\ 4\pi$.)

Note: on the “screen” (the plane on which the detector moves) is an *interference pattern of* alternating maxima and minima due to constructive and destructive interference.

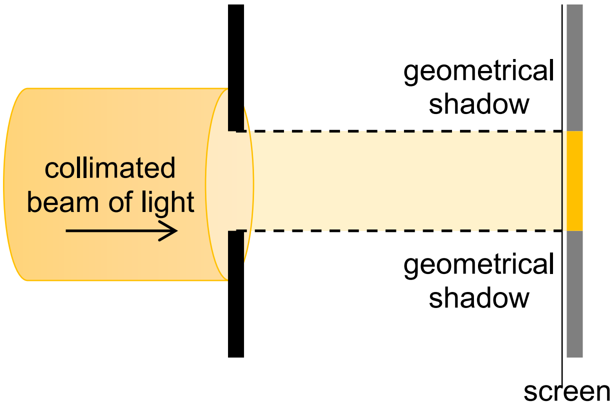
The formation of *interference patterns* is evidence that waves are being emitted by the sources and not particles. Particles will collide and bounce off each other, and thus unable to form interference patterns. Another property of waves that particles cannot have is that *waves can undergo diffraction*.

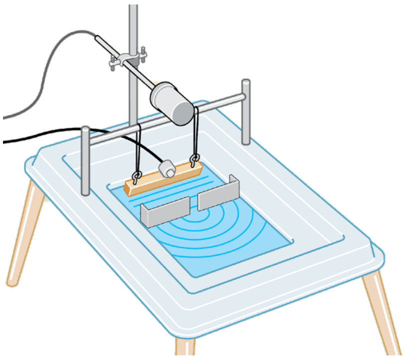
12.3 Diffraction

Diffraction is the

spreading of a wave
into the geometric shadow
when it passes through a slit
or past an edge of an obstacle

The *geometric shadow* is the shadow that *would have been cast* if diffraction effects are not present. Suppose a collimated beam of light falls onto a slit:

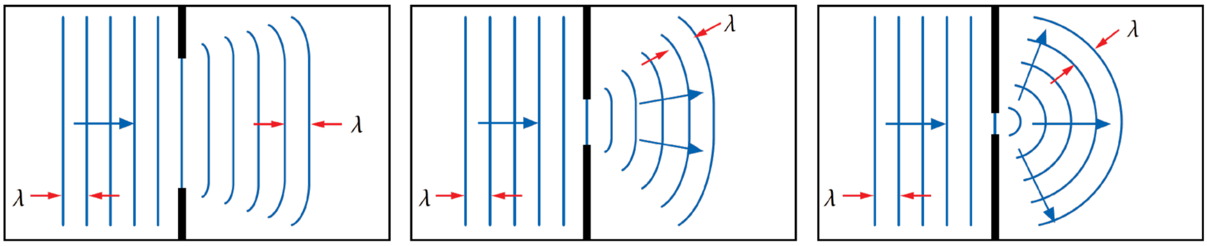




We can observe diffraction using a ripple tank. Plane waves are generated using an oscillating bar.

The plane waves move towards a slit in a barrier, through which the waves pass through and spread out.

We can do a visual inspection of the *extent* of diffraction when the slit width (relative to the wavelength) changes.



slit width >> λ
little diffraction.

slit width > λ
more extensive diffraction

slit width ≈ λ
most significant diffraction

In general,

diffraction is most significant when the size of the aperture (or obstacle) is of the same order of magnitude as the wavelength of the wave.

This is why we can hear sounds around doors and corridors - the wavelength of audible sound is in the same order of magnitude as the width of doors. So the sound waves can spread past edges of door frames significantly.

Conversely, light does not diffract significantly around household structures.

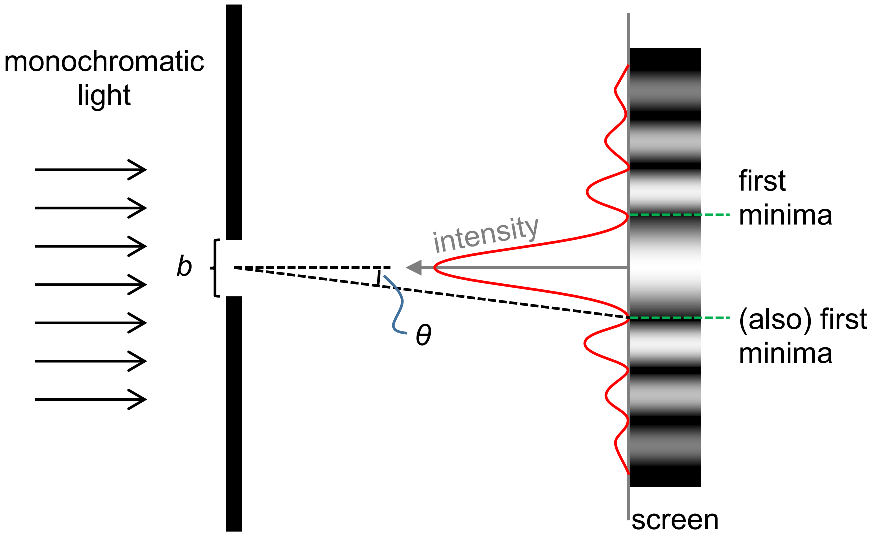
12.3.1 Single-Slit Diffraction

> In a single-slit diffraction, the position of first minimum is given by
>
> $$\sin\theta = \frac{\lambda}{b}$$

b is the slit width. Since the diffraction is significant, b is *in the order of* the wavelength of light λ.

When monochromatic light (light that has only a single wavelength – *chroma* has Greek origins referring to colour) is incident on the single slit, diffraction occurs and a pattern of a central bright fringe flanked by much weaker maxima with alternating dark fringes can be seen on a screen.

We will be dealing with bright/dark fringes (also referred to as maxima/ minima, regions of high intensity /low intensity).



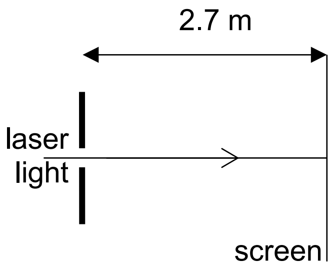
We can predict how the diffraction pattern will change when the set-up is modified.

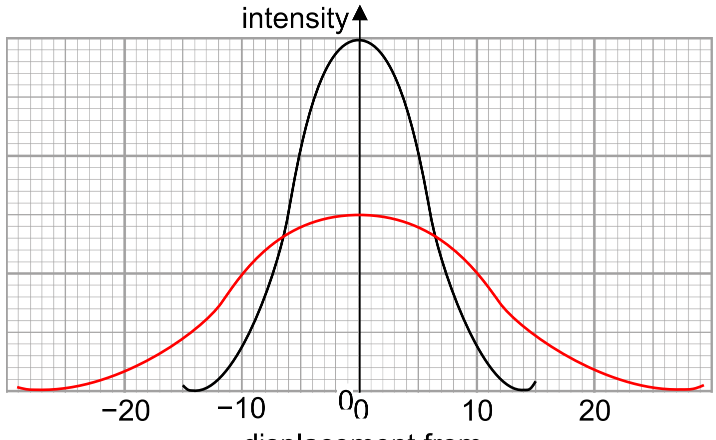
You may refer to the **SAO** structure to give complete responses to such questions.

For e.g., given the above set up, we explore how the pattern changes when the single slit is widened:

SAO	explainer	response to example
Separation of fringes	usually can refer to equation describes changes to the distance between bright or dark fringes	distance between first minima is decreased *less often seen alternative:* width of central maxima decreased
Amplitude of waves	addition of constituent amplitudes gives rise to maxima, subtraction of constituent amplitudes give minima since intensity ∝ (amplitude)^2, discuss intensity / energy of wave(s) reaching the screen	higher intensity at regions that are not minima wider slit allows more light to pass through
Overlap of diffracted wave(s)	discuss *extent* of diffraction at each slit	less spreading of the waves slit width is further away from the value of wavelength

Example 6
Laser light is incident on a single slit of width 12 μm. Part of the diffraction pattern is shown.
(a) Find the wavelength of the laser light.
(b) The slit is now replaced with a narrower slit. Sketch the new diffraction pattern.
(c) Explain the changes to the pattern when visible light of a shorter wavelength is used.
(d) Explain why the central fringe is white with coloured edges when white light is used.





Solution:

(a) distance from first minimum to central fringe $= 14.0$ cm

distance from slit to 1st minimum $= \sqrt{2.7^2 + 0.14^2}$

$$\sin\theta = \frac{0.14}{\sqrt{2.7^2 + 0.14^2}} = \frac{\lambda}{b}$$

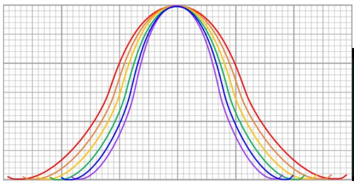
$$\lambda = \left(12 \times 10^{-6}\right)\left(\frac{0.14}{\sqrt{2.7^2 + 0.14^2}}\right) = 621 \text{ nm}$$

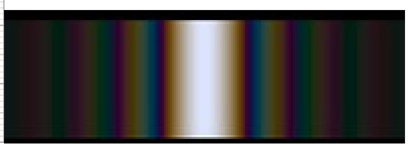
(b) slit width b decreased.[**S**] distance of first minimum from central bright fringe increases OR central maxima is broader as [**O**] there is more spreading of the waves. [**A**] lower overall intensity as less light passes through.

(c) wavelength λ decrease [**S**] distance of first minimum from central bright fringe decreases OR central maxima is narrower as [**O**] there is less spreading of the waves. [**A**] same amount of energy passes through slit of same width but is less spread out, so intensity at central bright fringe increases

(d) white light comprises wavelengths spanning 400 – 700 nm which overlap at central region to produce white. position of first minima for longer wavelengths is further away from central region than position for shorter wavelengths, so more reddish further away from central region.

Note: It is misconception to say "white at centre *due to no diffraction*". All wavelengths undergo diffraction, but they overlap at the centre.



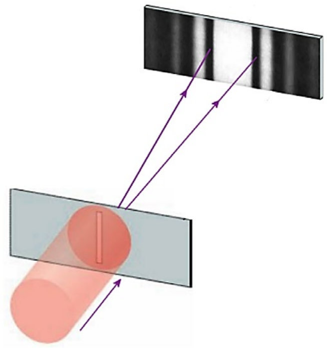


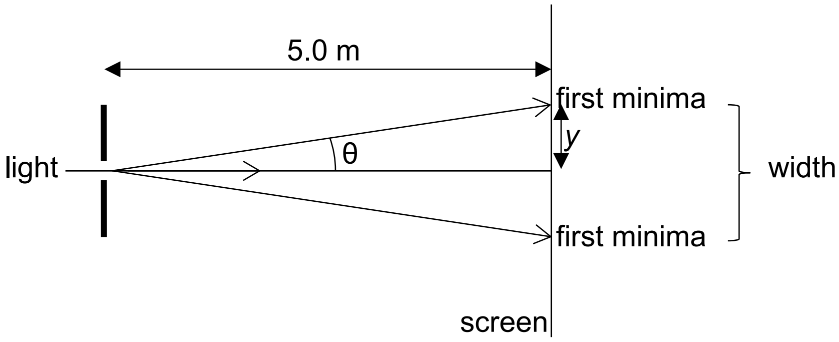
Example 7

A collimated beam of light wavelength 650 nm is incident on a surface and more than completely illuminates a single slit of width 0.10 mm.

(a) Find the width of the central maximum on a screen situated 5.0 m away from the slit.

(b) Describes the changes to the diffraction pattern when the slit width is doubled.





Solution:

(a) $\sin\theta = \dfrac{\lambda}{b}$

$$\tan\theta = \frac{y}{5.0}$$

$$\begin{aligned}\text{width} &= 2y \\ &= 2\ (5\tan\theta) \\ &= 2\left(5\tan\left(\sin^{-1}\frac{\lambda}{b}\right)\right) \\ &= 10\tan\left(\sin^{-1}\frac{650\times10^{-9}}{0.10\times10^{-3}}\right) = 0.0650\text{ m}\end{aligned}$$

(b) Since the angular position of the first minimum $\theta = \left(\sin^{-1}\dfrac{650\times10^{-9}}{0.10\times10^{-3}}\right) = 0.372^\circ$ is small,

$$\begin{aligned}\sin\theta &\approx \theta\ \text{(in rad)} \\ \theta &\propto \frac{1}{b}\end{aligned} \quad \text{and} \quad \begin{aligned}\tan\theta &\approx \theta \\ \text{width of central maximum} &\propto \theta\end{aligned}\ ;\ I = \frac{\text{power}}{\text{area}}$$

slit width *b* doubled, so twice the amount of energy passes through the slit

[**S**] width of central maxima halves as

[**O**] there is less spreading of the waves.

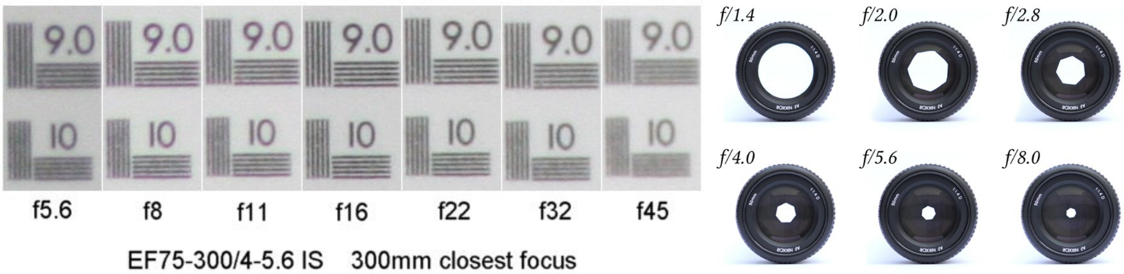
[**A**] twice the amount of energy is spread across half the area
so maximum intensity is quadrupled

Note that the single slit equation $\sin\theta = \frac{\lambda}{b}$ fails when the wavelength is larger than the slit as the sine function has a maximum value of 1. The physical meaning is that of *opacity* – waves with wavelengths that are significantly larger than the slit width(s) cannot be transmitted, and are reflected.

On the door of a microwave own, there are gaps in the metal mesh that are each a few millimetres wide. The wavelength of microwaves is about 12.5 cm, which is significantly larger. This design allows you to see the food inside the oven while preventing microwaves from escaping.

12.3.2 The Rayleigh Criterion

A camera, by means of the aperture and lens, is essentially a circular "slit" that allows light from the environment to reach an image sensor ("screen"). Therefore, they are also subject to the effects of single slit diffraction.



All else being equal, a camera system has a "diffraction limit". Like a pin-hole camera, usually the smaller the size of the aperture, the sharper the image becomes, as we can see moving from f5.6 to f16. Beyond the limit, diffraction effects become significant and the image starts to blur again.

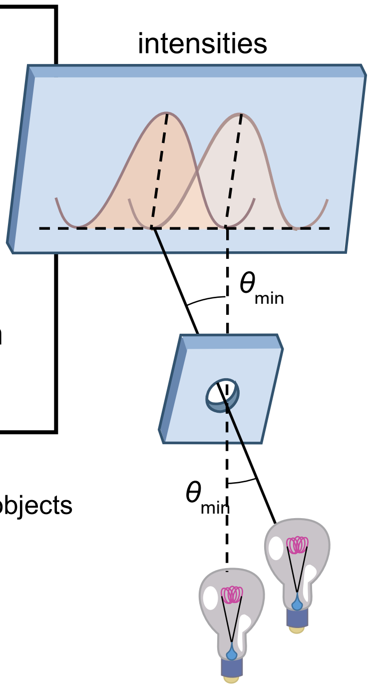
The ability to distinguish two lines or points in an object (each feature either giving off light on its own or reflecting light from it) is the *resolving power*, and the Rayleigh Criterion helps us find the limit:

The **Rayleigh Criterion** states that the

limit for which 2 sources of light can be just distinguished is when

the first minimum of the diffraction pattern of one source

coincides with the central maximum of the diffraction pattern of the other source

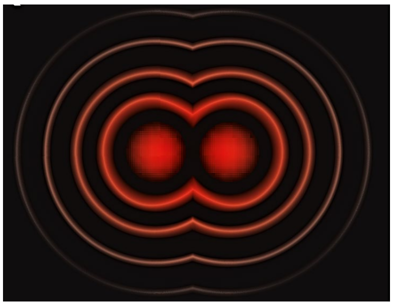


$$\theta \approx \frac{\lambda}{b}$$

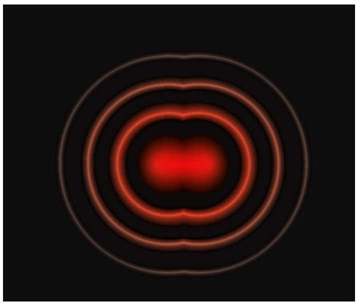
where θ = angular separation of the two objects
λ = wavelength
b = width of slit

Light from each object will undergo single slit diffraction as it passes through the aperture. Therefore, each light source will create its own diffraction pattern on the screen.

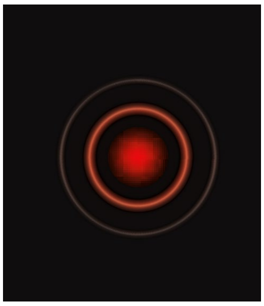
For diffraction patterns that are far apart, it is easy to distinguish the two sources. We say that the diffraction patterns are well-resolved. However, when the patterns are close to each other, the patterns start to overlap:



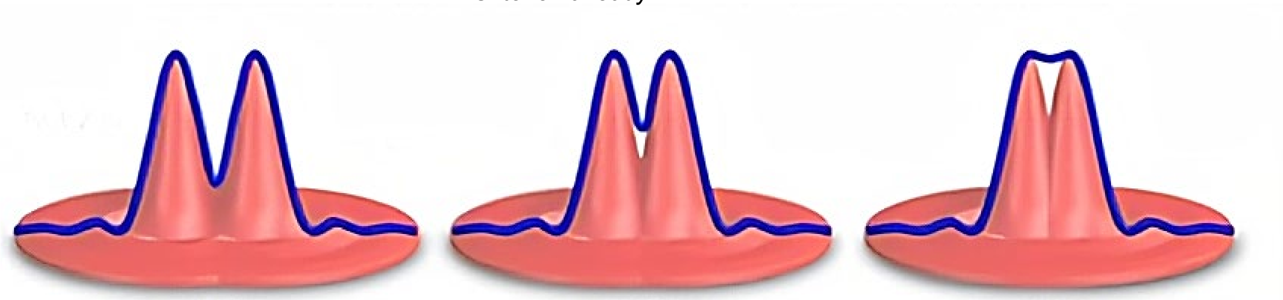
2 clearly distinct central maximas belonging to 2 sources of light

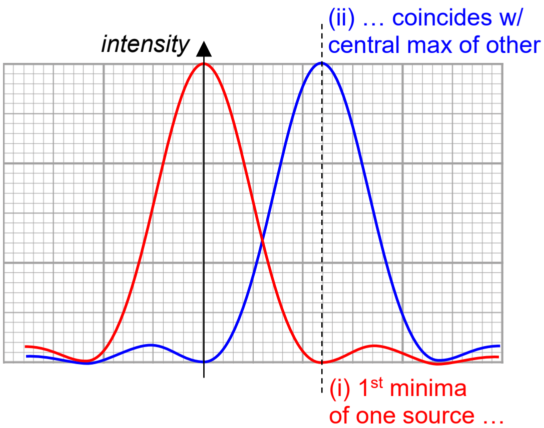


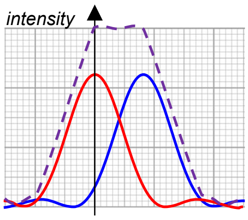
just barely distinguish 2 sources of light; at limit given by Rayleigh Criterion already



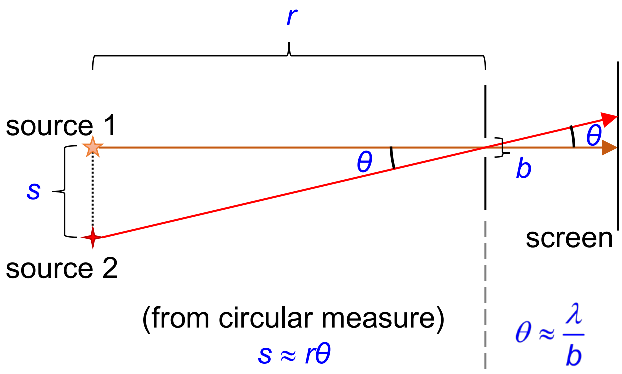
not well-resolved, looks like single source







Typical Geometry for Solving Rayleigh Criterion Questions



The equation for Rayleigh's Criterion resembles that of the single slit diffraction with small angle approximation applied $(\sin\theta \approx \theta)$.

The approximation is valid as we usually

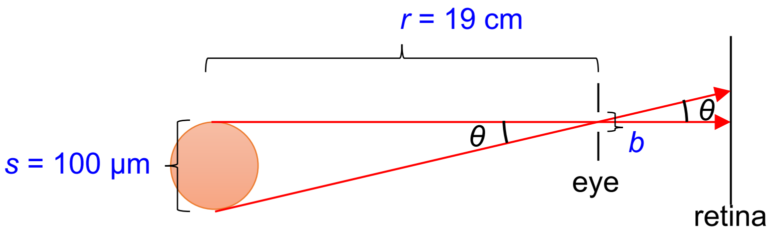
- work across a large distance (r) between the object and the slit
- are trying to "see" very small objects (distance s) where the top and bottom feature become the 2 sources of light.

Example 8

The maximum distance a particular human eye can resolve a grain of sand of radius 50 μm that is coloured red is 19 cm away from the sand. The grain of sand is then coloured blue. State the change in maximum distance, if any, for the eye to resolve the grain of blue sand.

Solution

$$\frac{s}{r} \approx \theta \approx \frac{\lambda}{b}$$
$$sb \approx r\lambda \approx (\text{constant})$$

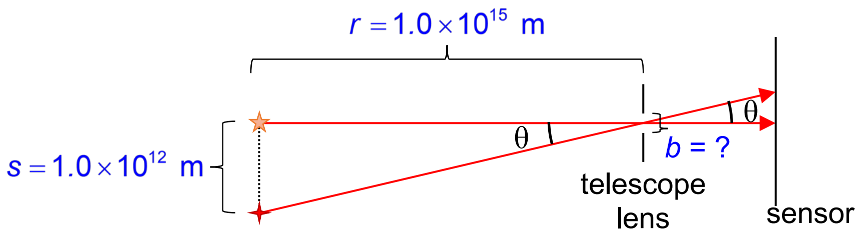


$$r_{\text{red}}\lambda_{\text{long}} \approx r_{\text{blue}}\lambda_{\text{blue}}$$

Wavelength has decreased (wavelength of blue is shorter than wavelength of red) so maximum distance has to increase. Eye has to be further than 19 cm from the sand.

Example 9

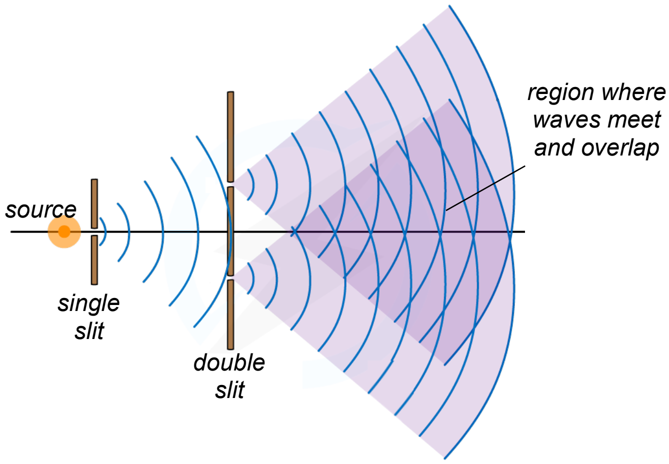
The collapsed hearts of dead stars, known as neutron stars, generate extremely strong magnetic fields that trap electrons, making them emit radio waves. Two dead stars are at a distance of 1.0×10^{15} m from Earth. The stars are separated by a distance of 1.0×10^{12} m and emit radio waves of wavelength 0.030 m. Find the diameter of a radio telescope on Earth that will just resolve the two stars.



Solution

$$\frac{s}{r} \approx \theta \approx \frac{\lambda}{b}$$
$$b \approx \frac{r\lambda}{s}$$
$$= \frac{(1.0 \times 10^{15})(0.03)}{1.0 \times 10^{12}}$$
$$= 30.0 \text{ m}$$

Diffraction through a single slit causes the same wave to spread out. We can then create two separate two sources of waves that are *coherent* to each other and make then interfere with each other.



The 2- source diffraction pattern allows us to

(i) confirm that the source is indeed a source of *waves* (because particles cannot interfere and form diffraction patterns) and

(ii) allows us to find the wavelength of the waves.

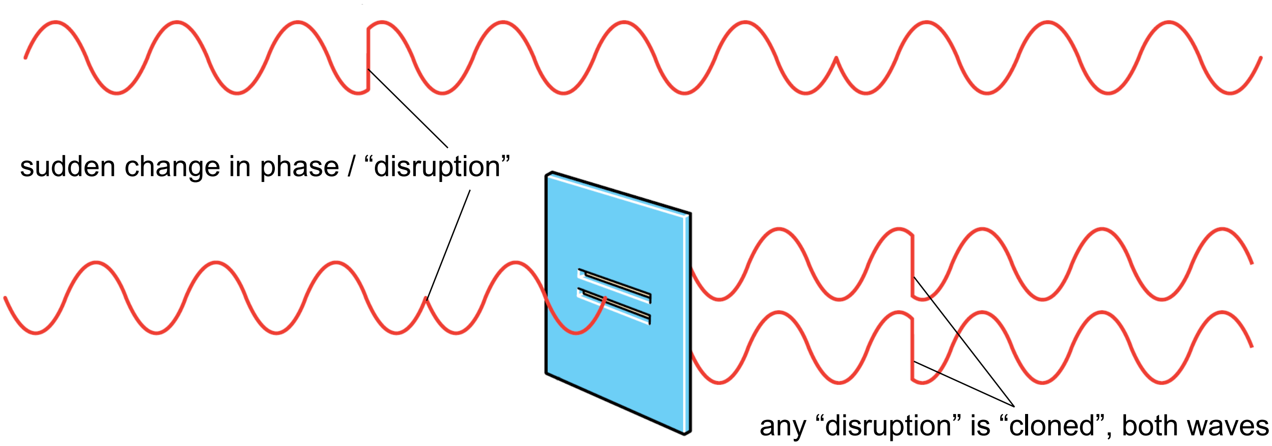
12.3.3 Coherent Sources

> If two waves are **coherent**,
>
> there is constant phase difference between the waves.

We have deliberately described the two wave sources in Examples 2 to 5 as "in-phase" or "synchronised" or "in anti-phase".

The formal description is that the wave sources are *coherent sources*.

We can use single-slit diffraction to "clone" the wave into 2 coherent sources.



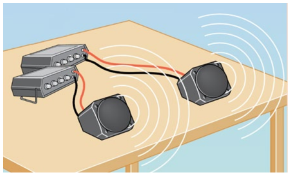
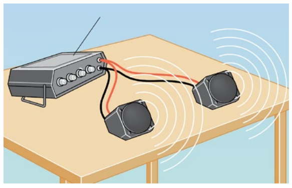
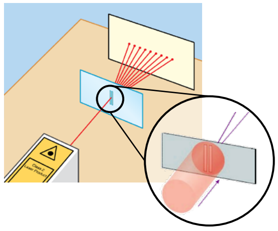
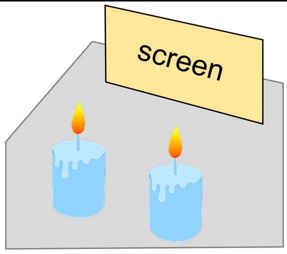
Real-world wave sources can exhibit sudden, random changes in phases.

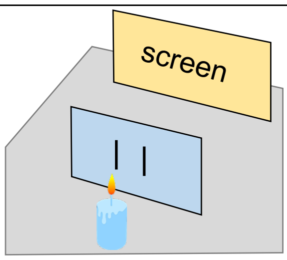
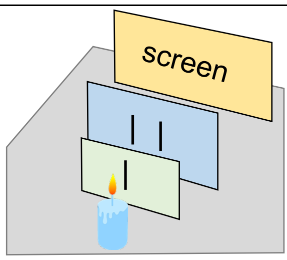
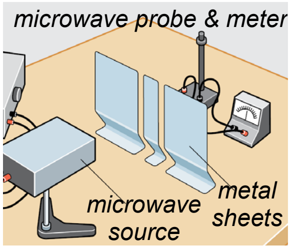
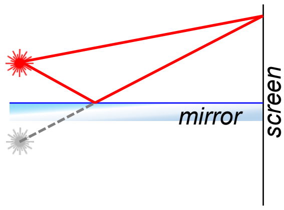
Therefore, we will not be able to get good, stable interference patterns with 2 separate real-world wave sources as they will not be *coherent*.

A bulb filament emits light by glowing hot. The process is random and spread across the whole filament and so different parts of the filament can randomly start or stop wave emission.

In a similar reasoning, light waves from all of a candle flame is not coherent because different fuel molecules combust with different oxygen molecules randomly in the vicinity, resulting in haphazard wave emission.

Try deciding if the following set-ups are coherent or otherwise:

2 speakers outputting same frequency	2 speakers connected to the same signal generator	1 laser beam shining on 2 slits	2 candles lighting up a screen
			
incoherent	coherent	coherent	incoherent

1 candle lighting up a double slit	1 candle on a single slit then a double slit	a microwave source on 2 slits	a light source and its reflection in a mirror
			
incoherent	coherent	coherent	coherent

Example 10
State the conditions required for an interference pattern of alternating maxima and minima to be formed by two sources of waves.

Solution
1. waves of the same type meet and overlap
2. waves from the sources have constant phase difference with each other
3. if waves are transverse, they are either unpolarised or polarized in the same plane

Note (i):
4. for good *contrast* (for maximas to be as high intensity as possible and minimas to be as low intensity as possible)
   waves from the sources should reach the screen with approximately same amplitude

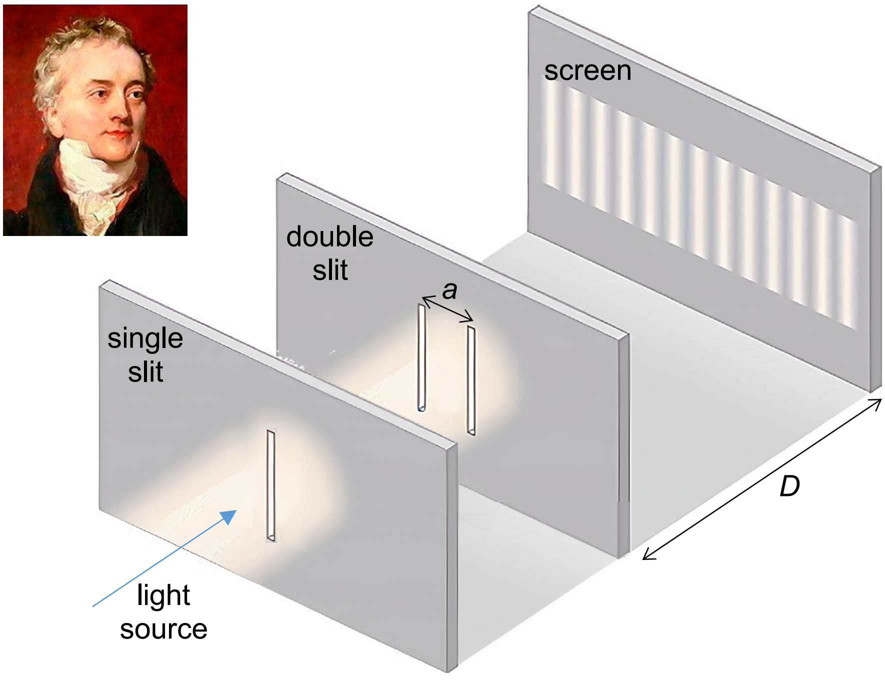
Note (ii): what should we expect to see if 2 sources are coherent but polarized at 90° to each other?

12.3.4 Young's Double Slit Experiment

English physicist and physician Thomas Young performed the double-slit experiment in 1801 to prove that light behaved as a wave.

He passed sunlight through a single slit and then a double slit to form an interference pattern.

If instead particles were passed through the 2 slits, there would simply be 2 lines (collections of particles) on the screen.



Earlier, Examples 2 to 5 showed the general concept concerning interference between 2 sources. The following double-slit equation applies as an approximation, when the distance between the double slit and the screen (D) is much larger than the separation between the double slits (a):

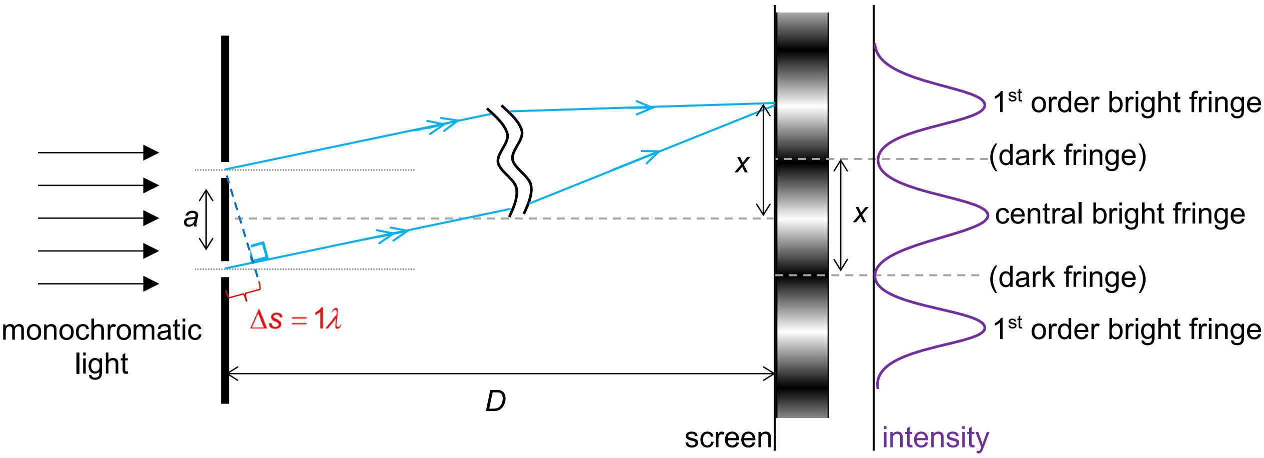
For double-slit interference, the distance between adjacent bright fringes (or adjacent dark fringes) is

$$x = \frac{\lambda D}{a}$$

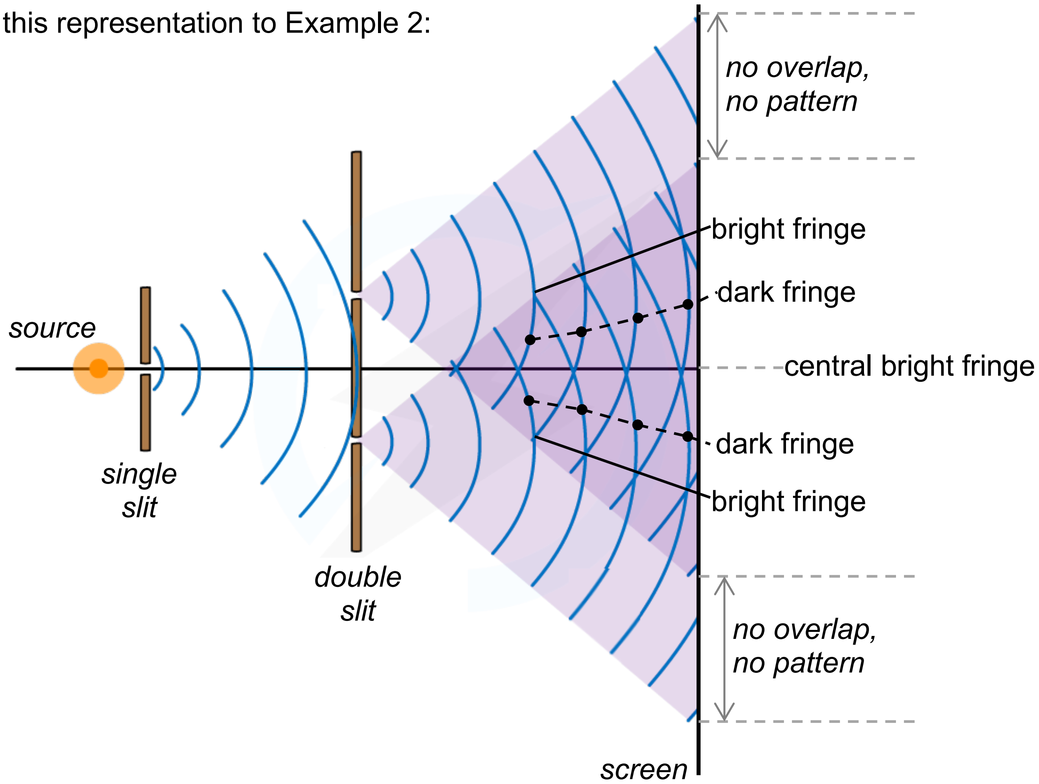
where
- x = fringe separation
- a = slit separation
- D = distance from double slits to screen
- λ = wavelength

and $D \gg a$

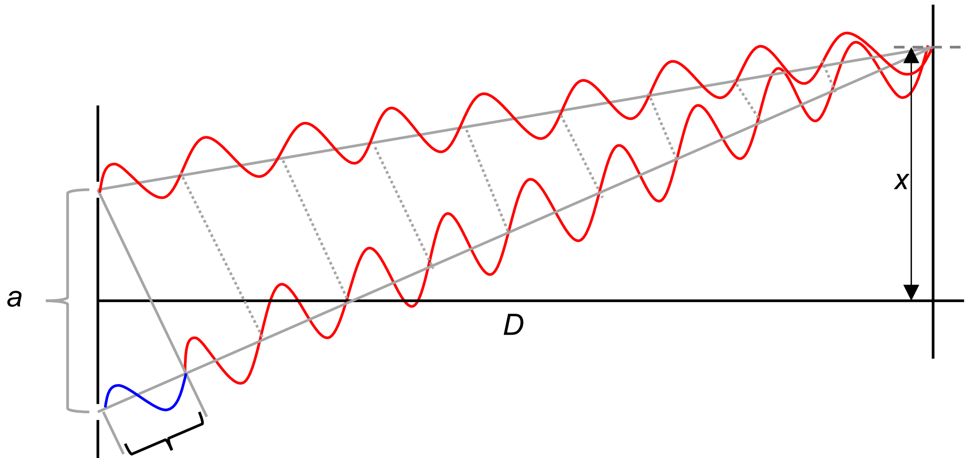
Note that fringes are evenly spaced out.



Compare this representation to Example 2:



Check back to pg 5 on explaining resultant intensity:



in general, for m^{th} order bright fringe: $\Delta s = m\lambda$

at 1st order maxima,

path difference is 1 wavelength $\Delta s = \lambda$

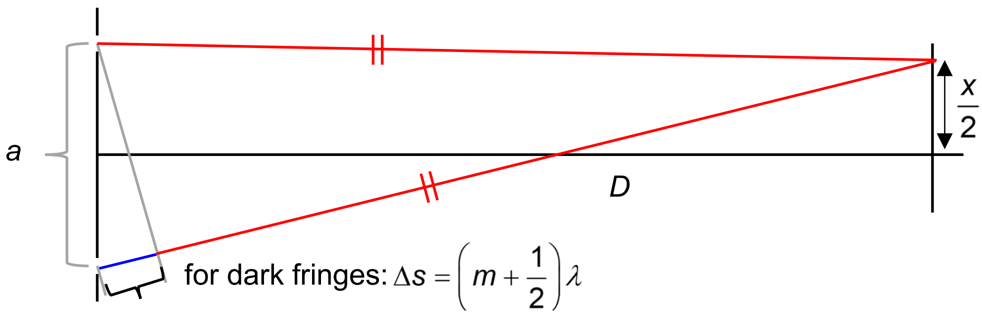
phase difference is $\Delta\phi = 2\pi$

waves from both sources meet and overlap in-phase

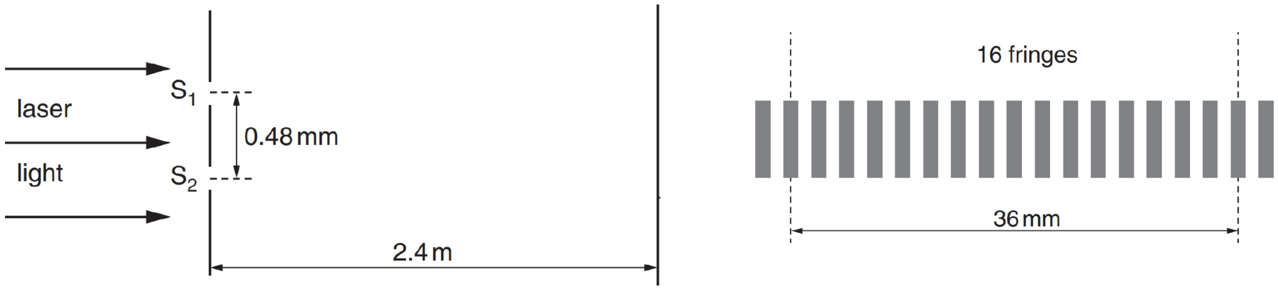
undergoes constructive interference

at the first minima, path difference is half a wavelength $\Delta s = \dfrac{\lambda}{2}$ and so phase difference $\Delta\phi = \pi$

waves from both sources meet and overlap in anti-phase so undergoes destructive interference



Example 11
The interference pattern of a double-slit experiment is shown. Find the laser wavelength..



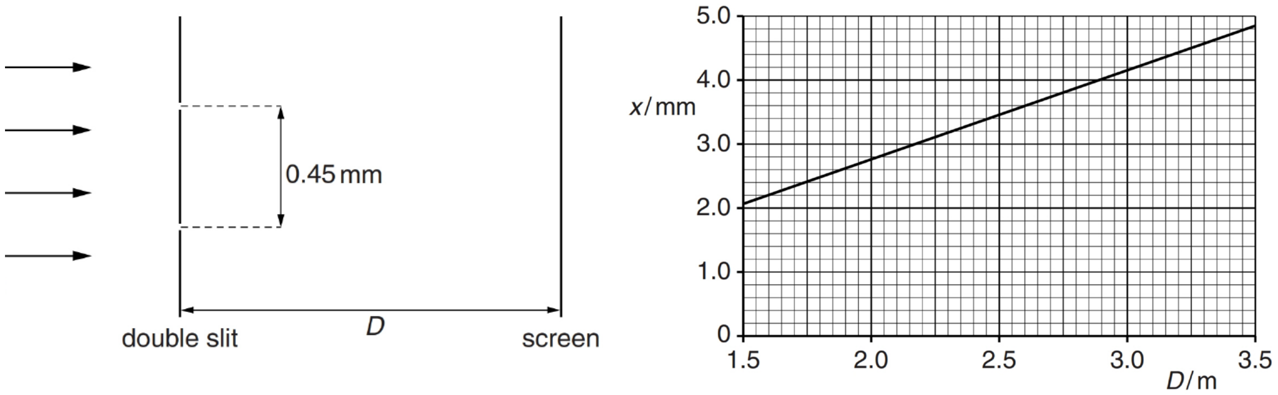
Solution

$$x = \frac{\lambda D}{a} = \frac{36\times10^{-3}}{16}$$

$$\lambda = \frac{ax}{D} = \frac{\left(0.48\times10^{-3}\right)}{2.4}\left(\frac{36\times10^{-3}}{16}\right)$$

$$= 450 \text{ nm}$$

Example 12
In a double-slit experiment, the fringe width x is measured as screen distance D is varied.
(i) Find the wavelength of the light source.
(ii) The slit separation is increased. State the effects on the graph.



Solution
(i)

$$x = \frac{\lambda D}{a}$$

$$\text{gradient of graph} = \frac{\lambda}{a}$$

$$\lambda = a(\text{gradient}) = a\left(\frac{x_1 - x_2}{D_1 - D_2}\right)$$

$$= \left(0.45\times10^{-3}\right)\left(\frac{(4.5-2.4)\times10^{-3}}{3.25-1.75}\right)$$

$$= 630 \text{ nm}$$

(ii) gradient of line is reduced.
The value of x will be reduced for all values of D; the new line is completely below the original line.

Example 13

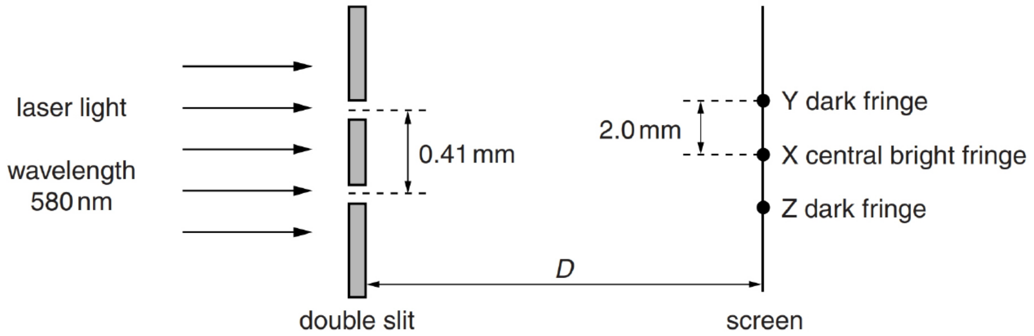
For the set-up shown below,

(i) explain why a bright fringe is formed at X,

(ii) state the difference in distance from each of the double slit to point Y, and

(iii) find distance *D*.

(iv) The intensity of the light incident on the two slits is constant. The width of one of the slits is now reduced. Compare the appearance of the fringes before and after the change of intensity.



Solution

(i) waves of constant phase difference from double slits meet and overlap at X. X is equidistance from both slits, path difference of waves is zero, waves arrive with no phase difference and undergo constructive interference.

(ii) at first dark fringe, the path difference is half wavelength so 290 nm.

(iii)

$$x = \frac{\lambda D}{a}$$

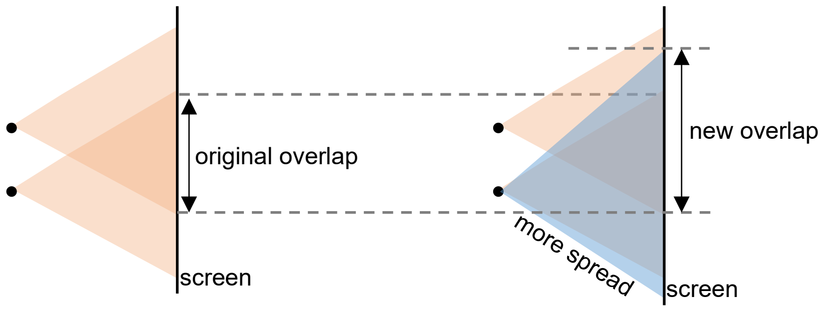
$$D = \frac{ax}{\lambda} = \frac{\left(0.41\times10^{-3}\right)\left(4.0\times10^{-3}\right)}{580\times10^{-9}}$$

$$= 2.83 \text{ m}$$

(iv) [**S**] fringe separation remains unchanged as slit separation unchanged.

[**A**] bright fringe is less bright while dark fringe is brighter so contrast is reduced.

[**O**] slit width more narrow so more significant spreading of waves. area of overlap where the interference pattern is visible is larger; more fringes visible.



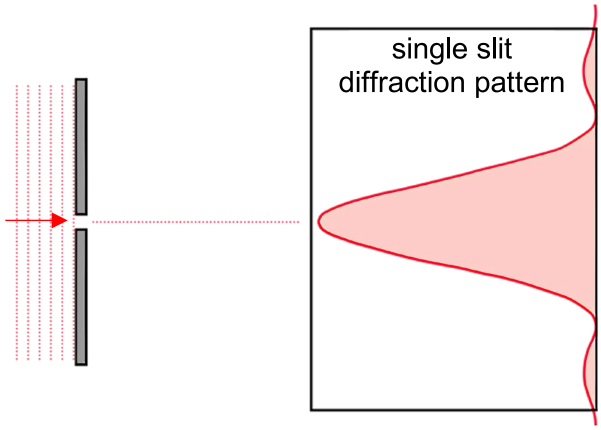
Note: Need to distinguish between slit *width b* and slit separation *a*. When discussing changes to the brightness/intensity of fringes under [**A**mplitude] of the [S**A**O] structure, state the changes to both the maxima and minima before commenting on the contrast.

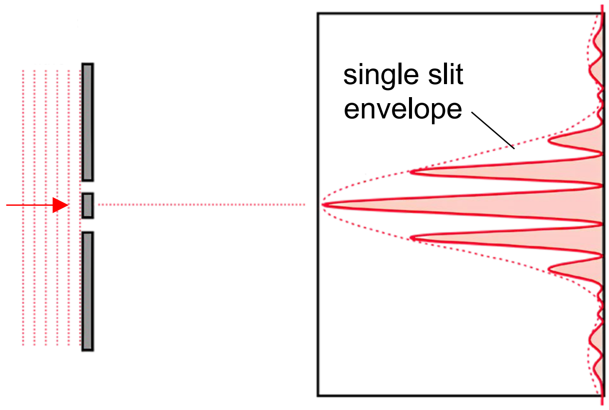
12.3.5 The Single Slit Envelope

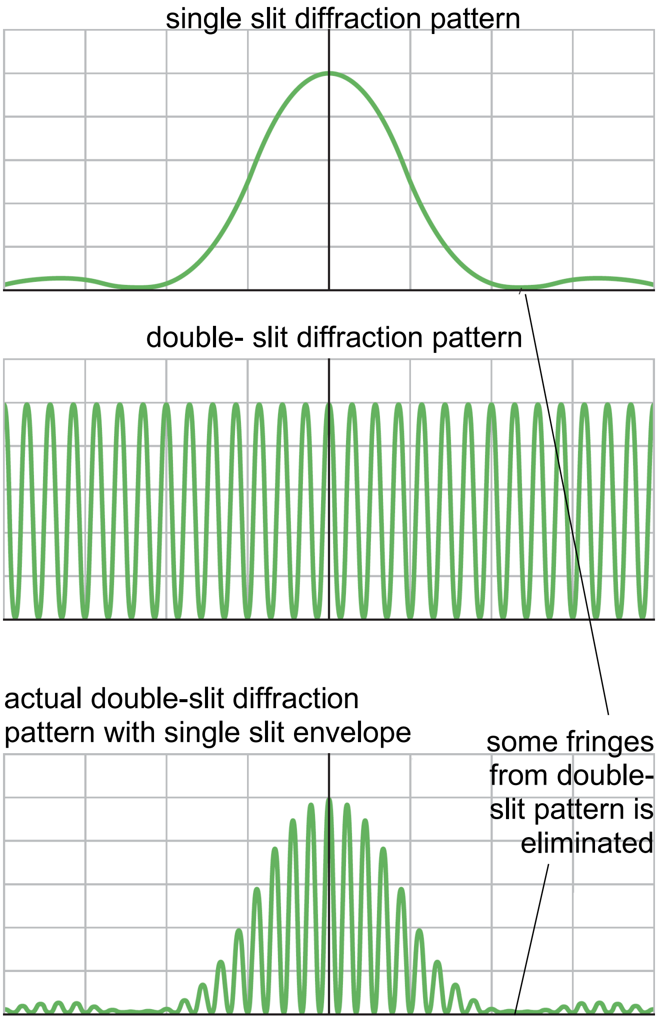
Take note that a double slit is simply two single slits placed in close proximity.

Physically, this implies that the *primary effects of single slit diffraction* will be observed *over and above* that of the double slit interference pattern.

We say that the overall intensity of a double-slit interference pattern is modified by an overarching single slit envelope.







If we do not account for the single slit envelope, the relative intensities of the bright fringes should be constant.

The envelope therefore results in single-slit minima being imposed onto a double-slit interference pattern.

The single slit envelope will continue to apply as we move from studying a double slit set-up to a multiple-slit set-up through a *diffraction grating*.

We also describe the phenomenon as the double slit interference pattern being *modulated* by the single slit.

Example 14

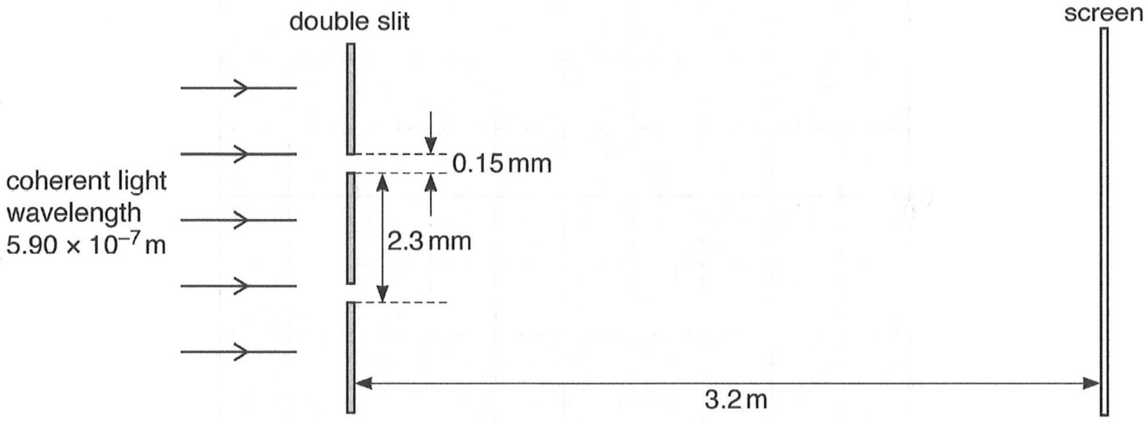
In the set-up shown below, one of the double slits is initially covered.

(a) Find the width of the central bright maximum.

(b) With both slits uncovered,

(i) use the Rayleigh criterion to explain if the two slits can be resolved on screen.

(ii) hence, estimate the number of fringes within the central bright maximum.



Solution

(a) let x be distance from first minimum to central bright maximum.

screen is very far away relative to slit width so apply small angle approximation

$$\tan\theta = \frac{x}{3.2}$$

$$\sin\theta = \frac{\lambda}{b}$$

$$\sin\theta \approx \theta \approx \tan\theta$$

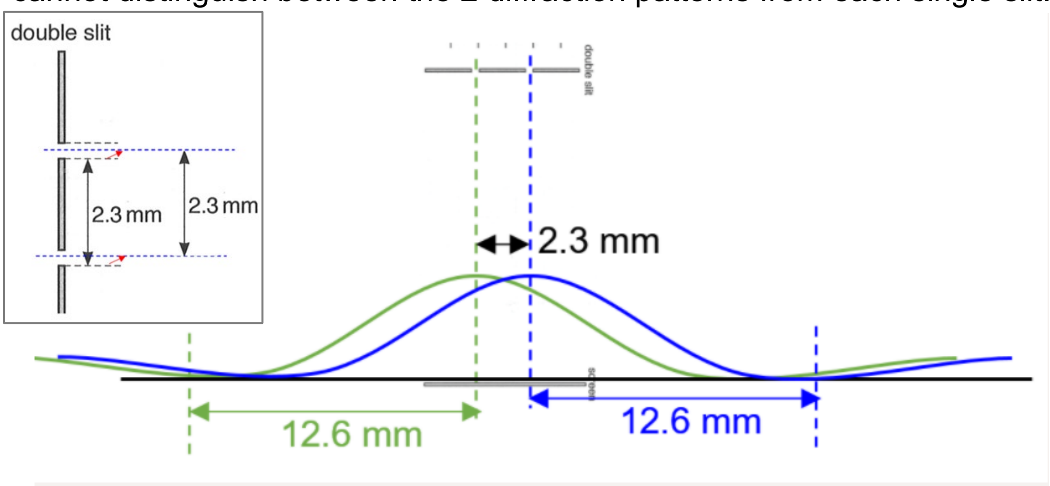
$$\frac{x}{3.2} \approx \frac{\lambda}{b}$$

width is $2x = 2(3.2)\left(\frac{5.9\times10^{-7}}{0.15\times10^{-3}}\right) = 0.0252$ m

(b)(i) distance from central bright maximum to first minima x = 12.6 mm

but the 2 central bright maxima's are 2.3 mm away from each other

cannot distinguish between the 2 diffraction patterns from each single slit.



(b)(ii) let y be fringe separation from double-slit effect

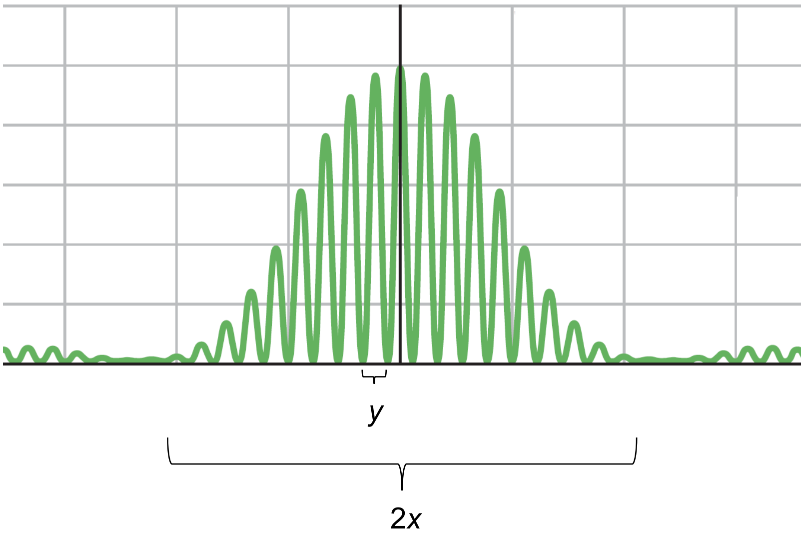
$$y = \frac{\lambda D}{a}$$

$$= \frac{\left(5.9 \times 10^{-7}\right)(3.2)}{2.3 \times 10^{-3}}$$

$$= 8.21 \times 10^{-4} \text{ m}$$

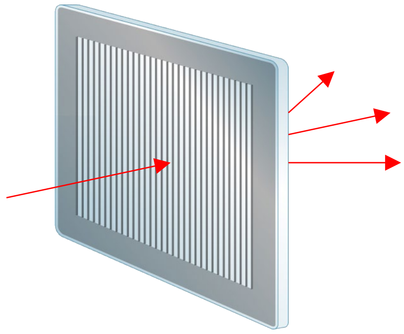
number of *fringes* that can fit into width of $2x$ (round down):

$$\frac{0.0252}{8.21 \times 10^{-4}} \approx 30$$



12.3.6 Diffraction Grating

A diffraction grating is like a double-slit, but instead of 2 slits it consists of a large number of equally spaced (opaque) lines. The slits between each of these lines is capable of diffracting incident light.



(above) perspective view of a typical diffraction grating where slits are cut at regular spacing

> For diffraction grating, angular position of n^{th} order maxima is
>
> $$d\ \sin\theta = n\lambda$$

where

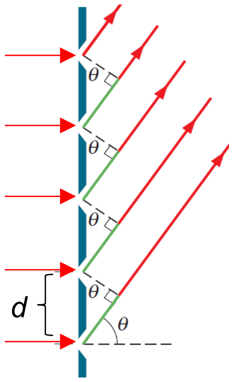
d = separation between adjacent slits

n = order of diffraction (an integer)

θ = angle between the n^{th} order beam and the normal to the grating

λ = wavelength of the incident beam

(right) top view of light passing through grating, path difference Δs in between adjacent slits is $d\ \sin\theta$

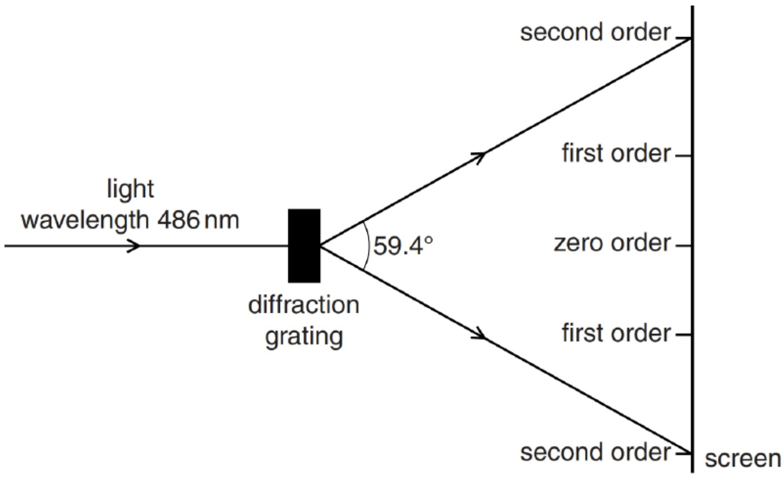


Diffraction gratings are usually specified by the number of lines per unit length for example N = 6000 / cm. To convert from N to d (in metres):

$$d = \frac{\text{unit length}}{N}$$

Example 15

In the set-up shown below, find the number of lines per millimetre of the grating.



Solution

$$d\ \sin\theta = n\lambda$$

$$\left(\frac{10^{-3}}{N}\right)\sin\left(\frac{59.4^\circ}{2}\right) = 2\left(486\times10^{-9}\right)$$

$$N = \frac{10^{-3}\left[\sin\left(\frac{59.4^\circ}{2}\right)\right]}{2\left(486\times10^{-9}\right)} = 510\ \text{mm}^{-1}$$

Example 16

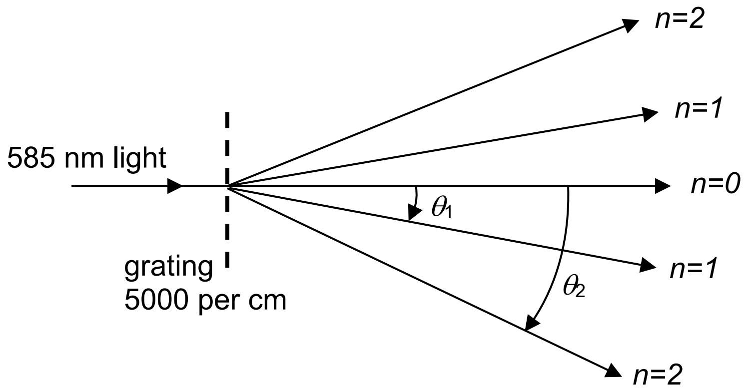
A diffraction grating can be used to find the wavelength of a light source. Explain how the second order maxima is formed.

Solution

waves spread into geometric shadow as they pass through each slit of diffraction grating
waves from each slit meet and overlap
with a path difference of two wavelengths
resulting in meeting with a phase difference of 4π
waves meet in phase so undergo constructive interference

Example 17

Find the largest possible angle at which a maxima is visible from the set up below.



Solution

$$d\sin\theta = n\lambda$$

$$\sin\theta = \frac{n\lambda}{d} = \frac{Nn\lambda}{10^{-2}}$$

max possible value implies $\sin\theta \to 1$

$$\frac{Nn\lambda}{10^{-2}} \le 1$$

$$n \le \frac{10^{-2}}{N\lambda} = \frac{10^{-2}}{(5000)(585\times10^{-9})}$$

$$n \le 3.42$$

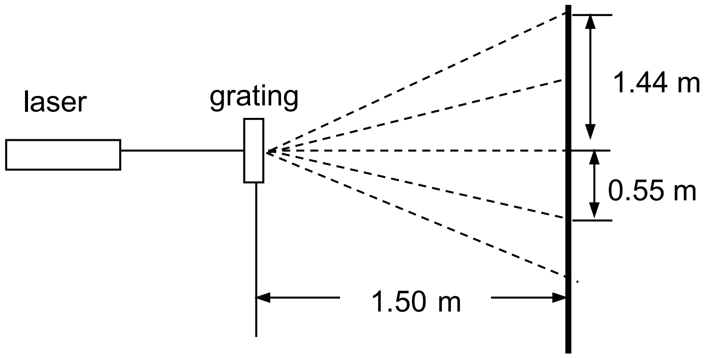
largest possible order $n = 3$

$$\theta_3 = \sin^{-1}\left(\frac{Nn\lambda}{10^{-2}}\right) = \sin^{-1}\left(\frac{(5000)(3)(585\times10^{-9})}{10^{-2}}\right)$$

$$= 61.3^\circ$$

Note: must distinguish between *n* (order of diffraction) and *N* (number of lines per unit length of grating) clearly.

Example 18
Laser light passes normally through a diffraction grating of 550 lines per mm as shown.



(a) Using measurements of the first-order diffraction, calculate the wavelength of the light.
(b) Suggest one advantage and one disadvantage of using observations of the second-order diffracted light rather than the first-order diffracted light.

Solution:

(a) $d \sin\theta = n\lambda$

$$\lambda = \left(\frac{10^{-3}}{550}\right)\sin\left(\tan^{-1}\left(\frac{0.55}{1.5}\right)\right)$$

$$= 626 \text{ nm}$$

(b) Advantage: 2^{nd} order image has larger angular position θ ($\theta_2 > \theta_1$).
results in longer distance measured from centre bright spot.
gives lower percentage uncertainty in the measurement of length
and in calculation for wavelength.

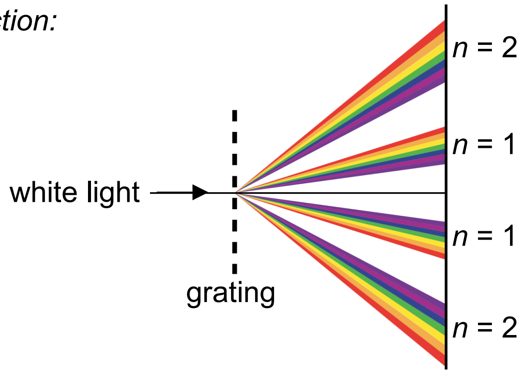
Disadvantage: 2^{nd} order image is less sharp and less bright
so finding location of 2^{nd} order bright spot is less accurate
less accuracy in the calculated value of wavelength.

12.3.7 Diffraction of White Light

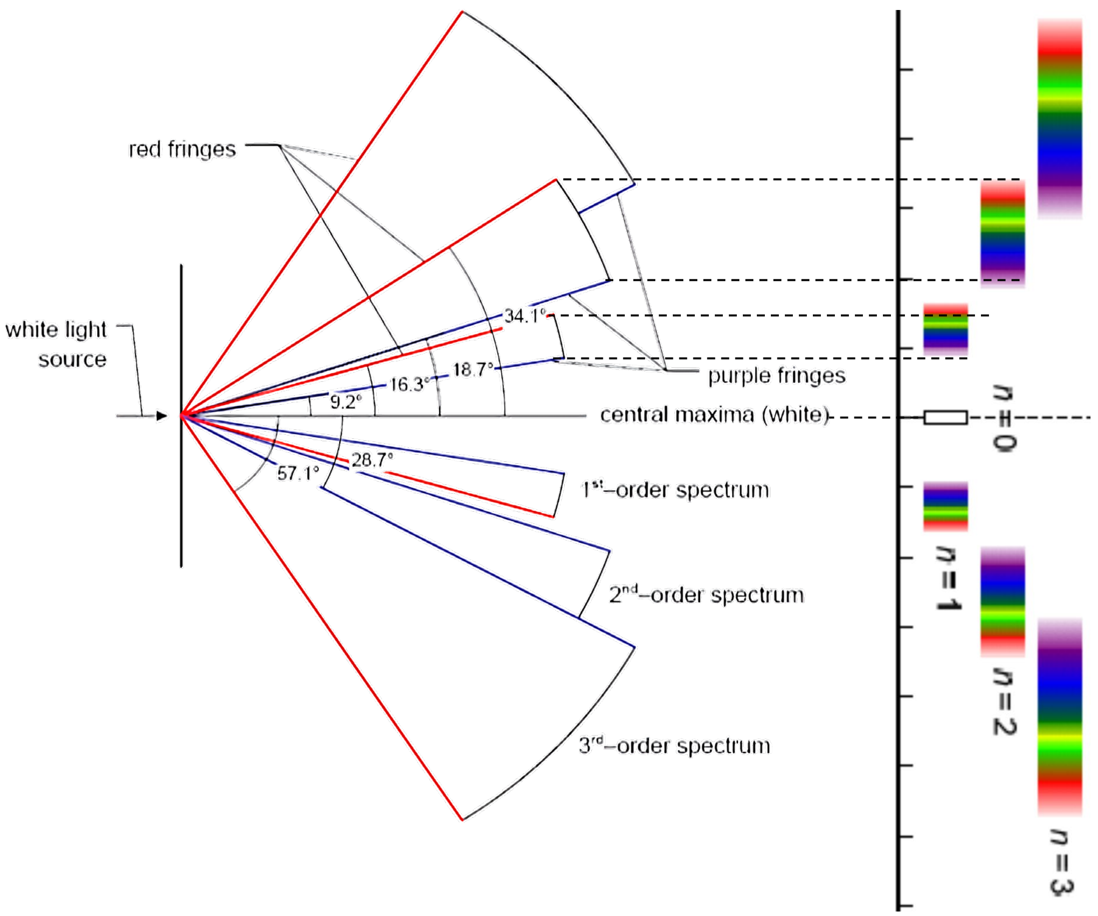
Since $d \sin\theta_n = n\lambda$, for the same order n, waves with longer wavelengths will be diffracted more than waves with a shorter wavelengths. Therefore, red light ($\lambda \approx 700$ nm) will have a larger diffraction angle compared to blue light ($\lambda \approx 400$ nm).

White light is a combination of light of different colours. When white light passes through a diffraction grating, the different wavelengths exhibit different angles of diffraction. There may even be overlapping spectra, e.g. the 2nd order red having a larger diffraction angle than the 3rd order violet.

Without overlapping orders of diffraction:



Overlapping orders of diffraction:



Example 19

In the spectrum of white light obtained using a certain diffraction grating, the second order and third order partially overlap. Find the 3rd order wavelength that coincides at the same angle with the 2nd order wavelength of 650 nm.

Solution:

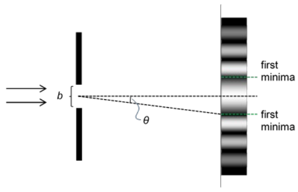
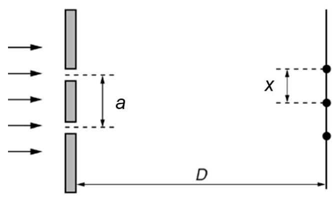
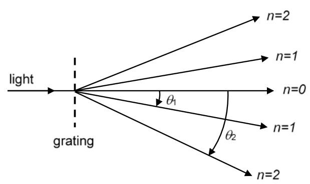
$$d\ \sin\theta = n\lambda$$

$$d\ \sin\theta = 3\lambda_3$$
$$d\ \sin\theta = 2\left(650\times10^{-9}\right)$$

$$\lambda_3 = \frac{2}{3}\left(650\times10^{-9}\right)$$
$$= 433\text{ nm}$$

The 2nd order red (650 nm) coincides with the 3rd order violet (433 nm).

12.3.8 Comparing Diffraction Patterns

	single slit	(Young's) double slit	diffraction grating
geometry			
equation	$\sin\theta = \frac{\lambda}{b}$	$x = \frac{\lambda D}{a}$	$d\ \sin\theta = n\lambda$
equation describes	position of 1st minima	separation between adjacent bright/dark fringes	position of n^{th} order maximas
fringe separation	not constant	constant	not constant

H212 Superposition will be continued…