Additional Notes

Topic 3: Dynamics

Content:

- Newton's laws of motion
- Linear momentum and its conservation

Learning Outcomes:

Candidates should be able to:

<u>Newton's laws of motion</u>

(a) state and apply each of Newton's laws of motion.

(b) show an understanding that mass is the property of a body which resists change in motion (inertia).

(c) describe and use the concept of weight as effect of a gravitational field on a mass.

<u>Linear momentum and its conservation</u>

(d) define and use linear momentum as the product of mass and velocity.

(e) define and use impulse as the product of force and time of impact.

(f) relate resultant force to the rate of change of momentum.

(g) recall and solve problems using the relationship $F = ma$, appreciating the resultant force and acceleration are always in the same direction.

(h) state the principle of conservation of momentum.

(i) apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension. (knowledge of the concept of coefficient of restitution is not required.)

(j) show an understanding that, for a (perfectly) elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation.

(k) show an understanding that, whilst the momentum of a system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

Additional Notes

Introduction

- In Kinematics, we described motion in terms of position, velocity and acceleration without considering what might cause that motion.
- In Dynamics, we consider the cause – what might cause one object to remain at rest and another object to accelerate? The two main factors are the mass of the body, and the forces acting on the body. Dynamics is therefore a study of the relationship between the forces acting on a body and the motion of that body.
- The study of dynamics is largely governed by Newton's Three Laws of Motion.

Newton's First Law (Law of inertia):
A body stays at rest or continues to move with constant velocity unless a resultant force acts on it.

Newton's Second Law:
The rate of change of momentum of a body is proportional to the resultant force acting on the body and takes place in the direction of the resultant force.

Newton's Third Law:
If body A exerts a force on body B, then body B will exert a force of the same type that is equal in magnitude and opposite in direction on body A.

For N2L defn, it is wrong to interchange the terms "rate of change of momentum" with "resultant force".

Nature of Science
Science is an evidence-based, model-building enterprise concerned with the natural world. In this topic, we can see how ***models such as free-body diagrams and vector diagrams*** *are used to represent real-life scenarios. Using models allow us to make predictions, which we can proceed to verify experimentally.*

Relating Science and Society
Examples of application of Newton's Law of Motion are found everywhere in everyday life. These can range from simple ones such as walking, playing tennis, to more complex applications such as car safety and rocket building.

A.1 Newton's 1st Law

memorise

A body stays at rest or continues to move with constant velocity unless a resultant force acts on it.

- Newton's 1st Law is also known as the law of inertia.
- The inertia of a body is the property of a body to resist a change in its motion.
- The mass of a body is a measure of the resistance of a body to a change in its motion i.e. a measure of its inertia.
- The greater the mass of a body, the lesser the body accelerates under the action of a given applied force.
- Unlike weight (see side note) which depends on gravitational field and therefore varies from place to place, mass is independent of gravitational forces, i.e. the mass of a body is the same on the Earth as on Moon. Mass is a scalar quantity while weight is a vector.

The weight W of a body is the gravitational force exerted on the body. If g is the acceleration of the body towards the centre of the Earth then we can substitute F (force accelerating the body) = W and $a = g$ in $F = ma$, hence $\boldsymbol{W = mg}$

Additional Notes

Worked Example 1

A passenger in a bus claimed that a bag flew forward when the bus brakes. Explain how this is possible.

- According to N1L, since no horizontal force acts on the bag during braking, it will continue to move forward with the same velocity as before.
- Since the bus slows down as it brakes while the bag maintains its horizontal velocity, the bag is seen by the passenger as flying forward.

A.2 Newton's 2nd Law

A.2.1 Linear Momentum

To understand Newton's 2nd Law, first, we need to know what linear momentum is.

Linear momentum is defined as the product of the mass of the body and its velocity.

memorise

Mathematically, it is $\boxed{p = mv}$

where p is the linear momentum, m is the mass, and v is the velocity of the body.

- Linear momentum is a vector quantity.
- Its direction is along its velocity v and its SI unit is kg m s^{-1} or N s.
- Momentum is a quantity that describes objects in motion. Imagine that you have intercepted a football and see two players A (of mass 80 kg) and B (of mass 130 kg) from the opposing team approaching you as you run with the ball. Both of the players are running towards you at 5 m s^{-1}. However, because the two players have different masses, intuitively you know that you rather collide with player A rather than player B.

A.2.2 Newton's 2nd Law

The rate of change of momentum of a body is proportional to the resultant force acting on the body and takes place in the direction of the resultant force.

memorise

- Mathematically, this translates to:

$F \propto \dfrac{mv - mu}{t}$ where v is the final velocity, and u is the initial velocity.

$F \propto m\,a$ if a is the acceleration of the body, then $a = (v - u) / t$

$F = \mathrm{k}\, ma$

where k is a constant which has no unit but whose size depends on the unit to be chosen for force.

Do note that Newton's 2nd Law is not defined as $F = ma$! That was good enough for O-Level but not good enough for A-Level.

Additional Notes

- With SI units, the **newton (N)** is the unit of force. One newton is defined as the force which gives a mass of 1 kg an acceleration of 1 m s^{-2}. Substituting F = 1 N, m = 1 kg and a = 1 m s^{-2} in F= k ma we obtain k = 1. Thus with these units k = 1 and ***F = ma.***
- Thus, a force can be measured by finding the acceleration it produces in a known mass.
- If **SI units are used**, Newton's second law may be written as :

Resultant force = rate of change of momentum

$$F_{net} = \frac{dp}{dt} = \frac{d(mv)}{dt}$$

- If the force is applied over a time interval of Δt :

Average resultant Force, $< F_{net} > = \frac{\Delta p}{\Delta t}$

In real life, the resultant force exerted on an object to change its momentum may change during the duration of contact. In such cases, we refer to the average resultant force exerted on the object. (*More on p15)*

<u>Worked Example 2</u>

A body of mass 6.0 kg initially has a velocity 4.0 m s^{-1} in the eastward direction. It suddenly changes velocity to 3.0 m s^{-1} in the southward direction in a time of 0.20 s.

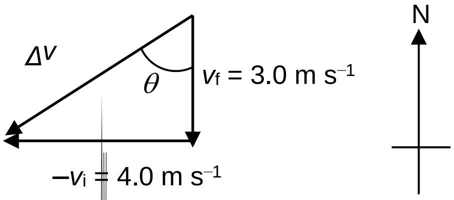
Calculate the average force acting on the body during the change of direction.

Remember all the change in velocity Δv questions from Topic 1 Measurements? Remember that Δv is a vector. Here is where you put your ability to do vector subtraction to good use.

Change in velocity, Δv = final velocity v_f – initial velocity v_i

$$\Delta v = v_f + (-v_i)$$

$$\Delta v = \sqrt{3.0^2 + 4.0^2}$$
$$= 5.0 \text{ m s}^{-1}$$



$\tan\theta = \frac{4.0}{3.0}$ => $\theta = 53.1°$.

By N2L, Average Force, $< F > = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t}$

$$<F> = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{(6.0)(5.0)}{0.20} = 150 \text{ N.}$$

Note that if the question specifies to calculate the "magnitude" of the average force, only the magnitude is required in your answer. However, if the question simply asks you for the "force", then both the magnitude and the direction are required.

A.3 Newton's 3rd Law of Motion

Additional Notes

If body A exerts a force on body B, then body B will exert a force of the same type that is equal in magnitude and opposite in direction on body A.

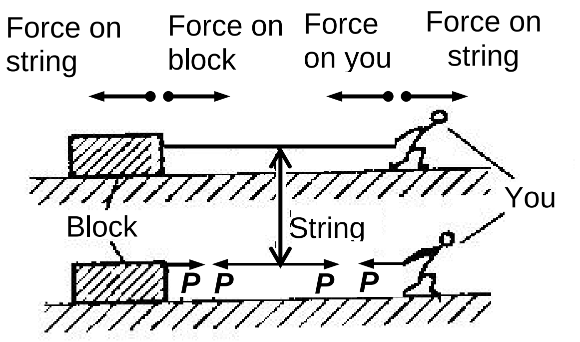
memorise

In this topic, we use F_{AB} to represent force by A on B, and F_{BA} to represent force by B on A.

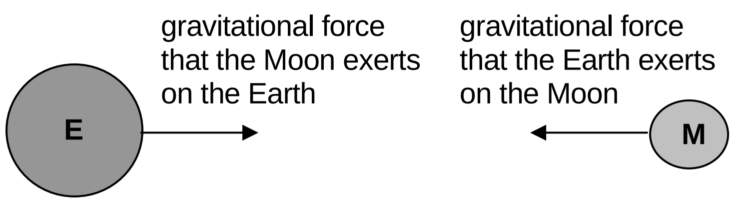
The force by body A on body B, and the force by body B on body A form an action-reaction pair. The forces happen at the same time, i.e. one force is not the cause of the other force. Some common examples of action-reaction pairs :

- If you slam your fist against the wall or kick a football with your bare foot, you can feel the force back on your fist or your foot.

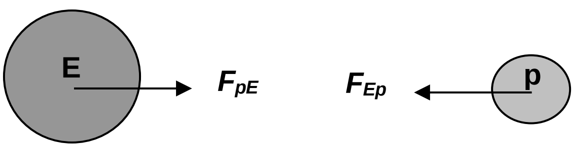
- If you pull a string attached to a block with a force P to the right as shown below, the string pulls you with an equal magnitude of force P to the left. Generally, we can assume the string transmits the force unchanged and so there is another pair of equal and opposite forces at the block. The string exerts a pull P to the right on the block and the block exerts an equal magnitude of pull P to the left on the string. The string is pulled outwards at both ends and is in a state of **tension**.



- The gravitational force that the Moon exerts on Earth is equal in magnitude and opposite in direction to the gravitational force that the Earth exert on Moon.



- The force acting on a freely falling projectile is the gravitational force exerted by the Earth on the projectile F_{Ep} where F_{Ep} means the force exerted by Earth (E) on projectile (p). The magnitude of F_{Ep} is mg. The reaction to this force is the gravitational force exerted by the projectile on the Earth F_{pE}. The reaction force F_{pE} must accelerate the Earth toward the projectile just as the action force F_{Ep} accelerates the projectile toward the Earth. However, because the Earth has such a large mass, its acceleration due to this reaction force is negligibly small.



Additional Notes

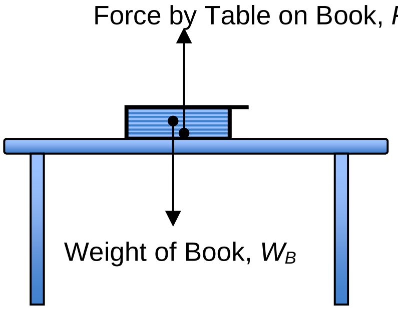
- **For two Forces to be Action-Reaction Pair, they must fulfil these three conditions.**

1. They are the same type of force, e.g. gravitational forces, contact forces.
2. They act on two different bodies.
3. They are equal in magnitude but opposite in direction to each other.

<u>Applying the conditions to identify action-reaction forces</u>

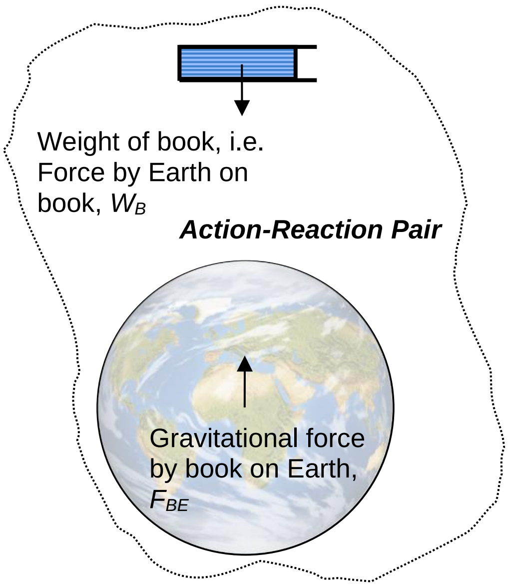
Note that both W_B and F_{TB} should act through the centre of gravity, CG of the book to achieve rotational equilibrium (more on this in the topic on Forces). In this diagram, they are drawn slightly displaced away from each other for clarity purpose.

a) Consider a stationary book which is lying flat on a table. Is the weight of the book and the force which the table acts on the book an action-reaction pair, and why?



No
1) They are not the same type of force.
W_B = gravitational (non-contact) force, and
F_{TB} = contact force.
2) W_B and F_{TB} are both acting on the same single body, i.e. the book itself!

b) So, what is the action-reaction pair for weight of the book, W_B?



By N3L, the action-reaction pair must be a *<u>gravitational force also</u>*, and it must act *<u>on a different body, which is the Earth in this case.</u>*

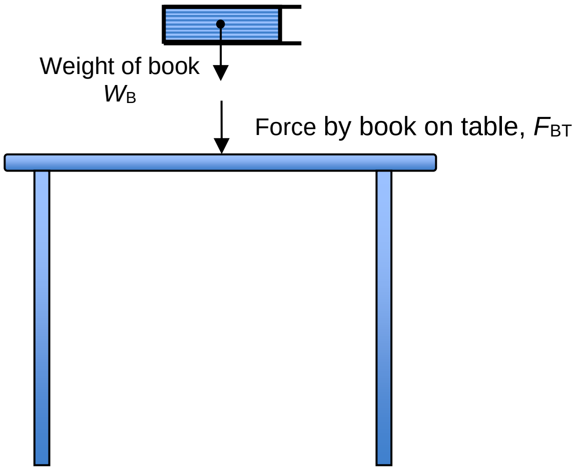
Thus, the action-reaction pair for W_B is actually the gravitational force by book on Earth, F_{BE}.

c) Next, what is the action-reaction pair for force by table on book, F_{TB}?

By N3L, the action-reaction pair must be a *<u>contact force also</u>*, i.e. contact force by book on table, F_{BT}

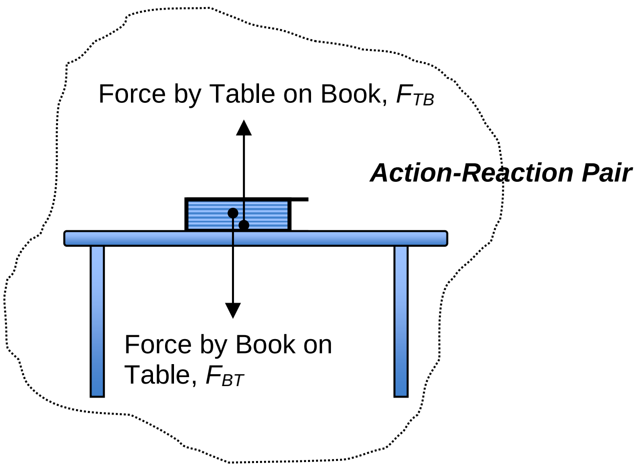
Additional Notes

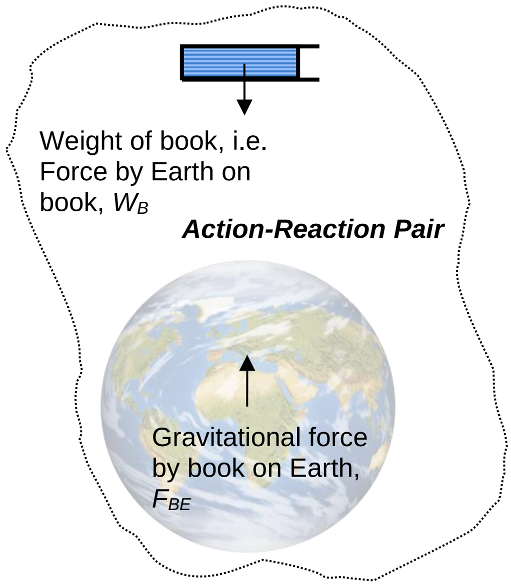
d) Are the weight of book, W_B and the force by book on table, F_{BT} the same force? Why?



No, W_B is not **<u>not</u>** **F_{BT}!**

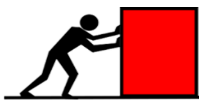
1) W_B is a *gravitational (non-contact) force* (i.e. by Earth on book, F_{EB}) but F_{BT} is a *contact force*
2) W_B forms action-reaction pair with the Earth (refer back to part (b)), while F_{BT} forms action-reaction pair with force by table on book, F_{TB}





<u>Worked Example 3</u>

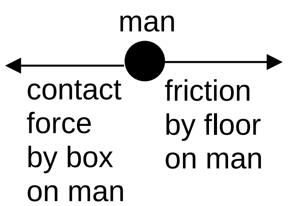
Scenario : A man is pushing a box which remains stationary.

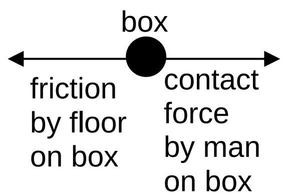


Explain whether the following statements are true.

1. The pushing force by man on box and the friction by floor on box form an action-reaction pair.
2. By Newton's Third Law, the contact force by box on man and the pushing force by man on box cancel each other out. Hence, the box remains stationary.
3. The box is stationary because the pushing force by man on box is equal in magnitude and opposite in direction to the friction by floor on box.

Start by considering the horizontal forces on the man and on the box:





Additional Notes

Statement 1:
The pushing (contact) force exerted by man on box, and the friction by floor on box are acting on the same body. This does not fulfil condition 2. In addition, these two forces are different types of forces, so they do not meet condition 1. Hence, they are not action-reaction pair.

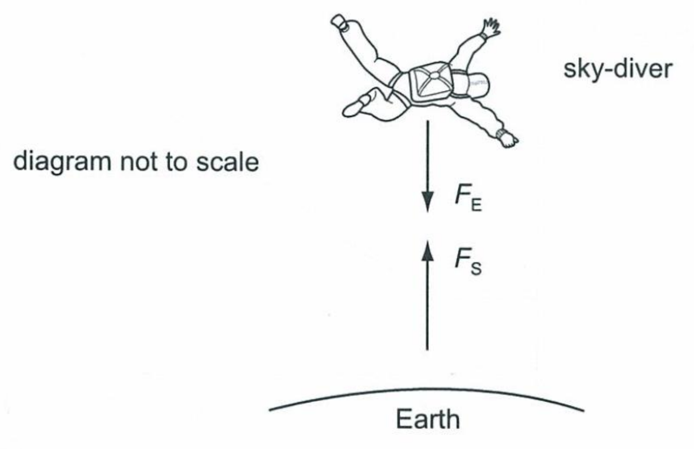
Statement 2:
The contact force by box on man and the pushing (contact) force by man on box form an action-reaction pair, because they act on different bodies (condition 2). So they do not cancel each other out. Only forces acting on the same body can cancel out if they are of same magnitude and act in opposite directions. Therefore, this is not the correct explanation for why the box remains stationary.

Statement 3:
True. Both the pushing (contact) force by man on box and the friction by floor on box act on the same body. The box does not move because these two forces are equal in magnitude and opposite in direction and hence cancel each other out.

(From statements 1 and 3, we can also see that two forces that are equal in magnitude and opposite in direction are not necessarily an action-reaction pair. Hence, always check against the three conditions.)

__Check Your Understanding 1__

A sky-diver jumps from an aircraft. The Earth exerts a downward force F_E on the sky-diver who also exerts an upward force F_S on the Earth.



Which statement is correct?

A. the magnitude of F_E > the magnitude of F_S

B. the magnitude of F_E < the magnitude of F_S

C. the magnitude of F_E = the magnitude of F_S and they cancel each other out

D. the magnitude of F_E = the magnitude of F_S and they do not cancel each other out

Go to page 25 to attempt Check Your Understanding 2 to 6.

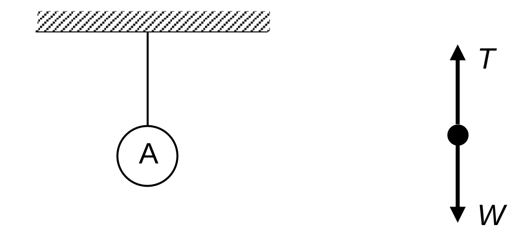
A.4 Solving Problems Using Newton's Laws of Motion

A **free-body diagram (FBD)** shows only the forces acting on ***one object***. The object can be simplified further by simply drawing a dot to represent a point mass.

Steps to draw a FBD,
1. Draw a dot or a small circle to represent the body.
2. Draw arrows with appropriate lengths to represent all the forces. (Start the arrows *from the dot*.)
3. Label the forces.

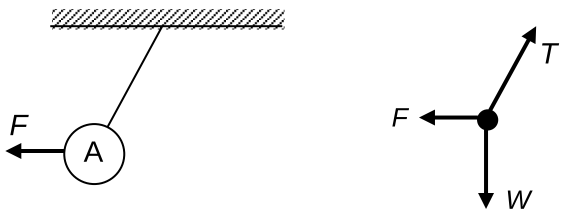
A few examples of FBD of body A:

a)



T – Tension
W – Weight of A

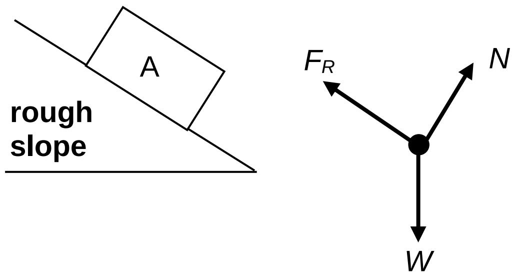
b)



T – Tension
W – Weight of A
F – Applied force

Note that there is only **one** gravitational force, which is the weight of body A.

c)



W – Weight of A
N – Normal contact Force by Surface
F_R – Frictional Force

For scenarios where the body involved has constant mass, we can derive the following equation from Newton's 2nd Law.

By definition, $F_{net} = \dfrac{dp}{dt} = \dfrac{d(mv)}{dt}$

For constant mass, $F_{net} = m\dfrac{dv}{dt} = ma$

or $\boxed{\langle F_{net} \rangle = m\dfrac{\Delta v}{\Delta t} = m \langle a \rangle}$

where $\langle F_{net} \rangle$ is the average net force acting on the object
m is the mass of the object
$\langle a \rangle$ is the average acceleration of the object

Additional Notes

It is usually easier to take the direction of acceleration as the positive direction.

General Approach to Solving Problems Involving Constant Mass

1. Draw a diagram of system of interacting bodies with all given information (e.g. force vectors touching bodies, acceleration/velocity vectors not touching bodies).
2. Draw a free body diagram of individual body with forces acting on the body.
3. Write out the equation for each body based on Newton's 2nd Law.
   (Note: In general, we take the direction of resultant force as positive)
4. The resultant force is the *vector sum* of all the forces acting on a body and is often referred to as the *total force*, *net force* or *unbalanced force*.
5. Both the resultant force and the acceleration are vectors and they are always along the same direction.

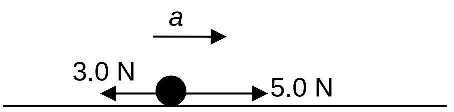
In the following examples, we will apply "General Approach to Solving Problem".

This seems like a standard O-Level question. However, do take note of how the formula is formed. It is formed considering the diagram of the forces acting on the object, i.e. the free body diagram.

<u>Worked Example 4</u>

A box of mass 2.0 kg is pushed horizontally on a horizontal surface with a force of 5.0 N. The frictional force that acts on the box is 3.0 N.

Calculate the acceleration of the box.



(Taking rightward to be positive)
By N2L, $F_{net} = ma$
$5.0 - 3.0 = ma$
$2.0 = 2.0\,(a)$
$a = 1.0 \text{ m s}^{-2}$

Do recognise that the weight is acting vertically downwards, not perpendicular to the slope.

To solve problems like this, your ability to resolve vectors is very important. Revisit Topic 1 on Vectors to recall how to resolve vectors.

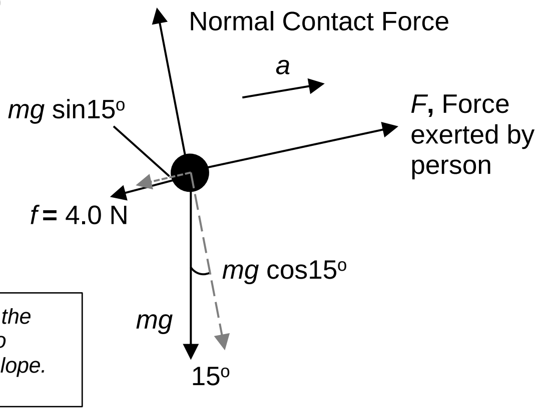
<u>Worked Example 5</u>

A box is pushed up a slope tilted at an angle of 15°. The mass of the box is 5.0 kg. The frictional force that is acting on the box is a constant at 4.0 N.

Calculate the force that a person must push on the box along the slope in order to keep it moving at an acceleration of 2.0 m s^{-2}.

(Taking upward along the slope to be positive)

By N2L , $F_{net} = ma$
$F - mg\sin 15^\circ - f = ma$



$F - mg\sin 15^\circ - f = 5.0\,(2.0)$
$F = 12.7 + 4.0 + 10.0$
$\quad = 26.7 \text{ N}$

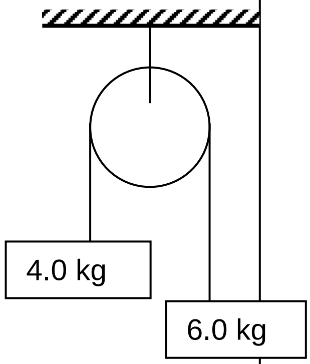
Note that N2L can be applied along the axis perpendicular to the slope as well. In this case, the resultant force perpendicular to slope is zero since there is no acceleration perpendicular to slope.

Additional Notes

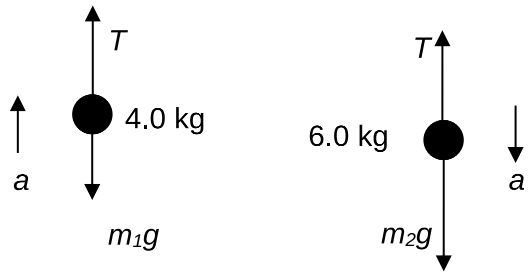
Worked Example 6

In the figure shown, masses 4.0 kg and 6.0 kg are connected by an inextensible thread looped over a smooth pulley.

Calculate the acceleration of the masses and the tension in the spring.



Free Body Diagrams



By Newton's Second Law,

for 4.0 kg mass: (taking upward positive):
$F_{net} = ma$
$T - (4.0)(9.81) = 4.0a$ … (1)

for 6.0 kg mass: (taking downward positive):
$F_{net} = ma$
$(6.0)(9.81) - T = 6.0a$ … (2)

Solving equations (1) and (2), $a = 1.96$ m s^{-2} and $T = 47.1$ N

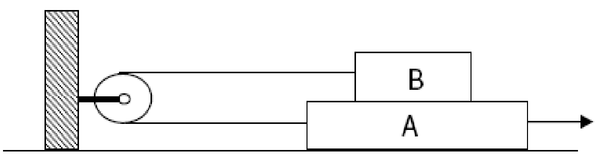
The directions of acceleration should also be "continuous", i.e., if m_1 accelerates upwards, m_2 accelerates downwards, and vice versa.

Note that tensional forces (force by a pulling string) always act away from the object considered.

Example 1

The diagram below shows a wooden block B resting on another block A. Both blocks are connected by an inelastic string which goes around a fixed smooth pulley. The friction between blocks A and B is 4.0 N, and block A is pulled at a constant velocity by a 20 N force.

By considering the horizontal forces acting on A and B, calculate the frictional force between block A and the ground.



Additional Notes

Note that static friction acts in the opposite direction of (intended) motion. So, draw the forces on the object in the proper direction.

The vertical forces (weight of A and B, normal contact force by ground on A, by A on B, by B on A) are ignored because the resultant motion is horizontal.

Solution:

Example 2 (Take $g = 10$ m s^{-2})

Determine the force that an 80.0 kg man exerts on the floor of an elevator when it

(a) is at rest;

(b) rises with a constant velocity of 2.0 m s^{-1};

(c) descends with a constant velocity. of 2.0 m s^{-1};

(d) rises and speeds up with a constant acceleration of 2.0 m s^{-2};

(e) descends and speeds up with a constant acceleration of 2.0 m s^{-2}.

In general, force man exerts on lift when lift is accelerating upwards is more than weight of man.

Force man exerts on lift when lift is accelerating downwards is less than weight of man.

Force man exerts on lift is equal to weight of man as long as *velocity remains* constant, including at rest.

Solution:

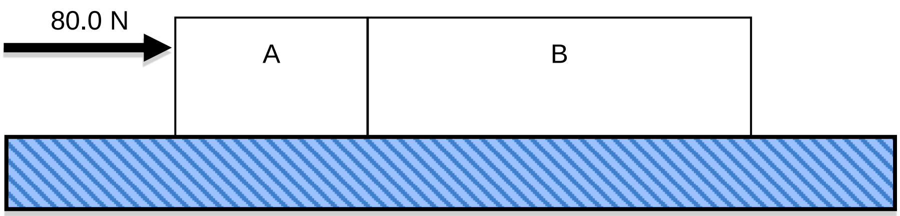
Additional Notes

Suppose the lift cable breaks, so that the lift and its contents are in free-fall. Its acceleration *a* is equal to *g* (acceleration of free fall). From (d), R_{lm}=0, that is, the man *appears* to be weightless.

Example 3

Boxes A and B of mass 2.0 kg and 5.0 kg respectively are being pushed along a rough, horizontal surface by a constant force of 80.0 N. The two boxes slide across the rough surface with a constant acceleration of 3.0 m s^{-2} to the right. The frictional force acting on box A is 35.0 N.

(a) Calculate the force exerted by box A on box B.
(b) Calculate the frictional force acting on B.



Solution:

Additional Notes

A.5 Impulse-Momentum

Take note that impulse is equal to the change in momentum and not the final momentum.

From Newton's 2nd Law, $F_{net} = \frac{dp}{dt}$

$\Rightarrow \int dp = \int F_{net} dt$

where $\int dp$ is the change in momentum

$\int F_{net} dt$ is called the impulse

Impulse is the product of the average force acting on the body and the time of impact, and is equal to the change in momentum of the body as a result of the application of the force.

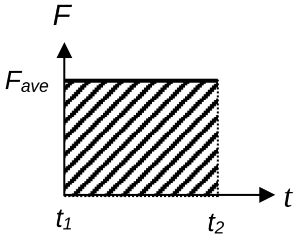
The unit of impulse is N s.

For average (or constant) force $<F_{net}>$, change in momentum, $\Delta p = <F_{net}> \Delta t$
where Δt is the duration acted upon by the net force

Take note that if the quantity represented by the values on the vertical axis is just one of the forces F acting on a body and not the net force, the area under F-t graph will not represent the change in momentum of the body.

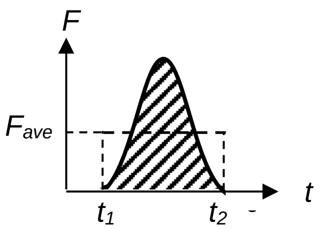
In general, for a net force F_{net} (including non-constant F_{net}),
change in momentum, $\Delta p =$ Area under a $F_{net} - t$ graph

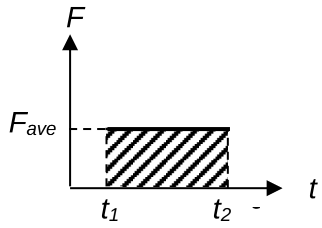
The force can be constant or varying (e.g. for colliding bodies). For a constant force as shown below, the change in momentum from t_1 to t_2 is given by the shaded area.
Recall $Ft = mv - mu$.



Additional Notes

For colliding bodies, the force is not constant but builds up to a maximum value as the deformation of the colliding body increases. When the force imparting an impulse vary with time, it is convenient to define an average force F_{ave}, such that $F_{ave}(t_2 - t_1)$ = area under the F-t graph for the same time interval $t_2 - t_1$.





i.e. the average force gives the same impulse to a particle as does the time varying force in the same time interval.

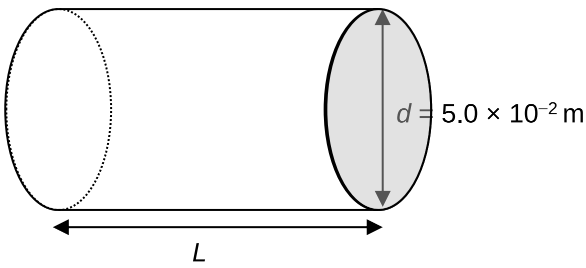
<u>Worked Example 7</u>

A jet of water leaves the nozzle of a hose of diameter 5.0×10^{-2} m with a speed 0.400 m s^{-1}. The water is directed perpendicularly to the wall and it can be assumed that the water does not rebound. The density of water is 1000 kg m^{-3}.

Calculate the force exerted on the wall by the water.

Consider a column of water that hits the wall in t = 1 s.

Given:
Diameter of hose, d =5.0×10^{-2} m
Speed of water, v = 0.400 m s^{-1}
Density of water, ρ = 1000 kg m^{-3}



Let L be the distance travelled by the column of water in 1 s. Hence, from the speed, L = 0.400 m.

Applying the Impulse-Momentum concept:

The wall exerts an impulse on the water and changes its horizontal momentum. Let $<F_{net}>$ be the average net force exerted by the wall on the water.

$$<F_{net}>\Delta t = \Delta p$$

$$<F_{net}>\times 1.0 = m(v_f - v_i) = \rho AL(v_f - v_i)$$

$$<F_{net}> = \rho\times\pi\left(\frac{d}{2}\right)^2\times L\times(v_f - v_i)$$

$$<F_{net}> = 1000\times\pi\left(\frac{5.0\times10^{-2}}{2}\right)^2\times 0.400\times(0 - 0.400) = -0.314 \text{ N}$$

(negative sign shows that the force exerted by the wall on the water is in opposite direction to the direction of the velocity of water).

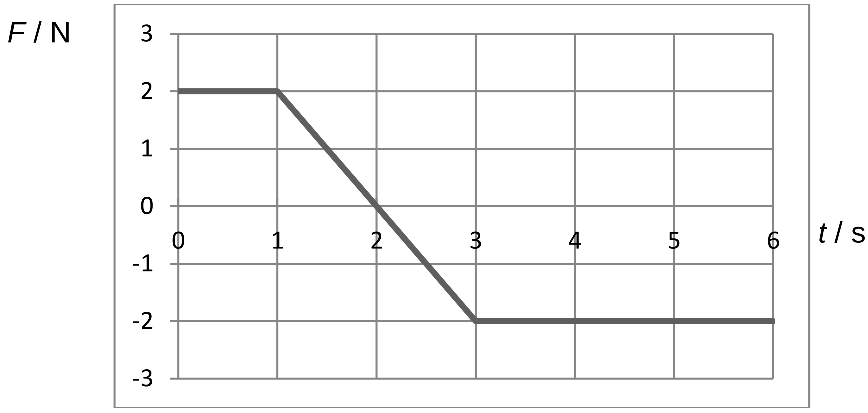
In Worked Example 7, *force on wall by water* forms an action-reaction pair with *force on water by wall*. The question is asking for *force on wall by water*. Since we can only apply N2L on the free body of the water to find *force on water by walll* as we do not have the mass of wall and all the forces acting on the wall, we have to apply N3L to find *force on wall by water* to complete the solution.

Additional Notes

By Newton's 3rd Law, force by water on wall has the same magnitude as force by wall on water.

Therefore, the force by water on wall = 0.314 N

<u>Worked Example 8</u> (Graph and Calculation)



The graph shows a resultant variable force acting on a body of mass 2.0 kg traveling in a straight line with an initial velocity of 1.0 m s^{-1} (in the positive direction).

(a) Calculate the change in momentum of the body from t = 0 to t = 6 s.
(b) Calculate the change in velocity of the body.
(c) Calculate the final velocity of the body.

A common careless mistake would be to forget that the area under the F-t graph refers to <u>Change in</u> momentum, not final momentum.

(a) Change in momentum of body = Impulse = Area under F-t graph.

Change in momentum of body $\Delta p = \frac{1}{2}(1+2)(2) - \frac{1}{2}(4+3)(2) = -4.0$ N s

(b) For constant mass, $\Delta p = m\Delta v$

$$\Delta v = \frac{\Delta p}{m}$$

$$= \frac{-4.0}{2} = -2.0 \text{ m s}^{-1}$$

(c) Change in velocity of body, $\Delta v = v_f - v_i$

$\therefore v_f = \Delta v + v_i$

$= -2.0 + 1.0 = -1.0$ m s^{-1}

Additional Notes

Worked Example 9 (Qualitative)

(a) A person who catches a fast ball usually draws back his hand. Explain how this reduces the average force on his hand.

(b) A person throwing a ball usually draws back his hand before throwing the ball forward. Explain how this increases the amount of change in momentum of the ball.

(a) Drawing back the hand increases the time of impact. For a given change in momentum (from a high value to zero), the increase in the time of impact will result in decrease in the average force acting on the hand.

(b) Drawing back the hand increases the time of impact. For a given average force exerted on the ball, the increase in the time of impact will result in increase in the amount of change in momentum of the ball. Thus for the same given average force, the ball will leave his hand with higher speed.

More information on Stretchable Seat Belts

http://hyperphysics.phy-astr.gsu.edu/hbase/seatb.html

Worked Example 10 (Qualitative)

Name two safety features of a car and explain how these safety features work to improve the safety of passengers.

Seatbelts

- During a collision, according to N1L, the passenger who was originally in motion will continue to be in motion.
- Without the seatbelts, the passenger may smash headlong through the windscreen as he continues in the forward direction while the car has already come to rest.

Video: Crumple Zone

https://tinyurl.com/vc9mv8da

Bodies of a car are designed to crumple upon collision

- This increases the duration of impact.
- By Impulse-Momentum Theorem, for the same change in momentum, this reduces the average impulsive force the passenger will experience.

Air bags

- Similar to crumple zones, the air bags work in a way such that there is an extension of the duration of impact, and thus reduces force experienced.

More information on Air Bags

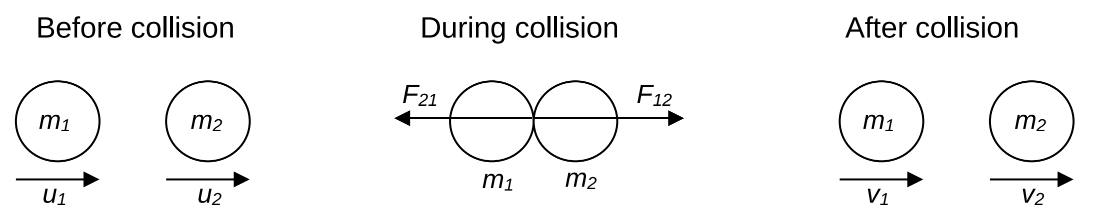
https://www.nhtsa.gov/equipment/air-bags

Additional Notes

B.1 Principle of Conservation of Linear Momentum

B.1.1 Derivation of the Principle of Conservation of Linear Momentum from Newton's Laws of Motion

When two bodies m_1 and m_2 collide (take rightwards as positive),



During collision,
m_1 acts on m_2 with a force of F_{12}
m_2 acts on m_1 with a force of F_{21}

According to Newton's 3rd Law, $F_{12} = -F_{21}$ … (1)
Taking the time of collision to be t,

Using Newton's 2nd Law, eqn (1) can be rewritten as,

$$\frac{m_2v_2 - m_2u_2}{t} = -\left(\frac{m_1v_1 - m_1u_1}{t}\right)$$

$$m_2v_2 - m_2u_2 = -m_1v_1 + m_1u_1$$

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

i.e. Total momentum after collision = Total momentum before collision

There have been A-Level questions on how to derive the principle of conservation of linear momentum from the Newton's Laws of Motion.

The above is the defining equation for conservation of linear momentum.

Definition: Principle of Conservation of Linear Momentum states that the net momentum of a **system** remains constant provided no resultant external force acts on the system.

- The principle of conservation of momentum can be used to analyse problems involving separation of bodies or collisions.
- It applies not only to collisions, but to any interaction between two or more bodies.
- For collisions, if no external resultant force acts on a system of colliding bodies, the total momentum of the bodies before collision = total momentum after collision.

Example of an Isolated system: A body falling towards the Earth increases its downward momentum but the body is interacting with the Earth which gains an equal amount of upward momentum from the attraction of the body on Earth. The complete **system consists of the body and the Earth,** and their total momentum remains constant. We can also say that the body and the Earth form an **isolated** system and the momentum of the system must be conserved.

Additional Notes

B.1.2 General Approach to Solving Problems involving Separations or Collisions

1. Identify whether the question is about a separation, elastic collision or inelastic collision (i.e. identify the isolated system)

2. Draw diagrams of what happens before and after the collision / separation (velocities and mass of body to be included).

3. Use the Principle of Conservation of Linear Momentum to form an equation. Take note that the direction of motion is important.

4. If collision is known to be elastic, use the fact that kinetic energy is conserved in the collision or that relative speed of separation = relative speed of approach to form another equation.

B.1.3 Collisions

- Note: In head-on collisions, the velocity/velocities of the colliding bodies are collinear (directed along the same straight line) both before and after the collision.

Understand the term "head-on", but there is no need to memorise the meaning.

- The momentum and the total energy of an isolated system are always conserved in any collision but the total kinetic energy of the system may or may not be conserved depending on the type of collision.

- There are two types of collisions:

    - An elastic collision is one in which the total kinetic energy remains the same.
    - An inelastic collision is one in which the total kinetic energy is not conserved.

- Real collisions are always inelastic.
- Sometimes, collisions can be such that $KE_f = 0$, i.e. the real colliding bodies come to a complete stop.
- Perfectly elastic collisions only occur between atomic and subatomic particles.

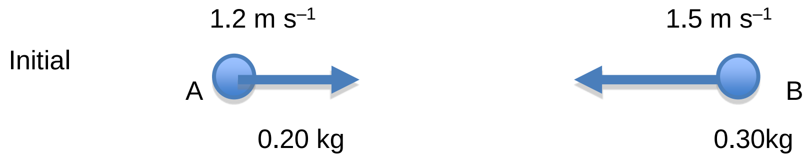
B.1.3.1 Elastic Collisions

- Both linear momentum and kinetic energy of the system are conserved.
- Although bodies which collide elastically always separate, but collisions that result in bodies separating need not necessarily mean the collision is elastic.
- Separation can refer to bodies moving away from each other, or moving in the same direction with the leading body moving faster than the lagging body.
- For elastic collisions, the relative speed of approach of the bodies is equal to the relative speed of separation of the bodies (this is the direct consequence of the conservation of kinetic energy).

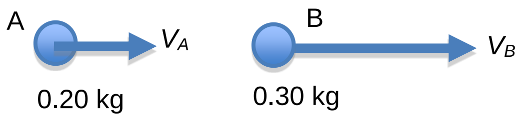
Additional Notes

Worked Example 11 (Using Conservation of Linear Momentum & Kinetic Energy)

Two balls A and B collide with each other head-on and elastically. Their masses and initial velocities are as shown:



Determine their velocities after collision.



Take note that direction of final velocities are randomly assumed to be to the right.

It is important to follow sign conventions strictly when applying COLM, since momentum is a vector.

Using principle of conservation of momentum (taking rightward as positive)

Sum of initial momentum = Sum of final momentum

$M_A U_A + M_B U_B = M_A V_A + M_B V_B$

$(0.20)(1.2) + (0.30)(–1.5) = (0.20)(V_A)+ (0.30)V_B$

$–0.21 = 0.20V_A + 0.30V_B$ … (1)

The addition symbols in the generic equation should not be changed. But, V_A and V_B adopted different signs based on their directions (or assumed directions) when substituted into the equation.

As Collision is Elastic, Kinetic energy is conserved

Sum of initial kinetic energy = Sum of final kinetic energy

Take note that Kinetic energy is a scalar, so the v used in ½ mv^2 refers to speed.

$\frac{1}{2} M_A U_A^2 + \frac{1}{2} M_B U_B^2 = \frac{1}{2} M V_A^2 + \frac{1}{2} M_B V_B^2$

$\frac{1}{2}(0.20)(1.2)^2 + \frac{1}{2} (0.30)(–1.5)^2 = \frac{1}{2} (0.20)V_A^2 + \frac{1}{2} (0.30)V_B^2$

$0.4815 = 0.10V_A^2 + 0.15V_B^2$ … (2)

Solving (1) and (2) simultaneously

$V_A = –2.04$ m s^{-1} (leftward)
$V_B = 0.660$ m s^{-1} (rightward)

Take note that if the assumed directions of velocities were wrong, the final values calculated will be negative. There is no need to re-calculate, just describe the direction as opposite to assumed direction, if necessary.

When final calculated velocity is negative, it need not mean body is travelling in negative direction. It means the body is moving opposite to your assumed direction.

Note: The solving of simultaneous equations involving Conservation of KE may lead to complicated quadratic equations. This can be avoided by using the method of Relative Speeds of Approach and Separation shown on the next page.

Relative Speed of Approach and Relative Speed of Separation

For an **elastic collision** where u_1 and u_2 are the initial velocities of two bodies (taking rightward positive) before collision, and v_1 and v_2 are the final velocities of the two bodies (taking rightward positive) after collision, the Conservation of Kinetic Energy can also be represented in the following relationship:

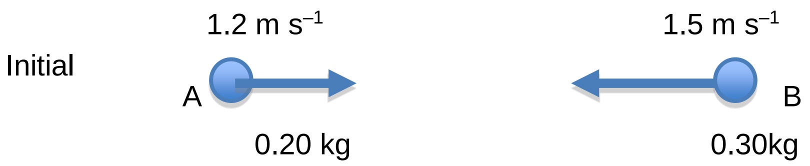
Relative Speed of Approach = Relative Speed of Separation

$$u_1 - u_2 = v_2 - v_1$$

Worked Example 12 (Using Conservation of Momentum & RSA = RSS)

For the same scenario in Worked Example 11, we now use the relative speed of separation = relative speed of approach method to form the second equation.

Two balls A and B collide with each other head-on and elastically. Their initial velocities are as shown:



Determine their velocities after collision.

Using principle of conservation of momentum,

Taking right direction to be positive

Sum of initial momentum = Sum of final momentum

$M_A\ U_A + M_B\ U_B = M_A\ V_A + M_B\ V_B$

$(0.20)(1.2) + (0.30)(-1.5) = (0.20)(V_A) + (0.30)V_B$

$-0.21 = 0.20V_A + 0.30V_B$ … (1)

The positive signs in the generic equation were not changed. But, V_A and V_B adopted different signs based on their directions when substituted into the equation.

Additional Notes

The relationship between Relative speed of approach (RSA) and Relative speed of separation (RSS) is very useful to solve MCQs quickly.

If RSA is not equal to RSS, the collision is not elastic in nature.

Note that this equation can only be used simultaneously with the equation for Conservation of Linear Momentum, but not with the equation for Conservation of Kinetic Energy.

Take note that directions of final velocities are randomly assumed.

For RSA = RSS, take note the sequence of which quantity subtracts which quantity is important. $u_1 - u_2 = v_2 - v_1$ is not the same as $u_1 - u_2 = v_1 - v_2$.

Additional Notes

See that it is easier to solve the algebraic equation involving RSS/RSA as compared to the equation for Conservation of KE.

Note that RSA = RSS is only valid for elastic collision.

As Collision is Elastic, we can use this equation

Relative speed of separation = Relative speed of approach

$V_B - V_A = 1.2 - (-1.5)$
$= 2.7$ … (2)

Solving Eqn (1) and (2) simultaneously

$V_A = -2.04$ m s^{-1} (leftward)
$V_B = 0.660$ m s^{-1} (rightward)

Take note that if the assumed directions of velocities were wrong, the final values calculated will be negative. There is no need to re-calculate, just describe the direction as opposite to assumed direction, if necessary.

B.1.3.2 Inelastic Collisions

Inelastic Collisions and Perfectly Inelastic Collisions are different. The latter is in fact a subset of the former.

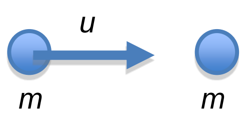
- For inelastic collisions, only conservation of momentum can be applied.
- Conservation of kinetic energy cannot be applied since some of the kinetic energy is transformed into internal energy and sound or other forms of energy.
- Similarly, the relationship of Relative speed of approach = Relative speed of separation cannot be applied.
- For perfectly inelastic collisions, the two bodies will stick together (sometimes the word coalesce is used) as one single body after collision, i.e. they move off together with a common velocity.

Worked Example 13 (Perfectly Inelastic Collision)

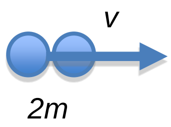
A body of mass m moving with speed u makes a head-on collision with an identical body which is initially at rest. The bodies coalesce and move off with a common velocity.

(a) Determine the common speed of the bodies after the collision.
(b) Determine the ratio of the kinetic energy of the system after the collision to that before.
(c) Explain what happens to the kinetic energy that is 'lost'.

Initial



Final



(a) Using the principle of conservation of linear momentum,
Sum of initial momentum = Sum of final momentum

$$mu = 2mv$$

$$v = \frac{1}{2}u$$

(b) Initial Kinetic Energy of system $= \frac{1}{2}mu^2$

Final Kinetic Energy of system

$$= \frac{1}{2}(2m)v^2$$

$$= mv^2$$

$$= m\left(\frac{1}{2}u\right)^2 \qquad \left(\text{as } v = \frac{1}{2}u\right)$$

$$= \frac{1}{4}mu^2$$

Ratio of Final KE to Initial KE $= \dfrac{\frac{1}{4}mu^2}{\frac{1}{2}mu^2} = \dfrac{1}{2}$

(c) This loss in kinetic energy of the system has become an increase in internal energy (molecular kinetic energy) of the bodies as well as sound.

B.1.4 Special Case: Separation of Bodies

- Problems involving separation of bodies involve a single body which separates, such as ‘explodes’ or splits up.

- By using the principle of conservation of momentum, the directions and the velocities of the different bodies may be analysed.

Additional Notes

Example 4 (Separation of Bodies)

a) Calculate the recoil velocity of a 5.0 kg rifle that shoots a 0.020 kg bullet out with a velocity of 620 m s^{-1}.

b) Explain whether the rifle or the bullet has a higher kinetic energy.

Solution:

In summary,

	Conservation of Linear Momentum	Conservation of Kinetic energy	Conservation of Total Energy
Elastic Collisions	✓	✓	✓
Inelastic Collisions	✓		✓

B.1.5 Real Life Applications of Elastic and Inelastic Collisions

- Most collisions in the real world are really inelastic in nature (even the example of two billiard balls colliding is not exactly elastic).
- Some of the kinetic energy before collision has been transformed into other forms of energy such as internal energy (heat) and sound.
- One can still approach these problems by approximating the collision to be elastic as such approximations do give a reasonable prediction of how the system will behave.

Additional Notes

Check Your Understanding 2

You slide a book from one end to another end across a smooth table top. After the book has left your hand, the book slides with a constant acceleration because of the force applied by you. (True/False)

Check Your Understanding 3

The normal reaction exerted by the table on the book is the reaction force to the weight of the book. (True/False)

Check Your Understanding 4

The attractive force that I exert on the Earth has the same magnitude as my weight. (True/False)

Check Your Understanding 5

When I push a box on a rough surface, the frictional force is the reaction force to my push force. (True/False)

Check Your Understanding 6

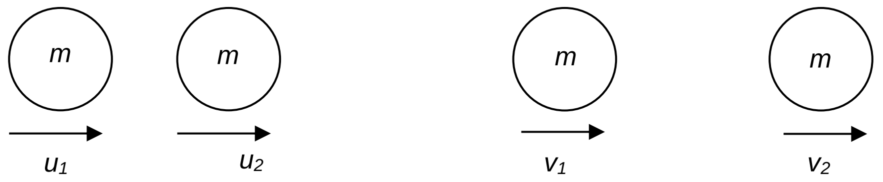
Since action and reaction pairs are always equal in magnitude and opposite in direction, the resultant force on any mass is always zero. (True/False)

APPENDIX

<u>Collision between two masses with the same mass</u>

Suppose that two bodies with the same mass m collide elastically and the body on the right has an initial velocity of zero,

Before collision *After collision*



By the Principle of Conservation of Linear Momentum (taking rightward as positive),

$$mu_1 = mv_1 + mv_2$$

$$u_1 = v_1 + v_2 \qquad \text{--- (1)}$$

Since the collision is elastic, kinetic energy is also conserved.

Relative speed of Approach = Relative speed of Separation

$$u_1 = v_2 - v_1 \qquad \text{--- (2)}$$

Subtracting equation (2) from equation (1),

$$2v_1 = 0$$

Therefore,

$$v_1 = 0$$

Sub $v_1 = 0$ into equation (1) or (2),

$$v_2 = u_1$$

This implies that the mass which was initially moving will come to a stop after collision and the mass that was initially stationary will move off with the same velocity as the mass that was initially moving. In other words, there is a complete transfer of kinetic energy.

Additional Notes

Summary

Concept	Definition	Important Concepts or Formulae
Newton's 1st Law	A body at rest remains at rest and a body at constant velocity continues its motion unless acted upon by a resultant force.	A body travelling with a constant velocity has no resultant force acting on it.
Newton's 2nd Law	The rate of change of momentum of a body is proportional to the resultant force acting on it, and the change of momentum takes place in the direction of the resultant force.	For average force <*F*> $\langle F\rangle=\dfrac{\Delta p}{\Delta t}$ For constant mass $<F>\dfrac{m\Delta v}{\Delta t}$ or $<F>=m<a>$ For constant velocity $<F>=\dfrac{v\Delta m}{\Delta t}$
Newton's 3rd Law	When body A exerts a force on body B, body B will exert a force of the same type, equal in magnitude but opposite in direction on body A.	NA
Linear Momentum	It is the product of the mass of the body and its velocity.	$p = mv$
Impulse	The product of the average force acting on the body and the time of impact.	Impulse =Change in momentum = Area under a *F*-*t* graph $=<F>t$
Principle of Conservation of Linear Momentum	The net momentum of a system remains constant provided no resultant external force is acting on the system.	It is derived from Newton's Laws of motion.

Additional Notes

Other Important Concepts

Term	Key Concepts	Relevant Equations
Elastic Collision	A collision where both the net momentum and the total kinetic energy of the system are conserved.	Equation 1 Sum of initial momentum = Sum of final momentum Equation 2(a) Sum of initial KE = Sum of final KE Equation 2(b) Relative speed of approach = Relative speed of separation *Note:* *Equations 2(a) and 2(b) should not be used simultaneously.*
Inelastic Collision	A collision whereby the net momentum of the system is conserved but the kinetic energy of the system is not conserved. Special Case: In a <u>Perfectly Inelastic</u> collision, the system of colliding bodies coalesce (stick together) after collision, i.e. they move off with a <u>common</u> velocity after collision.	Equation Sum of initial momentum = Sum of final momentum