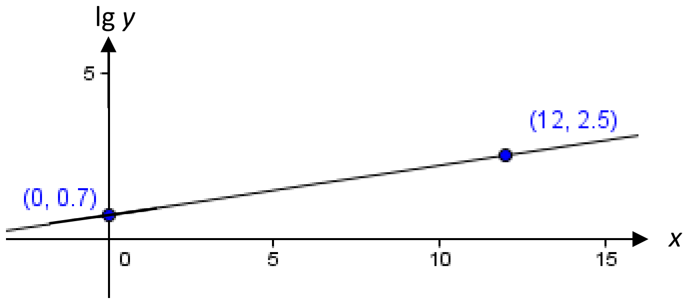
Topical Revision Paper 11

Term 3 Unit 11: Applications of straight line graphs

Question 1:

It is given that x and y are connected by $y = ab^x$ where a and b are constants. Experimental values of x and y were obtained and the straight line graph of $\lg y$ against x was plotted.

The diagram shows the points (0, 0.7) and (12, 2.5).



Estimate, to 2 significant figures, the value of a and of b. [5]

Solution:

$\lg y = \lg a + x \lg b$

$\lg a = 0.7$

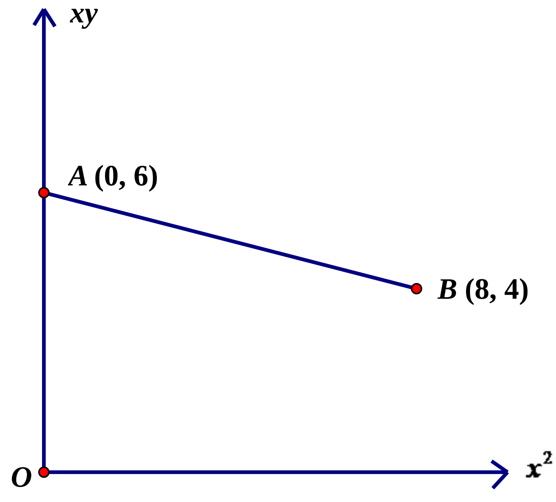
$a = 10^{0.7} = 5.0$ (2 s.f.)

$\lg b = \dfrac{2.5 - 0.7}{12 - 0}$

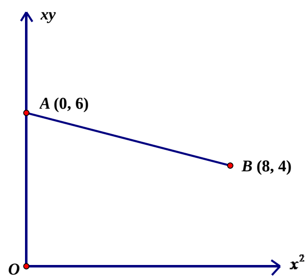
$b = 10^{0.15} = 1.4$ (2 s.f.)

Question 2:

The diagram shows the straight line graph of xy against x^2 passing through the points $A(0, 6)$ and $B(8, 4)$. Express y in terms of x. [3]



Solution:



xy – intercept is 6

gradient $\frac{6-4}{0-8} = -\frac{1}{4}$

Equation of straight line is of the form

" Y = mX + c "

i.e. $xy = mx^2 + c$

$$xy = -\frac{1}{4}x^2 + 6$$

Hence $y = -\frac{1}{4}x + \frac{6}{x}$ or $-\frac{x^2+24}{4x}$

Question 3:

The variables x and y are related by the equation $y = 5^{px+q}$ where p and q are constants. Given that the graph of $y = 5^{px+q}$ passes through $(0.5, 1)$ and $(2, 125)$, and $x \geq 0$,

(i) show that p = 2 and q = –1. [3]

(ii) Using the values p and q in **(i)**, express $y = 5^{px+q}$ in a linear form and hence, sketch the **linear** graph for the equation, indicating your axes and intercepts clearly. [3]

Solution:

(i)

$y = 5^{px+q}$

At $(0.5, 1)$,

$1 = 5^{\frac{1}{2}p+q}$

$5^0 = 5^{\frac{1}{2}p+q}$

$\frac{1}{2}p + q = 0 .... (eqn\ 1)$

$q = \frac{-1}{2}p$

At $(2, 125)$,

$125 = 5^{2p+q}$

$5^3 = 5^{2p+q}$

$2p + q = 3 .... (eqn\ 2)$

$\therefore 2p + \left(\frac{-1}{2}p\right) = 3$

$\frac{3}{2}p = 3$

$p = 2$

$\therefore q = \frac{-1}{2}(2) = -1$

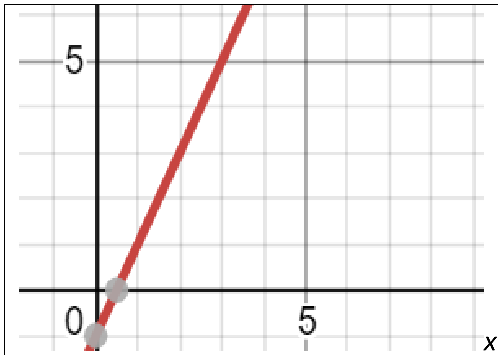
(ii) $y = 5^{px+q}$

$y = 5^{2x-1}$

$log_5 y = 2x - 1$

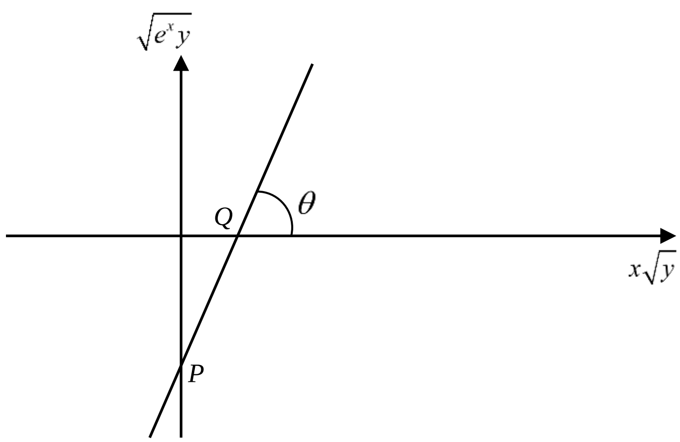
Alternatively use $\ln y = (2\ln 5)x - \ln 5$

$log_5 y$



Question 4:

Given that x and y are related by $\sqrt{e^x} = 4x - \dfrac{3}{\sqrt{y}}$ and are represented by a straight line shown in the diagram below. The straight line cuts the vertical and horizontal axes at P and Q respectively.



(i) Find the gradient of the line. [2]

(ii) Find the value of θ which is the acute angle in degree made by the straight line and the horizontal positive x-axis. [2]

(iii) Find the coordinates of P and Q. [2]

Solution:

(i)

$\sqrt{e^x} \;= 4x - \frac{3}{\sqrt{y}}$

$\sqrt{e^x}\sqrt{y} = 4x\sqrt{y} - 3$

$\sqrt{e^x y} = 4x\sqrt{y} - 3$

(ii)
$\tan\theta = 4$

$\theta = 76.0°$ (1d.p)

(iii)

$P(0,-3)$ and $Q(\frac{4}{3},0)$

Question 5:

Two variables x and y are known to be related by the equation $y = ax + \frac{b}{x} - 1$.

(i) Show that a straight line graph can be obtained by plotting the graph of $xy + x$ against x^2.

(ii) Hence, state its gradient and y-intercept.

Solution:

(i)

$y = ax + \frac{b}{x} - 1$

$xy = ax^2 + b - x$

$xy + x = ax^2 + b$

$Y \;\; = mX \; + c$

Thus, a straight line graph can be obtained by plotting the graph of $xy + x$ against x^2.

(ii) The gradient of the straight line graph is a and its y-intercept is b.