

- 1(a)** It is given in the formula list that the moment of inertia of rod about one of its ends is

$$I = \frac{1}{3}ML^2$$

uniform rod so CG is in middle of rod

$$\begin{aligned}\tau &= \left(\frac{L}{2} \sin \theta\right)(Mg) \\ &= \alpha I\end{aligned}$$

$$\left(\frac{L}{2} \sin \theta\right)(Mg) = \alpha \frac{1}{3}ML^2$$

$$\begin{aligned}\alpha &= \frac{3Lg \sin \theta}{2L^2} \\ &= \left(\frac{3}{2}\right) \frac{g \sin \theta}{L}\end{aligned}$$

$$k = 1.5$$

- 1(b)** by conservation of energy

$$\text{loss in GPE} = \tau\theta = \frac{1}{2}I\omega^2$$

$$Mg\left(\frac{L}{2} \cos \theta\right) = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

$$\omega = \sqrt{\frac{3g \cos \theta}{L}}$$

- 2(a)** the line integral of $B \cdot ds$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path

$$\oint B \cdot ds = \mu_0 I$$

- 2(b)(i)** in conductor

$$\begin{aligned} I &= (\text{area}) J_1 \\ &= \pi r_1^2 J_1 \end{aligned}$$

since there is an opposite current in metal braid,

$$\begin{aligned} J_2 &= -\frac{I}{\pi(r_3^2 - r_2^2)} \\ &= -\frac{\pi r_1^2 J_1}{\pi(r_3^2 - r_2^2)} \\ &= -\frac{r_1^2}{(r_3^2 - r_2^2)} J_1 \end{aligned}$$

- 2(b)(i)1.** insulator no current flow

$$\begin{aligned} \mu_c I &= \oint B \cdot ds \\ \mu_c \pi r_1^2 J_1 &= B(2\pi r_1) \\ B &= \mu_c \left(\frac{r_1}{2} \right) J_1 \end{aligned}$$

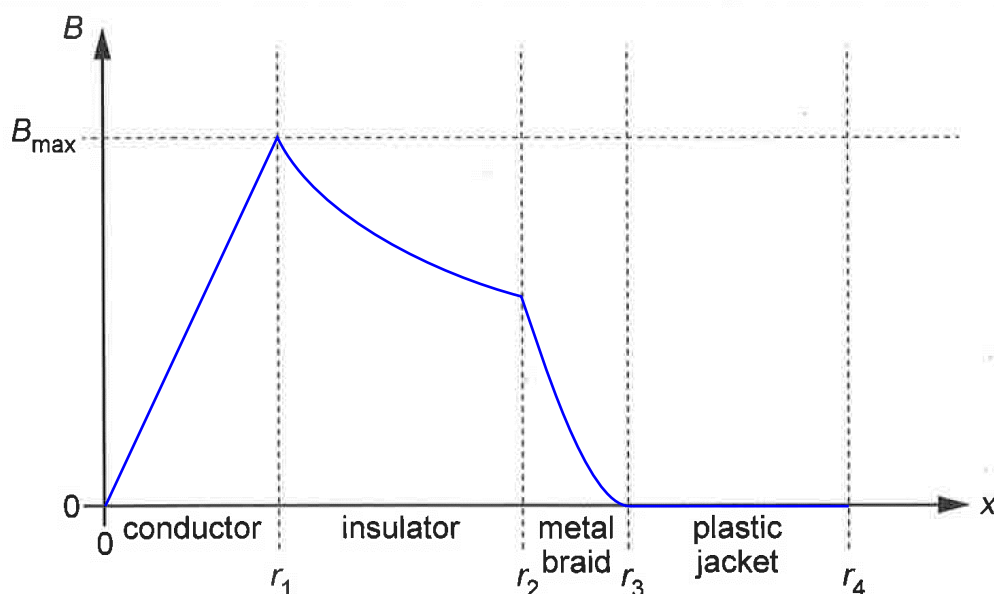
- 2(b)(i)2.**

$$\begin{aligned} \mu_i I &= \oint B \cdot ds \\ \mu_i (J_1 \pi r_1^2) &= B(2\pi r_2) \\ B &= -\left(\frac{\mu_i r_1^2}{2} \right) J_1 \end{aligned}$$

Note: at plastic jacket, no net current flow

$$B = 0$$

- 2(b)(iii)**



- 2(c)** When a current passes through a standard electrical transmission cable, the magnetic flux density in the vicinity of the cable will be nonzero. If high frequency signals are passing through, the rapidly changing magnetic field will lead to EM waves being generated. Hence there will be energy loss.

Note: Compare with the coaxial cable discussed in the previous sections. By design, the magnetic field outside a coaxial cable is zero.

3(a)

$$s_x = (u \cos \theta) t$$

$$s_y = (u \sin \theta) t - \frac{g}{2} t^2$$

$$0 = t \left((u \sin \theta) - \frac{g}{2} t \right)$$

$$u \sin \theta = \frac{g}{2} t$$

$$= \frac{g}{2} \left(\frac{s_x}{u \cos \theta} \right)$$

$$u^2 (2 \sin \theta \cos \theta) = g s_x$$

$$s_x = \frac{u^2}{g} \sin(2\theta)$$

$$= R$$

$$R = R(\theta)$$

$$\frac{dR}{d\theta} = \frac{u^2}{2g} \cos(2\theta)$$

$$R_{\max} \text{ is when } \frac{dR}{d\theta} = 0$$

$$\cos(2\theta) = 0$$

$$2\theta_{\max} = 90^\circ$$

$$\theta_{\max} = 45^\circ$$

$$R_{\max} = \frac{(150)^2}{9.81} = 2290 \text{ m} = 2.29 \text{ km}$$

3(b)(i), (ii), (iii), (iv)

$$R = (u \cos \theta) t$$

$$t = \frac{R}{u \cos \theta}$$

$$\tan \phi = \frac{h}{R}$$

$$-h = (u \sin \theta) t - \frac{g}{2} t^2$$

$$h = -(u \sin \theta) t + \frac{g}{2} t^2$$

$$= -(u \sin \theta) \left(\frac{R}{u \cos \theta} \right) + \frac{g}{2} \left(\frac{R}{u \cos \theta} \right)^2$$

$$= -R \tan \theta + \frac{gR^2}{2u^2 \cos^2 \theta}$$

$$\frac{h}{R} = \frac{gR}{2u^2 \cos^2 \theta} - \tan \theta$$

$$(\tan \phi + \tan \theta) \frac{u^2}{g} = \frac{R}{2 \cos^2 \theta} = \frac{R}{1 + \cos 2\theta}$$

$$R = \frac{u^2}{g} (\tan \phi + \tan \theta) (1 + \cos 2\theta)$$

$$= \frac{u^2}{g} \left(\tan \phi (1 + \cos 2\theta) + \frac{\sin \theta}{\cos \theta} (1 + \cos 2\theta) \right)$$

$$= \frac{u^2}{g} \left(\tan \phi (1 + \cos 2\theta) + \frac{1}{\cos \theta} \left(\frac{\sin 2\theta}{2 \cos \theta} \right) (1 + \cos 2\theta) \right)$$

$$= \frac{u^2}{g} \left(\tan \phi (1 + \cos 2\theta) + (\sin 2\theta) \left(\frac{1 + \cos 2\theta}{2 \cos^2 \theta} \right) \right)$$

$$= \frac{u^2}{g} (\tan \phi (1 + \cos 2\theta) + (\sin 2\theta))$$

when $\phi = 0$, equation reduces to:

$$R = \frac{u^2}{g} (0 + (\sin 2\theta))$$

3(b)(v)

$$R = \frac{u^2}{g} (\tan \phi + \tan \phi \cos 2\theta + \sin 2\theta)$$

$$\begin{aligned} \frac{dR}{d\theta} &= \frac{u^2}{g} \left(0 + \frac{1}{2} \tan \phi \sin 2\theta + \frac{1}{2} \cos 2\theta \right) \\ &= \frac{u^2}{2g} (\tan \phi \sin 2\theta + \cos 2\theta) \\ &= \frac{u^2}{2g} \left(\left(\sqrt{\tan^2 \phi + 1} \right) \sin \left(2\theta + \tan^{-1} \left(\frac{1}{\tan \phi} \right) \right) \right) \\ &= \frac{u^2}{2g} \left(\left(\sqrt{\tan^2 \phi + 1} \right) \sin \left(2\theta + \tan^{-1} \left(\tan \left(\frac{\pi}{2} - \phi \right) \right) \right) \right) \\ &= \frac{u^2}{2g} \left(\left(\sqrt{\tan^2 \phi + 1} \right) \sin \left(2\theta + \frac{\pi}{2} - \phi \right) \right) \\ &= \frac{u^2 \left(\sqrt{\tan^2 \phi + 1} \right)}{2g} \left(\sin \left(\frac{\pi}{2} + (2\theta - \phi) \right) \right) \\ &= \frac{u^2 \left(\sqrt{\tan^2 \phi + 1} \right)}{2g} (\cos(2\theta - \phi)) \end{aligned}$$

$$\text{set } \frac{dR}{d\theta} = 0$$

$$\cos(2\theta - \phi) = 0$$

$$2\theta - \phi = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4} - \frac{\phi}{2}$$

4(a)(i) each spring originally in 4.1 is subject to tension of $T = Mg$

In 4.2, each spring at equilibrium is subject to $\frac{1}{2}Mg$ but is being stretched by additional tension provided by hand. So will move up

4(a)(ii) In Fig. 4.2 at equilibrium, strings will be slack so effectively is 2 springs in parallel

Fig. 4.1 :

$$\frac{1}{k_{4.1, \text{eff}}} = \frac{2}{k}$$

$$k_{4.1, \text{eff}} = \frac{k}{2}$$

$$Mg = \frac{k}{2}(L - 2l_0) = k\left(\frac{L}{2} - l_0\right)$$

$$\frac{Mg}{k} = \frac{L}{2} - l_0$$

Fig. 4.2 :

$$k_{4.2, \text{eff}} = 2k$$

$$Mg = 2k\left(L_1 - \frac{L}{2} - l_0\right)$$

$$\begin{aligned} L_1 &= \frac{Mg}{2k} + l_0 + \frac{L}{2} \\ &= \frac{Mg}{2k} + l_0 + \frac{L}{2} + \left(\frac{Mg}{k} - \frac{Mg}{k}\right) \\ &= \frac{Mg}{2k} + l_0 + \frac{L}{2} + \left(\frac{Mg}{k} - \left(\frac{L}{2} - l_0\right)\right) \\ &= \frac{3Mg}{2k} + 2l_0 \end{aligned}$$

4(b)(i)

$$Mg = 2k \left(L_1 - \frac{L}{2} - l_0 \right)$$

$$\frac{5}{2}Mg = 2k \left(L_2 - \frac{L}{2} - l_0 \right)$$

$$\frac{3}{2}Mg = 2k (L_2 - L_1)$$

$$\frac{3}{4k}Mg = \frac{3}{14}L_1$$

$$\frac{14Mg}{4k} = L_1$$

$$= 2l_0 + \frac{3Mg}{2k} = 2l_0 + \frac{6Mg}{4k}$$

$$\frac{8Mg}{4k} = 2l_0$$

$$k = \frac{Mg}{l_0}$$

4(b)(ii)

$$k = \frac{Mg}{l_0}$$

$$\frac{Mg}{k} = \frac{L}{2} - l_0$$

$$\begin{aligned} L_1 &= 2l_0 + \frac{3Mg}{2k} \\ &= 2 \left(\frac{L}{2} - \frac{Mg}{k} \right) + \frac{3Mg}{2k} \\ &= L - \frac{2Mg}{k} + \frac{3Mg}{2k} \\ &= L + \frac{Mg}{2k} \\ &= L + \frac{Mg}{2 \left(\frac{Mg}{l_0} \right)} \\ &= L + \frac{l_0}{2} \end{aligned}$$

5(a)(i) consider moment of inertia

$$I = m \left(\frac{L}{2} \right)^2$$

consider restoring torque

$$\tau = k\theta = I\alpha$$

$$k\theta = I\alpha$$

$$= I(\omega^2\theta)$$

$$k = \frac{mL^2\omega^2}{2}$$

$$= \frac{mL^2}{2} \left(\frac{2\pi}{T} \right)^2$$

$$= \frac{4\pi^2 mL^2}{2T^2}$$

$$= \frac{2\pi^2 mL^2}{T^2}$$

5(a)(ii)

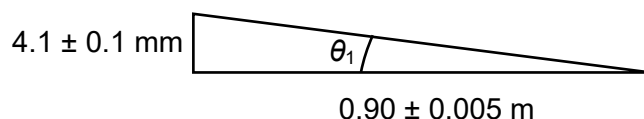
$$r = r_M + r_m + (\text{air gap})$$

$$= \frac{30 \times 10^{-2}}{2} + \frac{50 \times 10^{-3}}{2} + 10^{-3}$$

$$= 0.176 \text{ m}$$

5(a)(iii)

consider sector



$$s = r\theta$$

$$\theta_1 = \frac{s}{r} = \frac{4.1 \times 10^{-3}}{0.9} = \frac{41}{9000} = 0.00456 \text{ rad}$$

$$\frac{\Delta\theta_1}{\theta_1} = \frac{\Delta s}{s} + \frac{\Delta r}{r}$$

$$\Delta\theta_1 = \theta_1 \left(\frac{\Delta s}{s} + \frac{\Delta r}{r} \right) = \left(\frac{41}{9000} \right) \left(\frac{0.1}{4.1} + \frac{0.01}{1.8} \right) = 0.00013642 \approx 0.00014 \text{ rad}$$

$$\theta_1 = 0.0046 \pm 0.0002 \text{ rad}$$

5(a)(iii)

$$\tau = k\theta = I\alpha = LF_g$$

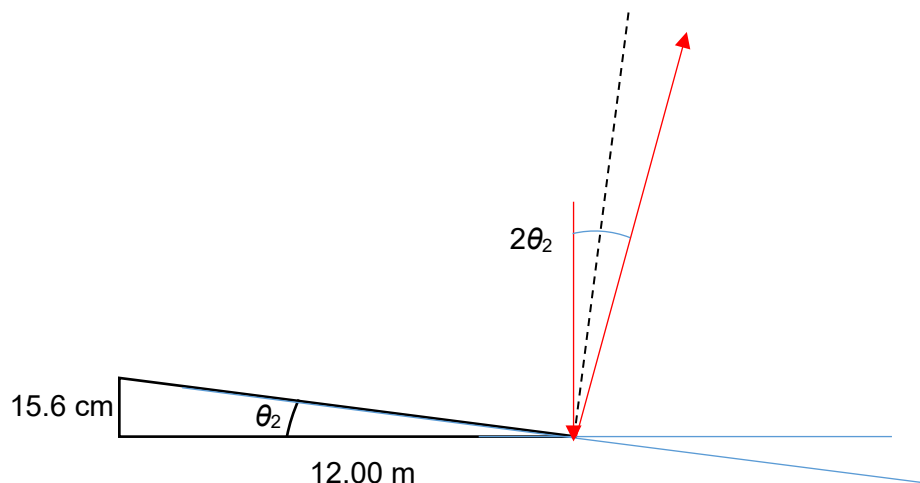
$$F_g = \frac{GMm}{r^2}$$

$$k\theta_1 = L \frac{GMm}{r^2}$$

$$\frac{2\pi^2 ML^3}{T^2} \theta_1 = L \frac{GMm}{r^2}$$

$$G = \frac{2\pi^2 L r^2 \theta_1}{MT^2}$$

5(b)(i) consider sector



$$s = r\theta$$

$$2\theta_2 = \frac{s}{r} = \frac{15.6 \times 10^{-2}}{12} = 0.013 \text{ rad} = 0.745^\circ$$

$$2\theta_2 = 0.0065 \text{ rad} = 0.3725^\circ$$

5(b)(ii) 10 min 22 sec is three quarters of a period

$$\frac{3}{4}T = 10(60) + 22 = 622 \text{ s}$$

$$T = \frac{4}{3}(622) = \frac{2488}{3} \text{ s}$$

$$G = \frac{2\pi^2 L r^2 \theta_2}{MT^2} = \frac{2(\pi^2)(1.8)(0.176)^2(0.0065)}{(158)\left(\frac{2488}{3}\right)^2}$$

$$= 6.58 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

5(b)(iii) The total time elapsed is $(n + \frac{1}{4})T$, n being an integer.

10 min 22 sec is approximately $\frac{3T}{4}$.

$$\frac{(n + 1/4)T}{3T/4} = \frac{4n + 1}{3} = \frac{13 \times 60^2 + 48 \times 60 + 18}{10 \times 60 + 22} = \frac{49698}{622} \approx 79.9 \Rightarrow n \approx 59.7$$

$$(60 + 1/4)T = 49698 \Rightarrow T = 824.9 \text{ s}$$

5(b)(iv) damping due to air resistance and hysteresis losses in torsional wire increased period and lowered value for G

- 6(a)** induced e.m.f. is directly proportional to the rate of change of magnetic flux linkage
 direction of induced e.m.f. produces effects to oppose the change causing it

6(b)(i) The total energy at any instant $U_T = U_C + U_L = \frac{1}{2} \frac{Q^2}{C} + \frac{1}{2} LI^2$

Assuming there are no energy losses: $\frac{dU_T}{dt} = 0$

$$\begin{aligned} \frac{d}{dt} \left(\frac{1}{2} \frac{Q^2}{C} + \frac{1}{2} LI^2 \right) &= 0 \\ \frac{d}{dt} \left(\frac{1}{2} \frac{Q^2}{C} + \frac{1}{2} L \left(\frac{dQ}{dt} \right)^2 \right) &= 0 \\ \frac{Q}{C} \frac{dQ}{dt} + L \left(\frac{dQ}{dt} \right) \frac{d^2Q}{dt^2} &= 0 \\ L \frac{d^2Q}{dt^2} + \frac{Q}{C} &= 0 \end{aligned}$$

- 6(b)(ii)** Comparing to the defining equation for a simple harmonic motion:

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

The general solution is $Q = Q_o \sin(\omega t + \phi)$ where $Q = Q_o$ at $t = 0$

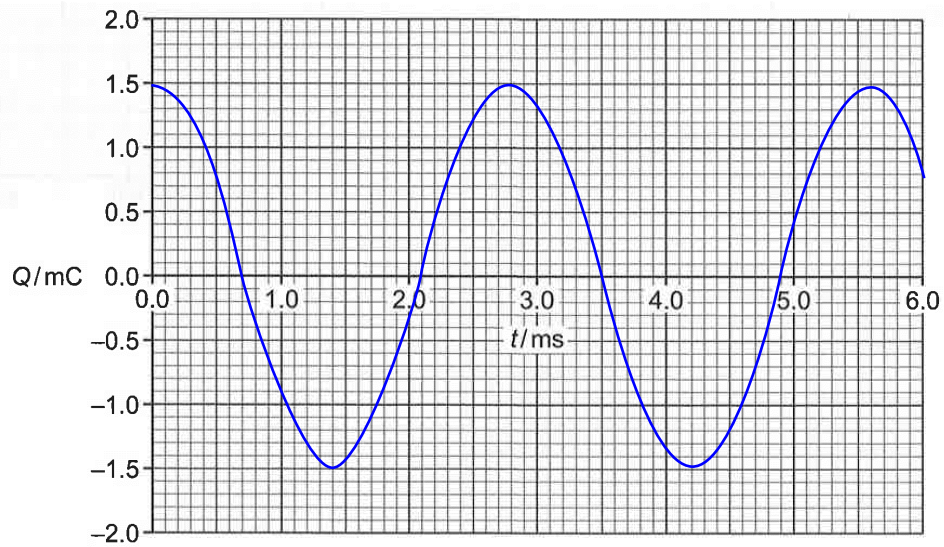
- 6(c)(i)**

$$\omega = \sqrt{\frac{1}{LC}} = \frac{2\pi}{T}$$

$$T = 2\pi \sqrt{(4 \times 10^{-3})(50 \times 10^{-6})} = 0.00281 \text{ s}$$

Capacitor fully charged half a period later so $t = 0.0014 \text{ s}$

6(c)(ii)



6(c)(iii) total energy of circuit remains constant

$$U_c = \frac{1}{2}(U_c)_{\max}$$

$$\frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \left(\left(\frac{1}{2} \right) \frac{Q_0^2}{C} \right)$$

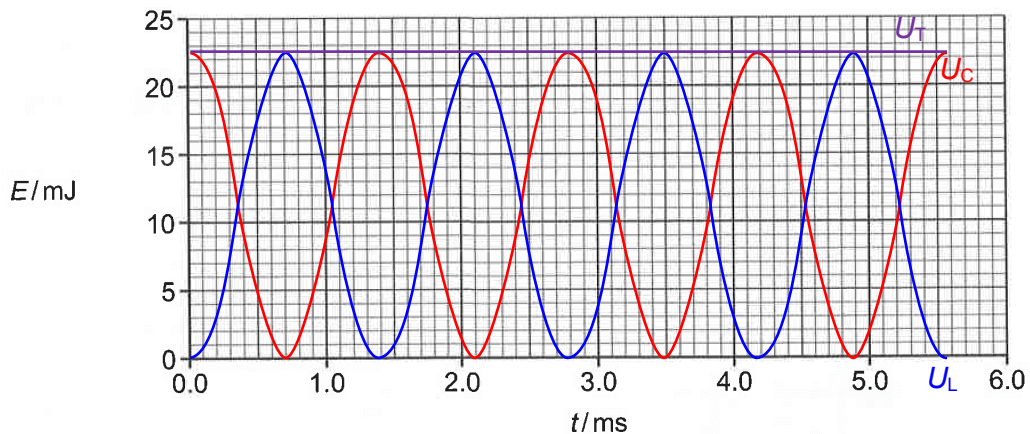
$$Q = \frac{1}{\sqrt{2}} Q_0$$

$$Q_0 \cos(\omega t) = \frac{1}{\sqrt{2}} Q_0$$

$$\left(\frac{2\pi}{T} \right) t = \frac{\pi}{4}$$

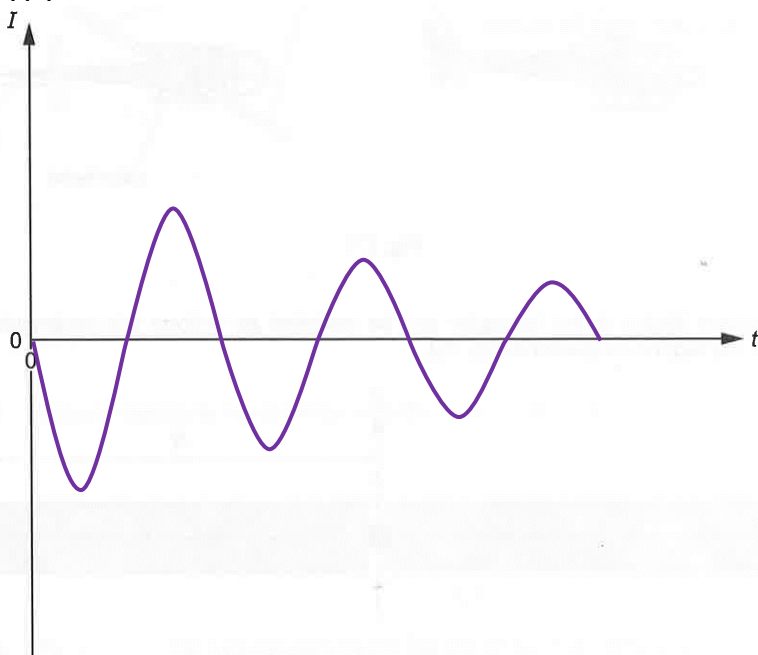
$$t = \frac{T}{8} = 0.35 \times 10^{-3} \text{ s}$$

6(c)(iv)



$$U_T = \left(\frac{1}{2}\right) \frac{Q_0^2}{C} = \left(\frac{1}{2}\right) \frac{(1.5 \times 10^{-3})^2}{50 \times 10^{-6}} = 22.5 \text{ mJ}$$

6(c)(v)1.

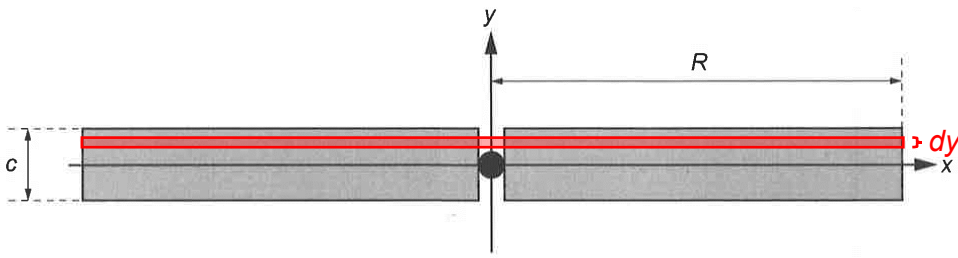


6(c)(v)2.

load resistor dissipates energy (through joule heating), results in damping of the oscillatory charging and discharging in circuit

period is slightly longer due to light damping

7(a)(i)



$$dm = \left(\frac{dy}{c} \right) M$$

$$\begin{aligned} I_x &= \int_{-c/2}^{c/2} y^2 dm \\ &= \frac{M}{c} \int_{-c/2}^{c/2} y^2 dy \\ &= \frac{M}{c} \left[\frac{y^3}{3} \right]_{-c/2}^{c/2} \\ &= \frac{1}{12} M c^2 \end{aligned}$$

7(a)(ii)

$$\begin{aligned} I_y &= \int_{-R}^R x^2 dm \\ &= \int_{-R}^R x^2 \left(\frac{dx}{2R} M \right) \\ &= \frac{M}{2R} \left[\frac{x^3}{3} \right]_{-R}^R \\ &= \frac{M}{2R} \left[\frac{2R^3}{3} \right] \\ &= \frac{1}{3} M R^2 \end{aligned}$$

7(a)(iii)

$$\begin{aligned}
 I_z &= I_x + I_y \\
 &= \frac{1}{12} M c^2 + \frac{1}{3} M R^2 \\
 &= \frac{1}{12} M (c^2 + 4R^2)
 \end{aligned}$$

7(a)(iv)

$$\begin{aligned}
 I_z &= \frac{1}{12} M (c^2 + 4R^2) \\
 &= \frac{1}{12} (\rho (2R) c t) (c^2 + 4R^2) \\
 &= \frac{1}{12} ((1470)(2(3.84))(0.20)(0.03)) (0.2^2 + 4(3.84)^2) \\
 &= 333 \text{ kg m}^2
 \end{aligned}$$

7(a)(v)

$$\begin{aligned}
 \tau &= I \frac{d\omega}{dt} \\
 &= (333) \frac{(320 - 20)(2\pi)}{9.0} \\
 &= 1.16 \times 10^3 \text{ N m}
 \end{aligned}$$

7(b)(i)

$$C_L = \frac{2(dL)}{\rho(r\omega)^2 c dr}$$

$$\text{units of } C_L = \frac{\text{kg m s}^{-2}}{(\text{kg m}^{-3})(\text{m s}^{-1})^2 (\text{m})(\text{m})} = 1$$

7(b)(ii)

$$\begin{aligned}
 L &= 2 \int dL \\
 &= C_L \rho \omega^2 c \int_0^R r^2 dr \\
 &= C_L \rho \omega^2 c \left[\frac{r^3}{3} \right]_0^R \\
 &= \frac{1}{3} C_L \rho \omega^2 c R^3
 \end{aligned}$$

7(c)(i)

The blades push air downwards, hence the air exerts an upward force on the blades, by Newton's 3rd Law. This upward force bends the blades upwards, hence the upward coning.

(The force at the end of the blades due to the rotor is downwards if the blades are bent upwards. This force balances the upward force due to the air.)

7(c)(ii)

$$L \cos \theta = Mg$$

$$\frac{1}{3} C_L \rho \omega^2 c R^3 \cos \theta = Mg$$

$$\frac{1}{3} (0.4) (125) \omega^2 0.23.84^3 \cos(10^\circ) = (2500)(9.81)$$

$$\omega = 115 \text{ rad s}^{-1}$$

$$= 1100 \text{ rpm}$$

7(d)

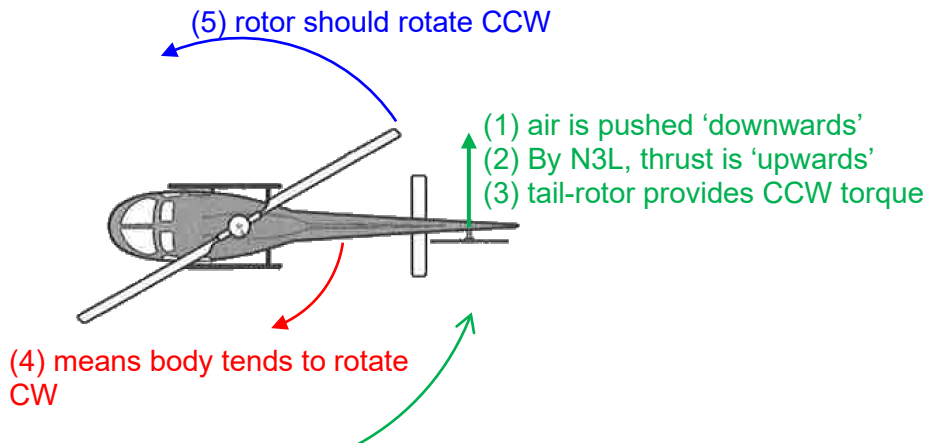
Helicopter needs to conserve zero total angular momentum during stable flight

When tail rotor rotates in a particular direction, helicopter body experiences torque in opposite direction

Tail rotor needs to provide counter-torque in reaction to the torque applied by rotor on body of helicopter

(see reasoning in order of brackets below)

Rotor spins counter-clockwise

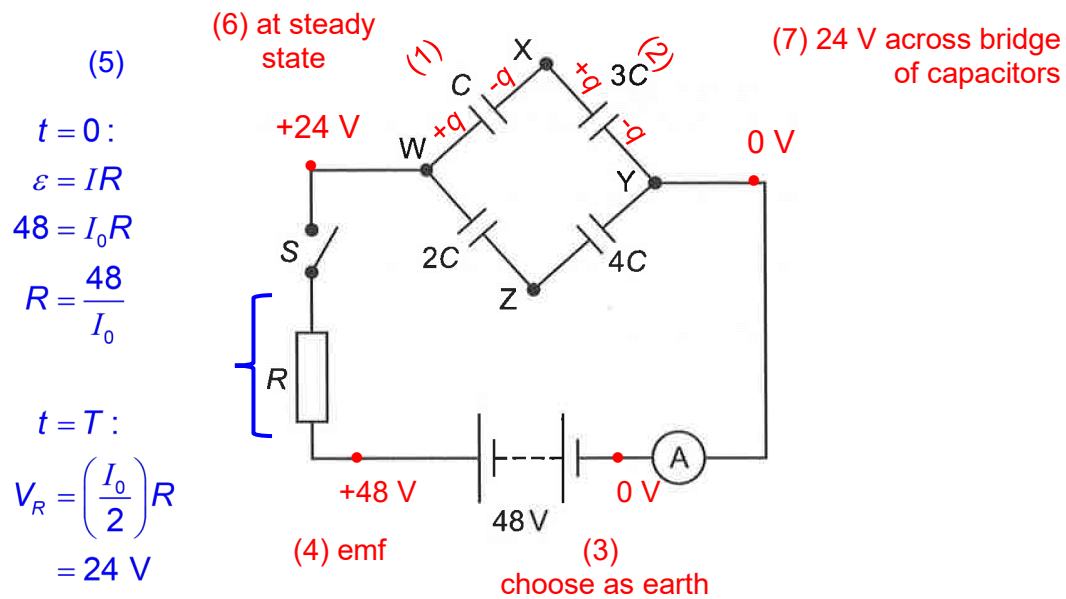
**7(e)**

One pair of blades rotate clockwise while the other pair of blades rotate anti-clockwise

No net torque on drone body

8(a)(i)

the numbers in brackets (1) show the order of deducing the values within the circuit



top branch :

$$Q = CV$$

$$q = CV_{WX} = (3C)V_{XY} = \left(\frac{1}{C} + \frac{1}{3C} \right)^{-1} (V_{WY})$$

$$\frac{q}{C} = V_{WX} = 3V_{XY} = 4V_{WY}$$

therefore the p.d.s are segmented into ratio of 3+1

$$V_{WX} = 18 \text{ V}$$

$$V_{XY} = 6 \text{ V}$$

bottom branch :

$$q_{\text{bottom}} = (2C)V_{WZ} = (4C)V_{ZY} = \left(\frac{1}{2C} + \frac{1}{4C} \right)^{-1} V_{WY}$$

$$\frac{q_{\text{bottom}}}{C} = 2V_{WZ} = 4V_{ZY} = \frac{4}{3} V_{WY}$$

therefore the p.d.s are segmented into ratio of 4+2

$$V_{WZ} = 16 \text{ V}$$

$$V_{ZY} = 8 \text{ V}$$

8(a)(ii)

$$\left(\frac{1}{C} + \frac{1}{3C}\right)^{-1} + \left(\frac{1}{2C} + \frac{1}{4C}\right)^{-1} = \frac{25}{12}C = (2.1)C$$

8(a)(iii)

$$\begin{aligned}\mathcal{E} &= V_R + V_C \\ &= \left(\frac{dQ}{dt}\right)R + \frac{Q}{C_{\text{eq}}}\end{aligned}$$

$$\text{as } t \rightarrow \infty, I \rightarrow 0, Q = Q_0$$

$$\mathcal{E} = \frac{Q_0}{C_{\text{eq}}} = \left(\frac{dQ}{dt}\right)R + \frac{Q}{C_{\text{eq}}}$$

$$\left(\frac{dQ}{dt}\right)R = \frac{1}{C_{\text{eq}}}(Q_0 - Q)$$

$$\int_0^t \frac{1}{RC_{\text{eq}}} dt = \int_0^Q \frac{1}{Q_0 - Q} dQ$$

$$\frac{t}{RC_{\text{eq}}} = -\left[\ln(Q_0 - Q)\right]_0^Q$$

$$-\frac{t}{RC_{\text{eq}}} = \ln\left(1 - \frac{Q}{Q_0}\right)$$

$$e^{-\frac{t}{RC_{\text{eq}}}} = 1 - \frac{Q}{Q_0}$$

$$Q = Q_0 \left(1 - e^{-\frac{t}{RC_{\text{eq}}}}\right)$$

8(a)(iv)

$$\frac{dQ}{dt} = Q \left(-e^{-\frac{t}{RC_{eq}}} \right) \left(-\frac{t}{RC_{eq}} \right)$$

$$I = \frac{Q_0}{RC_{eq}} \left(e^{-\frac{t}{RC_{eq}}} \right)$$

$$\propto e^{-\frac{t}{RC_{eq}}}$$

$$\frac{I_0 / 2}{I_0} = e^{-\frac{t}{RC_{eq}}}$$

$$\frac{T}{RC_{eq}} = \ln(0.5)$$

$$T = RC_{eq} \ln(2)$$

$$= (130 \times 10^3) \left(2.1 \times \frac{500 \times 10^6}{4} \right) \ln(2)$$

$$= 23.7 \text{ s}$$

8(a)(v)

$$q = CV$$

$$= \frac{500 \times 10^6}{4} (18)$$

$$= 2.25 \times 10^{-3} \text{ C}$$

8(a)(vi)

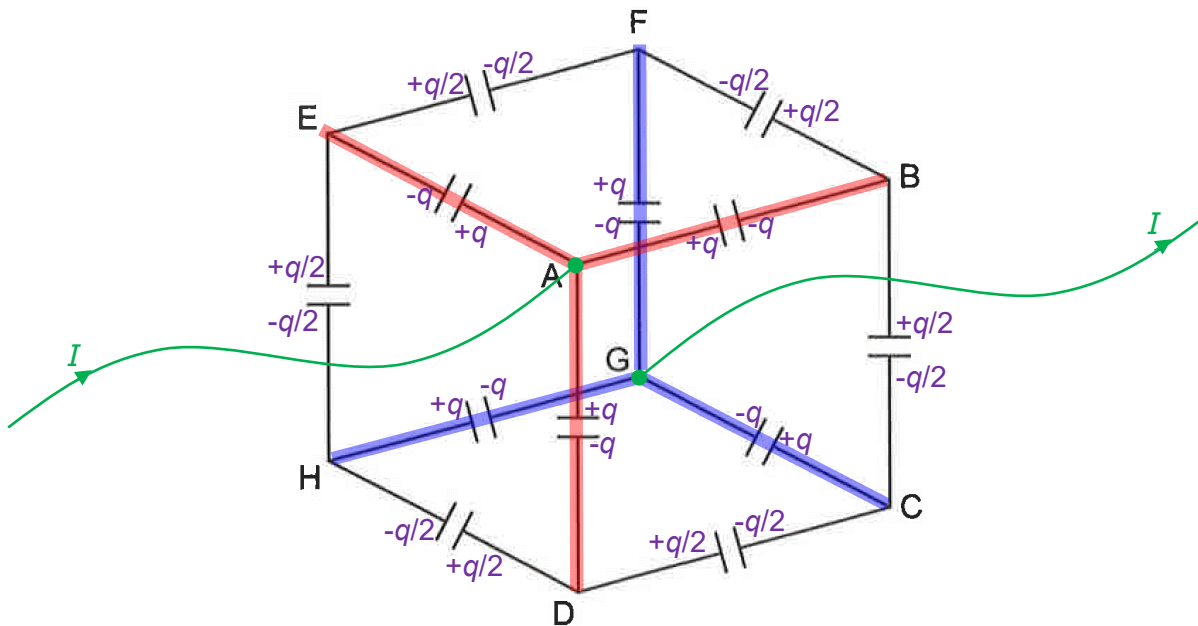
$$\text{energy} = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \left(\frac{500 \times 10^6}{4} \right) (24^2)$$

$$= 7.6 \times 10^{-2} \text{ J}$$

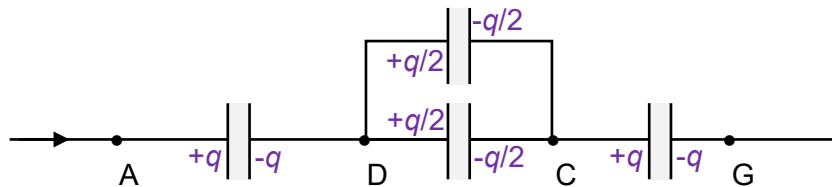
8(b)(i) (E, B, D) or (H, F, C)

8(b)(ii)



By symmetry, each of the following capacitors should store the same amount of charge: AE, AB, AD and GF, GH, GC.

Looking at for e.g. point D, it is connected to two other capacitors via DH and DC, so the charge is split in the middle of a 3 segment path across 3 parallel branches



Because each of the two terminals (A and G) are directly connected to three capacitors, a total charge of $Q_{\text{effective}} = 3q$ is directly supplied by the source of e.m.f..

By symmetry, the charges on each capacity will be distributed as per purple font above.

By Kirchhoff's voltage law,

$$V_{AG} = V_{AD} + V_{DC} + V_{CG} = \frac{q}{C} \left(1 + \frac{1}{2} + 1 \right) = \frac{5q}{2C}$$

$$C_T = \frac{Q_{\text{effective}}}{V_{AG}} = \frac{3q}{\frac{5q}{2C}} = 30 \mu F$$