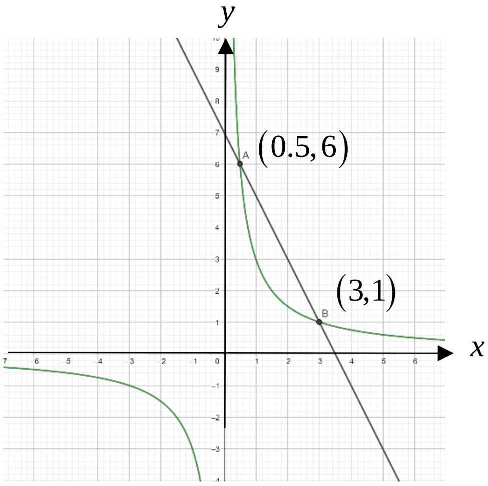
2018 H2 Mathematics Paper 1 (9758/01) – Suggested Solutions

1(i)	Use Quotient Rule, $y=\frac{1}{x}\ln x$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x\left(\frac{1}{x}\right)-\ln x}{x^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x^2}(1-\ln x)$ *Alternatively,* $y=\frac{1}{x}\ln x$ By Product Rule: $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{x^2}\ln x+\frac{1}{x}\left(\frac{1}{x}\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x^2}(1-\ln x)$
(ii)	From (i), $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1-\ln x}{x^2}=\frac{1}{x^2}-\frac{\ln x}{x^2}$ $\frac{\ln x}{x^2}=\frac{1}{x^2}-\frac{\mathrm{d}y}{\mathrm{d}x}$ Integrating both sides with respect to x: $\int_1^{\mathrm{e}}\frac{\ln x}{x^2}\,\mathrm{d}x=\int_1^{\mathrm{e}}\left(\frac{1}{x^2}-\frac{\mathrm{d}y}{\mathrm{d}x}\right)\mathrm{d}x$ $=\left[-\frac{1}{x}-y\right]_1^{\mathrm{e}}$ $=\left[\frac{1}{x}+y\right]_{\mathrm{e}}^{1}$ $=\left[\frac{1+\ln x}{x}\right]_{\mathrm{e}}^{1}$ $=\frac{1+\ln 1}{1}-\frac{1+\ln \mathrm{e}}{\mathrm{e}}$ $=1-\frac{2}{\mathrm{e}}$

	(Otherwise method– Not advised) Using integration by parts, $\int_1^e \frac{\ln x}{x^2}\,dx = \left[-\frac{1}{x}\ln x\right]_1^e - \int_1^e \left(-\frac{1}{x^2}\right)dx$ $= \left(-\frac{1}{e}\ln e + \frac{1}{1}\ln 1\right) + \int_1^e \left(\frac{1}{x^2}\right)dx$ $u = \ln x$ $\frac{dv}{dx} = \frac{1}{x^2}$ $= -\frac{1}{e} + \left[-\frac{1}{x}\right]_1^e$ $\frac{du}{dx} = \frac{1}{x}$ $v = -\frac{1}{x}$ $= -\frac{1}{e} - \frac{1}{e} + 1$ $= 1 - \frac{2}{e}$
2(i)	Given $y = \frac{3}{x}$ ....(1) and $y + 2x = 7$ ....(2) intersect, Substitute (1) into (2) : $\frac{3}{x} + 2x = 7$ $2x^2 - 7x + 3 = 0$ $(2x-1)(x-3) = 0$ $x = \frac{1}{2}$ or $x = 3$ $\frac{3}{x} + 2x = 7$ $2x^2 - 7x + 3 = 0$ $(2x-1)(x-3) = 0$ $x = \frac{1}{2}$ or $x = 3$ $\therefore$ The x-coordinates of A and B are $x = \frac{1}{2}$ or $x = 3$

(ii)



Volume required

$$= \pi\int_{0.5}^{3} y_1^{\,2} dx - \pi\int_{0.5}^{3} y_2^{\,2}\ \mathrm{d}x$$

$$= \pi\int_{0.5}^{3}\left[\left(-2x+7\right)^2 - \left(\frac{3}{x}\right)^2\right]\mathrm{d}x$$

$$= \pi\int_{0.5}^{3}\left[\left(-2x+7\right)^2 - \left(\frac{9}{x^2}\right)\right]\mathrm{d}x$$

$$= \pi\left[\frac{\left(-2x+7\right)^3}{(-2)(3)} + \left(\frac{9}{x}\right)\right]_{0.5}^{3}$$

$$= \frac{125}{6}\pi \text{ units}^3$$

3(i)

$$x\frac{\mathrm{d}y}{\mathrm{d}x} = 2y - 6 \qquad ....(1)$$

Given $y = ux^2$ $\quad .....(2)$

Differentiate (2) with respect to x:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{du}}{\mathrm{d}x}x^2 + 2ux \qquad .....(3)$$

Substitute (3) into (1):

$$\Rightarrow x\left(\frac{\mathrm{du}}{\mathrm{d}x}x^2 + 2ux\right) = 2\left(ux^2\right) - 6$$

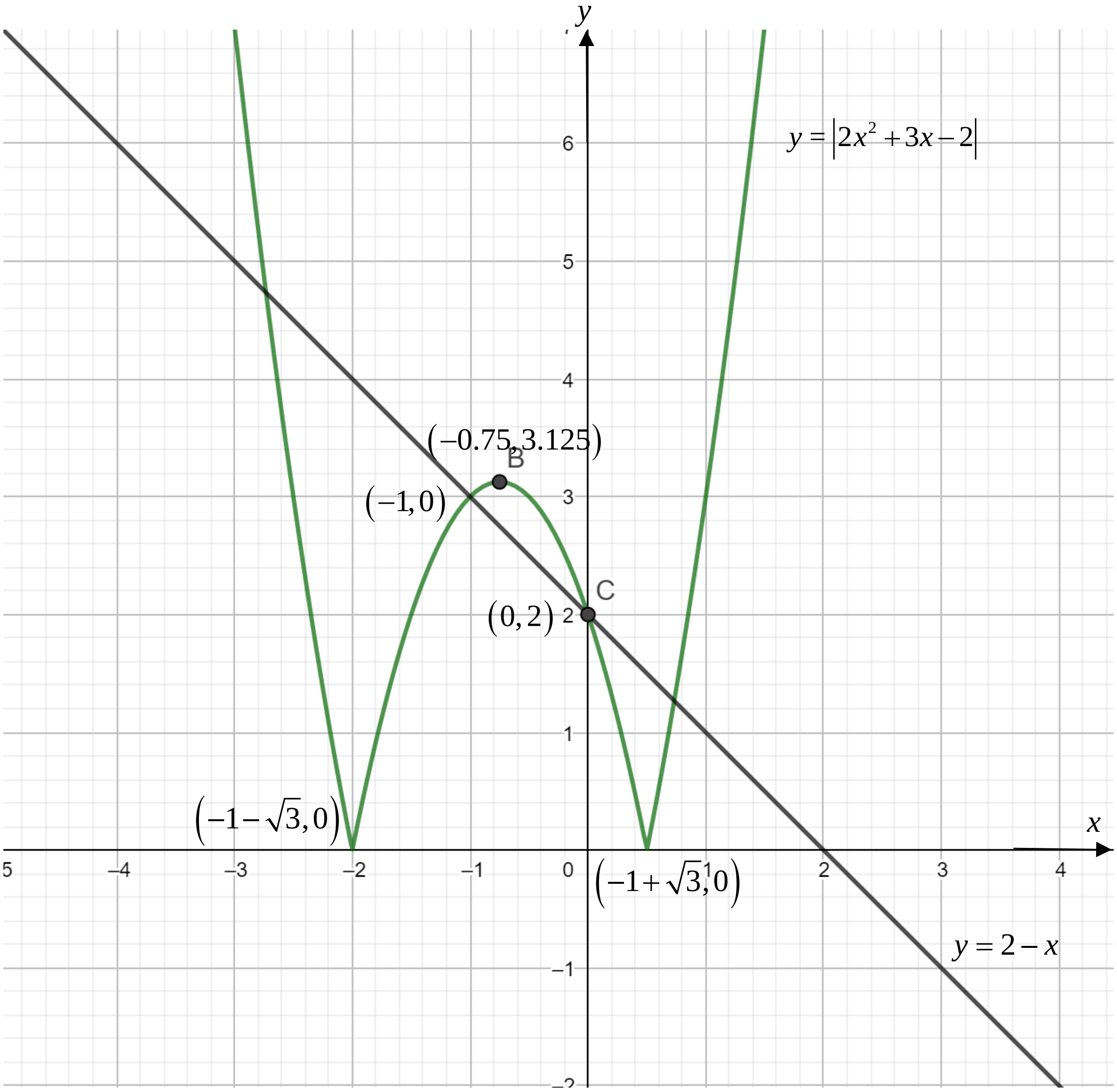
$$\Rightarrow \frac{\mathrm{du}}{\mathrm{d}x}x^3 + 2ux^2 = 2ux^2 - 6$$

$$\frac{\mathrm{du}}{\mathrm{d}x} = -\frac{6}{x^3} \qquad .....(4)$$

$$\therefore \mathrm{f}(x) = -\frac{6}{x^3}$$

(ii)	To solve (1), we solve (4) Integrate (4) with respect to x : $u = \frac{3}{x^2} + C$, where C is an arbitrary constant $\frac{y}{x^2} = \frac{3}{x^2} + C$, where C is an arbitrary constant $y = 3 + Cx^2$ When $x = 1, y = 2$, $C = -1$ $\therefore\ y = 3 - x^2$
4(i)	$\left\|2x^2 - 3x - 2\right\| = \begin{cases} 2x^2 - 3x - 2 & , x \le -2 \text{ or } x \ge \frac{1}{2} \\ -\left(2x^2 - 3x - 2\right) & , -2 < x < \frac{1}{2} \end{cases}$ $\left\|2x^2 - 3x - 2\right\| = 2 - x$ For $x \le -2$ or $x \ge \frac{1}{2}$, $2x^2 + 3x - 2 = 2 - x$ $2x^2 + 4x - 4 = 0$ $x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-4)}}{2(2)} = -1 \pm \sqrt{3}$ For $-2 < x < \frac{1}{2}$, $-\left(2x^2 + 3x - 2\right) = 2 - x$ $2x^2 + 3x - 2 = x - 2$ $2x^2 + 2x = 0$ $x = 0$ or $x = -1$ $\therefore\ x = -1 - \sqrt{3}$ or $-1 + \sqrt{3}$ or -1 or 0

(ii)



Solving

$|2x^2 - 3x - 2| < 2 - x$, using the graph above:

$-1-\sqrt{3} < x < -1$ or $0 < x < -1+\sqrt{3}$

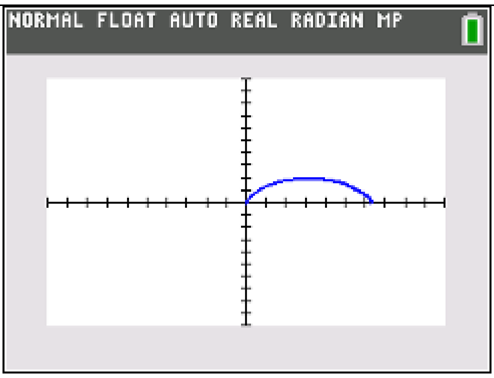
5	$\mathrm{f}(x)=\dfrac{x+a}{x+b}$ ff(x) = g(x) = x, $\Rightarrow x=\mathrm{f}^2(x)$ $\dfrac{\frac{x+a}{x+b}+a}{\frac{x+a}{x+b}+b}=x$ $\dfrac{(x+a)+a(x+b)}{(x+a)+b(x+b)}=x$ $x+a+a(x+b)=x(x+a)+bx(x+b)$ $(x+b)(a-bx)=(x+a)(x-1)$ $ax-bx^2+ab-b^2x=x^2-x+ax-a$ $-bx^2+ab-b^2x-x^2+x+a=0$ $(-b-1)x^2+(1-b^2)x+a(1+b)=0$ By comparing coefficients : $x^2:-b-1=0$ $b=-1$ Thus, $b=-1$. For ff(x) = g(x) = x, $\Rightarrow x=\mathrm{f}^2(x)$, $\mathrm{f}(x)$ must be a self-inverse function $\therefore \mathrm{f}(x)=\mathrm{f}^{-1}(x)$ Hence, $\mathrm{f}^{-1}(x)=\dfrac{x+a}{x-1}$
6(i)	Given that $\mathbf{a}\times 3\mathbf{b}$ **=** $2\mathbf{a}\times\mathbf{c}$ $\mathbf{a}\times 3\mathbf{b}-2\mathbf{a}\times\mathbf{c}$ **=** $\mathbf{0}$ $\mathbf{a}\times(3\mathbf{b}-2\mathbf{c})$ **=** $\mathbf{0}$ Either $(3\mathbf{b}-2\mathbf{c})=\mathbf{0}$ **or** $\mathbf{a}=\mathbf{0}$ or $(3\mathbf{b}-2\mathbf{c})$ is parallel to $\mathbf{a}$ As $\mathbf{a}\neq\mathbf{0}$, $(3\mathbf{b}-2\mathbf{c})$ is parallel to $\mathbf{a}$. $\therefore$ $(3\mathbf{b}-2\mathbf{c})=\lambda\,\mathbf{a}$ for some $\lambda\in\mathbb{R}$. (shown)

(ii)	Given that **a** and **c** are unit vectors, $\|\mathbf{a}\| = \|\mathbf{c}\| = 1$, $\|\mathbf{b}\| = 4$ and angle between **b** and **c** $= 60°$ From (i), $3\mathbf{b} - 2\mathbf{c} = \lambda\mathbf{a}$ for some $\lambda \in \mathbb{R}$ $(3\mathbf{b} - 2\mathbf{c})\bullet(3\mathbf{b} - 2\mathbf{c}) = (\lambda\mathbf{a})\bullet(\lambda\mathbf{a})$ $\Rightarrow \|3\mathbf{b} - 2\mathbf{c}\|^2 = \lambda^2 \|\mathbf{a}\|^2$ $(3\mathbf{b} - 2\mathbf{c})\bullet(3\mathbf{b} - 2\mathbf{c}) = \lambda^2 (1)^2$ $9\mathbf{b}\bullet\mathbf{b} - 12\mathbf{b}\bullet\mathbf{c} + 4\mathbf{c}\bullet\mathbf{c} = \lambda^2$ $9\|\mathbf{b}\|^2 - 12(2) + 4\|\mathbf{c}\|^2 = \lambda^2$ $9(4)^2 - 12(2) + 4(1)^2 = \lambda^2$ $\lambda^2 = 124$ $\lambda = \pm\sqrt{124} = \pm 2\sqrt{31}$
7(i)	$\dfrac{x^2 - 4y^2}{x^2 + xy^2} = \dfrac{1}{2}$, $2x^2 - 8y^2 = x^2 + xy^2 \quad ...(1)$ Differentiating (1) with respect to x, $4x - 16y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x + y^2 + x\left(2y\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}(2xy + 16y) = 4x - 2x - y^2$ $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2x - y^2}{2xy + 16y}$ (shown) $\quad ...(2)$
(ii)	When $x = 1$, From (1), $2 - 8y^2 = 1 + y^2$ $\Rightarrow y = \dfrac{1}{3}$ or $-\dfrac{1}{3}$ At point $\left(1, \dfrac{1}{3}\right)$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2 - \frac{1}{9}}{2\left(\frac{1}{3}\right) + 16\left(\frac{1}{3}\right)} = \dfrac{17}{54}$.

	At point $\left(1,-\frac{1}{3}\right)$, $\frac{dy}{dx}=\frac{2-\frac{1}{9}}{2\left(-\frac{1}{3}\right)+16\left(-\frac{1}{3}\right)}=-\frac{17}{54}$ Tangent of equation at point $\left(1,\frac{1}{3}\right)$: $y-\frac{1}{3}=\frac{17}{54}(x-1)$ ...(1) Tangent of equation at point $\left(1,-\frac{1}{3}\right)$: $y+\frac{1}{3}=-\frac{17}{54}(x-1)$ ...(2) $(1)-(2)$, $-\frac{2}{3}=\frac{17}{54}(x-1)+\frac{17}{54}(x-1)$ $-\frac{2}{3}=\frac{17}{27}(x-1)$ $x=-\frac{1}{17}$ $y=\frac{17}{54}\left(-\frac{1}{17}-1\right)+\frac{1}{3}=0$ $\therefore N\left(-\frac{1}{17},0\right)$
8(a)	Given $u_{n+1}=2u_n+A_n$, where $A\,is\,a\,cons\tan t$, $n\geq 1$, $u_1=5$, $u_2=15$, At $n=1$, $u_2=2u_1+A$ $15=2(5)+A$ $\Rightarrow A=5$ At $n=2$, $u_3=2u_2+2A=$ $=2(15)+2(5)$ $=40$

(ii)	$u_n = a\left(2^n\right) + bn + c$ $n = 1,$ $u_1 = 2a + b + c = 5$ .....(1) $n = 2,$ $u_2 = 4a + 2b + c = 15$ .....(2) $n = 3,$ $u_3 = 8a + 3b + c = 140$ ....(3) Using GC, solving $(1), (2)$ and (3), $a = \frac{15}{2}, b = -5, c = -5$
(iii)	$\sum_{r=1}^{n} u_r$ $= \sum_{r=1}^{n} \left[\frac{15}{2}\left(2^r\right) - 5r - 5 \right]$ $= \sum_{r=1}^{n} \frac{15}{2}\left(2^r\right) - 5\sum_{r=1}^{n} r - 5\sum_{r=1}^{n} 1$ $= \left(\frac{15}{2}\right)\left[\frac{2\left(2^n - 1\right)}{2-1}\right] - 5\left[\frac{n(n+1)}{2}\right] - 5n$ $= 15\left(2^n - 1\right) - \frac{5}{2}n(n+1) - 5n$ $\left[= 15\left(2^n - 1\right) - \frac{15}{2}n - \frac{5}{2}n^2 \right]$

9(i)	$x = 2\theta - \sin 2\theta \quad , \quad y = 2\sin^2\theta \quad , 0 \le \theta \le 2\pi$ $\frac{dx}{d\theta} = 2 - 2\cos 2\theta \qquad \frac{dy}{d\theta} = 4\sin\theta\cos\theta$ $\frac{dy}{dx} = \frac{4\sin\theta\cos\theta}{2-2\cos 2\theta}$ $= \frac{4\sin\theta\cos\theta}{2-2(2\cos^2\theta-1)}$ $= \frac{4\sin\theta\cos\theta}{4(1-\cos^2\theta)}$ $= \frac{4\sin\theta\cos\theta}{4\sin^2\theta}$ $= \frac{\cos\theta}{\sin\theta}$ $= \cot\theta$ (shown) Alternative, $x = 2\theta - \sin 2\theta \quad , \quad y = 2\sin^2\theta \quad , 0 \le \theta \le 2\pi$ $\frac{dx}{d\theta} = 2 - 2\cos 2\theta \qquad \frac{dy}{d\theta} = 4\sin\theta\cos\theta$ $\frac{dy}{dx} = \frac{4\sin\theta\cos\theta}{2-2\cos 2\theta}$ $= \frac{4\sin\theta\cos\theta}{2(1-\cos 2\theta)}$ $= \frac{4\sin\theta\cos\theta}{2(2\sin^2\theta)}$ $= \frac{4\sin\theta\cos\theta}{4\sin^2\theta}$ $= \frac{\cos\theta}{\sin\theta}$ $= \cot\theta$ (shown)

9(ii)	When $\theta = \alpha$, $x = 2\alpha - \sin 2\alpha$, $y = 2\sin^2 \alpha$ $\frac{dy}{dx} = \cot\alpha$ Gradient of normal to the curve $= -\frac{1}{\cot\alpha} = -\tan\alpha$ Equation of normal to the curve $y - 2\sin^2\alpha = -\tan\alpha\left[x - (2\alpha - \sin 2\alpha)\right]$ At point A where the normal meets the x-axis ($y = 0$) $-2\sin^2\alpha = -\frac{\sin\alpha}{\cos\alpha}\left[x - (2\alpha - \sin 2\alpha)\right]$ $2\sin^2\alpha\cos\alpha = x\sin\alpha - 2\alpha(\sin\alpha) + \sin 2\alpha(\sin\alpha)$ $x\sin\alpha = 2\alpha(\sin\alpha) + \sin 2\alpha(\sin\alpha) - 2\sin^2\alpha(\cos\alpha)$ $\sin\alpha\left[x - 2\alpha - \sin 2\alpha + 2\sin\alpha\cos\alpha\right] = 0$ $\sin\alpha\left[x - 2\alpha - \sin 2\alpha + \sin 2\alpha\right] = 0$ $\sin\alpha(x - 2\alpha) = 0$ $x = 2\alpha$ or $\sin\alpha = 0$ $\alpha = 0$ or π If $\alpha = 0$ or π, $\tan\alpha = 0$ $\Rightarrow$ normal is a horizontal line, moreover $y = 0$ $\Rightarrow$ normal is the x-axis (impossible) Hence, $x = 2\alpha$ $\therefore k = 2$
9(iii)	 From the graph, the arc length of C is also the total length of C, taking $\beta = 0$, $\gamma = \pi$.

	Total length of C $=\int_0^{\pi}\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\frac{\mathrm{dy}}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$ $=\int_0^{\pi}\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\frac{\mathrm{dy}}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$ $=\int_0^{\pi}\sqrt{\left(2-2\cos 2\theta\right)^2+\left(4\sin\theta\cos\theta\right)^2}\,\mathrm{d}\theta$ $=\int_0^{\pi}\sqrt{\left(4\sin^2\theta\right)^2+\left(4\sin\theta\cos\theta\right)^2}\,\mathrm{d}\theta$ $=\int_0^{\pi}\sqrt{16\sin^4\theta+16\sin^2\theta\cos^2\theta}\ \mathrm{d}\theta$ $=\int_0^{\pi}\sqrt{\left(16\sin^2\theta\right)\left(\sin^2\theta+\cos^2\theta\right)}\ \mathrm{d}\theta$ $=\int_0^{\pi}\sqrt{\left(16\sin^2\theta\right)}\ \mathrm{d}\theta \quad ,\because \left(\sin^2\theta+\cos^2\theta\right)=1$ $=\int_0^{\pi}4\sin\theta\ \mathrm{d}\theta$ $=\left[-4\cos\theta\right]_0^{\pi}$ $=(-4\cos\pi)-(-4\cos 0)$ $=4+4$ $=8$ units
10(i)	$L\frac{\mathrm{d}I}{\mathrm{d}t}+RI+\frac{q}{C}=V \qquad ...(1)$ Given R, C and L are constant, Differentiating (1) with respect to t , $L\frac{\mathrm{d}^2I}{\mathrm{d}t^2}+R\frac{\mathrm{d}I}{\mathrm{d}t}+\frac{1}{C}\frac{\mathrm{dq}}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}$ since $I=\frac{\mathrm{dq}}{\mathrm{d}t}$ and assuming that V is a constant, $\Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t}=0$ $\Rightarrow L\frac{\mathrm{d}^2I}{\mathrm{d}t^2}+R\frac{\mathrm{d}I}{\mathrm{d}t}+\frac{I}{C}=0$ (shown) $\quad ...(2)$

(ii)	**Method 1:** $I = Ate^{-\frac{Rt}{2L}}$ Let $B = -\dfrac{R}{2L}$ ln I = $\ln Ate^{Bt}$ ln I = $\left(\ln At + Bt\right)$ ln I = $\left(\ln At + Bt\right)$ Differentiate w.r.t t $\dfrac{1}{I}\dfrac{\mathrm{d}I}{\mathrm{d}t} = \dfrac{1}{t} + B$ $\dfrac{\mathrm{d}I}{\mathrm{d}t} = I\left[\dfrac{1}{t} + B\right]$ $\dfrac{\mathrm{d}I}{\mathrm{d}t} = Ate^{Bt}\left(\dfrac{1}{t} + B\right)$ $\dfrac{\mathrm{d}I}{\mathrm{d}t} = Ae^{Bt} + BAte^{Bt} = Ae^{Bt}[1 + Bt]$ $\dfrac{\mathrm{d}^2I}{\mathrm{d}t^2} = BAe^{Bt} + BA[e^{Bt} + Bte^{Bt}]$ $= BAe^{Bt}[2 + Bt]$ Given that $I = Ate^{-\frac{Rt}{2L}}$ is a solution to the differential equation, $L\dfrac{\mathrm{d}^2I}{\mathrm{d}t^2} + R\dfrac{\mathrm{d}I}{\mathrm{d}t} + \dfrac{I}{C} = 0$ $LBAe^{Bt}[2 + Bt] + RAe^{Bt}[1 + Bt] + \dfrac{Ate^{Bt}}{C} = 0$ $LB[2 + Bt] + R[1 + Bt] + \dfrac{t}{C} = 0$

	$C=-\dfrac{t}{LB[2+Bt]+R[1+Bt]}$ $=-\dfrac{t}{L\left(-\dfrac{R}{2L}\right)\left(2-\dfrac{Rt}{2L}\right)+R\left(1-\dfrac{Rt}{2L}\right)}$ $=-\dfrac{t}{\left(-\dfrac{R}{2}\right)\left(2-\dfrac{Rt}{2L}\right)+R\left(1-\dfrac{Rt}{2L}\right)}$ $=-\dfrac{t}{-R+\dfrac{R^2t}{4L}+R-\dfrac{R^2t}{2L}}$ $=-\dfrac{t}{-R+\dfrac{R^2t}{4L}+R-\dfrac{2R^2t}{4L}}$ $=-\dfrac{t}{-\dfrac{R^2t}{4L}}$ $=\dfrac{4L}{R^2}$ (shown) **Method 2:** $I=Ate^{-\frac{Rt}{2L}}$ $\dfrac{\mathrm{d}I}{\mathrm{d}t}=Ae^{-\frac{Rt}{2L}}+\left(-\dfrac{R}{2L}\right)Ate^{-\frac{Rt}{2L}}$ $=Ae^{-\frac{Rt}{2L}}-\dfrac{R}{2L}I$ $\dfrac{\mathrm{d}^2I}{\mathrm{d}t^2}=\left(-\dfrac{R}{2L}\right)Ae^{-\frac{Rt}{2L}}-\dfrac{R}{2L}\dfrac{\mathrm{d}I}{\mathrm{d}t}$ Given that $I=Ate^{-\frac{Rt}{2L}}$ is a solution to the differential equation, $L\dfrac{\mathrm{d}^2I}{\mathrm{d}t^2}+R\dfrac{\mathrm{d}I}{\mathrm{d}t}+\dfrac{I}{C}=0$ $\mathrm{L}\left[\left(-\dfrac{R}{2L}\right)Ae^{-\frac{Rt}{2L}}-\dfrac{R}{2L}\dfrac{\mathrm{d}I}{\mathrm{d}t}\right]+R\left[\dfrac{\mathrm{d}I}{\mathrm{d}t}\right]+\dfrac{Ate^{-\frac{Rt}{2L}}}{C}=0$ $\left[\left(-\dfrac{R}{2}\right)Ae^{-\frac{Rt}{2L}}-\dfrac{R}{2}\left(\dfrac{\mathrm{d}I}{\mathrm{d}t}\right)\right]+\left[R\dfrac{\mathrm{d}I}{\mathrm{d}t}\right]+\dfrac{Ate^{-\frac{Rt}{2L}}}{C}=0$ $\left(-\dfrac{R}{2}\right)Ae^{-\frac{Rt}{2L}}+\dfrac{R}{2}\dfrac{\mathrm{d}I}{\mathrm{d}t}=-\dfrac{Ate^{-\frac{Rt}{2L}}}{C}$

$$C = -\frac{Ate^{-\frac{Rt}{2L}}}{\left(-\frac{R}{2}\right)Ae^{-\frac{Rt}{2L}} + \frac{R}{2}\frac{\mathrm{d}I}{\mathrm{d}t}}$$

$$= -\frac{Ate^{-\frac{Rt}{2L}}}{\left(-\frac{R}{2}\right)Ae^{-\frac{Rt}{2L}} + \frac{R}{2}\left(Ae^{-\frac{Rt}{2L}} + \left(-\frac{R}{2L}\right)Ate^{-\frac{Rt}{2L}}\right)}$$

$$= -\frac{Ate^{-\frac{Rt}{2L}}}{\left(Ae^{-\frac{Rt}{2L}}\right)\left[\left(-\frac{R}{2}\right) + \frac{R}{2}\left(1 + \left(-\frac{R}{2L}\right)t\right)\right]}$$

$$= -\frac{Ate^{-\frac{Rt}{2L}}}{\left(Ae^{-\frac{Rt}{2L}}\right)\left[\left(-\frac{R}{2}\right) + \frac{R}{2} - \frac{R^2}{4L}t\right]}$$

$$= -\frac{t}{\left(-\frac{R^2}{4L}\right)t}$$

$$= \frac{4L}{R^2} \text{ (shown)}$$

Method 3:

Given $I = Ate^{-\frac{Rt}{2L}} = Ate^{Kt}$ where $K = -\frac{R}{2L}$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = Ae^{Kt} + At(Ke^{Kt}) = A(1+Kt)e^{Kt}$$

$$\frac{\mathrm{d}^2I}{\mathrm{d}t^2} = A\left[Ke^{Kt} + (1+Kt)Ke^{Kt}\right] = A(2K + K^2t)e^{Kt}$$

Substitute all these equations into (2):

$$LA(2K + K^2t)e^{Kt} + RA(1+Kt)e^{Kt} + \frac{Ate^{Kt}}{C} = 0$$

$$\div Ae^{Kt}:\ L(2K + K^2t) + R(1+Kt) + \frac{t}{C} = 0$$

$$L\left[2\left(-\frac{R}{2L}\right) + \left(\frac{R^2}{4L^2}\right)t\right] + R\left[1 + \left(-\frac{R}{2L}\right)t\right] + \frac{t}{C} = 0$$

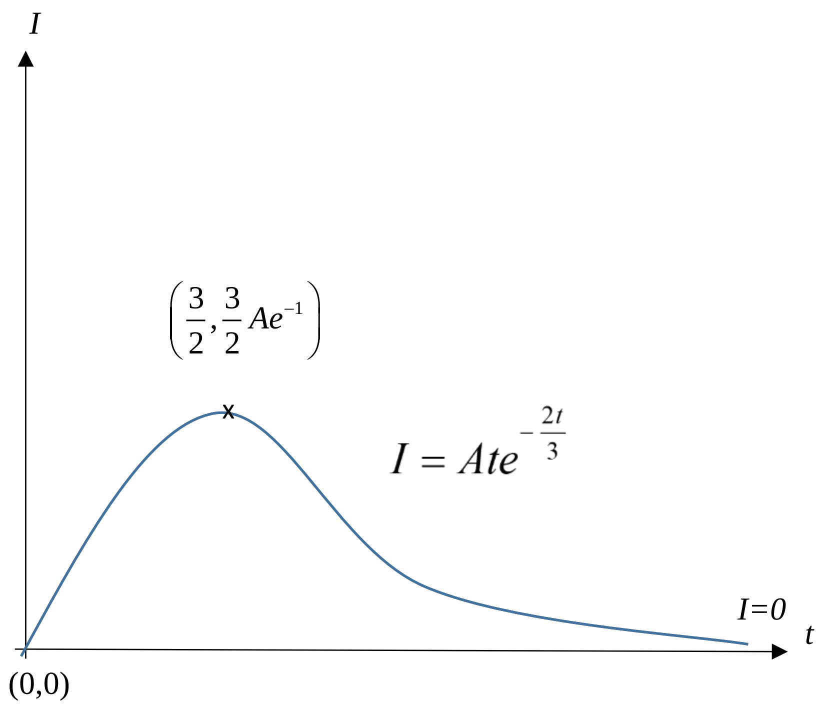
$$-R + \frac{R^2}{4L}t + R - \frac{R^2}{2L}t + \frac{t}{C} = 0$$

$$\frac{t}{C} = \frac{R^2}{4L}t$$

As this equation holds for all $t > 0$, $\frac{1}{C} = \frac{R^2}{4L}$

$C = \frac{4L}{R^2}$ (shown)

(iii)	Given that $R=4,\ L=3$ and $C=0.75$, $\frac{4L}{R^2}=\frac{4\times 3}{4^2}=\frac{3}{4}=0.75=C$ $I=Ate^{-\left(\frac{Rt}{2L}\right)}$, $\frac{\mathrm{d}I}{\mathrm{d}t}=Ae^{-\frac{Rt}{2L}}+\left(-\frac{R}{2L}\right)Ate^{-\frac{Rt}{2L}}$ At stationary values of I, $\frac{\mathrm{d}I}{\mathrm{d}t}=0.$ $0=Ae^{-\frac{4t}{6}}-\frac{4}{6}Ate^{-\frac{4t}{6}}$ $Ae^{-\frac{4t}{6}}\left(1-\frac{4}{6}t\right)=0$ $\text{Sin}\,ce\ A>0$ and $e^{-\frac{4t}{6}}>0$ for all $t\geq 0$, $\therefore Ae^{-\frac{4t}{6}}=0$ or $1-\frac{4}{6}t=0$ no solution since $A>0$ and $e^{-\frac{4t}{6}}>0$ for all $t\in\mathbb{R}$ $t=\frac{3}{2}$ when $t=\frac{3}{2}$, $I=A\left(\frac{3}{2}\right)e^{-\frac{4\left(\frac{3}{2}\right)}{2(3)}}$ $=\frac{3}{2}Ae^{-1}$ $\frac{\mathrm{d}^2I}{\mathrm{d}t^2}=\left(-\frac{R}{2L}\right)Ae^{-\frac{Rt}{2L}}-\frac{R}{2L}\frac{\mathrm{d}I}{\mathrm{d}t}$ $=-\frac{R}{2L}\left(Ae^{-\frac{Rt}{2L}}+\frac{\mathrm{d}I}{\mathrm{d}t}\right)$ At stationary values of I, $\frac{\mathrm{d}I}{\mathrm{d}t}=0$, $t=\frac{3}{2}$, $\frac{\mathrm{d}^2I}{\mathrm{d}t^2}=-\frac{4}{6}\left(Ae^{-\frac{4t}{6}}\right)<0$ since $A>0$ and $e^{-\frac{4t}{6}}>0$ for all $t\in\mathbb{R}$ hence it is a maximum. $\therefore$ I is maximum at $\frac{3}{2}Ae^{-1}$ when $t=\frac{3}{2}$. **From the Method 3 in (ii)** In this particular circuit,

	$I = Ate^{-\frac{2}{3}t}$, $\dfrac{dI}{dt} = A(1-\frac{2}{3}t)e^{-\frac{2}{3}t}$ and $3\dfrac{d^2I}{dt^2} + 4\dfrac{dI}{dt} + \dfrac{I}{0.75} = 0$ For maximum value of I, $\dfrac{dI}{dt} = 0$: $1-\frac{2}{3}t = 0 \Rightarrow t = \frac{3}{2}$ $I_{\max} = \dfrac{3}{2}Ae^{-1} = \dfrac{3A}{2e}$ For maximum value of I, $\dfrac{dI}{dt} = 0$ $3\dfrac{d^2I}{dt^2} + 4(0) + \dfrac{1}{0.75}I = 0$ $\dfrac{d^2I}{dt^2} = -\dfrac{1}{2.25}I = -\dfrac{1}{2.25}\left(\dfrac{3A}{2e}\right)$ $= -\dfrac{2A}{3e} < 0 \quad (\because A > 0)$ $\therefore$ I is maximum at $\dfrac{3}{2}Ae^{-1}$ when $t = \dfrac{3}{2}$.
(iv)	

11(i) (a)

Amount the first \$100 is worth at the end of 31 Dec 2016

$= 100(1.002)^{12}$

$\approx \$102.43$ (2 d.p.)

(i) (b)

n	Amount at start of the nth month (\$)	Amount at the end of the nth month (\$)
1	100	$100(1.002)$
2	$100(1.002)+100$	$100(1.002)^2+100(1.002)$
3	$100(1.002)^2+100(1.002)+100$	$100(1.002)^3+100(1.002)^2+100(1.002)$
	..	..
	..	..
N		$100(1.002)^N+100(1.002)^{N-1}+......+100(1.002)$

At the end of 31 Dec 2016, $N = 12$,

Total amount at the end of 31 Dec 2016 $= 100(1.002)^{12}+100(1.002)^{11}+......+100(1.002)$

$$= \frac{100(1.002)(1.002^{12}-1)}{1.002-1}$$

$$\approx \$1215.71$$

(i) (c)

$100(1.002)^N+100(1.002)^{N-1}+......+100(1.002) > 3000$

$$\frac{100(1.002)(1.002^N-1)}{1.002-1} > 3000$$

$50100(1.002^N-1) > 3000$

using GC,

29	2988.6 < 3000
30	3094.8 > 3000
31	3201.2 > 3000

On the last day of the 29th month, he will have \$2988.6 in his account. Since he deposit \$100 at the start of every month, he will exceed \$3000 on the first day of the 30th month, i.e. 1st day of June 2018.

(ii) (a)

Amount the first \$100 is worth at the end of 31 Dec 2016

$= \$(100+12b)$

(ii) (b)

n	Amount at start of the nth month (\$)	Amount at the end of the nth month (\$)
1	100	$100+b$
2	$100+b+100$	$2(100)+b+2b$
3	$2(100)+2(b)+100$	$3(100)+b+2b+3b$
	..	..
	..	..
N		$N(100)+b+2b+3b+...+Nb$

	When $N = 24$, $24(100)+\frac{24}{2}\left(2b+(23)b\right)=2800$ $2400+12(25b)=2800$ $b=\frac{4}{3}$
(iii)	Given that $a=1$ for plan P, Total amount at the end of 31 Dec 2016 = $100(1.01)^{N}+100(1.01)^{N-1}+......+100(1.01)$ $=\frac{100(1.01)(1.01^{60}-1)}{1.01-1}$ $\approx \$8248.6367$ For Plan Q to have the same amount as Plan P at the end of the 60th month $60(100)+b+2b+3b+...+60b=8248.6366$ $6000+\frac{60}{2}\left(2b+(59b)\right)=8248.6366$ $61b=74.954555$ $b\approx 1.23$ (3 s.f.)