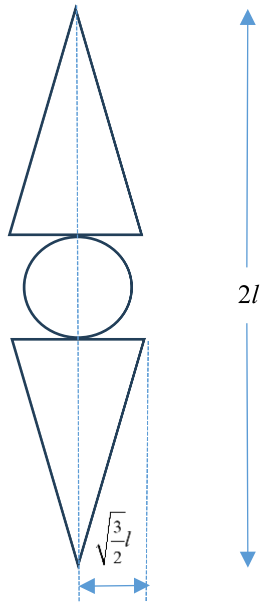
Q1	Suggested Answers
	Since C has an oblique asymptote with gradient 1.5, $p=1.5$ C contains the point (3, 7): $7=\dfrac{1.5(3)^2+q(3)+r}{3-1}$ $r=0.5-3q$ $\therefore y=\dfrac{1.5x^2+qx+0.5-3q}{x-1}$ Turning point at (3, 7): $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(3x+q)(x-1)-(1.5x^2+qx+0.5-3q)(1)}{(x-1)^2}$ $0=\dfrac{(3(3)+q)(3-1)-(1.5(3)^2+q(3)+0.5-3q)}{(3-1)^2}$ $(9+q)(2)-14=0$ $\therefore q=-2$ $\therefore r=0.5-3(-2)=6.5$

Q2	Suggested Answers

Method 1:

Let the radius of the sphere be r and the height of the cone be h.

$$2h+2r=2l \Rightarrow h=l-r$$



Total volume, $V=2\left(\frac{1}{3}\pi h\times\frac{3}{2}l^2\right)+\frac{4}{3}\pi r^3$

$$=\pi l^2\left(l-r\right)+\frac{4}{3}\pi r^3$$

$$\frac{\mathrm{d}V}{\mathrm{d}r}=\pi l^2(-1)+4\pi r^2$$

$$\frac{\mathrm{d}V}{\mathrm{d}r}=0\Rightarrow 4\pi r^2=\pi l^2$$

Since $r>0,\ r=\frac{l}{2}$

$\frac{\mathrm{d}^2V}{\mathrm{d}r^2}=8\pi r>0$ for $r=\frac{l}{2}$, ie minimum volume when $r=\frac{l}{2}$.

Method 2:

Let the radius of the sphere be r and the height of the cone be h.

$$2h+2r=2l \Rightarrow h=l-r$$

Total volume, $V=2\left(\frac{1}{3}\pi h\times\frac{3}{2}l^2\right)+\frac{4}{3}\pi r^3=\pi l^2h+\frac{4}{3}\pi\left(l-h\right)^3$

$$\frac{\mathrm{d}V}{\mathrm{d}h}=\pi l^2-4\pi\left(l-h\right)^2=0$$

$$l^2=4\left(l-h\right)^2$$

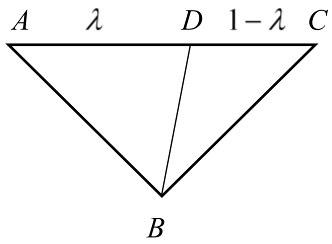
$l=2\left(l-h\right)$ or $l=2\left(h-l\right)$

$l=2l-2h$ or $3l=2h$

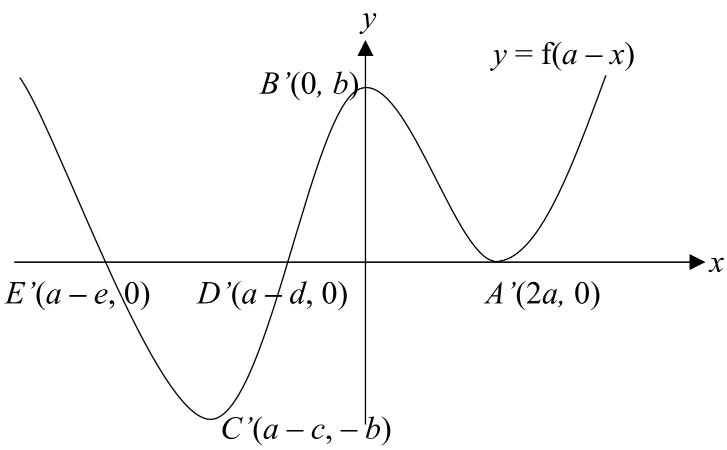
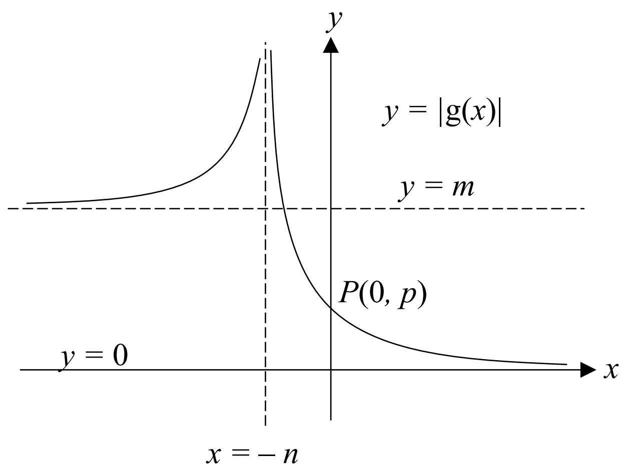
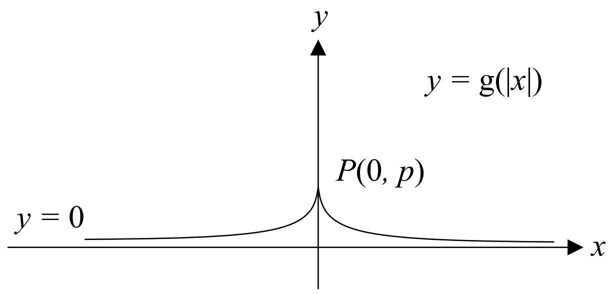
$h=\frac{l}{2}$ or $h=\frac{3l}{2}\left(\text{reject} \because l>h\right)$

$$\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=8\pi\left(l-h\right)$$

When $h=\frac{l}{2}$, $\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=4\pi l>0$, ie minimum volume when $r=\frac{l}{2}$.

Q3	Suggested Answers
(a)	**Method 1** $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}\Rightarrow\left\|\overrightarrow{AB}+\overrightarrow{BC}\right\|^2=\left\|\overrightarrow{AC}\right\|^2$ $\Rightarrow\left(\overrightarrow{AB}+\overrightarrow{BC}\right)\bullet\left(\overrightarrow{AB}+\overrightarrow{BC}\right)=\overrightarrow{AC}\bullet\overrightarrow{AC}$ ($\because \left\|\mathbf{v}\right\|^2=\mathbf{v}\bullet\mathbf{v}$) $\Rightarrow\overrightarrow{AB}\bullet\overrightarrow{AB}+2\left(\overrightarrow{AB}\bullet\overrightarrow{BC}\right)+\overrightarrow{BC}\bullet\overrightarrow{BC}=\overrightarrow{AC}\bullet\overrightarrow{AC}$ $\Rightarrow AB^2+BC^2+2\left(\overrightarrow{AB}\bullet\overrightarrow{BC}\right)=AC^2$ $\Rightarrow AC^2+2\left(\overrightarrow{AB}\bullet\overrightarrow{BC}\right)=AC^2$ since $AB^2+BC^2=AC^2$ $\Rightarrow\overrightarrow{AB}\bullet\overrightarrow{BC}=0$ So AB is perpendicular to BC and hence $\angle ABC=90°$.
	Method 2 $AB^2+BC^2=AC^2\Rightarrow\left\|\overrightarrow{AB}\right\|^2+\left\|\overrightarrow{BC}\right\|^2=\left\|\overrightarrow{AC}\right\|^2$ $\Rightarrow\left\|\overrightarrow{AB}\right\|^2+\left\|\overrightarrow{BC}\right\|^2=\left\|\overrightarrow{AB}+\overrightarrow{BC}\right\|^2$ $\Rightarrow\left\|\overrightarrow{AB}\right\|^2+\left\|\overrightarrow{BC}\right\|^2=\left(\overrightarrow{AB}+\overrightarrow{BC}\right)\bullet\left(\overrightarrow{AB}+\overrightarrow{BC}\right)$ $\Rightarrow\left\|\overrightarrow{AB}\right\|^2+\left\|\overrightarrow{BC}\right\|^2=\overrightarrow{AB}\bullet\overrightarrow{AB}+\overrightarrow{BC}\bullet\overrightarrow{BC}+2\left(\overrightarrow{AB}\bullet\overrightarrow{BC}\right)$ $\Rightarrow\left\|\overrightarrow{AB}\right\|^2+\left\|\overrightarrow{BC}\right\|^2=\left\|\overrightarrow{AB}\right\|^2+\left\|\overrightarrow{BC}\right\|^2+2\left(\overrightarrow{AB}\bullet\overrightarrow{BC}\right)$ $\Rightarrow\overrightarrow{AB}\bullet\overrightarrow{BC}=0$ So AB is perpendicular to BC and hence $\angle ABC=90°$.
(b)	**Method 1** By ratio theorem, $\mathbf{d}=\overrightarrow{OD}=(1-\lambda)\overrightarrow{OA}+\lambda\overrightarrow{OC}=(1-\lambda)\mathbf{a}+\lambda\mathbf{c}$. Area of triangle ABD  $=\frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{BD}\right\|$ $=\frac{1}{2}\left\|(\mathbf{b}-\mathbf{a})\times(\mathbf{d}-\mathbf{b})\right\|$ $=\frac{1}{2}\left\|(\mathbf{b}-\mathbf{a})\times((1-\lambda)\mathbf{a}+\lambda\mathbf{c}-\mathbf{b})\right\|$ $=\frac{1}{2}\left\|(1-\lambda)(\mathbf{b}\times\mathbf{a})+\lambda(\mathbf{b}\times\mathbf{c})-\lambda(\mathbf{a}\times\mathbf{c})+\mathbf{a}\times\mathbf{b}\right\|$ $=\frac{1}{2}\left\|(\lambda-1)(\mathbf{a}\times\mathbf{b})+\lambda(\mathbf{b}\times\mathbf{c})-\lambda(\mathbf{a}\times\mathbf{c})+\mathbf{a}\times\mathbf{b}\right\|$ since $\mathbf{b}\times\mathbf{a}=-\mathbf{a}\times\mathbf{b}$. $=\frac{1}{2}\left\|\lambda(\mathbf{a}\times\mathbf{b})+\lambda(\mathbf{b}\times\mathbf{c})-\lambda(\mathbf{a}\times\mathbf{c})\right\|$ $=\frac{\lambda}{2}\left\|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}\right\|$ since $0<\lambda<1$. So $k=\frac{\lambda}{2}$.

	Method 2 Since triangles ABD and ABC share the same height, Area of triangle ABD $=\dfrac{\lambda}{\lambda+(1-\lambda)}(\text{Area of triangle } ABC)$ $=\lambda\left(\dfrac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|\right)$ $=\dfrac{\lambda}{2}\left\|(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})\right\|$ $=\dfrac{\lambda}{2}\left\|\mathbf{b}\times\mathbf{c}-\mathbf{b}\times\mathbf{a}-\mathbf{a}\times\mathbf{c}\right\|$ $=\dfrac{\lambda}{2}\left\|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}\right\|$ So $k=\dfrac{\lambda}{2}$.

Q4	Suggested Answers
(a)	
(b)(i)	
(b)(ii)	

Q5	Suggested Answers

(a)

$$\sum_{r=m+1}^{n}\left(3r^2-17r+17\right)$$

$$=3\sum_{r=m+1}^{n} r^2+\sum_{r=m+1}^{n}(-17r+17)$$

$$=3\left(\sum_{r=1}^{n} r^2-\sum_{r=1}^{m} r^2\right)+17\sum_{r=m+1}^{n}(-r+1)$$

$$=3\left[\frac{n}{6}(n+1)(2n+1)-\frac{m}{6}(m+1)(2m+1)\right]$$

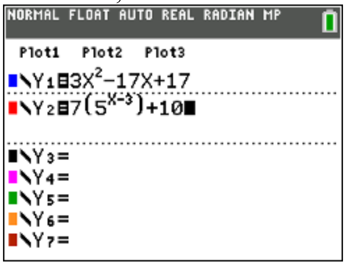
$$+17\left[\frac{n-m}{2}\left(-m+(-n+1)\right)\right]$$

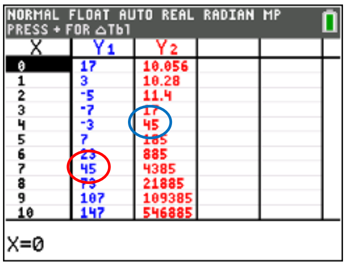
$$=\frac{n}{2}(n+1)(2n+1)-\frac{m}{2}(m+1)(2m+1)$$

$$+17\left[\frac{n-m}{2}(1-m-n)\right]$$

(b)

From GC, the term 45 can be found in both series.





(c)

The nth term of J,

$$j_n=\frac{b}{2}(3^n-1)-\frac{b}{2}(3^{n-1}-1)$$

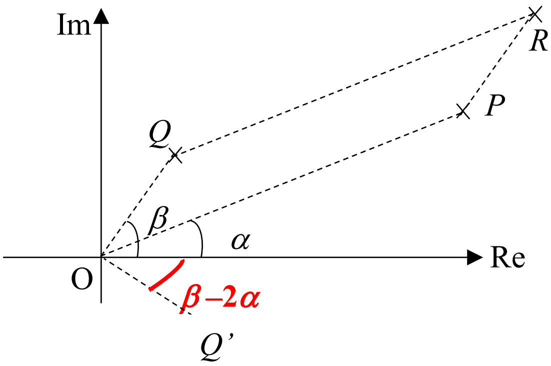
$$=\frac{b}{2}(3^n-3^{n-1})$$

$$=\frac{b}{2}(3^{n-1})(3-1)$$

$$=b3^{n-1}$$

Given b is a positive odd integer, then j_n is a positive odd integer since 3^{n-1} is odd.

Since the arithmetic progression is 1, 3, 5, …, the set of **all** positive odd integers, each of j_n must be a part of the arithmetic progression.

Q6	**Suggested Answers**
(a)	If $\lvert p\rvert=\lvert q\rvert$, $OPRQ$ forms a rhombus. $\arg(r)=\frac{1}{2}(\beta-\alpha)+\alpha=\frac{1}{2}(\alpha+\beta)$
(b)	 $\angle POQ'=\beta-\alpha$
(b)(i)	**Method 1:** $\arg(q')=-(\beta-2\alpha)$ since $\beta>2\alpha$ **Method 2:** $\arg(q')=\alpha-(\beta-\alpha)=2\alpha-\beta$
(b)(ii)	$\lvert q'\rvert=\lvert q\rvert$ Let F be the foot of perpendicular from Q' to the real axis. Real part of $q'=\lvert q\rvert\cos(-\beta+2\alpha)$ (Adjacent side of $\Delta OQ'F$) Imaginary part of $q'=[\lvert q\rvert\sin(-\beta+2\alpha)]$ (Opposite side of $\Delta OQ'F$ but negative) $\therefore q'=\lvert q\rvert\cos(-\beta+2\alpha)+\mathrm{i}\lvert q\rvert\sin(-\beta+2\alpha)$ Alternative: $q'=\lvert q\rvert\cos(\beta-2\alpha)-\mathrm{i}\lvert q\rvert\sin(\beta-2\alpha)$

Q7	Suggested Answers
(a)	$x=\sin^2\theta$ $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=2\sin\theta\cos\theta$ When $x=0, \theta=0$. When $x=\dfrac{1}{2}, \theta=\dfrac{\pi}{4}$ $\displaystyle\int_0^{\frac{1}{2}}\sqrt{\frac{16x}{1-x}}\ \mathrm{d}x=\int_0^{\frac{\pi}{4}}\sqrt{\frac{16\sin^2\theta}{1-\sin^2\theta}}\left(2\sin\theta\cos\theta\right)\ \mathrm{d}\theta$ $\displaystyle=\int_0^{\frac{\pi}{4}}\sqrt{\frac{16\sin^2\theta}{\cos^2\theta}}\left(2\sin\theta\cos\theta\right)\ \mathrm{d}\theta$ $\displaystyle=\int_0^{\frac{\pi}{4}}\frac{4\sin\theta}{\cos\theta}\left(2\sin\theta\cos\theta\right)\ \mathrm{d}\theta \quad \left(\because 0\le\theta\le\frac{\pi}{2}, \sqrt{\cos^2\theta}=\cos\theta\right)$ $\displaystyle=\int_0^{\frac{\pi}{4}}8\sin^2\theta\ \mathrm{d}\theta$ $\displaystyle=4\int_0^{\frac{\pi}{4}}1-\cos 2\theta\ \mathrm{d}\theta$ $=\left[4\theta-2\sin 2\theta\right]_0^{\frac{\pi}{4}}$ $=\pi-2$
(b)	$\displaystyle\int_a^b\frac{\lvert x-1\rvert}{\sqrt{\frac{1}{2}x^2-x+1}}\ \mathrm{d}x$ $\displaystyle=\int_a^1\frac{-(x-1)}{\sqrt{\frac{1}{2}x^2-x+1}}\ \mathrm{d}x+\int_1^b\frac{(x-1)}{\sqrt{\frac{1}{2}x^2-x+1}}\ \mathrm{d}x$ $\displaystyle=-\left[\frac{\left(\frac{1}{2}x^2-x+1\right)^{\frac{1}{2}}}{\frac{1}{2}}\right]_a^1+\left[\frac{\left(\frac{1}{2}x^2-x+1\right)^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^b$ $\displaystyle=-\left[2\left(\frac{1}{2}x^2-x+1\right)^{\frac{1}{2}}\right]_a^1+\left[2\left(\frac{1}{2}x^2-x+1\right)^{\frac{1}{2}}\right]_1^b$ $\displaystyle=-\left[2\left(\frac{1}{2}\right)^{\frac{1}{2}}-2\left(\frac{1}{2}a^2-a+1\right)^{\frac{1}{2}}\right]+\left[2\left(\frac{1}{2}b^2-b+1\right)^{\frac{1}{2}}-2\left(\frac{1}{2}\right)^{\frac{1}{2}}\right]$ $\displaystyle=-2\sqrt{2}+2\left(\frac{1}{2}a^2-a+1\right)^{\frac{1}{2}}+2\left(\frac{1}{2}b^2-b+1\right)^{\frac{1}{2}}$

Q8	Suggested Answers
(a)	Note that the sequence is an arithmetic progression with common difference 1.5 $u_{50} = u_1 + 49(1.5)$ $99u_1 = u_1 + 73.5$ $u_1 = 0.75$
(b)	**Method 1** $\sum_{k=1}^{n} u_{2k} = u_2 + u_4 + \ldots + u_{2n}$ is a sum of n terms of an arithmetic sequence with first term $u_2 = 0.75 + 1.5 = 2.25$ and common difference $2(1.5) = 3$ $u_2 + u_4 + \ldots + u_{2n} = \frac{n}{2}\left(2(2.25) + (n-1)(3)\right)$ $= \frac{n}{2}\left(1.5 + 3n\right)$ From GC, for $\frac{n}{2}\left(1.5+3n\right) > 2025$, least $n = 37$
	Method 2 $\sum_{k=1}^{n} u_{2k} = \sum_{k=1}^{n}\left(2.25 + \left(2k-1\right)3\right) > 2025$ Using GC, least $n = 37$
(c)(i)	$\frac{u_k}{u_{25}} = \frac{u_5}{u_k}$ $u_k^2 = u_5 u_{25}$ $\left[0.75 + (k-1)(1.5)\right]^2 = \left[0.75 + (5-1)(1.5)\right]\left[0.75 + (25-1)(1.5)\right]$ $0.75 + (k-1)(1.5) = \sqrt{6.75 \times 36.75} = 15.75$ $k = 11$
(c)(ii)	**Method 1** Common ratio $= \frac{15.75}{36.75} = \frac{3}{7}$ $S_{13} = \frac{36.75\left(1 - (\frac{3}{7})^{13}\right)}{1 - \frac{3}{7}}$ $= 64.31144$ $= 64.311$
	Method 2

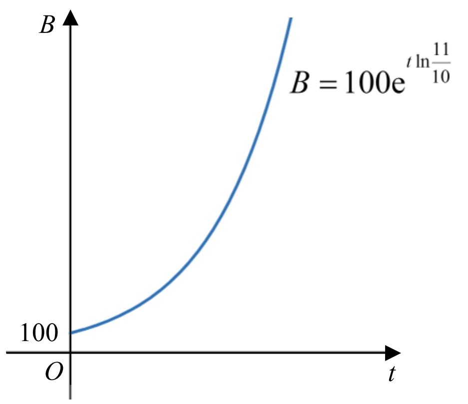
	Using GC, $\sum_{r=1}^{13}\left(36.75\left(\frac{3}{7}\right)^{r-1}\right)=64.311$

Q9	Suggested Answers
(a)	$k=\frac{3}{2}$ When $x=\frac{3}{2}$, the function will be undefined.
(b)	$\mathrm{D_f}=\mathbb{R}\setminus\left\{\frac{3}{2}\right\}$ $\mathrm{R_f}=\mathbb{R}\setminus\left\{\frac{1}{2}\right\}$ Since $\mathrm{R_f}\not\subset\mathrm{D_f}$, f^2 does not exist.
(c)	$y=\frac{x+1}{2x-3}$ $(2x-3)y=x+1$ $2xy-3y=x+1$ $2xy-x=3y+1$ $x=\frac{3y+1}{2y-1}$ $\mathrm{f}^{-1}(x)=\frac{3x+1}{2x-1}$
(d)	**<u>Method 1:</u>** $\mathrm{fh}(x)=\frac{3x-7}{2x-1}$ $\mathrm{f}^{-1}\mathrm{fh}(x)=\mathrm{f}^{-1}\left(\frac{3x-7}{2x-1}\right)$ $\mathrm{h}(x)=\mathrm{f}^{-1}\left(\frac{3x-7}{2x-1}\right)$ $=\frac{3\left(\frac{3x-7}{2x-1}\right)+1}{2\left(\frac{3x-7}{2x-1}\right)-1}$ $=\frac{9x-21+2x-1}{6x-14-2x+1}$ $=\frac{11x-22}{4x-13}$

	Method 2: $\mathrm{fh}(x)=\dfrac{3x-7}{2x-1}$ $\mathrm{fh}(x)=\dfrac{\mathrm{h}(x)+1}{2\mathrm{h}(x)-3}=\dfrac{3x-7}{2x-1}$ $(2x-1)(\mathrm{h}(x)+1)=(3x-7)(2\mathrm{h}(x)-3)$ $2x\mathrm{h}(x)+2x-\mathrm{h}(x)-1=6x\mathrm{h}(x)-9x-14\mathrm{h}(x)+21$ $-4x\mathrm{h}(x)+13\mathrm{h}(x)=-11x+22$ $\mathrm{h}(x)=\dfrac{11x-22}{4x-13}$

Q10	Suggested Answers
(a)	$\cos(t^3)=1-\frac{(t^3)^2}{2!}+\frac{(t^3)^4}{4!}+\cdots=1-\frac{t^6}{2}+\frac{t^{12}}{24}+\cdots$ $\int_0^x t^2\cos(t^3)\mathrm{d}t$ $\approx\int_0^x t^2(1-\frac{t^6}{2}+\frac{t^{12}}{24})\mathrm{d}t$ $=\int_0^x t^2-\frac{t^8}{2}+\frac{t^{14}}{24}\mathrm{d}t$ $=[\frac{t^3}{3}-\frac{t^9}{18}+\frac{t^{15}}{360}]_0^x$ $=\frac{x^3}{3}-\frac{x^9}{18}+\frac{x^{15}}{360}$
(b)	$\int_0^{0.1} t^2\cos(t^3)\,\mathrm{d}t\approx\frac{0.1^3}{3}-\frac{0.1^9}{18}+\frac{0.1^{15}}{360}\approx 0.00033\,(5\text{d.p.})$
(c)	$\int_0^x t^2\cos(t^3)\mathrm{d}t=\frac{1}{3}[\sin(t^3)]_0^x=\frac{1}{3}\sin(x^3)$ $\int_0^{0.1} t^2\cos(t^3)\,\mathrm{d}t=\frac{1}{3}\sin(0.1^3)=0.00033\ (5\text{ d.p.})$
(d)	The approximation was accurate up to 5 d.p. Approximation was good since $x=0.1$ is close to zero.

Q11	Suggested Answers
(a)	$x^2 - y^2 = 16 \Rightarrow y = \sqrt{x^2 - 16}\ (\because y \geq 0)$ Area required $= \int_5^8 \sqrt{x^2 - 16} - \left(-\frac{1}{2}x + \frac{11}{2}\right) \mathrm{d}x$ $= 8.4723$ $= 8.47 \text{units}^2$
(b)	Volume required $= \pi \int_4^5 x^2 - 16 \ \mathrm{d}x + \frac{1}{3}\pi(3)^2(6)$ $= \pi\left[\frac{x^3}{3} - 16x\right]_4^5 + 18\pi$ $= \pi\left[-\frac{115}{3} - \left(-\frac{128}{3}\right)\right] + 18\pi$ $= \frac{67}{3}\pi \text{units}^3$
(c)	Volume required $= \pi(5)^2(3) - \pi\int_0^3 y^2 + 16 \ \mathrm{d}y$ $= 75\pi - \pi\left[\frac{y^3}{3} + 16y\right]_0^3$ $= 75\pi - 57\pi$ $= 18\pi \text{ units}^3$

Q12	Suggested Answers
(a)	$\dfrac{dB}{dt}=kB$, where $k\in\mathbb{R}$.
(b)	$\dfrac{1}{B}\dfrac{dB}{dt}=k$ Integrating both sides w.r.t. t, $\int\dfrac{1}{B}\,dB=\int k\,dt$ $\ln B=kt+C$, where C is an arbitrary constant (note that $B\geq 0$ since it refers to mass) $B=e^{kt+C}=De^{kt}$ Given $B=100$ when $t=0$: $100=D$ Given $B=110$ when $t=1$: $110=100e^{k}$ $k=\ln\frac{11}{10}$ $\therefore B=100e^{t\ln\frac{11}{10}}$ or $100\left(\dfrac{11}{10}\right)^{t}$
(c)	 The biomass would grow indefinitely if it was left on its own.
(d)	$\dfrac{dB}{dt}=mB\dfrac{1}{3+2t^2}$, where $m\in\mathbb{R}$. $\dfrac{1}{B}\dfrac{dB}{dt}=\dfrac{m}{3+2t^2}$ Integrating both sides w.r.t.t: $\int\dfrac{1}{B}\,dB=\int\dfrac{m}{3+2t^2}\,dt$ $=\dfrac{m}{2}\int\dfrac{1}{\frac{3}{2}+t^2}\,dt$ $\ln B=\dfrac{m}{2}\sqrt{\dfrac{2}{3}}\tan^{-1}\left(\sqrt{\dfrac{2}{3}}t\right)+C$

	$B = \mathrm{e}^{\frac{m}{\sqrt{6}}\tan^{-1}\left(\sqrt{\frac{2}{3}}t\right)+C}$ $= D\mathrm{e}^{\frac{m}{\sqrt{6}}\tan^{-1}\left(\sqrt{\frac{2}{3}}t\right)}$ Given $B=100$ when $t=0$: $100 = D$ Given $B=101$ when $t=1$: $101 = 100\mathrm{e}^{\frac{m}{\sqrt{6}}\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)}$ $m = \left(\ln\frac{101}{100}\right)\frac{\sqrt{6}}{\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)} = 0.03559595$ $B = 100\mathrm{e}^{\frac{0.03559595}{\sqrt{6}}\tan^{-1}\left(\sqrt{\frac{2}{3}}t\right)}$ $= 100\mathrm{e}^{0.014532\tan^{-1}\left(\sqrt{\frac{2}{3}}t\right)}$ $= 100\mathrm{e}^{0.0145\tan^{-1}\left(\sqrt{\frac{2}{3}}t\right)}$
(e)	As $t \to \infty$, $\tan^{-1}\left(\sqrt{\frac{2}{3}}t\right) \to \frac{\pi}{2}$ $\therefore B \to 100\mathrm{e}^{0.014532\left(\frac{\pi}{2}\right)} = 102.30893$ grams