JURONG PIONEER JUNIOR COLLEGE
9478 H2 PHYSICS

PROJECTILE MOTION TUTORIAL SOLUTIONS

Self-Check Questions

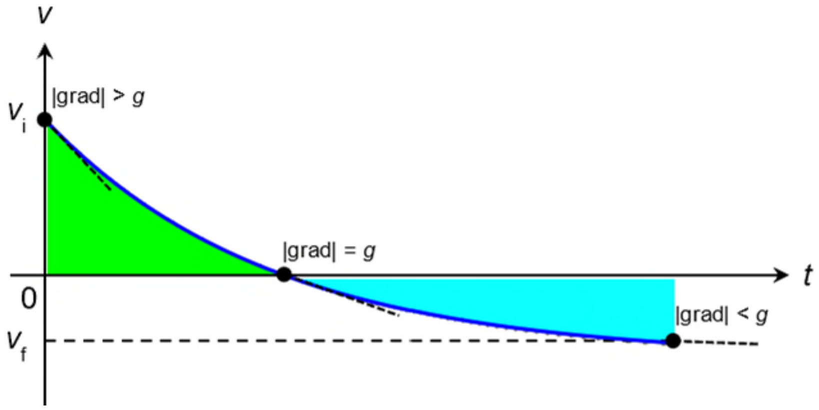
S1 The horizontal component of velocity.

S2 The vertical component of velocity is zero, the acceleration = g = 9.81 ms^{-1} downwards.

S3 All the kinematics quantities are independent of the mass.

S4 When the resultant force on the object is zero. For a falling object it means that air resistance force pointing upwards is equal to the weight pointing downward.

S5 Decreasing gradient magnitude, longer time to fall, lower speed when returned.



Self-Practice Questions

1 Horizontal velocity and acceleration are constant. Vertical velocity is zero at highest point [Ans: **D**]

2 Option A has no effect. Options B will result in smaller range and smaller maximum height. Option D will result in a larger range and larger maximum height.

$$Range\ R = (u\cos\theta)t$$

$$t = 2(u\sin\theta)/g,\ \text{using } s_y = (u\sin\theta)t - \frac{1}{2}gt^2 = 0$$

$$R = 2u^2\cos\theta\sin\theta/g = u^2\sin(2\theta)/g$$

R is maximum when θ=45. Other angles reduces R.

Max height $H = (u\sin\theta)^2/(2g)$. H increases with θ

[Ans: **C**]

3 The time of flight will increase because the downward acceleration is lower. The range will be lower because the horizontal speed decreases continuously. [Ans: **A**]

4 The resultant force Y reduces to zero when the air resistance X increases to its maximum value, which is equal to the weight of the object. [Ans: **A**]

5 The vertical initial velocity and acceleration are the same for X and Y. Horizontal velocity does not affect the time taken to travel the same vertical height. [Ans: **C**]

Discussion Questions

1 (a) Take upward to be positive.

$$u_y = 15\sin(40°) = 9.64 \text{ m s}^{-1}$$

$$s_y = u_y t - \frac{1}{2}gt^2$$

$$-45 = 9.64t - 4.905t^2$$

$$0 = -45 - 9.64t + 4.905t^2$$

solving:

$$t = \frac{9.64 \pm \sqrt{9.64^2 + 4(45)(4.905)}}{2(4.905)} = 4.17 \text{ s, ignore '-' solution}$$

(b)

$$s_x = 15[\cos(40°)]t$$

$$= 15[\cos(40°)](4.17)$$

$$= 47.9 \text{ m}$$

(c)

$$v_y = 15\sin(40°) - 9.81(4.17)$$

$$= -31.27 \text{ m s}^{-1}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{[15\cos(40°)]^2 + 31.27^2}$$

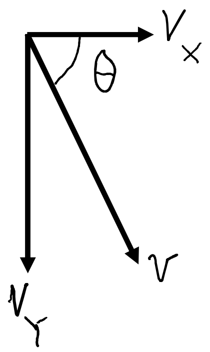
$$= 33.3 \text{ m s}^{-1}$$

$$\tan\theta = \frac{v_y}{v_x}$$

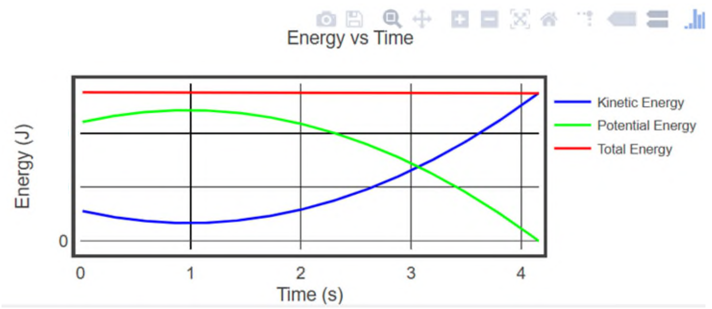
$$= \frac{31.27}{15\cos(40°)}$$

$$= 2.72$$

$$\theta = 69.8^o \text{ below horizontal.}$$



(d)



KE decreases until minimum and increases from minimum value until it hits the ground.
The variation of GPE is opposite to that of KE. It increases until it reaches the maximum value at the highest point. It decreases thereafter.
KE + GPE = constant.

2 **(a)** vertical component:
Take upward to be positive

$$s_y = 3.0 - 2.3 = 0.7 \text{ m}$$

$$v_y = 6.5\sin(50°) = 4.98 \text{ m s}^{-1}$$

$$s = ut - \frac{1}{2}gt^2$$

$$0.7 = 4.98t - 4.905t^2$$

$$4.905t^2 - 4.98t + 0.7 = 0$$

$$t = \frac{4.98 \pm \sqrt{4.98^2 - 4(4.905)(0.7)}}{2(4.905)} = 0.169 \text{ s or } 0.847 \text{ s}$$

taking the longer time, t = 0.847 s
(The shorter time refers to the time when the ball is rising at 3.0 m)

(b) Maximum height is reached when the vertical velocity is zero

$$v_y{}^2 = u_y{}^2 - 2a_y s_y$$

$$0 = [6.5\sin(50)]^2 - 2(9.81)s_y$$

$$s_y = 1.26 \text{ m}$$

$$\text{height} = 1.26 + 2.3 = 3.56 \text{ m}$$

(c) Ball is moving with horizontal velocity at the top, v_x=6.5cos(50°)

$$KE = \frac{1}{2}mv_x^2$$

$$= \frac{1}{2}(0.600)(6.5\cos(50°))^2$$

$$= 5.24 \text{ J}$$

(d) Use the horizontal component to calculate the horizontal distance

$$s_x = 6.5[\cos(50)]t, \qquad t = 0.847 \text{ s from } (a)$$

$$s_x = 3.54 \text{ m}$$

(e) Calculate the horizontal and vertical velocities separately, then combine them to calculate the final velocity.

$$v_x = 6.5\cos(50°) = 4.18 \text{ m s}^{-1}$$

$$v_y = 6.5\sin(50°) - 9.81(0.847) = -3.33 \text{ m s}^{-1}$$

(-ve sign means directed downward)

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{4.18^2 + 3.33^2}$$

$$= 5.34 \text{ m s}^{-1}$$

angle θ given by $\tan\theta = \dfrac{v_y}{v_x} = \dfrac{3.33}{4.18} = 0.7966$

$\theta = 38.5^o$ below the horizontal.

3(a) Assume that there is no air resistance and the kick is at an optimal angle of 45^0.

time of flight:

$$0 = 30(\sin 45°)t - \frac{1}{2}(9.81)t^2$$

$$t = \frac{2(30\sin 45°)}{9.81}$$

$$= 4.32 \text{ s}$$

Horizontal distance = (30 cos45)4.32

= 91.6 m < 95 m.

The initial velocity is not high enough.

3(b) Assume optimal angle of launch is 45° above horizontal.

time of flight:

$$0 = 9.5(\sin 45°)t - \frac{1}{2}(9.81)t^2$$

$$t = \frac{2(9.5\sin 45°)}{9.81}$$

$$= 1.37 \text{ s}$$

Horizontal distance = (9.5 cos45°)1.37

= 9.2 m

The world record is within the theoretical limit.

3(c) Let the initial velocity be u and time of flight = *t*. Take upward to be positive.

horizontal: $24.77 = u\cos(38°)t$ ----- (1)

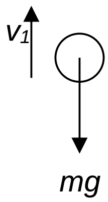
vertical: $-2.10 = u\sin(38°)t - \frac{1}{2}(9.81)t^2$ ----- (2)

Substitute t from (1) into (2)

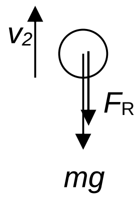
$$-2.10 = 24.77\ \tan(38°) - \frac{9.81}{2}\left(\frac{24.77}{u\cos 38°}\right)^2$$

Solving for u, u = 15.0 m s^{-1}

4(a) Air resistance always opposes motion of ball.



Ball moving upward under free fall

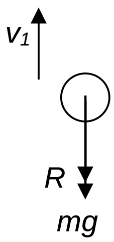


Ball moving upward in the presence of air resistance

(i) With air resistance, there are now two forces decelerating the ball. The ball hence experiences greater deceleration. As the deceleration is greater, the time taken for velocity to decrease to zero, at maximum height, is shorter.

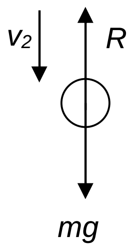
(ii) Since both Earth's gravitational pull and air resistance are pointing downwards, this causes the object's velocity to decrease at a faster rate. The maximum height reached by the object is hence lower than the case without air resistance.

4(b)



Ball moving upward in the presence of air resistance R

Net deceleration
$a_u = g + a_r$

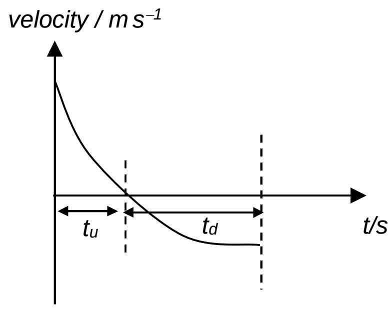


Ball moving downward in the presence of air resistance R

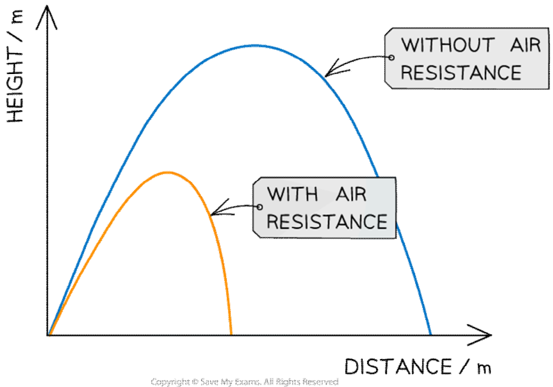
Net acceleration
$a_d = g - a_r$

*a_r has a non constant magnitude. $a_r = \dfrac{R}{m}$

For both cases, the distance travelled by the ball is the same, which means the area of the *v-t* graph is the same for upward and downward motion. Since $a_d < a_u$, the decrease in speed on the rise is more than the increase in speed on the fall. Therefore, t_d is greater than t_u since the two areas are the same.



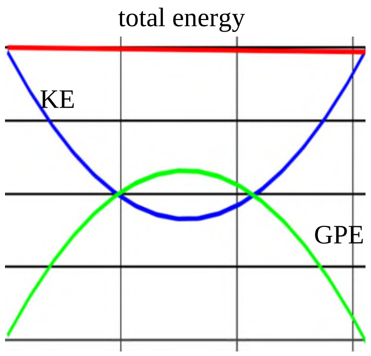
4(c)



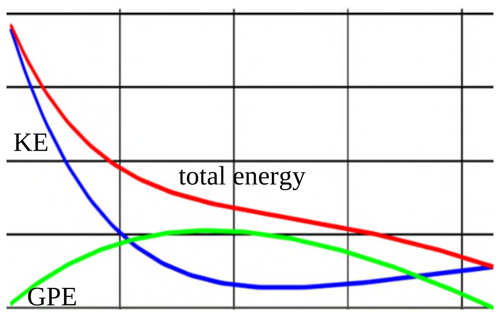
Key differences: Lower maximum height, shorter range, non-symmetrical path about the highest point.

4(d) Without air resistance

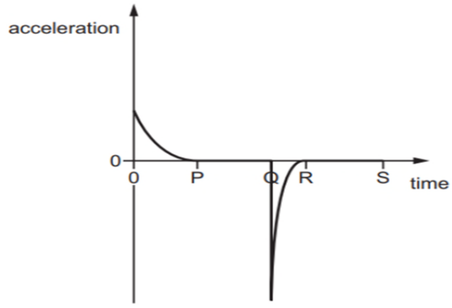
KE decreases to a minimum as GPE increases to a maximum. The sum of KE and GPE is a constant.



With air resistance, the total sum of KE and GPE decreases continuously. The decrease in KE is more than the increase in GPE on the rise, and the decrease in GPE is more than the increase in KE on the fall.



5(a)

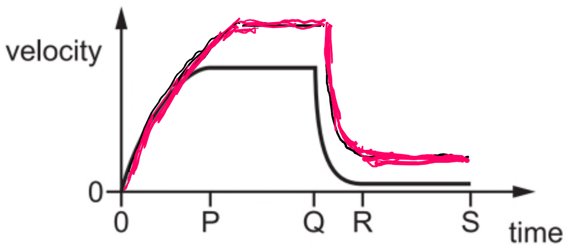


(downward is positive direction)
The gradient of *v*-*t* graph gives *a*-*t* graph

5(b) From 0 to P, the acceleration decreases from *g* because the velocity and hence the air resistance directed upward increases. The net force (*mg* – air resistance) and hence the acceleration decreases, causing the velocity to increase at a slower rate, until it reaches the maximum terminal velocity at P. At this point the air resistance = *mg*, the acceleration is zero. From P to Q, it maintains the terminal velocity.

At Q the air resistance increases greatly due to the opening of the parachute. The net force is now pointing upward, creating an upward acceleration, hence reducing the downward velocity. The velocity will continue to decrease until the air resistance is again equal to the weight at point R. From R to S, the parachutist moves with a lower terminal velocity.

5(c)



Higher terminal velocities due to higher weight.

6(a) As the object moves downward, it encounters air resistance which is directed upward and increases with the speed of the object. At high enough speed, the air resistance will be equal to the weight of the object. The air resistance cancels out the weight, the resultant force and hence the acceleration becomes zero. The velocity reaches a maximum constant value from that instance.

(b) The net force is $mg - F$. Applying Newton's 2nd law

$$mg - F = ma$$

$$mg - kv = ma$$

$$m(g-a) = kv$$

$$g - a = kv/m$$

(c)

velocity v/m s^{-1}	acceleration a/m s^{-2}	$(g - a)$/m s^{-2}
0	9.8	0.0
20	8.2	1.6
30	5.7	4.1
40	0.0	9.8

At $v = 0$, $a = g =$ 9.81 m s^{-2}, $g - a = 0$

At $v = 40$ m s^{-1}, graph shows the object is already moving with terminal velocity, so $a = 0$
$g - a = 9.8$ m s^{-2}

At v= 30 m s^{-1}, tangent to graph at 30 m s^{-1} gives a. $a = \dfrac{41.0 - 12.0}{5.50 - 0.40} = 5.7 \text{ m s}^{-2}$

$g - a = 9.8 - 5.7 = 4.1$ m s^{-2}

(d)

From the equation

$$g-a=\frac{kv}{m}$$

$$\frac{g-a}{v}=\frac{k}{m}$$

RHS is a constant ratio, so the LHS must also be constant if the relatioship is true.

for v= 20 m s^{-1}, $\frac{g-a}{v}=\frac{1.6}{20}=0.08$

for v = 30 m s^{-1}, $\frac{g-a}{v}=\frac{4.1}{30}=0.137$

The 2 ratios on the LHS differ by a big percentage, the suggestion $F = kv$ is incorrect.

(Try $F = kv^2$, the ratio $\frac{g-a}{v^2}$ becomes 0.0040 and 0.0046, much better assumption)

7 (a) The gradient of the graph, which represents acceleration of the ball, varies with speed. Hence, the resultant force on the ball varies. Since weight is constant, the air resistance experienced by the ball must be changing.

(b) When the velocity of the ball is zero, it is at the highest point of its motion. Air resistance is zero at this instant, only weight acts on the ball at this point hence it experiences acceleration of free fall. The magnitude can be obtained from the gradient of the tangent of the graph at v=0.

(c) (i) Maximum height, H = Area under graph

H may be determined (approximately) by dividing the area into 3 sections:

(1) t = 0 to 0.5 s, (2) t = 0.5 s to 1.0 s, (3) t = 1.0 s to 1.75 s

$$H=\frac{1}{2}(0.5)(15.25+25)+\frac{1}{2}(0.5)(8.25+15.25)+\frac{1}{2}(0.75)(8.25)$$

$$= 19 \text{ m}$$

(ii) $$\frac{\text{energy lost}}{\text{initial KE}}=\frac{\frac{1}{2}m(25)^2-m(9.81)(19)}{\frac{1}{2}m(25)^2}=0.40$$

(d) (i) $a=$ gradient of tangent

$$=-\frac{21.5}{1.6}$$

$$=-13.4 \qquad \text{[refer graph in } \textbf{(e)} \text{]}$$

$$\approx -13 \text{ m s}^{-2}$$

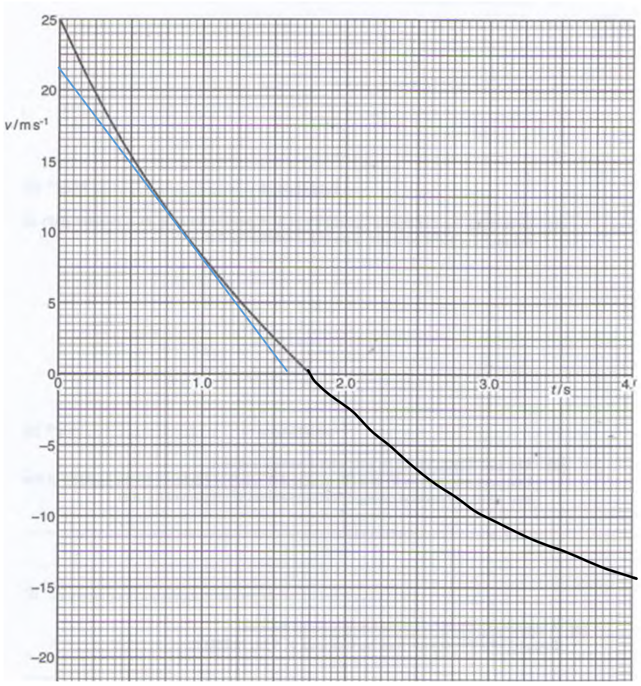
(ii) $mg + R = ma$ (a is positive here, directed downward)

$R = ma - mg = m(a - g)$

$= (0.350)(13 - 9.81)$

$= 1.1$ N

(e)



Points to note:
- The gradient must continue to decrease
- When the ball is back at ground level, the area under the graph in the two sections must be equal.
- The final speed is lower than the initial speed, the time to rise is shorter than the time to fall.

8 (a) Let its initial velocity be u, projected at an angle of θ above the horizontal.

Take rightward as positive,
$u_x = u \cos \theta$
$s_x = u_x t = (u \cos \theta)t$
$40 = (u \cos \theta)(3.0)$
$u \cos \theta = 13.333$ - - - (1)

Take upward as positive,
$u_y = u \sin \theta$
$s_y = u_y t + \frac{1}{2} a_y t^2 = (u \sin \theta)t + \frac{1}{2}(-9.81)t^2$
$-10 = (u \sin \theta)(3.0) + \frac{1}{2}(-9.81)(3.0)^2$
$u \sin \theta = 11.382$ - - - (2)

$\frac{(2)}{(1)}: \quad \frac{u \sin\theta}{u \cos\theta} = \frac{11.382}{13.333}$ → $\tan \theta = 0.854$ → $\theta = 40.5°$

(b) Launch speed, $u = \sqrt{u_x^2 + u_y^2} = \sqrt{(13.333)^2 + (11.382)^2} = 17.5 \text{ m s}^{-1}$

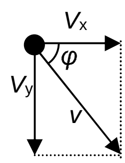
(c) Let its final velocity be v, at an angle of φ from the horizontal.

Horizontally, $v_x = u_x = 13.333 \text{ m s}^{-1}$

Vertically, $v_y = u_y + at = 11.382 + (-9.81)(3.0) = -18.05 \text{ m s}^{-1}$

$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(13.333)^2 + (18.05)^2} = 22.4 \text{ m s}^{-1}$,

$\tan\varphi = \frac{v_y}{v_x} = \frac{18.05}{13.333}$ → $\varphi = 53.5°$ (below the horizontal)



9 (a) (i) Taking upward as positive,
$u_y = 25\sin 45° = 17.7 \text{ m s}^{-1}$

(ii) ${v_y}^2 = {u_y}^2 + 2as_y$

At maximum height, $v_y = 0$.

$0 = (25\sin 45°)^2 + 2(-9.81)s_y$ → $s_y = 15.9 \approx 16$ m (shown)

(iii) 1. At height 16 m (maximum height), velocity is horizontal,
i.e. $v = u_x = u \cos 45°$

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\left(u\cos 45°\right)^2 = \frac{1}{2}mu^2\left(\cos 45°\right)^2$$
$$= K\ (0.5) = 0.5K$$

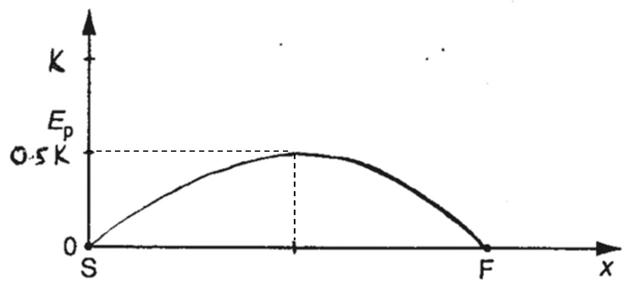
gain in *G.P.E.* = loss in *K.E.*
$G.P.E._{\text{at 16m}} - 0 = K - 0.5K$
$G.P.E._{\text{at 16m}} = 0.5K$

2. At height 8.0 m, $G.P.E._{\text{at 8m}} = \frac{1}{2}G.P.E._{\text{at 16m}}$ (since change in *G.P.E.* = *mgh*)

$$= \frac{1}{2}(0.5K) = 0.25K$$

loss in *K.E.* = gain in *G.P.E.*
$K - K.E._{\text{at 8.0m}} = 0.25K - 0 \rightarrow K.E._{\text{at 8.0m}} = 0.75K$

(b) (i)



(ii)