RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Additional Practice Questions for Chapter 4: Functions

1 The function g is defined by

$$\mathrm{g}: x \mapsto 1-\frac{1}{1-x}, \text{ where } x \in \mathbb{R}, x \neq a.$$

(i) State the value of a and explain why this value has to be excluded from the domain of g. **[2]**

(ii) Find $\mathrm{g}^2(x)$ and $\mathrm{g}^{-1}(x)$, giving your answers in simplified form. **[4]**

(iii) Find the values of b such that $\mathrm{g}^2(b)=\mathrm{g}^{-1}(b)$. **[2]**

2 The function f is defined by

$$\mathrm{f}: x \mapsto \ln(8-x)^3, \quad x<8.$$

(a) Solve the inequality $\mathrm{f}(x) \leq x$. **[3]**

(b) Find, in a similar form, the inverse function f^{-1}. **[3]**

3 **(a)** The function h is defined by $\mathrm{h}: x \rightarrow \frac{1}{2}x^2+3$, for $x \in \mathbb{R}$.

The function g is defined by $\mathrm{g}: x \rightarrow \frac{x+1}{5x-1}$, for $x \in \mathbb{R}, x \neq 0.2$.

(i) Find $\mathrm{gh}(2)$. **[2]**

(ii) Find the value of x for which $\mathrm{g}(x)=1.4$. **[1]**

(b) The function f is defined by $\mathrm{f}: x \rightarrow \frac{x+a}{2x+b}$, for $x \in \mathbb{R}, x \neq k$.

(i) Give an expression for k and explain why this value of x has to be excluded from the domain of f. **[2]**

The function f is such that $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$ for all x in the domain of f.

(ii) Determine the possible values of a and of b. **[3]**

(iii) Find an expression for $\mathrm{f}^{-1}(-4)$. **[1]**

4 The function f is defined by $\mathrm{f}: x \mapsto |2x-1|+4, \quad x \in \mathbb{R}$.

(i) Show that the composite function ff exists and give the corresponding definition of ff. State the range of ff.

(ii) Solve the equation $\mathrm{f}(x)=100$.

(iii) The function g is defined by $\mathrm{g}: x \mapsto |2x-1|+4, \quad x<k$.

Find the greatest value of k for g^{-1} to exist. Using this value of k, find g^{-1}.

5 Functions g and h are defined by

$$\mathrm{g}: x \mapsto x^2 + mx + 2, \quad x \in \mathbb{R} \text{ where } m \text{ is a constant,}$$

$$\mathrm{h}: x \mapsto \ln(x-1) + 2, \quad x > 1.$$

(i) Sketch the graph of $y = \mathrm{h}(x)$, stating the exact coordinates of the point of intersection with the axis and the equation of the asymptote. **[2]**

(ii) Find $\mathrm{h}^{-1}(x)$ and write down the domain of h^{-1}. **[3]**

(iii) On the same diagram as in part **(i)**, sketch the graph of $y = \mathrm{h}^{-1}(x)$.

Find the solution of the equation $\mathrm{h}(x) = \mathrm{h}^{-1}(x)$. **[3]**

State the condition for the composite function hg to exist. Find the maximum range of values of m such that hg exists. **[3]**

6 The functions f , g and h are defined as follows:

$$\mathrm{f}: x \mapsto x^2 + 4x + 1, \quad x \geq k$$

$$\mathrm{g}: x \mapsto 1 - \frac{1}{x}, \qquad x \in \mathbb{R} \setminus \{0\}$$

$$\mathrm{h}: x \mapsto \mathrm{e}^{2x}, \qquad x \in \mathbb{R}$$

(i) State the smallest value of k such that f^{-1} exists and express f^{-1} in a similar form. **[3]**

(ii) Express each of the following functions in terms of f, g and h as appropriate.

(a) $x \mapsto \ln\sqrt{x}$, $x > 0$ **(b)** $x \mapsto 1 - \frac{1}{\mathrm{e}^{2x}}$, $x \in \mathbb{R}$ **(c)** $x \mapsto x$, $x \geq -3$ **[4]**

7 Functions f and g are defined by

$$\mathrm{f}: x \mapsto x^2 - 2x + a \quad \text{for } x \in \mathbb{R}, \text{ where } a \text{ is an unknown constant}$$

$$\mathrm{g}: x \mapsto \tan x \qquad \text{for } x \in \mathbb{R}, \ -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

(i) Find the range of f in terms of a. **[2]**

(ii) Show that the composite function fg^{-1} exists. Find the range of fg^{-1} in terms of a. **[3]**

(iii) It is given that $a = -3$. The domain of f is now restricted to $\{x \in \mathbb{R} : x \geq 1\}$.

For the new domain, give a definition (including the domain) of f^{-1}. **[3]**

8 The functions f and g are defined by

$$\mathrm{f}: x \mapsto 2x^3 + 1, \qquad x > 0,$$

$$\mathrm{g}: x \mapsto \ln(x - a), \quad x > a.$$

(i) Give a reason why f^{-1} exists. Hence find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. **[3]**

(ii) Only one of the composite functions fg and gf exists. State the greatest possible value of a such that this composite function exists and using this value of a, give a definition of this composite function. Write down the range of the composite function. Explain why the other composite function does not exist. **[5]**

(iii) By sketching suitable graphs or otherwise, find the set of values of x such that $\mathrm{f}^{-1}\mathrm{f}(x) = \mathrm{ff}^{-1}(x)$. **[2]**

9 The functions f and g are defined as follows:

$$\mathrm{f}: x \mapsto \frac{5-x}{1-x}, \qquad x \in \mathbb{R}, x \neq 1,$$
$$\mathrm{g}: x \mapsto 2x^2+4x+\lambda, \quad x \in \mathbb{R}, x > -2.$$

It is given that range of f is $(-\infty,1) \cup (1,\infty)$.

(i) Show that $\mathrm{f}^{-1} = \mathrm{f}$.

(ii) Evaluate $\mathrm{f}^{51}(4)$.

(iii) Find the range of values of λ such that fg exists.
For these values of λ, find the range of fg in terms of λ.

10 The function f is defined by $\mathrm{f}: x \mapsto (ax+b)^2$, $x \in \mathbb{R}$, $x \geq -\dfrac{b}{a}$ where a and b are positive constants.

(i) Show that f^{-1} exists. **[2]**

(ii) State the geometrical relationship between the graphs of f and f^{-1}.
Given that the graphs of f and f^{-1} intersect, show that $a \leq \dfrac{1}{4b}$. **[4]**

(iii) Show that the composite function ff is defined. **[2]**

(iv) Solve the equation $\mathrm{ff}(2) = b^2$. **[3]**

11 The functions f and g are defined by

$$\mathrm{f}: x \mapsto 50-(2x-4)^2, \quad -k \leq x \leq k,$$
$$\mathrm{g}: x \mapsto \frac{3x-3}{x^2-8x+18}, \qquad x \in \mathbb{R},$$

where k is a positive real number.

(a) Without using a calculator, solve the inequality $\mathrm{g}(x) > 2$. **[3]**

(b) It is given that the range of f is $[-50, 50]$.

(i) Find the value of k. **[2]**

(ii) Without the use of a calculator, solve the inequality $\mathrm{gf}(x) > 0$. **[4]**

12 The functions f and fg^{-1} are defined as follows:

$$\mathrm{f}: x \mapsto x+5, \quad x \in \mathbb{R},$$
$$\mathrm{fg}^{-1}: x \mapsto x^2, \quad x \in \mathbb{R}, \quad x > \sqrt{5}$$

(i) Find the functions g and fg.

(ii) The function h is defined such that its inverse exists and the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{h}^{-1}(3x+1)$ are symmetrical about the line $y = x$. Find $\mathrm{h}(x)$.

13 Functions f and g are defined by

$$\mathrm{f} : x \mapsto (4+2x)^{\frac{1}{2}}, \quad x \in \mathbb{R},\ 0 \le x \le 16$$

$$\mathrm{g} : x \mapsto 3x+1, \qquad x \in \mathbb{R}$$

(i) State the range of f. **[1]**

(ii) With the aid of a diagram, show that f^{-1} exists and define f^{-1} in a similar form. **[4]**

(iii) On the same diagram as in part **(ii)**, sketch the graphs of f^{-1} and $\mathrm{f}^{-1}\mathrm{f}$, indicating their endpoints. **[3]**

(iv) Explain why the x-coordinates of the point(s) of intersection between the graphs in part **(iii)** satisfies the equation $x^2-2x-4=0$. **[1]**

(v) State whether the composite function fg exists, justifying your answer. **[2]**

(vi) **Not in syllabus (For enrichment)**
Find the largest possible domain of g in the form [m, n], m, n $\in \mathbb{R}$, for which the composite function fg exists.

14 The function f and g are defined by

$$\mathrm{f}(x) = \mathrm{e}^{2x}-4, \quad x \in \mathbb{R},$$

$$\mathrm{g}(x) = x+2, \quad x \in \mathbb{R}.$$

(i) Find $\mathrm{f}^{-1}(x)$ and state its domain **[3]**

(ii) Find the exact solution of $\mathrm{fg}(x)=5$, giving your answer in its simplest form. **[3]**