Topic: Trigonometry

Question 1 [2008 EOY, Q5]	marks
Solve for $0^\circ \le x < 360^\circ$ when $2\cos(2x - 40^\circ) + 1 = 0$.	[5]

Question 2 [2008 EOY, Q8]	marks
Prove the identity $\dfrac{\sin x}{1 - \cos x} + \dfrac{1 - \cos x}{\sin x} \equiv 2\operatorname{cosec} x$.	[3]

Question 3 [2008 EOY, Q16]	marks
Given that $\tan x = -3$ and $\sin x$ is positive, without using the calculator, find the exact values of (i) $\cos x + \sin x$,	[2]
(ii) $\sec x - \operatorname{cosec} x$.	[2]

Question 4 [2009 EOY, Q3]	marks
Prove the identity $\dfrac{2\cot\theta + \operatorname{cosec}^2\theta}{(\sin\theta + \cos\theta)^2} \equiv \operatorname{cosec}^2\theta$.	[4]

Question 5 [2009 EOY, Q5]	marks
(a) Given that $\cos A = -\frac{2}{3}$ and $180^\circ < A < 360^\circ$. Without solving for angle *A*, find the value of (i) $\cot A$,	[2]
(ii) $\operatorname{cosec}(180^\circ + A)$.	[2]
(b) Solve $\tan(2x - 65^o) = -1$ for $0^\circ \le x \le 360^\circ$.	[4]

Question 6 [2009 EOY, Q12]	marks
(i) Show that ($2\sin x - 1$) is a factor of the expression $6\sin^3 x + \sin^2 x - 4\sin x + 1$.	[2]
(ii) Factorise the expression $6\sin^3 x + \sin^2 x - 4\sin x + 1$ completely.	[3]
(iii) Hence find all values of *x* between 0° and 360° inclusive for which $6\sin^3 x + \sin^2 x - 4\sin x + 1 = 0$.	[4]

Question 7 [2009 EOY, Q13]	marks
(i) Sketch, on the same diagram, the graphs of $y = 2\sin x + 1$ and $y = 3\cos 2x$ for the domain $0^\circ \leq x \leq 360^\circ$.	
(ii) Hence, deduce the smallest possible positive value of *m* for which the equation $2\sin x + 1 = 3\cos 2x + m$ has 3 solutions.	[1]

Question 8 [2010 EOY, Q10]	marks
(i) Solve, for $0° \le x \le 180°$, the equation $2\cos 2x = 3\cos 2x \sin^2 2x$.	[4]
(ii) Sketch the graph of $y = 2\|\tan x\| - 1$ for $0° \le x \le 360°$.	[3]

Question 9 [2010 EOY, Q11]	marks
Prove the identity $\dfrac{1-\operatorname{cosec} x}{\tan x-\sec x} \equiv \cot x$.	[3]

Question 10 [2010 EOY, Q16]	marks
Simplify $\dfrac{\sin(-135^\circ)}{\sec 240^\circ}+\dfrac{\tan 200^\circ \tan 300^\circ}{\cot(-290^\circ)}$, leaving your answer in exact form.	[3]

Question 11 [2011 EOY, Q2]	marks
Sketch the graph of $y = 3\sin 2x - 2$ for $0^\circ < x < 360^\circ$,	[3]

Question 12 [2011 EOY, Q9]	marks
Solve the following equations for angles between 0° and 360°. (i) $\cos x = 4\sin^2 x \cos x$,	[5]

(ii) $\tan\left(2y - 85^{\circ}\right) = -5.$	[4]
Question 13 [2011 EOY, Q12]	marks
(i) Given that $\dfrac{2\cos^2 x}{1+3\sin^2 x} = \dfrac{8}{13}$, where $90^{\circ} < x < 180^{\circ}$. Without using a calculator, find the value of $\dfrac{2\cos x}{1+3\sin x}$.	[4]

(ii) Prove the identity $\dfrac{\cot\theta}{1+\operatorname{cosec}\theta}-\dfrac{\cot\theta}{1-\operatorname{cosec}\theta}\equiv 2\sec\theta$ . .	[3]
Question 14 [2012 EOY, Q6]	marks
The function f is defined by $\mathrm{f}: x \mapsto 1-2\sin\left(\dfrac{x}{3}\right)$. (i) State the amplitude and the period of f.	[2]

(ii) Sketch the graph of $y = \mathrm{f}(x)$ for $0 \le x \le 1080^\circ$.	[4]
Question 15 [2012 EOY, Q7]	marks
(i) Solve the equation $\tan\left(\frac{x}{2} - 50^\circ\right) + 1 = 0$ for angles between 0° and 360°.	[4]

(ii) Given that $\dfrac{\cos x}{\tan x - 1} + \dfrac{\sin x}{\cot x - 1} = k(\sin x + \cos x)$ for all values of x, find the value of k.	[3]
Question 16 [2013 EOY, Q2]	marks
Angles A and B are in different quadrants such that $\tan A = \dfrac{3}{4}$ and $\sin B = -\dfrac{4}{5}$. Find, without using calculator, the possible values of $(2\sin A - \tan B)$ in the exact form.	[6]

Question 17 [2013 EOY, Q7]	marks
Sketch the graph $y = 1 - 4\cos 2x$ for $0° \le x \le 360°$.	[3]
Question 18 [2013 EOY, Q8]	marks
Solve the equations for $0° \le x \le 180°$. (i) $\sin 3x = \cos 2x$,	 [3]

(ii) $\tan(2x - 40^{\circ}) = -2.718$.	[3]
Question 19 [2013 EOY, Q9]	marks
Prove the identities. (i) $\sin^4 x + \sin^2 x \cos^2 x + \cos^2 x \equiv 1$,	[4]

(ii)

$$\frac{2 - \operatorname{cosec}^2 x}{\operatorname{cosec}^2 x + 2\cot x} \equiv \frac{\sin x - \cos x}{\sin x + \cos x}.$$

[4]

Question 20 [2014 EOY, Q9]	marks
(i) Solve the trigonometric equation $6-4\cos^2\left(\frac{x}{2}+30^\circ\right)=9\sin\left(\frac{x}{2}+30^\circ\right)$ for $0^\circ \le x \le 720^\circ$.	[6]

(ii) Prove that $2\sin^2 x - \dfrac{\tan x - \cot x}{\tan x + \cot x} = 1$.	[3]
Question 21 [2014 EOY, Q12]	marks
The function f is given by $\mathrm{f} : x \mapsto 2 - 3\cos 2x$ for the domain $0^\circ \le x \le 180^\circ$. Sketch the graph $y = \mathrm{f}(x)$. Hence, state the range of the function for its given domain.	[4]

Question 22 [2014 EOY, Q15]	marks
Given that θ is an angle that lies in the 4th quadrant, simplify the expression $\dfrac{1}{\cos\theta\sqrt{1+\tan^2\theta}}+\dfrac{2\cot\theta}{\sqrt{\dfrac{1}{\sin^2\theta}-1}}$.	[3]

Question 23 [2015 EOY, Q13]	marks
Given that $\sec x+\tan^2 x=0$, show that $\cos x=-\dfrac{2}{1+\sqrt{5}}$.	[4]

(i) Prove that $\dfrac{\operatorname{cosec} x - \sec x}{\operatorname{cosec} x + 1} + \dfrac{\operatorname{cosec} x + \sec x}{\operatorname{cosec} x - 1} = 2\sec^2 x + 2\sec x \tan^2 x$.	[3]
(ii) Hence, solve for $0° < x < 360°$ the equation $\dfrac{\operatorname{cosec} x - \sec x}{1 + \operatorname{cosec} x} = \dfrac{\operatorname{cosec} x + \sec x}{1 - \operatorname{cosec} x}$.	[4]

Question 24 [2015 EOY, Q14]	marks
Sketch the graph of $y = \tan\frac{x}{2}$ for $0° \le x \le 360°$, showing clearly any asymptotes and intercepts with the axes. On the same axes, sketch the graph of $y = 2\lvert\sin x\rvert$. Hence state the number of solutions between $0°$ and $360°$ inclusive when $\tan\frac{x}{2} = 2\lvert\sin x\rvert$.	[4]