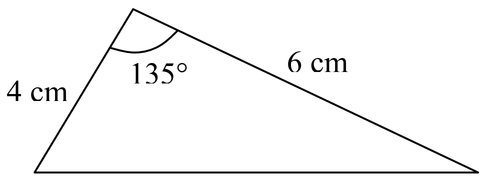
1(a) If $\tan A = -\frac{7}{24}$, such that $90° < A < 180°$, find, without using a calculator, the value of

(i) $\sin A$, [2]

(ii) $\cos A$, [1]

(iii) $\tan (A - 90°)$. [1]

(b) Find the area of the following triangle. Express your answer in the form $a\sqrt{b}$, where a and b are positive integers. [2]



2 On separate axes, sketch the graph of

(i) $y^2 + x = 0$, [2]

(ii) $xy = 1$, for $x < 0$, [2]

(iii) $y = kx^3$, where k is a negative constant. [2]

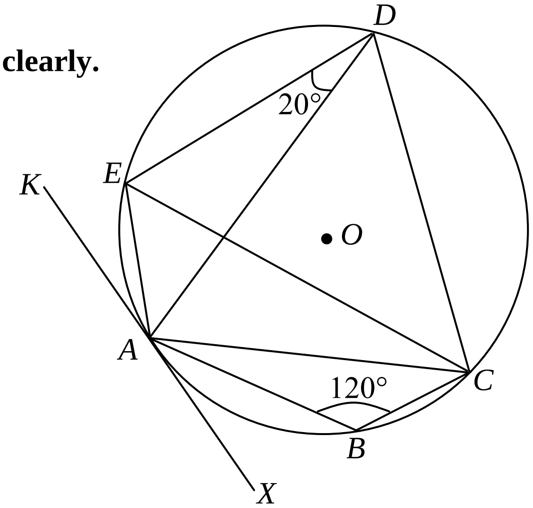
3 A, B, C, D and E are points on the circumference of a circle, centre O. KAX is a tangent to the circle. Given that $\angle ADE = 20°$ and $\angle ABC = 120°$, calculate

(i) $\angle ACE$, [1]

(ii) $\angle CAX$, [2]

(iii) $\angle OEC$. [4]

State all the reasons clearly.

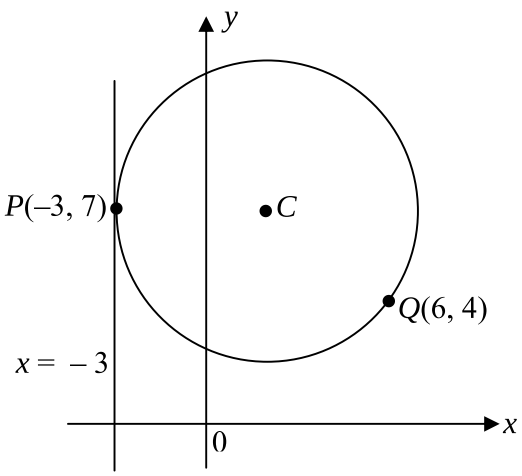


[Turn Over

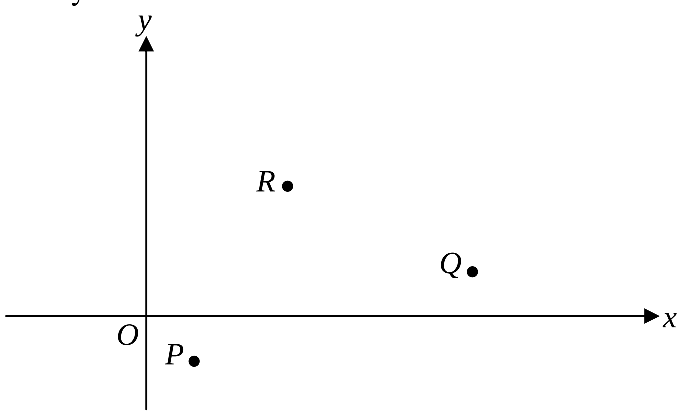
4 In the diagram, the circle, with centre C, passes through the points $P(–3, 7)$ and $Q(6 , 4)$. The vertical line $x = –3$ is a tangent to the circle at P.

(i) Explain why the y-coordinate of C is 7. [2]

(ii) Find the equation of the circle. [4]



5 In the diagram, the coordinates of P, R and Q are $(1, –1)$, $(3, 3)$ and $(7, 1)$ respectively.



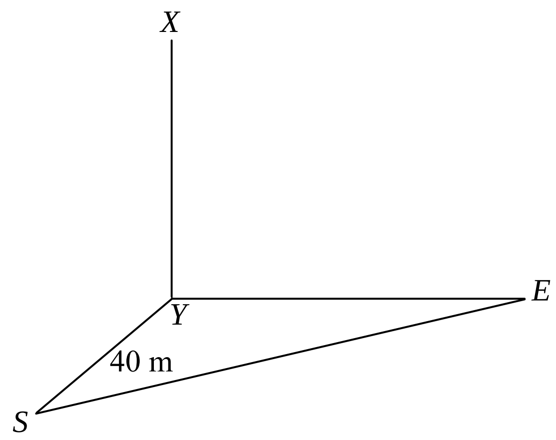
(a) T is a point on the line segment PR such that $PT : PR$ is $3 : 5$. Find

(i) the coordinates of T, [2]

(ii) the area of triangle PQR. [2]

(b) Find the coordinates of S if $PQRS$ is a parallelogram. [2]

6 Three points S, Y and E, are on level ground. Y is the foot of a building XY. S is 40 m due south of Y and E is due east of Y.

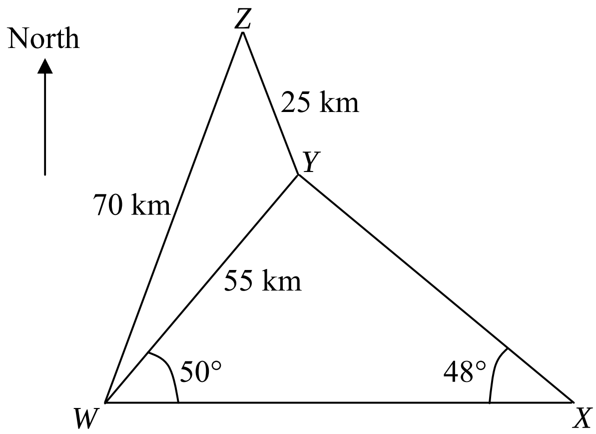


(a) The angle of depression of S from X is 55°. Calculate the height of the building. [2]

(b) The bearing of E from S is 064°. Calculate the distance ES. [2]

(c) A man walks from E to S and stops at a point from which the angle of elevation of X is the greatest. Find the size of this angle. [4]

7 W, X, Y and Z are four points on level ground with W due west of X. It is given that $WY = 55$ km, $ZY = 25$ km, $WZ = 70$ km, $\angle YWX = 50°$ and $\angle YXW = 48°$.



Find

(i) the length of WX, [3]

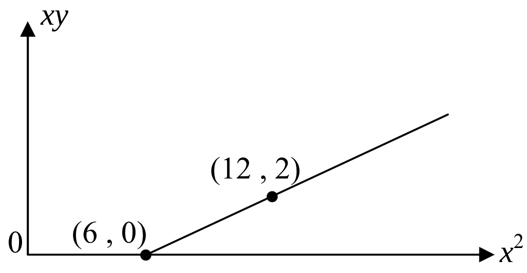
(ii) $\angle WYZ$, [2]

(iii) the bearing of W from Y, [2]

(iv) the bearing of Z from W, [3]

(v) how far W is west of Y. [2]

8 The diagram below shows part of a straight line graph drawn to represent the equation $x + \frac{r}{x} = ys$, where r and s are constants.



(a) Calculate the value of

(i) s, [3]

(ii) r. [2]

(b) A point (9, k) lies on the straight line. Find the value of k. [2]

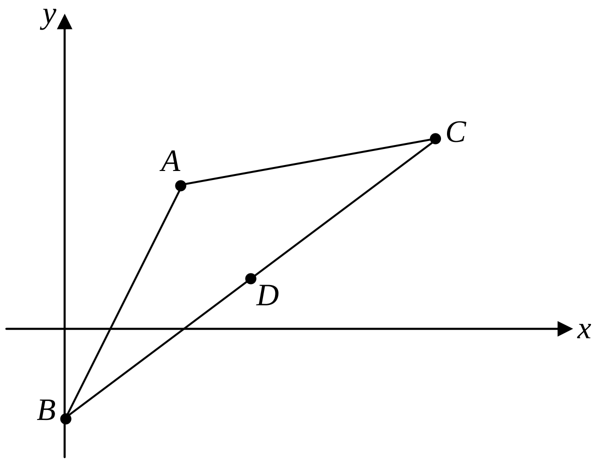
(c) Find the values of x when $y = 5$, giving your answers to 2 decimal places. [2]

(d) Find the gradient and the vertical intercept of the straight line that can be drawn on the diagram to solve the simultaneous equations

$x + \frac{r}{x} = ys$, $x = \frac{20}{2x + y}$. [2]

9 **The solution of this question by accurate drawing will not be accepted.**

In the diagram, the coordinates of A and B are $(5\ ,\ 6)$ and $(0\ ,\ –4)$ respectively.
AD is the perpendicular bisector of BC.



(i) Explain why $AB = AC$. [1]

(ii) Given that the gradient of BC is $\frac{3}{4}$, find the equation of BC. [1]

(iii) Show that the equation of AD is $3y + 4x = 38$. [2]

(iv) Hence, find the coordinates of D. [3]

(v) Find the length of AD. [2]

10 **Answer the whole of this question on the INSERT provided.**

The graph of $y = 8 - x - \frac{4}{x}$ is partially shown on the INSERT.

(a) By drawing a tangent, estimate the gradient of the graph at the point where $x = 1$. [3]

(b) Find the coordinates of the point on the curve at which the gradient is 0. [1]

(c) By drawing suitable straight lines on the graph, find

(i) the range of values of x for which $1 \le y \le 3$, [2]

(ii) the solutions of $2x^2 - 11x + 4 = 0$. [3]

Bonus Question

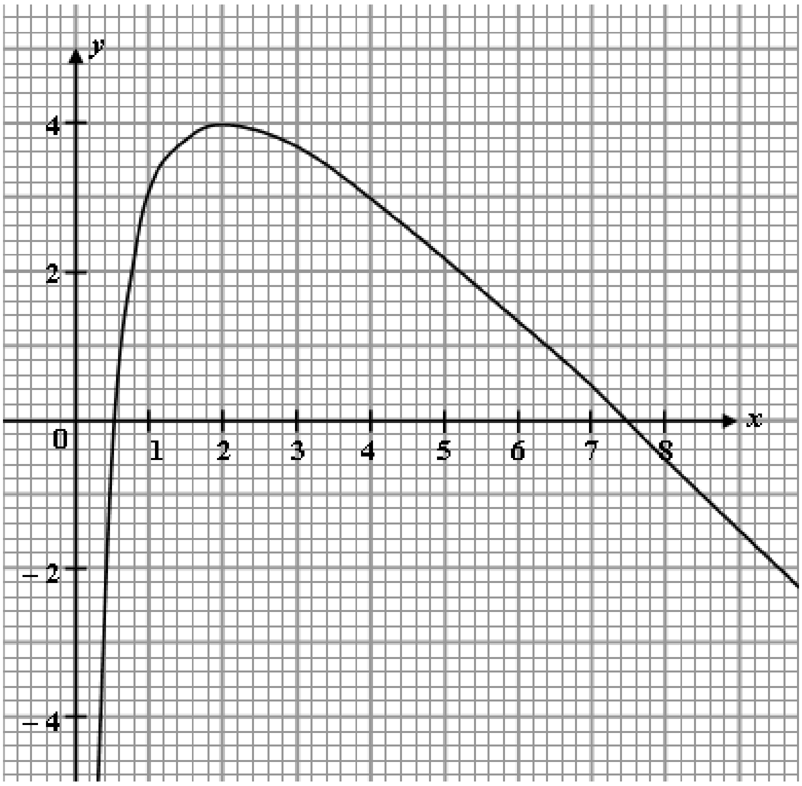
11 In triangle *ABC*, the lengths of the sides are 3 consecutive positive integers and the largest angle is twice the smallest angle. Find the length of the longest side. *Hint: sin* $2x = 2$ *sin x cos x.* [3]

End of Paper

INSERT

<u>Question 10</u>

Tie this page behind the last sheet of your answer paper.



(a)

Ans: ________________________

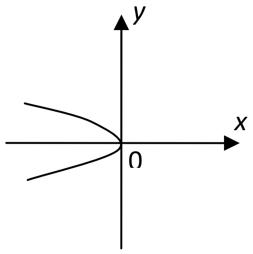
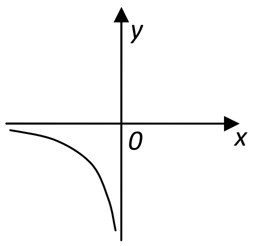
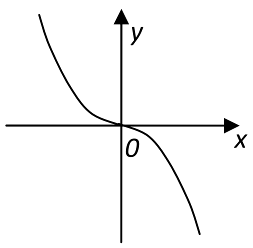
(b) Ans: ________________________

(c)(i)

Ans: ________________________

(c)(ii)

Ans: ________________________

Qn	Solution	
1ai	$7^2 + 24^2 = 625 = 25^2$ $\sin A = \dfrac{7}{25}$	
1aii	$\cos A = -\dfrac{24}{25}$	
1aiii	$\tan(A - 90°) = \dfrac{24}{7}$	
1b	Area $= 0.5(4)(6)\sin 135°$ $= 12 \sin 45°$ $= 12 \times \dfrac{1}{\sqrt{2}}$ $= 6\sqrt{2}\ \text{cm}^2$	
2a	$y^2 = -x$ 	
2b	$y = \dfrac{1}{x}, x < 0$ 	
2c	$y = kx^3$, k is a negative constant 	

Qn	Solution	
3i	$\angle ACE = 20°$ *(∠s in the same segmt)*	
3ii	$\angle ADC = 180° - 120°$ (*∠s in opp segmt*) $= 60°$ $\angle CAX = 60°$ (*alt. segmt theorem*)	
3iii	$\angle EOC = (20° + 60°) \times 2$ $= 160°$ *(∠ at centre = 2∠ at circumference)* $\angle OEC = \dfrac{180° - 160°}{2}$ (*∠ sum of Δ, base ∠s of isos.Δ*) $= 10°$	
4	(i) The radius, *PC*, is perpendicular to the tangent $x = -3$, (or <u>radius perpendicular to tangent</u>) Since $x = -3$ is a vertical line, <u>*PC* is horizontal or parallel to the *x*-axis</u>, so *C* has the same *y*-coord as *P*. (ii) $(6 - a)^2 + (4 - 7)^2 = (a + 3)^2$ $36 - 12a + a^2 + 9 = a^2 + 6a + 9$ $36 = 18a$ $a = 2$ $r = 2 + 3 = 5$ or $r^2 = 25$ Eqn: $(x - 2)^2 + (y - 7)^2 = 25$ or $x^2 + y^2 - 4x - 14y + 28 = 0$	
5ai	$T(x,y) = \left(\dfrac{2(1)+3(3)}{5}, \dfrac{3(3)+2(-1)}{5}\right)$ $= \left(\dfrac{11}{5}, \dfrac{7}{5}\right)$ or (2.2 , 1.4)	
5aii	Area $= \dfrac{1}{2}\begin{vmatrix} 1 & 7 & 3 & 1 \\ -1 & 1 & 3 & -1 \end{vmatrix}$ $=$ 10 square units	
5b	$S(-3, 1)$	

Qn	Solution	
6a	Height of building = 40 tan 55° = 57.1 m (3sf)	
6b	$ES = \dfrac{40}{\cos 64°} = 91.2$ m (3sf)	
6c	Let d be the shortest distance from Y to the line segment ES. $\dfrac{d}{40} = \sin 64°$ $d = 40 \sin 64°$ or 35.95 (4sf) Let $x°$ be the greatest angle of elevation. $\tan x = \dfrac{40 \tan 55°}{40 \sin 64°}$ *or* $\dfrac{57.13}{35.95}$ $x \approx 57.8°$ (1 dp)	
7i	$\angle WYX = 180° - 50° - 48° = 82°$ $\dfrac{WX}{\sin 82°} = \dfrac{55}{\sin 48°}$ $WX = 73.3$ km (3sf)	
7ii	$\cos \angle WYZ = \dfrac{25^2 + 55^2 - 70^2}{2(25)(55)}$ $\angle WYZ = 117.0°$ (1dp)	
7iii	180° + 40° = 220° Bearing of W from Y = 220°	
7iv	$\dfrac{70}{\sin 117.0357°} = \dfrac{25}{\sin \angle ZWY}$ $\angle ZWY = 18.549°$ 90° – 18.549° – 50° = 21.451° Bearing of Z from W = 021.5°	
7v	Let K be the point such that it is due south of Y and due east of W. $\cos 50° = \dfrac{WK}{55}$ $WK \approx 35.4\ km$	

Qn	Solution	
8ai	Gradient = $\dfrac{2-0}{12-6}$ $= \dfrac{1}{3}$ $x + \dfrac{r}{x} = ys$ $x^2 + r = xys$ $xy = \left(\dfrac{1}{s}\right)x^2 + \dfrac{r}{s}$ $\dfrac{1}{s} = \dfrac{1}{3}$ ➔ $s = 3$	
8aii	Sub (6,0) and $m = \dfrac{1}{3}$ into $xy = m(x^2) + c$ to find the value of c. $c = -2$ $\dfrac{r}{s} = -2$ ➔ $r = -2(3) = -6$	
8b	$k = \dfrac{1}{3}(9) - 2$ $= 1$ **OR** Solve for $k = 1$ from $\dfrac{1}{3} = \dfrac{k-0}{9-6}$	
8c	$5x = \left(\dfrac{1}{3}\right)x^2 - 2$ $x^2 - 15x - 6 = 0$ $x = \dfrac{-(-15) \pm \sqrt{(-15)^2 - 4(1)(-6)}}{2(1)}$ $x \approx 15.39 \;\; or \;\; -0.39$	
8d	$x = \dfrac{20}{2x+y}$ $2x^2 + xy = 20$ $xy = -2x^2 + 20$ Gradient = –2, vertical intercept = 20.	

Qn	Solution	
9ai	All the points on the perpendicular bisector of *BC* are equidistant to *B* & *C*, so *AB* = *AC*. OR Prove Triangle *ABD* is congruent to Triangle *ACD*.	
9aii	$y = \frac{3}{4}x - 4$	
9aiii	Gradient of *AD* = $-1 \div \frac{3}{4}$ $= -\frac{4}{3}$ Sub *A*(5,6) into $y = -\frac{4}{3}x + c$ to get the value of $c = \frac{38}{3}$. Eqn of *AD*: $y = -\frac{4}{3}x + \frac{38}{3}$. Therefore the eqn of *AD* is $3y + 4x = 38$ (shown).	
9aiv	$y = -\frac{4}{3}x + \frac{38}{3}$ ---- (1) $y = \frac{3}{4}x - 4$ ---- (2) (1) = (2) : $\frac{3}{4}x - 4 = -\frac{4}{3}x + \frac{38}{3}$ $\frac{3}{4}x + \frac{4}{3}x = 4 + \frac{38}{3}$ $\frac{25}{12}x = \frac{50}{3}$ $x = 8, y = 2$ *D*(8,2)	
9av	$AD = \sqrt{(5-8)^2 + (6-2)^2}$ = 5 units	

Qn	Solution	
10a	Gradient $\approx$ 2.5 to 4	
10b	(2, 4)	
10ci	$0.6 \pm 0.1 \le x \le 1 \pm 0.1$, $4 \pm 0.1 \le x \le 6.4 \pm 0.1$	
10cii	Eqn of str line: $y = x - 3$ $x \approx 0.4 \pm 0.1$ or 5.1 ± 0.1	
BQ 11	Let the length of the shortest side by y units. Using sine rule, $\dfrac{y+2}{\sin 2x} = \dfrac{y}{\sin x}$ $\dfrac{y+2}{2\sin x \cos x} = \dfrac{y}{\sin x}$ $\dfrac{y+2}{2\cos x} = \dfrac{y}{1}$ ----- (1) Using cosine rule, $y^2 = (y+1)^2 + (y+2)^2 - 2(y+1)(y+2)\cos x$ after simplification, $\cos x = \dfrac{y+5}{2(y+2)}$ ----- (2) Sub (2) into (1) & simplify to get: $y^2 + 4y + 4 = y^2 + 5y$ $y = 4$ Length of longest side = 6 units	