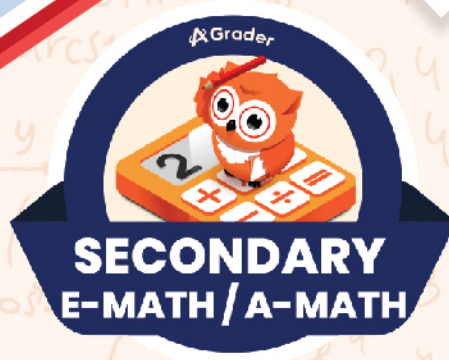




SEC 1-4 MATH REVISION NOTES

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Secondary 1

Topic 1: Prime, Factors and Multiples

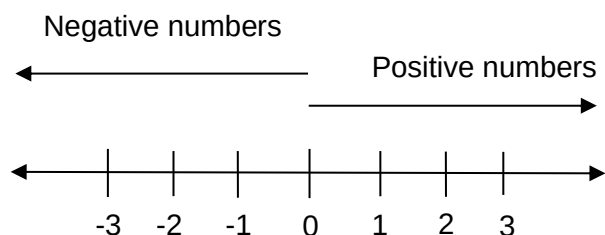
1. A **prime number** is a whole number (greater than 1) that has **exactly 2 different factors** which can be **divided by only 1 or itself**.
 - Examples of prime numbers: 2, 3, 5, 7, 11, 13, and 17.
2. A **composite number** is a whole number that has **more than 2 different factors**.
 - Examples of composite numbers: 4, 6, 8, 10, 12, 14 and 15.
3. **Prime factors** are **factors that are prime numbers**.
4. **Highest Common Factor (HCF)**: multiply the **lowest power of each common prime factors** of the given numbers.
5. **Lowest Common Multiple (LCM)**: multiply the **highest power of every prime factor** of the given numbers.
6. A **perfect square** is a number whose **square roots are whole numbers**.
 - Examples of perfect squares: 1,4,9,16,25,36 ...
 - The index of the prime factors is always a multiple of two.
7. A **perfect cube** is a number whose **cube roots are whole numbers**
 - Example of perfect cubes: 1,8,27,64,125,216 ...
 - The index of the prime factors is always a multiple of three.
8. A number P is a **multiple** of the number Q if $\frac{P}{Q} = \text{integer}$.
9. 0 and 1 are neither prime numbers nor composite numbers.



Topic 2: Real Numbers

1. Number line

- moving **towards left** → **decreasing** in value
- moving **towards right** → **increasing** in value



2. Inequality signs ($<$, $>$, $\leq$, $\geq$)

- “ $<$ ” means ‘is lesser than’
- “ $>$ ” means ‘is greater than’
- “ $\leq$ ” means ‘is lesser than or equal to’
- “ $\geq$ ” means ‘is greater than or equal to’

3. Four operations (Positive and Negative Numbers)

- $1 + 2 = 3$
- $1 - 2 = -1$
- $1 + (-2) = -1$
- $1 - (-2) = 3$
- $(-1) \times 2 = -2$
- $1 \times (-2) = -2$
- $(-1) \times (-2) = 2$
- $(-2) \div 1 = -2$
- $2 \div (-1) = -2$
- $(-2) \div (-1) = 2$

4. Order of Operations: BODMAS (Bracket, Order, Division and Multiplication (from left to right), Addition and Subtraction (from left to right)).

5. Rational Numbers: Numbers that can be expressed as a fraction

- **Integers (Whole numbers: 0 and Positive integers; Negative integers)**
- **Fractions**
- **Terminating decimals (0.52, -0.1635)**
- **Recurring decimals (0.333..., 0.6)**

6. Irrational Numbers

- $\sqrt{2}$, π , $\sqrt[3]{10}$

7. Zero is neither positive nor negative number.



Topic 3: Approximation and Estimation

1. Approximation and Estimation

- **Approximation** is the process of rounding off a **given** number to give an approximate value depending on the required degree of accuracy according to the following:
 - A given place value
 - A given number of decimal places
 - A given number of significant figures
- **Estimation** is the process of guessing the value of an **unknown** quantity in real life by giving an estimated value whereby the quantity is approximately a certain value.

2. To round off a number to a **specific place value** or **decimal place**:

Steps	Specified Place Value	Specified Decimal Place
1	Look at the digit that is just on the right of the specified place value.	Look at the digit that is just on the right of the specified decimal place.
2	If the digit is less than 5 , no changes to the digit at the specified place value. Replace all the digits on the right of the specified place value with zeros.	If the digit is less than 5 , drop/ remove this digit (as well as all the digits that is on the right of the specified decimal place) and no changes to the digit at the specified decimal place.
	If the digit is 5 or more , add 1 to the digit in the specified place value (regroup when necessary). Replace all the digits on the right of the specified place value with zeros.	If the digit is 5 or more , drop/ remove this digit (as well as all the digits that is on the right of the specified decimal place) and add 1 to the digit in the specified decimal place (regroup when necessary).
3	After rounding off a number to the required degree of accuracy, the degree of accuracy needs to be presented in brackets.	
	For example, 4690 $\approx$ 5000 (nearest 1000)	For example, 89.995 $\approx$ 90.00 (2 decimal places or 2d.p.)

When asked to round off to a specific decimal place, your answer should be in the same number of decimal places.

- #### 3. Significant figures
- are the number of digits used to denote an exact value to a specific degree of accuracy, where a number is more accurate when it is given to a greater number of significant figures.
- **5 Rules** to identify the significant digits:
 - **Rule 1:** All **non-zero** digit(s) are **significant**.
 - **Rule 2:** All **zero(s)** between **non-zero** digits are **significant**.
 - **Rule 3:** **Zero(s)** that comes before the **first non-zero** digit are **not significant**.
 - **Rule 4:** **Zero(s)** following a **non-zero** digit after the decimal point are **significant**.
 - **Rule 5:** **Zero(s)** following a **non-zero** digit in a whole number **may or may not be significant**, depending on how the number is rounded off.
- #### 4. A number should be rounded off from its original value instead from the rounded values.



5. **Rounding Off Error** is the difference between its true value and the rounded off figure.
- Calculators do not always give exact values due to the number of digits stored that can be displayed on the calculator screen as well as the capacity of the calculator.
 - **Rounding off non-exact values in intermediate working** may result in a **follow-through error**.
 - **Follow-through errors** can occur when **non-exact values** in **intermediate working** are not rounded off to a **sufficient degree of accuracy**.
 - To reduce **follow-through errors**, the **degree of accuracy** in the **intermediate steps** should be given at least **1 digit greater than the required degree of accuracy** in the final answer.
 - For example, if we want the final answer to be accurate to 3 decimal places, the intermediate steps should be given to at least 4 decimal places.
 - For accuracy purpose, we use values stored in the calculator to compute the subsequent steps of the working.



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Topic 4: Basic Algebra and Algebraic Manipulations

1. Basic Algebraic Notation and Terminology

- **Letters** (e.g. x, y, ab , etc) are used to represent:
 - **Specific unknown numbers** – to solve for unknown
 - **Generalised numbers** – to express a general law
 - **Variables** – to express a relationship between two or more quantities where the unknowns vary
 - **Algebraic notations**

Algebraic Notations	Meaning
ab	$a \times b$
$3a$	$3 \times a$ or $a + a + a$
$\frac{a}{b}$	$a \div b$ or $a \times \frac{1}{b}$
a^2	$a \times a$
a^3	$a \times a \times a$
$(2a)^3$	$2a \times 2a \times 2a = 8a^3$

- **Common mistakes** (to take note):

- ❖ $3x \neq 3 + x$ but $3x = 3 \times x = x + x + x$
- ❖ $x^2 \neq 2x$ but $x^2 = x \times x$

- **Algebraic Expression** involves numbers, variables, and operational signs (+, −, ×, ÷). It has **no equal sign** and is made up of different parts.

Algebraic Expression: $x^2 + 7xy - 3y + 4x - 8$		
<u>Terminology</u>	<u>Definition</u>	<u>Illustration</u>
Variable	A letter that represents a number and its value can be varied.	x and y
Term	+ and − signs separate terms from one another. It can be a number, a variable or the product or quotient of numbers and variables.	5 terms: x^2 , $7xy$, $-3y$, $4x$ and -8
Constant term	A term with fixed value that does not contain any variables.	-8
Coefficient	The number that is multiplied to the variable.	Coefficient of x^2 is 1. Coefficient of xy is 7. Coefficient of y is -3 . Coefficient of x is 4.
Degree	The highest index in the algebraic expression.	Highest index is 2 as observed in x^2 . Degree of the expression is 2.

- Evaluating algebraic expressions** is the process of finding the value of the expression by replacing/ substituting the variables in the expression with the given values.
- Linear expression** is an algebraic expression where the **degree** of the expression is **1**.
Quadratic expression is an algebraic expression where the **degree** of the expression is **2**.
 - When an expression contains a term that is made up of more than 1 variables, e.g. xy , that expression is a non-linear expression.



4. Addition and Subtraction of Linear Terms

- **Like terms** contains the same variable(s) with the same degree.
 - For example, $2x$ and $-3x$, $5xy$ and $9xy$, $3y^2$ and $-4y^2$.
- **Unlike terms** are variables that are not like terms.
 - For example, $2x$ and $3y$, x^2y and xy^2 .
- To add/ subtract **like terms**, we add/ subtract their coefficients and their variable remains the same.
 - For example, $2x + 4x = 6x$, $8y - 4y = 4y$.
- We **cannot** simplify the addition and subtraction of **unlike terms**.
 - For example, $2g - 4h - g + 7h = g + 3h$
- **Note:** $x + y = y + x$ (Cumulative Law)
 $x - y \neq y - x$

5. Multiplication and Division of Algebraic Expressions

- **Coefficients** and **Variables** of the respective terms are multiplied or divided on their own.
 - For example, $4ab^2 \times (-3ab) = -12a^2b^3$ and $\frac{4b^2}{2ab} = \frac{2b}{a}$
- There will be no "×" or "÷" sign in the result of multiplication and division of algebraic expressions.
- $xy = yx$ (Commutative Law)

6. Expansion of Algebraic Expressions

- Follows the rules and priorities in **order of operations**.
- When an expression in bracket is multiplied by a **constant number or a variable**, each term in the bracket must be **multiplied** by this **constant number or a variable** when the bracket is **removed**.

$$x(y + z) = xy + xz \text{ (Distributive Law)}$$

7. Factorisation of Algebraic Expressions is the process of expressing an algebraic expression as the **product** of two or more factors. It is the **reverse process** of **expansion**.

- To factorise an algebraic expression completely, factorise out (take out) the greatest common factor(s) to ensure that the factorisation is complete.

8. Linear Algebraic Expressions with Fractional Coefficients is an algebraic term with **fractional coefficient** or an algebraic fraction that has as **algebraic expression in its numerator** with an **integral denominator**. For example, $\frac{2a}{7}$ or $\frac{2}{7}a$.

- **Addition/ Subtraction** of algebraic fractions is done by using **equivalent fractions** to make **denominator** of the algebraic fractions the **same** (applying **LCM**) before proceeding with the addition or subtraction.



Topic 5: Linear Equations

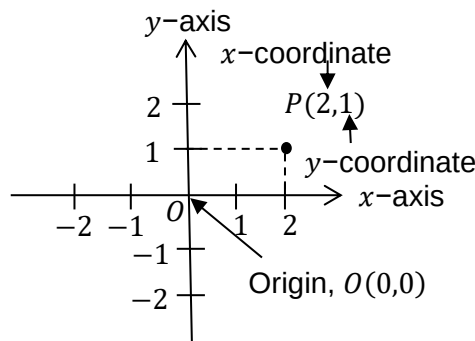
1. **Equation** is a statement in which two mathematical expressions are equal, where it contains an equal sign (=).
2. **Solving Linear Equations:** To find the value of the unknown variable in the equation. This value found is called the **solution** or **root** of the equation.
 - As the expression on the Left-Hand Side (LHS) of the equation is equal to the Right-Hand Side (RHS) of the equation, the equation must be **balanced/ equivalent**.
 - Always **maintain the balance** when **manipulating** the equation.
 - **Isolate** the unknown on one side of the equation to solve for the unknown variable.
 - **Steps to solve linear equations**
 - **Step 1:** Simplifying the given equation where possible
 - **Step 2:** Group the unknown variable(s) to one side of the equation by adding, subtracting, multiplying, or dividing **both sides** of the equation with the **same quantity**.
 - Check your solution by substituting the value of the unknown that you have found into the original equation to see if it satisfies the equation where $LHS=RHS$.
3. **Solving Fractional Equations** with fractional coefficients and fractional equations.
 - **Fractional equation** is an equation that contains one or more **fractional expressions**.
 - **Convert** simple fractional equations into linear equations with integer coefficients by multiplying.
 - **Cross Multiplication**
 - **Multiply by Lowest Common Multiple**
4. **Problem-Solving using Algebra**
 - **Step 1:** Let the unknown variable that is required to be found be x .
 - **Step 2:** Form an equation using the information given in the question using the unknown variable x .
 - **Step 3:** Solve the equation by finding the value(s) of the unknown variable x to answer the question.
5. **Formula** is an equation that expresses the **relationship** between **certain quantities or variables** where it expresses a rule in algebraic terms.



Topic 6: Linear Functions and Graphs

1. Cartesian Coordinate System consists of:

- Vertical-axis, **y-axis**
- Horizontal-axis, **x-axis**
- Origin, **O**. It is the point of intersection of x -axis and y -axis with the coordinates $(0,0)$.
- Coordinates of a point, P , is written in a fixed order as (x,y) , where x is the x -coordinate of P and y is the y -coordinate of P .
- **x -coordinate** represents the horizontal distance to the left or right of the y -axis.
- **y -coordinate** represents the vertical distance above or below the x -axis.



2. Functions: Relationship between two variables, an independent variable x (input) that produces exactly one dependent variable y (output). It can be represented using words, an equation, a table of values or a graph.

- Every set of coordinates of point (x,y) can be represented as a point on the graph.
- **Linear function:** A set of points conform to a linear graph where the function of the graph is known as the **equation of the straight line**.

3. Equation of straight line: A formula connecting the x -coordinate and y -coordinate of all the points on the line.

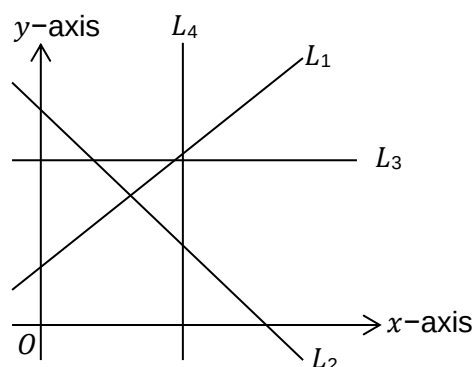
- **Equation of a straight line:** $y = mx + c$, where x and y are **Point (x,y)** on the line, constant m is the **gradient of the line** and the constant c is the **y -intercept** (the line that crosses the y -axis).

- **Gradient of a straight line, m ,** measures the degree of steepness of the slope of the line.

➤ **Gradient, $m = \frac{\text{Vertical change}}{\text{Horizontal change}}$ or $\frac{\text{rise}}{\text{run}}$** [change refers to the change in number of units]

4. Types of Straight Lines

- **Positive gradient, L_1 :** Positive slope slant “uphill” from left to right.
- **Negative gradient, L_2 :** Negative slope slant “downhill” from left to right.
- **Zero gradient, L_3 :** Horizontal line
- **Undefined gradient, L_4 :** Vertical line



Topic 7: Number Patterns

- Number sequence** is a set of numbers that are formed and arranged in a specific manner governed by a **specific rule**. Each number in a number sequence is called a **term**.
 - The first number is called **1st term**, second number is called the **2nd term**, and the n^{th} number is called the **n^{th} term**. The terms can be represented by $T_1, T_2, \dots, T_n$.
 - The n^{th} term of the sequence, T_n , is also known as the **general term**. It can be represented by an algebraic expression.
 - For example, 2, 5, 8, 11, 14, ...
 - The terms are 2, 5, 8, 11 and 14.
 - $T_1 = 2, T_2 = 5, T_3 = 8, T_4 = 11$ and $T_5 = 14$.
- Types of number patterns:**
 - Number sequence**
 - Line pattern**
 - Diagram pattern**
- Common types of Number sequence**
 - Involving common difference**
 - Start with 2, then add 3 to get to the next term. (Common difference of +3).
For example: 2, 5, 8, 11, 14, ...
 - Start with 10, then subtract 4 to get to the next term. (Common difference of -4).
For example: 10, 6, 2, -2, -6, ...
 - Involving common ratio**
 - Start with 1, then multiply by 2 to get to the next term. (Common ratio of $\times 2$)
For example: 1, 2, 4, 8, 16, ...
 - Start with 81, then divide by 3 to get to the next term. (Common ratio of $\div 3$)
For example: 81, 27, 9, 3, 1, $\frac{1}{3}$, ...
 - Special sequence**
 - Square numbers. For example: 1, 4, 9, 16, 25, ...
 - Cube numbers. For example: 1, 8, 27, 64, 125, ...
 - Prime numbers. For example: 2, 3, 5, 7, 11, 13, ...
 - There are many different types of number patterns that are not listed above. Observation is required to identify the pattern in the number sequence.
- General formula of a sequence**
 - To find the **general term** of the sequence, identify the pattern in the sequence and form an algebraic expression that fits the number pattern.
 - For example, "2, 5, 8, 11, 14, ..." is governed by the rule of "start with 2, then add 3 to get to the next term". This can be written in algebraic expression of " $2 + 3(n - 1)$ " where n represents the position in the sequence. Then, simplify the general term to " $3n - 1$ ".



Topic 8: Percentage

- Percent (%)** means “out of every hundred”. **Percentage** is a fraction with a denominator of 100. It allows a good comparison of different quantities.
 - In general, $x\% = \frac{x}{100}$
- Converting Fraction/ Decimal into a Percentage**
 - To convert a **Fraction** into a **Percentage**, multiply it with 100%. If the fraction is in mixed number, convert it to an improper fraction first before multiplying it with 100%.
 - To convert a **Decimal** into a **Percentage**, multiply it with 100% or shift the decimal point two places to the right and write % behind.
- Expressing One Quantity as a Percentage of another Quantity**
 - Expressing quantity a as a percentage of b (ensure that a and b are in the same unit)
 - Convert the fraction $\frac{a}{b}$ into a percentage, $\frac{a}{b} \times 100\%$
- Percentage Change of a Quantity, Percentage Points and Reverse Percentage**
 - Change in value of a quantity can be expressed as a percentage increase/ decrease in the original value.
 - Percentage increase** = $\frac{\text{New value} - \text{Original value}}{\text{Original value}} \times 100\%$
 - Percentage decrease** = $\frac{\text{Original value} - \text{New value}}{\text{Original value}} \times 100\%$
 - Increase/ Decrease** = % increase/decrease $\times$ original value
 - New Value** = final % $\times$ original value
 - Percentage points refers to the numerical value change in the percentage
 - For example, the increase from 7% to 9% is two percentage points (not 2%).
 - Reverse percentage involves the use of comparison of proportions to find the original value (100%).
- Take note: Never add** percentages or take the average of percentages when they do not have the same base.
- Percentage in real-world contexts**
 - Profit and Loss**
 - Profit** = Selling Price – Cost Price
 - Loss** = Cost Price – Selling price
 - Profit/ Loss** can be expressed as a percentage of the cost price.
 - % Profit/ Loss = $\frac{\text{Profit/Loss}}{\text{Cost Price}} \times 100\%$
 - Profit/ Loss** can be expressed as a percentage of the selling price.
 - $\frac{\text{Profit/Loss}}{\text{Selling Price}} \times 100\%$
 - Discount** = Marked Price (original price) – Sale Price (discounted price)
 - Percentage discount** = $\frac{\text{discount}}{\text{marked price}} \times 100\%$
 - Goods and Services Tax (GST)** is a tax on consumption, paid in addition to the price of goods and services. GST is also payable on service charge.
 - Commission** is the amount of payment an agent receives for selling/ providing a service on behalf of another party.



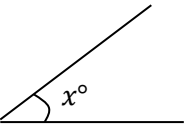
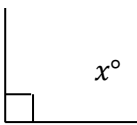
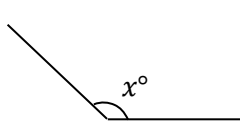
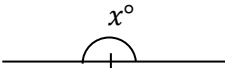
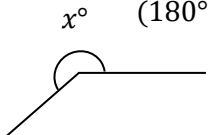
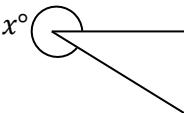
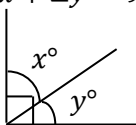
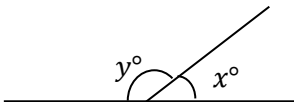
Topic 9: Rate, Ratio and Speed

- Ratio** is used to compare two or more quantities of the same kind that either have no units or are measured in the same units.
 - The order in which the ratio is expressed is important as $x:y \neq y:x$.
 - The ratio of x to y is written as $x:y$ or $\frac{x}{y}$.
 - Equivalent ratios** have the same value, for example, 1:4 and 2:8. Ratios are equivalent when they have the same simplest form.
- Rate** is a measure of how one quantity changes with respect to another quantity. Rates over a period of time can be constant or varying.
 - Simple Interest**, $I = \frac{PRT}{100}$, where the principal amount of money, \$ P , is deposited into a bank at a simple interest rate of $R\%$ per annum (p.a.) for T years.
 - Hire Purchase** is where part of the purchase price is paid first (deposit/ downpayment) and the remaining amount is paid in monthly instalments (including interest).
 - Currency Exchange** rates of different currencies fluctuate constantly due to the daily **demand** and **supply** of the currency. Banks or licensed money changers earn money through either imposing a direct commission or the exchange or indirectly through buying and selling at different rates of exchange.
 - Income Tax** is imposed on all working individuals or businesses to fund expenditure in the public sector.
- Speed** is the rate of change of distance travelled with time.
 - $\text{Speed} = \frac{\text{Distance}}{\text{Time taken}}$
 - Constant speed** is when the speed of a moving object does not change throughout the journey.
 - Average speed** is the speed of an object that travelled a certain amount of distance over a period of time. Average speed is used as a moving object typically has varying speeds at different times.
 - $\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$
 - Note:** Average speed $\neq$ finding the average of speeds
 - Units Conversion for Speed** can be done by converting the units individually.
For example:
 - $1 \text{ km/h} = \frac{1000\text{m}}{60\text{min}} = \frac{1000}{60} \text{ m/min}$
 - $1 \text{ km/h} = \frac{1000\text{m}}{60 \times 60\text{s}} = \frac{1000}{3600} \text{ m/s}$

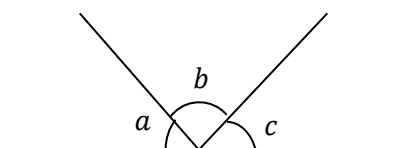
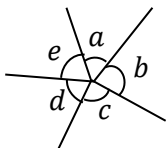
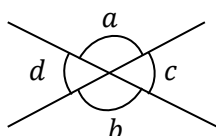
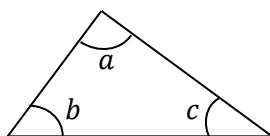
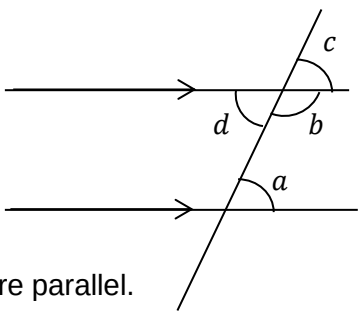


Topic 10: Basic Geometry

1. Types of Angles

Acute angle ($0^\circ < x^\circ < 90^\circ$) 	Right angle ($x^\circ = 90^\circ$) 	Obtuse angle ($90^\circ < x^\circ < 180^\circ$) 
Straight angle ($x^\circ = 180^\circ$) 	Reflex angle ($180^\circ < x^\circ < 360^\circ$)  	
Complementary angles $\angle x + \angle y = 90^\circ$ 	Supplementary angles $\angle x + \angle y = 180^\circ$ 	

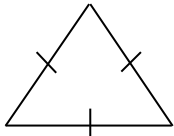
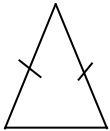
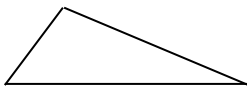
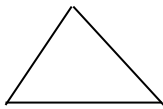
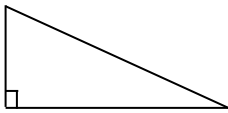
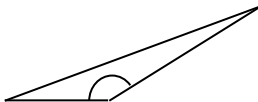
2. Properties of Angles

Adjacent angles on a straight line $\angle a + \angle b + \angle c = 180^\circ$ 	Angles at a point $\angle a + \angle b + \angle c + \angle d + \angle e = 360^\circ$ 
Vertically opposite angles $\angle a = \angle b$ and $\angle c = \angle d$ 	Sum of angles in a triangle $\angle a + \angle b + \angle c = 180^\circ$ 
Properties of angles formed by two parallel lines and transversal - Corresponding angles are equal ➤ $\angle a = \angle c$ - Alternate angles are equal ➤ $\angle a = \angle d$ - Interior angles are supplementary ➤ $\angle a + \angle b = 180^\circ$ Note: Transversal is any line that cuts 2 lines. We can use the above properties to prove 2 lines are parallel. 	



Topic 11: Polygons and Geometrical Constructions

1. Classification of Triangles

Types of triangles	Figure	Properties / Remarks
<u>Equilateral triangle</u> A triangle with 3 equal sides.		All angles in an equilateral triangle are equal to 60° . ($\angle$ s of equilateral Δ)
<u>Isosceles triangle</u> A triangle with at least 2 equal sides.		The base angles of an isosceles triangle are equal. (base $\angle$ s of isos. Δ)
<u>Scalene triangle</u> A triangle with no equal sides.		All the angles in a scalene triangle are different.
<u>Acute-angled triangle</u> A triangle with 3 acute angles.		For example, scalene, equilateral and isosceles triangles. (acute angles are angles that are less than 90°)
<u>Right-angled triangle</u> A triangle with a right angle.		There are many types of right-angled triangles that are scalene, but there is only one special type of right-angled triangle that is isosceles triangle with angles 45° , 45° and 90° .
<u>Obtuse-angled triangle</u> A triangle with an obtuse angle		For example, scalene or isosceles triangles. Obtuse angles are angles that are more than 90° but less than 180° .

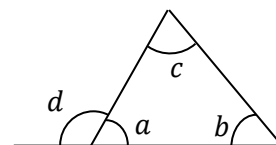
2. Triangles

• Basic Properties of Triangles

- Largest angle of a triangle is opposite the longest side, smallest angle is opposite the shortest side.
- Sum of the lengths of any two shorter sides of a triangle must be greater than the length of the third side.

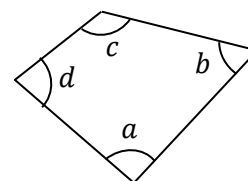
• Angle Properties of Triangles

- Sum of the interior angles of a triangle is 180° .
 $\angle a + \angle b + \angle c = 180^\circ$ ($\angle$ sum of Δ)
- Exterior angle formed by extending a side of the triangle is equal to the sum of its interior opposite angles.
 $\angle d = \angle c + \angle b$ (ext. $\angle$ of Δ)

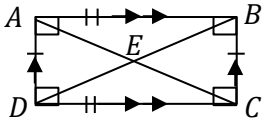
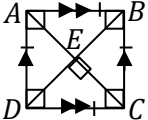
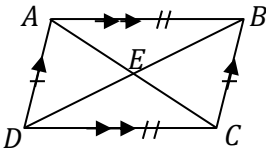
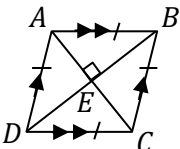
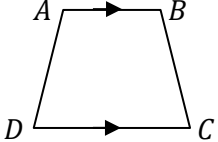
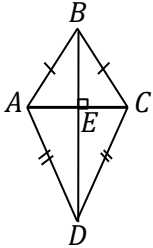


3. Angle Properties of Quadrilaterals (4-sided figure)

- Sum of interior angles of a quadrilateral is 360° .
 $\angle a + \angle b + \angle c + \angle d = 360^\circ$ ($\angle$ sum of quad)

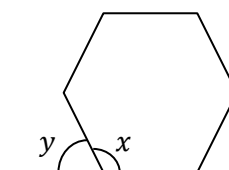


4. Properties of Special Quadrilaterals

Quadrilaterals	Sides	Interior angles	Diagonals
<u>Rectangle</u> 	2 pairs of parallel sides. Opposite sides are equal.	All 4 corner angles at the vertices are right angles, 90° .	2 diagonals are equal in length. ($AC = BD$) Diagonals bisect each other. ($AE = EC, DE = EB$)
<u>Square</u> 	2 pairs of parallel sides. All 4 sides are equal in length.	All 4 corner angles at the vertices are right angles, 90° .	2 diagonals are equal in length. ($AC = BD$) Diagonals bisect each other at right angles at E. ($AE = EC, DE = EB$) Diagonals bisect the interior angles. ($\angle ABD = \angle CBD = 45^\circ$)
<u>Parallelogram</u> 	2 pairs of parallel sides. Opposite sides are equal.	Opposite angles are equal. ($\angle ABC = \angle ADC$, $\angle BAD = \angle BCD$) $\angle ABC + \angle BCD = 180^\circ$ (int. $\angle$ s)	Diagonals bisect each other at E. ($AE = EC$, $BE = ED$)
<u>Rhombus</u> 	2 pairs of parallel sides. All 4 sides are equal in length.	Opposite angles are equal. ($\angle ABC = \angle ADC$, $\angle BAD = \angle BCD$) $\angle ABC + \angle BCD = 180^\circ$ (int. $\angle$ s) $\angle BAC = \angle BCA$, $\angle DAC = \angle DCA$	Diagonals bisect each other at right angles. ($AE = EC, BE = ED$) and $\angle AEB = \angle BEC = \angle AED = \angle CED = 90^\circ$ Diagonals bisect the interior angles. ($\angle ACD = \angle ACB$, $\angle ADB = \angle BDC$)
<u>Trapezium</u> 	At least 1 pair of parallel lines.	$\angle ABC + \angle BCD = 180^\circ$ (int. $\angle$ s)	
<u>Kite</u> 	At least 2 pairs of equal adjacent sides. ($AB = BC$, $AD = CD$) $\triangle ABC$ and $\triangle ACD$ are isosceles.	$\angle BAC = \angle BCA$, $\angle DAC = \angle DCA$	Diagonals cut each other at right angles at E (not bisect), $\angle AEB = \angle BEC = \angle CED = \angle AED = 90^\circ$. The longer diagonal BD bisects the shorter diagonal AC.

5. Angle Properties of Polygons

- Sum of interior angle and its corresponding angle is 180° .
➤ $\angle x + \angle y = 180^\circ$
- Sum of exterior angles of a n -sided polygon is 360° .
➤ Each exterior angle of a n -sided regular polygon, $\angle y = \frac{360^\circ}{n}$
- Sum of interior angles of a n -sided polygon = $180^\circ(n - 2)$.
➤ Each interior angle of a n -sided regular polygon, $\angle x = \frac{180^\circ(n-2)}{n}$



Note: **Regular polygon** is a polygon with all equal sides and all equal interior angles.



6. Common Polygons

Name of Polygons	Number of sides (n)	Sum of Interior Angles $180^\circ(n - 2)$
Triangle	3	$180^\circ(3 - 2) = 180^\circ$
Quadrilateral	4	$180^\circ(4 - 2) = 360^\circ$
Pentagon	5	$180^\circ(5 - 2) = 540^\circ$
Hexagon	6	$180^\circ(6 - 2) = 720^\circ$
Heptagon	7	$180^\circ(7 - 2) = 900^\circ$
Octagon	8	$180^\circ(8 - 2) = 1080^\circ$
Nonagon	9	$180^\circ(9 - 2) = 1260^\circ$
Decagon	10	$180^\circ(10 - 2) = 1440^\circ$



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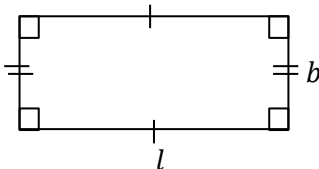
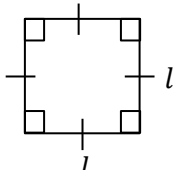
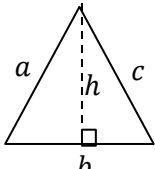
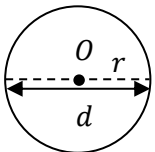
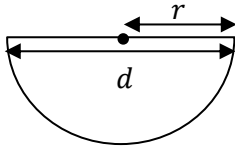
Topic 12: Perimeter and Area of Plane Figures

1. Conversion of Units

- $1 \text{ m}^2 = (100 \text{ cm})^2 = 10\,000 \text{ cm}^2$
- $1 \text{ cm}^2 = (10 \text{ mm})^2 = 100 \text{ mm}^2$

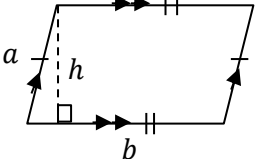
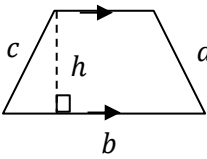
2. Perimeter and Area of Basic Plane Figures (RECAP)

- **Perimeter** refers to the length of the outline of the two-dimensional figure, that enclosed the figure.
- **Area** refers to the amount of surface that covers the two-dimensional figure.
- The following table shows the formulae of the area and perimeter of some two-dimensional figures.

Figure	Formulae	
	Area (square units)	Perimeter (units)
Rectangle 	$l \times b$	$2(l + b)$
Square 	l^2	$4l$
Triangle 	$\frac{1}{2} \times b \times h$	$a + b + c$
Circle 	πr^2	$2\pi r = \pi d$
Semicircle 	$\frac{1}{2} \pi r^2$	$\pi r + 2r$ or $\frac{1}{2} \pi d + d$



3. Perimeter and Area of Parallelograms and Trapeziums

Figure	Formulae	
	Area (square units)	Perimeter (units)
Parallelogram 	$b \times h$	$2(a + b)$
Trapezium 	$\frac{1}{2}(h)(a + b)$	$a + b + c + d$



Topic 13: Volume and Surface Area of Prisms and Cylinders

1. Conversion of Units

- $1\text{ ml} = 1\text{ cm}^3$
- $1\text{ l} = 1000\text{ ml} = 1000\text{ cm}^3$
- $1\text{ m}^3 = (100\text{ cm})^3 = 1000\,000\text{ cm}^3$

2. Density is a measure of the heaviness of an object with respect to its volume. It is defined as **mass per unit volume**.

- $\text{Density} = \frac{\text{Mass}}{\text{Volume}}$
- The units of density can be in kg/m^3 or g/cm^3

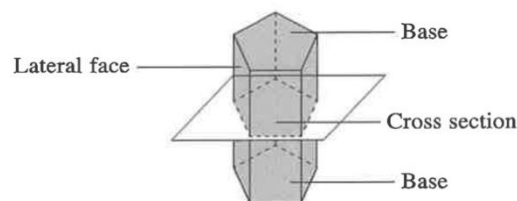
3. Cubes and Cuboids (RECAP)

- **Cubes**
 - Volume of cube = length $\times$ length $\times$ length = l^3
 - Surface area of cube = $6 \times \text{length} \times \text{length} = 6l^2$
- **Cuboids**
 - Volume of cuboid = length $\times$ breadth $\times$ height = lbh
 - Surface area of cuboid = $2(\text{length} \times \text{breadth} + \text{breadth} \times \text{height} + \text{length} \times \text{height})$
 $= 2(lb + bh + lh)$

4. Prisms are solid objects that has two identical bases and flat sides with **uniform cross section**. Any horizontal cross section of a prism is parallel to the bases with identical faces/ lateral faces.

- A **Prism** has uniform polygonal cross section.
- Some examples of prisms are:

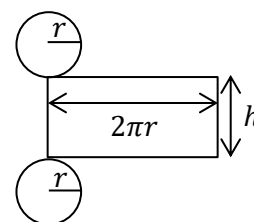
- Triangular prism
- Quadrilateral prism
- Pentagonal prism
- Hexagonal prism
- Trapezoidal prism
- Cube/ Cuboid



- **Volume of prism** = Area of cross section $\times$ distance between cross-sectional bases
 $= \text{Base Area} \times \text{Height}$
- **Total Surface Area of prism** = Total area of the lateral faces + $2 \times$ base area
 $= \text{Perimeter of the base} \times \text{Height} + 2 \times \text{base area}$

5. Cylinder is a 3-dimensional figure with straight parallel sides and a **uniform circular cross section**.

- **Volume of Cylinder**
 $= \text{Area of cross section} \times \text{distance between cross-sectional bases}$
 $= \text{Base Area} \times \text{Height}$
 $= \pi r^2 h$
- **Surface Area of Cylinder** = $2 \times \text{base area} + \text{curved surface area}$
 $= 2\pi r^2 + 2\pi rh$



Topic 14: Statistical Data Handling

1. Steps in Statistical Data Handling

- Step 1: **Collection** of Data
- Step 2: **Organisation** of Data
 - Using the Frequency Table with Tally (Multiples of 5)

Number of books read by the students in a week

Student	Tally	Frequency
A	////	4
B	//// ///	8
C	////	5

- Step 3: **Presentation** of Data
 - **Pictogram**: Using pictures or icons to represent 1 or more units of data.
 - **Bar Graph**: Vertical bar graph, Horizontal bar graph, Double bar graph
 - **Pie Chart**: Using sectors of a circle, where the angle of a sector is proportional to the frequency of data that it is representing.
 - **Line Graph**: Formed by joining the points with the data collected using straight lines
- Step 4: **Analyse and Interpretation** of Data

2. Advantages and Disadvantages of the Different Types of Statistical Diagrams

Statistical Diagram	Advantages	Disadvantages
Frequency Table	• Organise data collected • Easy to read the exact frequency for each category	• Presentation of data not as visual and appealing as statistical graphs • Difficult to compare frequencies across different categories as compared to some statistical graphs
Pictogram	• Visually appealing • Easy to read • Handles large data set easily	• Difficult to draw pictures and to quantify a partial picture • Time consuming to draw identical picture (shape and size) • Only suitable for small number of categories • Low degree of accuracy of data presentation
Bar Graph	• Easier to draw than pictogram • Visually clear	• Less colourful and appealing • More abstract as frequency is represented by length of each bar • Data may be distorted/ misinterpreted if frequency axis does not start from 0.
Pie Chart	• Easier to compare relative size of each category with the whole • Shows percentage of each category	• Difficult to compare two sectors of about the same size • No exact numerical data • Will look very cluttered with many categories • Harder to construct than bar graph



2. Advantages and Disadvantages of the Different Types of Statistical Diagrams (Con't)

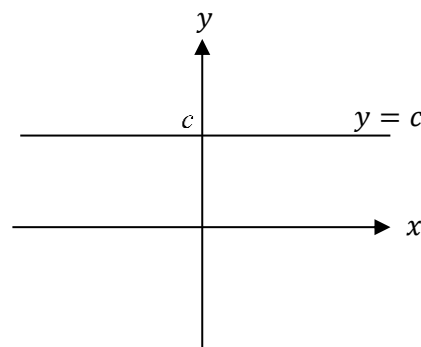
Statistical Diagram	Advantages	Disadvantages
Line Graph	- Shows trend in data relationship – observe rising or falling trend of a set of numerical data over a period of time - Allows projection of future values - Retains exact data values - Shows minimum and maximum values	- Must not read in between plotted points unlike linear function graphs - Data may be distorted and misinterpreted if vertical axis does not start from 0. - Data with small fluctuations (flat trend lines between 2 values) may have no significant meaning/ prevent observation of these minor changes (if vertical axis starts from 0.)



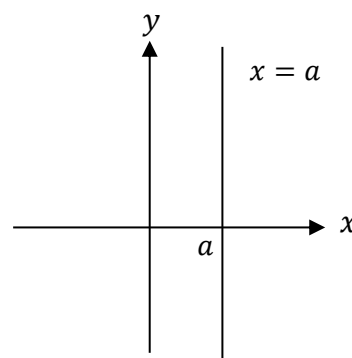
Secondary 2

Topic 1: Linear Graphs

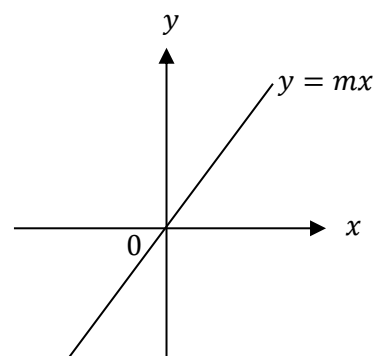
1. Graphs of linear equations are **straight lines**.
2. A straight-line graph is expressed as an equation in the form of $y = mx + c$.
 - where **m is the gradient**. **Gradient** = $\frac{\text{Vertical Change}}{\text{Horizontal Change}}$
 - and **c is the y -intercept**.
3. Graph of $y = c$: A **horizontal straight line parallel to x -axis**.



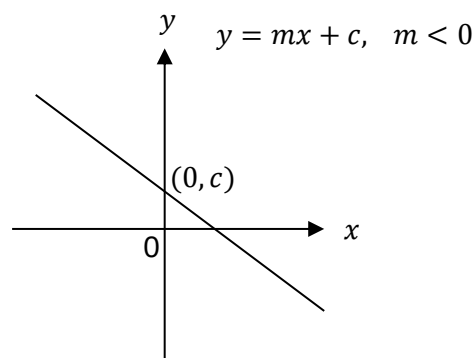
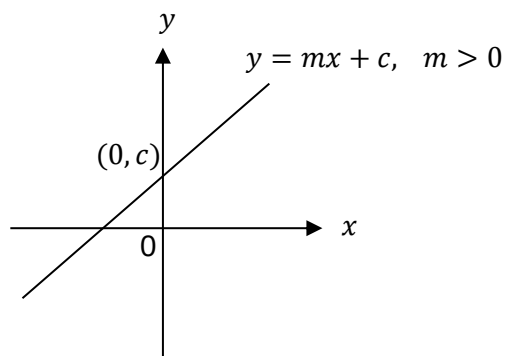
4. Graph of $x = a$: A **vertical straight line parallel to y -axis**.



5. Graph of $y = mx$: A **straight line that passes through the origin and has the gradient m** .



6. Graph of $y = mx + c$: A straight line that **cuts the y-axis (known as y-intercept) at $(0, c)$ and has the gradient m**



7. To **draw a linear graph**, select at least 3 different points for plotting.
8. To **solve for a pair of simultaneous linear equations graphically**, find the **coordinates of the intersection point** between the two graphs.
9. When two straight lines are **parallel to each other**, they have the **same gradient but different y-intercepts**. In such situation, there will be **no solutions** as the two lines will not intersect.
10. When two straight lines are the **same line**, they have the **same gradient and the same y-intercept**. In such situation, there will be **infinitely many solutions**.



Topic 2: Simultaneous Linear Equations

1. In general, n equations are needed to solve for the values of n unknowns.
2. A pair of simultaneous linear equations is needed to solve for 2 unknowns.
3. **Two main methods of solving simultaneous linear equations without graph:**
 - **Elimination Method**
 - **Step 1:** Compare the coefficients of the unknown (x and y) we want to eliminate. Usually look out for the coefficients that are 1 or -1 .
 - **Step 2:** If the numerical values of the coefficients of one of the unknown are the same in both equations (ignore the positive or negative sign), eliminate the unknown either by adding or subtracting the two equations.
If the numerical values of the coefficients of the unknown are not the same in both equations, manipulate one/both equations first to obtain new equation(s) whereby the numerical values of the coefficients of the variable are the same.
Tip: Choose the coefficient of the unknown that has smaller LCM when considering which unknown to eliminate.
 - **Step 3:** Solve the resulting linear equation with one unknown.
 - **Step 4:** Substitute the value obtained for the unknown into any one of the original equations to solve for the other unknown.
 - **Substitution Method**
 - **Step 1:** Select one of the two equations and rearrange it to make one unknown the subject, in terms of the other. Look out for the coefficient of the unknown that is 1 or -1 and use this equation.
 - **Step 2:** Substitute the equation in **Step 1** into the other equation (do not have to rearrange the other equation) whereby the resulting equation will only have one unknown. Solve the equation to find the value of this unknown.
 - **Step 3:** Substitute the solution found in **Step 2** into the equation in **Step 1** to find the value of the other unknown.
4. When the question did not specify which method to use, use the method that is deemed easier to solve for unknowns.
5. Check the solution by substituting the values of the 2 unknowns which you have found into the original equations. (Ensure that LHS = RHS for each equation.)



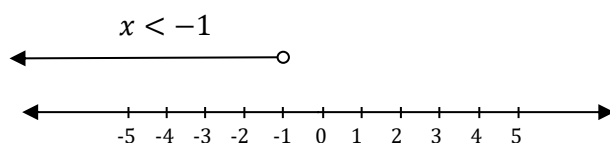
Topic 3: Linear Inequalities

1. Inequality signs

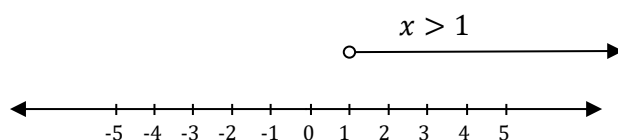
- $x < a$ refers to x is **lesser** than the constant value a .
- $x > a$ refers to x is **greater** than the constant value a .
- $x \leq a$ refers to x is **equal or lesser** than the constant value a .
- $x \geq a$ refers to x is **equal or greater** than the constant value a .

2. Number Line Representation for Inequalities

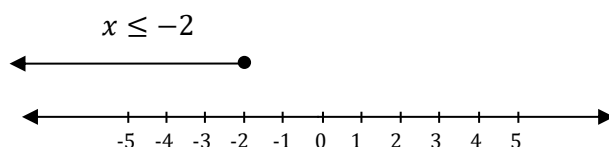
- For example, if $x < -1$



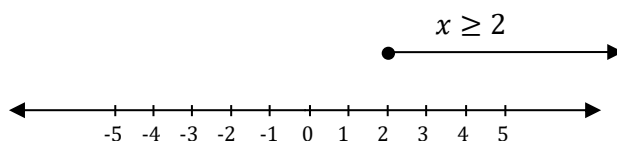
- if $x > 1$



- if $x \leq -2$



- if $x \geq 2$



Note: A circle $\bigcirc$ indicates that x cannot take on a particular value while a solid dot $\bullet$ indicates that x can take on the particular value.



3. Arithmetic Operation during Inequalities

- We can **add a positive or negative number** to both sides of an inequality **without changing** the inequality sign.

$$x > 3$$

$$x + 2 > 3 + 2$$

$$x + 2 > 5$$

- We can **subtract a positive or negative number** to both sides of an inequality **without changing** the inequality sign.

$$x > 3$$

$$x - 5 > 3 - 5$$

$$x - 5 > -2$$

- We can **multiply/divide by a positive number** to both sides of an inequality **without changing** the inequality sign.

$$x > 3$$

$$x \times 2 > 3 \times 2$$

$$2x > 6$$

- However, when we **multiply/divide by a negative number** to both sides of an inequality, we **MUST REVERSE** the inequality sign.

$$x > 3$$

$$x \times (-3) < 3 \times (-3)$$

$$-3x < -9$$



Topic 4: Expansion of Algebraic Expression

1. Simplification/ Expansion of Algebraic Expressions using The Distributive Law

- $a(b + c) = ab + ac$ (1 term x 2 terms = 2 terms)
- $a(b + c + d) = ab + ac + ad$ (1 term x 3 terms = 3 terms)
- $(a + b)(c + d) = a(c + d) + b(c + d) = ac + ad + bc + bd$ (2 terms x 2 terms = 4 terms)
- $(a + b)(c + d + e) = ac + ad + ae + bc + bd + be$ (2 terms x 3 terms = 6 terms)

2. Special Algebraic Identities

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$



Topic 5: Factorisation of Algebraic Expressions

- Factorisation:** Process of expressing an algebraic expression as a product of two or more factors. It is the reverse process of expansion.
- Factorisation by Extracting Common Factors**
 - Check for common factor of the terms and extract them before applying other methods of factorization.
 - The common factor can be in a form of 1 term or an expression.
 - For example: $ab + ac = a(b + c)$
- Factorisation by Grouping** to identify common factors
 - Arrange the terms into **two groups** to find the common factors (especially when the expression has 4 terms).
 - Change the sign of the factor when necessary (by factorising the negative sign).
 - For example: $ac + ad + bc + bd = a(c + d) + b(c + d) = (a + b)(c + d)$
- Factorisation of Quadratic Expressions using “Multiplication Frame” Method**
 - Example: Factorise $x^2 + 4x - 77$
 - **Step 1:** Write the 1st term, x^2 , in the top-left corner and 3rd term (constant term), -77 in the bottom-right corner.

×		
	x^2	
		-77

- **Step 2:** Consider and select the appropriate factors of the 1st term and 3rd term. Write the selected pair of factors in the first row and the first column.

×	x	11
x	x^2	
-7		-77

[Note: Since the overall coefficient of the term in x is 4, we select the pair of factors of -77 to be -7 and 11 as $11x - 7x = 4x$.]

- **Step 3:** Complete the multiplication frame by multiplying to obtain the coefficient of the term in x . Ensure that the result matches the given term in x .

×	x	11
x	x^2	$11x$
-7	$-7x$	-77

Therefore, $x^2 + 4x - 77 = (x + 11)(x - 7)$.

- The above method can be done by either using **Trial and Error** or **Reasoning** when selecting the pairs of factors.
 - It can be used to factorise:
 - Quadratic expressions of the form $x^2 + bx + c$,
 - Quadratic expressions of the form $ax^2 + bx + c$, and
 - Algebraic expressions involving **two variables**
- Factorising using Special Algebraic Identities**
 - Difference of two squares**
 - $a^2 - b^2 = (a - b)(a + b)$
 - Perfect square**
 - $a^2 + 2ab + b^2 = (a + b)^2$
 - $a^2 - 2ab + b^2 = (a - b)^2$



Topic 6: Quadratic Equations and Graphs

1. Quadratic Equations

- Equation in the form of $ax^2 + bx + c$, where $a \neq 0$, constants a and b are the coefficients of x^2 and x respectively, and c is the constant term.
- To solve quadratic equation in the form of $ax^2 + bx + c = 0$, we need to find the value(s) of x that satisfies the equation, which is known as the **roots** of the equation.

2. Methods to solve Quadratic Equations

- Factorisation and Graphical Method** (Secondary 2)
- Completing the Square and Quadratic Formula** (Secondary 3)

3. Solving Quadratic Equations by Factorisation

- Step 1:** Rewrite the given equation in the general form, $ax^2 + bx + c = 0$ (if necessary).
- Step 2:** Factorise the expression on the LHS of the equation.
- Step 3:** By using the **Zero Product Principle**, set each factor equal to 0.

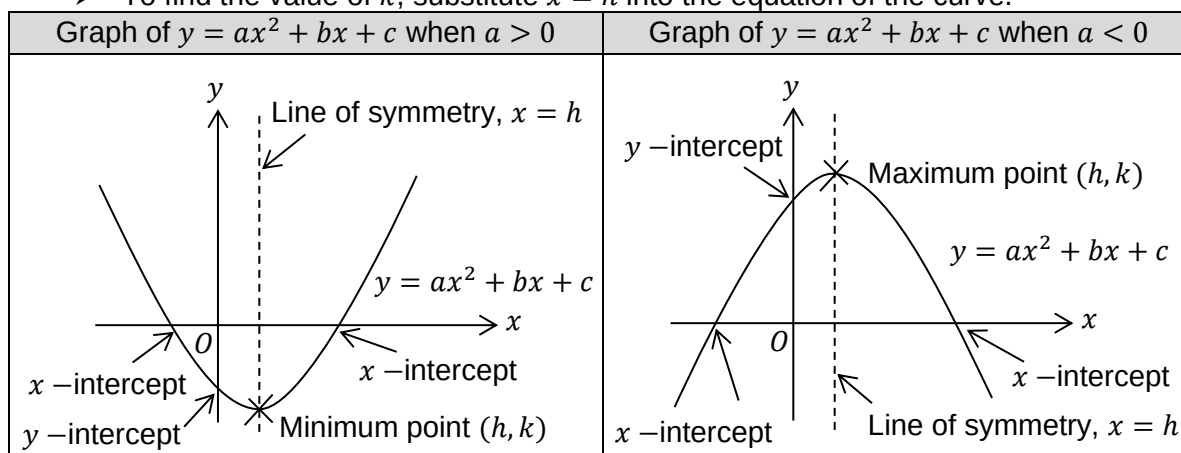
Zero Product Principle (Product of 0)

If p and q are real numbers where $pq = 0$, then $p = 0$ or $q = 0$. Therefore, if P and Q are factors of the quadratic algebraic expression such that $PQ = 0$, then $P = 0$ or $Q = 0$.

- Step 4:** Solve the linear equation(s) in Step 3 to obtain the solution(s)/ roots of the equation.
- Step 5:** Check your answers by substituting the solution(s) into the original equation.

4. Graphs of Quadratic Functions

- Quadratic graphs are graphs of the equation form $y = ax^2 + bx + c$, where $a \neq 0$ and a, b and c are constants.
- Shape of a quadratic graph is a **parabola**, either a **U-shaped** (smiley face) or **n-shaped** (sad face) curve, depending on the coefficient of x^2 , a .
 - If $a > 0$, the quadratic graph is **U-shaped** and has a **minimum point**.
 - If $a < 0$, the quadratic graph is **n-shaped** and has a **maximum point**.
 - The **minimum point** and **maximum point** are called the **turning points**.
- To find **y-intercept**, substitute $x = 0$ into the equation to find the value of y .
- To find **x-intercept**, substitute $y = 0$ into the equation to find the value of x .
- Quadratic curve is symmetrical about the line of symmetry. **Line of symmetry** is a vertical line in the form of $x = h$ which passes through either the **maximum** or **minimum point** (h, k) of the curve.
 - Equation of line of symmetry, $x = h = \frac{x_1 + x_2}{2}$, where x_1 and x_2 are the x -intercepts.
 - To find the value of k , substitute $x = h$ into the equation of the curve.



Topic 7: Algebraic Fractions and Formulae

1. Algebraic Fractions

- Fraction in the form of $\frac{A}{B}$ (where $B \neq 0$), and A and/ or B are expressions that contains algebraic terms.
- One important rule: **Value of the fraction** remains the **same** if both its numerator and denominator are multiplied or divided by the identical non-zero number/ expression.
 - $\frac{a}{b} = \frac{a \times c}{b \times c}$ and $\frac{a}{b} = \frac{a \div c}{b \div c}$ where $b, c \neq 0$.
- To simplify an algebraic fraction, we divide both the numerator and denominator by their common factors. The numerator and denominator in the final answer should not have any common factors other than 1.

2. Multiplication and Division of Algebraic Fractions

- **Multiplication of two algebraic fractions**
 - Factorise both numerator and denominator where possible.
 - Divide/ Cancel out the common factors for both the numerator and denominator.
 - Multiply the remaining terms within the numerator and denominator respectively. (It works the same way as numeric fractions.)
 - $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$, where b and $d \neq 0$.
- **Division of two algebraic fractions**
 - Factorise both numerator and denominator where possible.
 - Convert the division into multiplication by reciprocating the second fraction.
 - Divide/ Cancel out the common factors for both the numerator and denominator.
 - Multiply the remaining terms within the numerator and denominator respectively.
 - $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

3. Addition and Subtraction of Algebraic Fractions

- **Algebraic Fractions can be added/ subtracted the same way as numeric fractions**
 - Factorise both numerator and denominator where possible.
 - Divide/ Cancel out the common factors for both numerator and denominator.
 - Make the denominators of the fractions the same by finding the lowest common multiple (LCM) so that the algebraic fractions can be expressed as like fractions.
 - Add/ Subtract the numerators of the fractions (similar to adding/ subtracting numeric fractions) while leaving the common denominators the same. Take note to the change of operation signs when expanding negative terms.

4. Solving Equations involving Algebraic Fractions

- **Only 2 algebraic fractions/ numerals within the equation**
 - Use cross-multiplication method
- **3 or more algebraic fractions/ numerals within the equation**
 - **Method 1:** Express as like fractions by ensuring each side of the equation are expressed as a single fraction before performing cross-multiplication
 - **Method 2:** Multiply throughout individual fractions/ numerals by the LCM of denominators.
- Always substitute the solutions back into the original equation to check for validity and ensure that the denominators are always non-zero.

5. Changing the Subject of a Formula

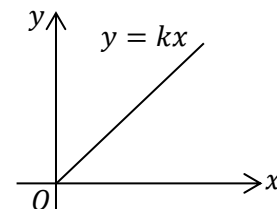
- **Formula** describes the mathematical relationship between one quantity and the other quantities. **Subject** of a formula appears alone on the **LHS** of the equation.
- Perform algebraic manipulations such as expansion, taking out common factor, etc, to make the variable (the **subject**) appear on the LHS of the formula.



Topic 8: Direct and Inverse Proportions

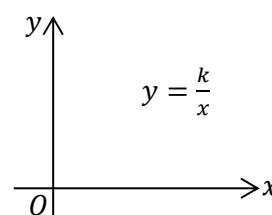
1. Direct Proportion

- Variable y is directly proportional to variable x if the ratio of $y : x$ remains unchanged for all values. For example, $\frac{y_1}{x_1} = \frac{y_2}{x_2}$ or $\frac{y_1}{y_2} = \frac{x_1}{x_2}$
- General equation: $y = kx$, where k is a constant and $k \neq 0$ where it can be determined by a known ordered pair, e.g. (x_1, y_1) .
- Graphical representation of $y = kx$ is a linear relationship, represented by a straight line with gradient k that passes through the origin.
- When x increases, y increases and when x decreases, y decreases.



2. Inverse Proportion

- Variable y is inversely proportional to variable x when y is directly proportional to $\frac{1}{x}$. For example, $x_1y_1 = x_2y_2$
- General equation: $y = \frac{k}{x}$ or $k = xy$, where k is a constant and $k \neq 0$.
- Graphical representation of $y = \frac{k}{x}$ is a hyperbola.
- Graph of y against $\frac{1}{x}$ is a straight line starts from the origin ($x \neq 0$).
- When x increases, y decreases, and when x decreases, y increases.



3. Percentage change

- Increase/ decrease by:** New value = Original value + / - amount increase/ decrease
- Increase/ decrease to:** New value = Amount increase/ decrease to
- Percentage change** = $\frac{\text{Change in Value}}{\text{Original Value}} \times 100\%$



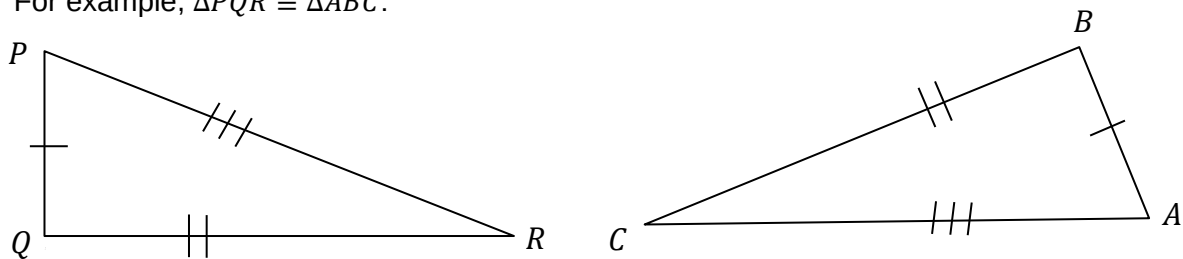
Topic 9: Congruence and Similarity

1. **Transformation** refers to a **change** in either the **position** or the **size** of an object. An **image** is obtained when a figure undergoes a transformation.

- Types of transformation:
 - **Translation** by 'sliding' the figure
 - **Rotation** by 'turning' the figure
 - **Reflection** by 'flipping' the figure on a mirror line
 - **Enlargement/ Reduction** by 'resizing' the figure

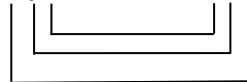
2. Congruent Figures

- Two figures are **congruent** if they have exactly the **same shape** and **same size**.
- They can be mapped onto each other under **translation, rotation** and **reflection**.
- **Symbol for congruency** is " $\equiv$ ", which represents "is congruent to".
- If two polygons are congruent,
 - all their corresponding **angles** are **equal** (same shape), **and**
 - all their corresponding **sides** are **equal** (same size)
- For example, $\triangle PQR \equiv \triangle ABC$:



- $\angle P = \angle A$, $\angle Q = \angle B$ and $\angle R = \angle C$, (all corresponding angles are equal)
- $PQ = AB$, $QR = BC$ and $PR = AC$. (all corresponding sides are equal)
- Note that the symbol " $\equiv$ " expresses that the way the two polygons are named have to be in a concise and precise manner.

This means that we write $\triangle PQR \equiv \triangle ABC$ in a specific manner, whereby the **order**

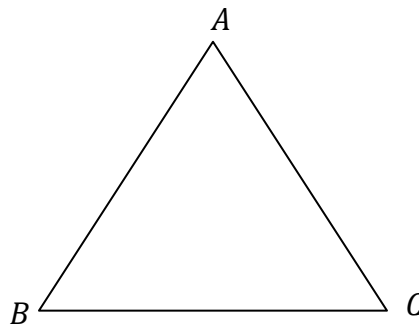
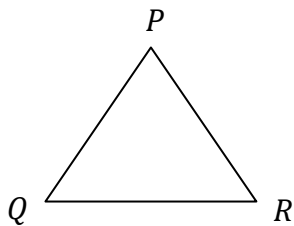


of the vertices of one polygon **must correspond** to the other of the vertices of the other polygon.



3. Similar Figures

- Two figures are **similar** if they have exactly the **same shape** but **not** necessarily the same size.
- If two similar figures also have exactly the same size, they are **congruent** to one another. Therefore, a pair of congruent figures is also a pair of similar figures, **but** a pair of similar figures may not be congruent to one another. **Congruency** is a **special case** of **similarity**.
- There is **no special symbol** for similarity.
- If two polygons are similar,
 - all their corresponding **angles** are **equal**, and
 - all the **ratios** of the lengths of their corresponding **sides** are **equal**.
- Enlargement/ Reduction** is a transformation that **changes** the **size** of an object **without changing** its **shape**. **Image** obtained is either enlarged or reduced in size as compared to the object. The **object** and **corresponding image** are similar.
- An **enlargement** is determined by the center of **enlargement** and a **scale factor**, k .
- The **scale factor**, k , of an enlargement is the ratio of the length of a side of the image to the corresponding length of an object.
 - Scale factor**, $k = \frac{\text{Length of a side of the image}}{\text{Length of the corresponding side of object}}$.
 - When $0 < k < 1$, the image produced is diminished (size of object > size of image).
 - When $k = 1$, the image and the object have the same size, whereby they are **congruent**.
 - When $k > 1$, the image is enlarged (size of object < size of image).
- For example, $\triangle PQR$ is similar to $\triangle ABC$:



- $\angle P = \angle A$, $\angle Q = \angle B$ and $\angle R = \angle C$, (all corresponding angles are equal)
- $\frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC} = k$ (all the ratios of their corresponding sides are equal)

4. Map Scale Drawing is an enlarged or reduced drawing of actual figures as the actual objects might be too large or too small to draw on paper.

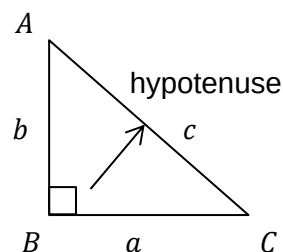
- Scale factor** of a scale drawing is the **ratio** of the length of a **drawing** to the corresponding length of the **actual** object. It is usually represented as a **linear scale** in the form of $1:n$, where 1 unit in length on the scale drawing represents the actual length of n units.
- If the **linear scale** on map is $1:n$, then the **area scale** of the map is $1:n^2$.
- Note:** There are no units in scale factor, whereby the units in the map scale in the form of fraction $\frac{1}{n}$ or ratio $1:n$ must be the same.
 - For example, map scale of $\frac{1}{12500}$ or $1:12500$ means that 1 cm on the map represents 12500 cm on the actual ground.



Topic 10: Pythagoras' Theorem

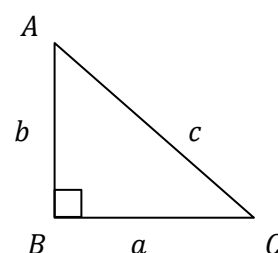
1. Pythagoras' Theorem

- In a right-angled triangle, the longest side of the triangle (opposite to the right angle), is called the **hypotenuse**, c .
- Pythagoras' Theorem** states that for any **right-angled** triangle, the **square** of the length of the **hypotenuse** is always equal to the **sum of the squares** of the lengths of the **other two sides**.
 - In a right-angled triangle ABC , $AC^2 = AB^2 + BC^2$, where AC is the length of the hypotenuse of the triangle.
i.e. $c^2 = a^2 + b^2$
 - Note: When the lengths of any two sides of a right-angled triangle are given, the length of the third side can be found using **Pythagoras' Theorem**.



2. Converse of Pythagoras' Theorem

- Use to show that a particular triangle is a right-angled triangle.
- In $\triangle ABC$, if $a^2 + b^2 = c^2$, we can conclude that
 - $\triangle ABC$ is a right-angled triangle
 - $\angle ABC = 90^\circ$



Topic 11: Trigonometric Ratios

1. Trigonometric Ratios

- In a **right-angled triangle** ABC in which $\angle C = 90^\circ$, the trigonometric ratios of $\angle A$ are:

➤ **Sine** of $\angle A$, or $\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AB}$

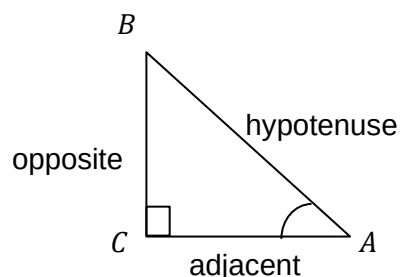
SOH

➤ **Cosine** of $\angle A$, or $\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{AC}{AB}$

CAH

➤ **Tangent** of $\angle A$, or $\tan A = \frac{\text{opp}}{\text{adj}} = \frac{BC}{AC}$

TOA



- Trigonometric ratios are only applicable to right-angled triangles.
- Sine, Cosine and Tangents are functions and not algebraic terms. They always come with angles.
- In any right-angled triangle, the hypotenuse remains the same while the opposite and adjacent sides change with reference to the angle used.
- To memorise the three trigonometric ratios, we use “**TOA CAH SOH**”

2. Use of **Calculator** to evaluate trigonometric ratios

- When computing angles in degrees, always remember to check/ set the mode of the calculator to “**DEG**”.



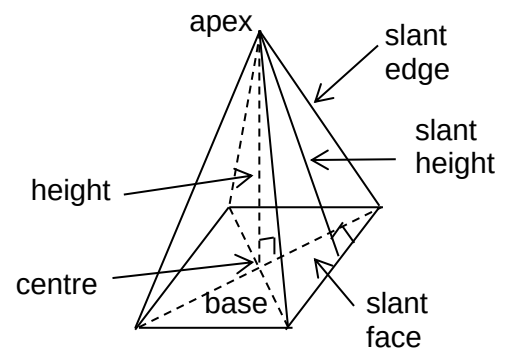
Topic 12: Volume and Surface Area of Pyramids, Cones and Spheres

1. **Pyramids** is a solid with **polygonal base** and its **slanted faces** are **triangles** joined to the sides of the polygonal base.

- **Vertices** are the corner points of the pyramid.
- **Apex** is where the vertex of all slanted faces meet.

- **Types of pyramids**

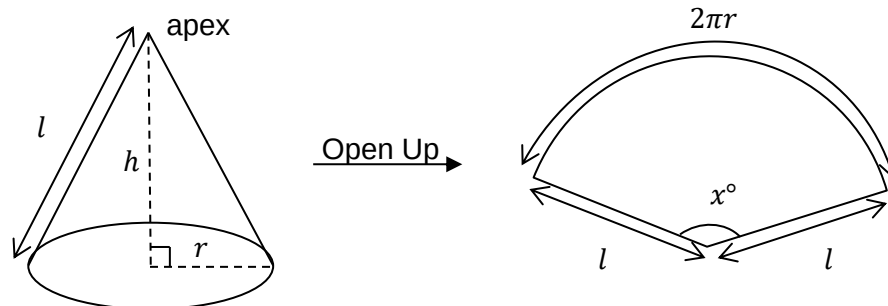
- **Right pyramid** is a pyramid with its apex perpendicularly above the centre of the base. For example, square/ rectangular pyramid.
- **Regular pyramid** is a **right pyramid** with regular polygon base.
- **Tetrahedron** is a triangular pyramid.



- **Volume of Pyramid** = $\frac{1}{3} \times \text{Base Area} \times \text{Height}$
- **Total Surface Area of Pyramid** = **Total Area of All Faces**
= **Area of Base** + **Area of All Lateral Faces**
- **Area of One of the Slant Face (Lateral faces)** = $\frac{1}{2} \times \text{Base Length} \times \text{Slant Height}$

2. **Cones** are not pyramids but like pyramids with circular bases.

- **Volume of Cone** = $\frac{1}{3} \times \text{Base Area} \times \text{Height} = \frac{1}{3} \pi r^2 h$
- **Curved Surface Area of Cone** = $\pi r l$, where l is the slant height of the cone.
- **Surface Area of Cone** = **Base Area** + **Curved Surface Area** = $\pi r^2 + \pi r l$



3. **Spheres** are 3D shapes that are perfectly round and symmetrical. Every points on its surface are **equidistant** from the centre of the sphere.

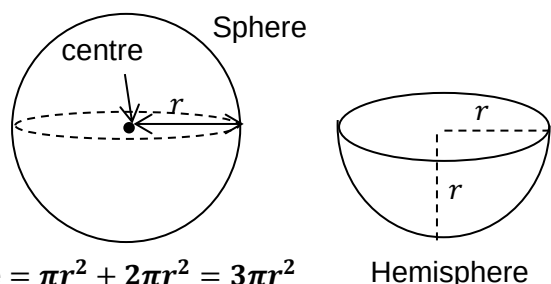
- **Hemisphere** is a half sphere which has a circular base and a curved surface that is half of that of the corresponding sphere.

- **Sphere**

- **Volume of Sphere** = $\frac{4}{3} \pi r^3$
- **Surface Area of Sphere** = $4 \pi r^2$

- **Hemisphere**

- **Volume of Hemisphere** = $\frac{2}{3} \pi r^3$
- **Total Surface Area of Closed Hemisphere** = $\pi r^2 + 2 \pi r^2 = 3 \pi r^2$



Topic 13: Probability of Single Events

1. Probability and Sample Space

- **Probability** is the **measure of chance/** the **likelihood** of an event happening, which has the value between 0 and 1.
- **Sample space** is the collection of **all possible outcomes** of a probability experiment. It is usually denoted by the letter " S ", and written using the set notation, $\{ \}$.

2. Probability of Single Events

- In general, the probability experiment with equally likely outcomes, the probability of an event E , $P(E)$, happening is given by:
 - $P(E) = \frac{\text{number of favourable outcomes for event } E}{\text{total number of possible outcomes}} = \frac{n(E)}{n(S)}$
 - $P(\text{not } E) = 1 - P(E)$
- For any event E to occur, $0 \leq P(E) \leq 1$.
 - If $P(E) = 0$, the event E is an impossible event, whereby it will **never** occur.
 - If $P(E) = 1$, the event E is a certain event, whereby it will **definitely** occur.



Topic 14: Statistical Diagrams

1. Dot diagrams

- Dot diagrams involving number line must be drawn with **equal intervals** to represent the range of values.
- A set of data is represented by using dots, placing them above a horizontal number line. The number of dots indicates the value of the data points.
- **Advantages:** Suitable to display a small set of numerical data
- **Disadvantages:**
 - Not suitable to display a large set of numerical data (each dot represents one unit).
 - Not suitable to display a data set with many different values/ a large range.

2. Histograms for Ungrouped and Grouped Data

- Similar to vertical bar chart except that there are no spaces between the adjacent bars as the horizontal axis in histogram is a number line.
- In Histogram, the area of the bar denotes the value of frequency, unlike bar graphs where height denotes the frequency. If the widths of the bars in histogram are equal, then only the heights need to be considered.
- **Advantages:**
 - Suitable for displaying a large set of numerical data.
 - Suitable to display a data set (grouped data) when individual data values are not important.
- **Disadvantages:**
 - More abstract as frequency is represented by area of each column.
 - Data may be misinterpreted if the frequency axis does not start from zero.
 - For grouped data, individual data values are lost.

3. Stem-and-Leaf Diagrams

- Used to display grouped numerical data while preserving the values of each data point.
- Data values are split into two parts – the stem and the leaves.
- Data are grouped into **equal class intervals** by the stems.
- Stems must be arranged in ascending order from the top to bottom.
- Leaves must be recorded in ascending order separated with spacing, from left to right (when the leaf is on the right of the stem).
- Total number of leaves must tally with the total number of data.
- **Back-to-back stem-and-leaf diagram** is used to compare two data sets. The two data sets will share a common stem with one group of the data set represented by the leaves on the left and the other on the right.
- **Advantages:**
 - Suitable to display a data set with many different values by grouping the values into equal class intervals.
 - Individual data values are retained. (represented by the leaves)
 - Can display two sets of data for easy comparison.
- **Disadvantages:**
 - Tedious to construct if there are too many data values.
 - Limitations to grouping of class intervals. Example, “21-30”, “31-40”, due to the stem.



Topic 15: Averages of Statistical Data

1. **Mean** is the average of the values, obtained by dividing the sum of all values by the number of values.
 - **Mean** = $\frac{\text{sum of data values}}{\text{number of data values}}$
 - **Mean of frequency distribution**, $\bar{x} = \frac{\sum fx}{\sum f}$, where x is a data value and f is the frequency of the data value.
 - **Estimated mean of grouped data**, $\bar{x} = \frac{\sum fx}{\sum f}$, where x is the mid-value and f is the frequency of the class interval.
2. **Median** is the middle value that divides the data into 2 equal parts.
 - To find the median in a data set, the data set has to be arranged in ascending order. The 2 possible cases to consider when finding the median are:
 - **Case 1: When n is odd**, where n is the number of data.
The median occurs exactly at the middle position, given by $\left(\frac{n+1}{2}\right)^{\text{th}}$ position.
 - **Case 2: When n is even**, where n is the number of data.
The median is the average of the two middle values.
For example, if the position of median is 3.5,
then the median = mean of 3rd value and 4th value.
3. **Mode** is the value that occurs most frequently in a data set. If there is no number in a data set that appears more than once, this data set has no mode.
 - There can be more than 1 mode in a data set.
 - When the data is presented in a frequency table, the mode is the data value with the highest frequency.
 - In grouped data, the class interval with the highest frequency is known as the **modal class**.
4. **Measures of Central Tendency (Mean, Median, Mode)**
 - **Appropriate use of average**
 - **Mean** is preferred when there are no extreme values as mean takes into account all the data values in the calculation.
 - **Median** is preferred when there are extreme values that affect the mean.
 - **Mode** is preferred when we are interested in the data value that is the most common or popular.



Secondary 3 - Emaths

Topic 1: Quadratic Equations

1. **Factorisation** by using Zero Product Principle (taught in Secondary 2).
2. **Completing the square** to change the algebraic expression from $ax^2 + bx + c$ into the form of $a(x + h)^2 + k$.
 - Coefficient of x^2 must be 1. Factorise out the negative/ positive coefficient of x^2 from the equation to make the coefficient of x^2 to be 1.
 - Complete the square for $x^2 + bx$ by adding $\left(\frac{b}{2}\right)^2$ so that the completed square will be $\left(x + \frac{b}{2}\right)^2$.
 - To ensure that the equation still has the same value, subtract $\left(\frac{b}{2}\right)^2$ from the equation after adding.
3. **General formula:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $ax^2 + bx + c = 0$, where a , b and c are real numbers and $a \neq 0$.
4. **Graphical method** (taught in Secondary 2).
5. **Completing the Square:** $y = \pm x^2 + bx + c$ can be written in the form of $y = \pm(x - h)^2 + k$

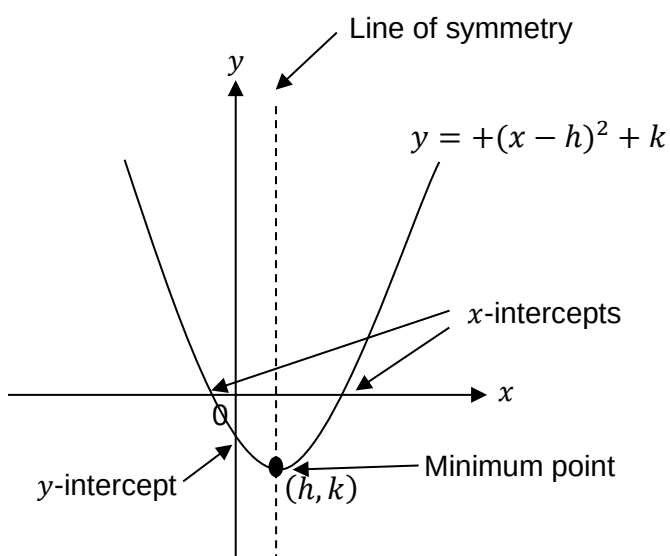
When sketching a quadratic curve using $y = \pm(x - h)^2 + k$ by completing the square, (h, k) is the vertex, where $h = \frac{-b}{2a}$ and $f(h) = k$.

$-x^2$ coefficient (negative) $\rightarrow$ Quadratic Curve with Maximum Point (sad-face curve).

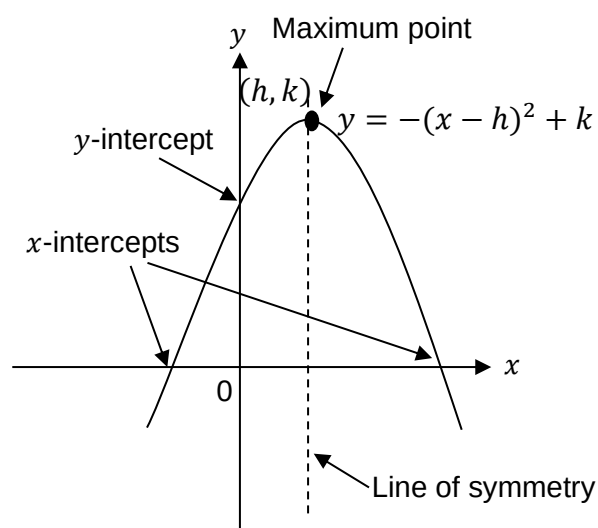
x^2 coefficient (positive) $\rightarrow$ Quadratic Curve with Minimum Point (smiley-face curve).

Note: Coefficient of $x^2 \neq 0$.

Line of symmetry: a straight line across the turning (maximum or minimum) point (h, k) .



Curve with minimum point



Curve with maximum point



Things to **take note** when sketching a quadratic curve:

- Label all **x -intercepts** (if any) by substituting $y = 0$ into the equation.
- Label all **y -intercepts** by substituting $x = 0$ into the equation.
- Label the **Line of symmetry** ($x = h$).
- Label the **Turning Point** (maximum/ minimum point) by using the line of symmetry.
- Label the **equation** of the curve.
- Label the **x -axis** and **y -axis**.

6. **Factorising:** $= \pm x^2 + bx + c$ can be also written in the form of $y = \pm(x - p)(x - q)$

When sketching a quadratic curve using $y = \pm(x - p)(x - q)$, p and q are the **x -intercepts**.
For y -intercept, substitute $x = 0$ to find y .

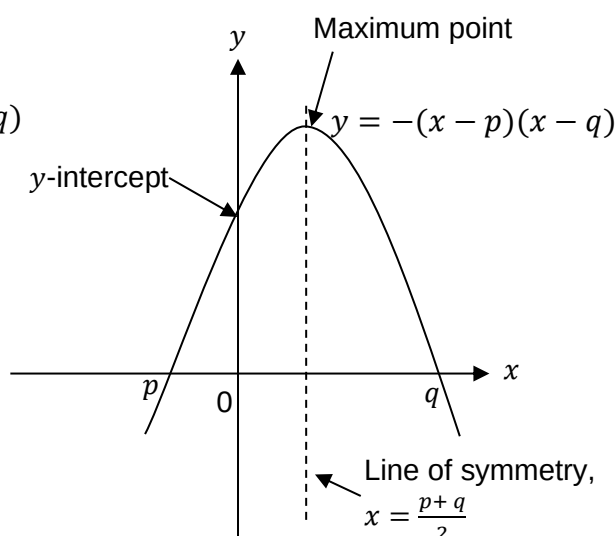
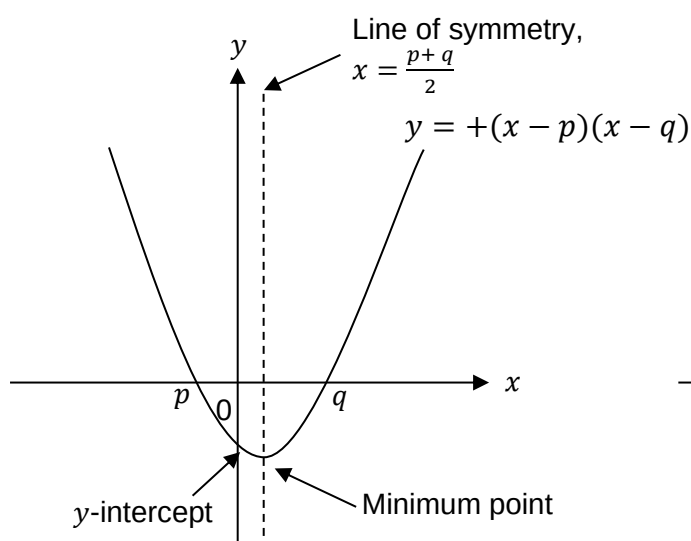
Determine the turning points (maximum/ minimum point)

- **Line of symmetry: Mid-point of the two x -intercepts** ($x = \frac{p+q}{2}$), which is also the x -coordinate of the turning point.
- Substitute the x -coordinate to find the y -coordinate of the turning point.

$-x^2$ coefficient (negative) $\rightarrow$ Maximum Point (sad-face curve).

x^2 coefficient (positive) $\rightarrow$ Minimum Point (smiley-face curve).

Note: Coefficient of $x^2 \neq 0$.



7. Using **Graphical Method** to Solve $ax^2 + bx + c = 0$

- Construct a table of values of x with its associated value of y for the equation of the quadratic curve. (Ensure that the points are well spread out.)
- Choose an appropriate scale for both the x -axis and y -axis to allow the largest possible graph to be drawn on the graph paper (for more accurate results) unless given by the question.
- Plot the points on the graph paper and connect them to form a smooth curve.
- Ensure that all the labeling are written on the graph paper.
- The solutions of the equation of $ax^2 + bx + c = 0$ will be at the x -intercepts (where $y = 0$).



Topic 2: Simultaneous Linear Inequalities

1. Addition Property

- We can **add a positive or negative number** from both sides of an inequality **without changing** the inequality sign.

$$\begin{aligned}x &> 3 \\x + 2 &> 3 + 2 \\x + 2 &> 5\end{aligned}$$

2. Subtraction Property

- We can **subtract a positive or negative number** from both sides of an inequality **without changing** the inequality sign.

$$\begin{aligned}x &> 3 \\x - 5 &> 3 - 5 \\x - 5 &> -2\end{aligned}$$

3. Multiplication/ Division Property

- We can **multiply/ divide a positive number** from both sides of an inequality **without changing** the inequality sign.

$$\begin{aligned}x &> 3 \\x \times 2 &> 3 \times 2 \\2x &> 6\end{aligned}$$

- However**, when we **multiply/ divide a negative number** from both sides of an inequality, we **MUST change** the inequality sign.

$$\begin{aligned}x &> 3 \\x \times (-3) &< 3 \times (-3) \\-3x &< -9\end{aligned}$$

4. Transitive Property

- If $x < y$ and $y < z$, then $x < z$.

$$\begin{aligned}x &< 3 \\3 &< z \\ \text{Therefore, } x &< z.\end{aligned}$$

5. We can solve linear inequalities the same way as linear equations **BUT** we need to pay attention to the changing of inequality signs when multiplying or dividing a negative number.

6. Given that $f(x)$, $g(x)$ and $h(x)$ are linear functions of x .

- To solve linear inequalities of $f(x) < g(x)$ **AND** $g(x) < h(x)$, we need to solve them individually and find the **range of solution that satisfies both the inequalities**.
- To solve linear inequalities simultaneously in the form of $f(x) < g(x) < h(x)$, we first need to find the solution(s) to each inequality separately by **splitting the inequality into two inequalities** in the form of $f(x) < g(x)$ and $g(x) < h(x)$ before solving them.

After that, we consider only the common solutions of the inequalities, which is the **overlapping/ intersection of the solutions** for the individual inequality. We can represent the inequalities on a number line to identify the overlapping region that represents the range of values which satisfies both inequalities.

- When there is **no overlapping region** on the number line, there is **no solution** that satisfies both inequalities.



7. It is always good to maintain a good habit of presenting your final answer of your inequality with the subject on the left-hand side. This will allow you to read the inequality sign better.

For example: $x < 3$, $x \geq -5$



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Topic 3: Indices and Standard Form

1. **Index Notation:** $a \times a \times a \times a \dots \times a$ (n times) = a^n where a is called the base and n is called the exponent or index. It is read as “a to the power of n”.

2. **Laws of Indices**

Law 1: $a^m \times a^n = a^{m+n}$

Law 2: $a^m \div a^n = a^{m-n}$

Law 3: $a^n \times b^n = (ab)^n$

Law 4: $a^n \div b^n = \left(\frac{a}{b}\right)^n$

Law 5: $(a^m)^n = a^{m \times n} = a^{mn}$

Law 6: $a^0 = 1$

Law 7: $a^{-m} = \frac{1}{a^m}$

Law 8: $a^{\frac{1}{n}} = \sqrt[n]{a}$

Law 9: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Rules for same base

Rules for same index

The denominator of the index is always the root.

3. **Equations involving Indices**

- If $a^x = a^n$, then by comparing the power, $x = n$ as they have the same base, a , where $a \neq -1, 0$ or 1 .
- If $a^x = b^x$, then by comparing the base, $a = b$ as they have the same power, x .
- An equation that contains a variable in index form is called an indicial/ exponential equation.
- To solve an exponential equation in the form of $a^x = b$, we express b in the form of a^n (to make both sides of the equation have the same base) so that we can compare the power whereby $x = n$.

4. **Simple Interest (Recap):** Simple interest is based on the principal amount.

$$\text{Simple Interest} = \frac{\text{Principal Sum} \times \text{Interest Rate per period} \times \text{Number of periods}}{100}$$

5. **Compound Interest:** Compound interest is based on the principal amount **AND** interest that accumulates in every period of time.

Total Amount (over a period of time) =

$$\text{Principal Sum} \left(1 + \frac{\text{Interest rate per period}}{100} \right)^{\text{Number of periods}}$$

6. For $\sqrt[n]{a}$, the value of a cannot be negative when the power, n , is an even number.
7. Denominator of a fraction cannot be 0 as it will be undefined.
8. **Standard Form:** Very large/ small numbers that are usually written in the standard form of,

$$A \times 10^n, \text{ where } n \text{ is an integer and } 1 \leq A < 10$$

- When expressing a very large number in standard form, the decimal point will be shifted to the left and the power of 10 will increase according to number of places that the decimal point is shifted.
- When expressing a very small number in standard form, the decimal point will be shifted to the right and the power of 10 will decrease according to the number of places that the decimal point is shifted.



9. Common Prefixes

Numerical Value	Powers of 10	English word	SI Prefix	Symbol
1 000 000 000 000	10^{12}	trillion	tera	T
1 000 000 000	10^9	billion	giga	G
1 000 000	10^6	million	mega	M
1 000	10^3	thousand	kilo	k
0.01	10^{-2}	hundredth	centi	c
0.001	10^{-3}	thousandth	milli	m
0.000 001	10^{-6}	millionth	micro	μ
0.000 000 001	10^{-9}	billionth	nano	n
0.000 000 000 001	10^{-12}	trillionth	pico	p



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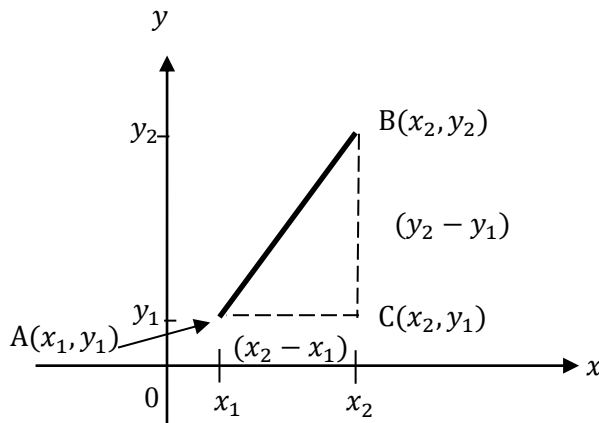
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Topic 4: Coordinate Geometry

1. Length of a Line Segment/ Distance between two points on Cartesian Coordinates

- **Length of Line AB** $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 - Derive from **Pythagoras' Theorem**, where
Length of AB $= \sqrt{AC^2 + BC^2}$



2. Gradient of a Line Segment/ Straight Line: Measure of the degree of steepness of the slope of the line.

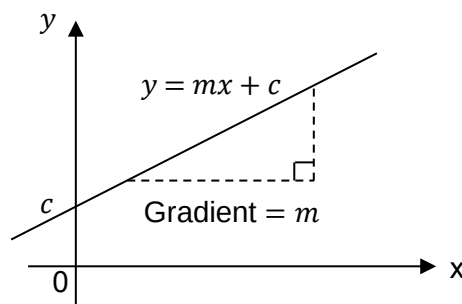
- **Gradient** $= \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$
- **Gradient of Line AB**, $m = \frac{y_2 - y_1}{x_2 - x_1}$
- **Different forms of Gradient of Straight Line**
 - **Positive** (upward sloping): When $(y_2 - y_1)$ and $(x_2 - x_1)$ have the same sign.
 - **Negative** (downward sloping): When $(y_2 - y_1)$ and $(x_2 - x_1)$ have opposite sign.
 - **Zero** (horizontal line): When $y_1 = y_2$ or $y_2 - y_1 = 0$, as the numerator (rise) is 0.
 - **Undefined** (vertical line): When $x_1 = x_2$ or $x_2 - x_1 = 0$, as the denominator (run) is 0.

3. Equation of a Line Segment/ Straight Line:

$$y = mx + c$$

where

- m is the gradient of the line, and
- c is the y -intercept of a line (crosses the y -axis).



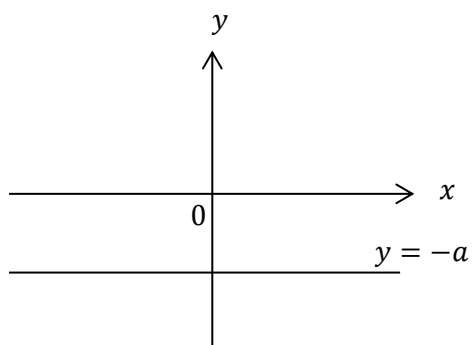
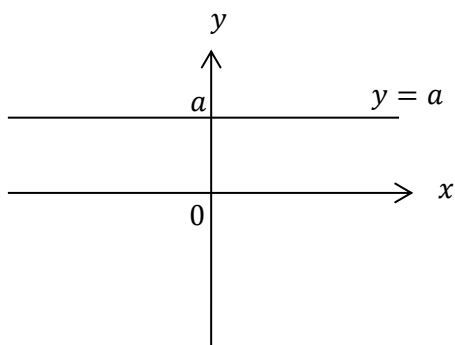
- If a straight line with gradient m , passes through a point (x_1, y_1) , then the equation of the straight line can be given by:
 - $m = \frac{y - y_1}{x - x_1} \rightarrow y - y_1 = m(x - x_1)$
- Equation of a Line Segment/ Straight Line can be formed when given the following information:
 - Two coordinate points on the straight line, or
 - One coordinate point on the straight line **and** the gradient of straight line.
- Equation of **Horizontal Line**:
 - $y = y\text{-intercept} = c$
- Equation of **Vertical Line**:
 - $x = x\text{-intercept}$



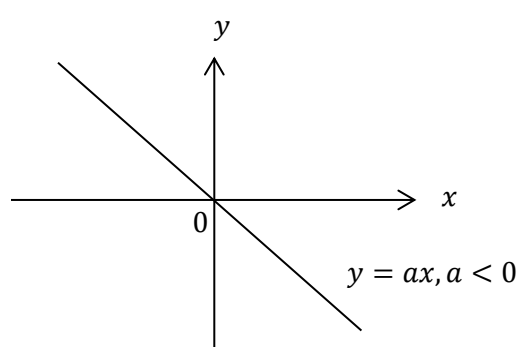
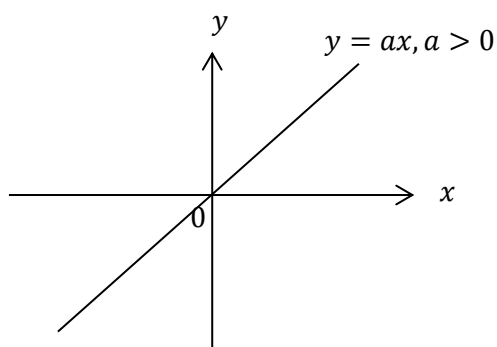
Topic 5: Graphs of Functions and Graphical Solution

1. Graphs of $y = ax^n$

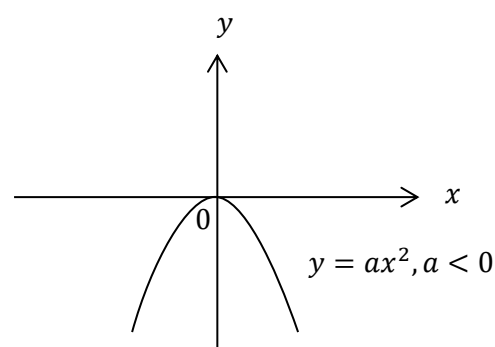
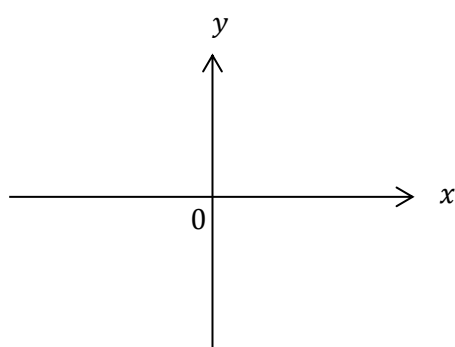
- When $n = 0$,



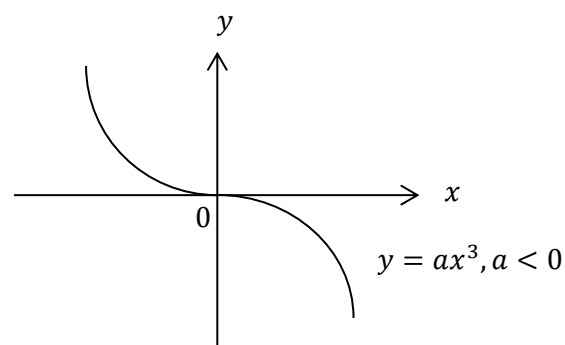
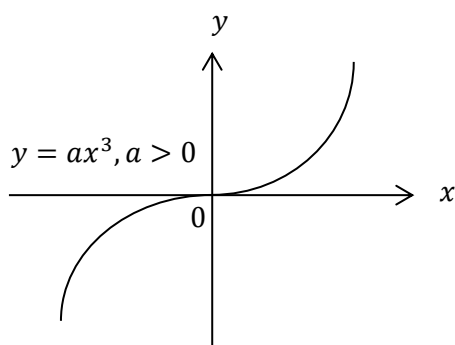
- When $n = 1$,



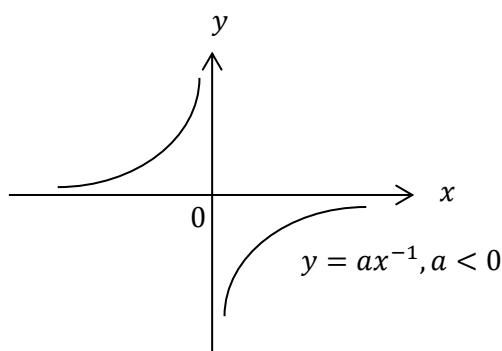
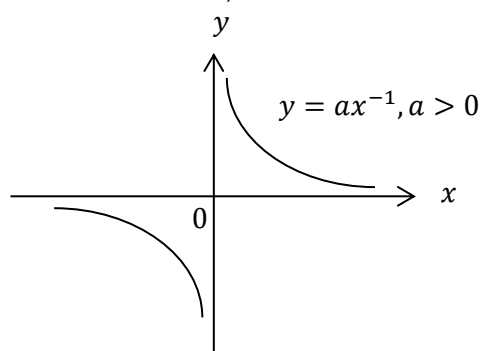
- When $n = 2$,



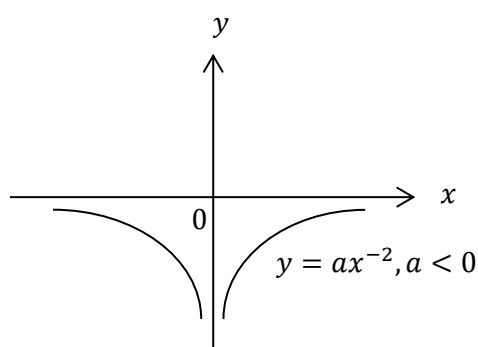
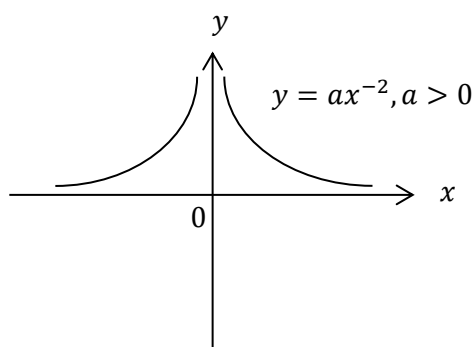
- When $n = 3$,



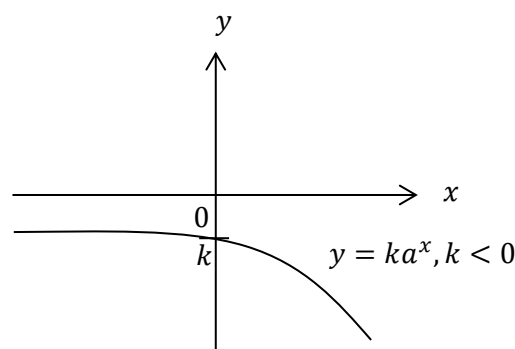
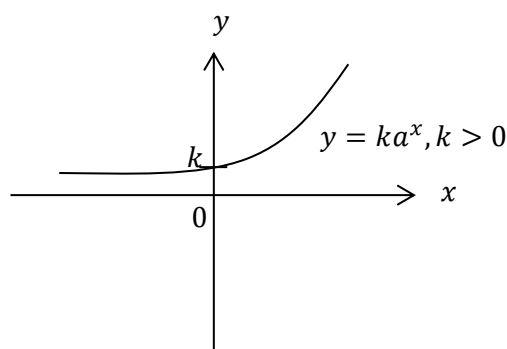
- When $n = -1$,



- When $n = -2$,



2. Graph of $y = ka^x$, where $a > 0$.



3. Graphical Solution

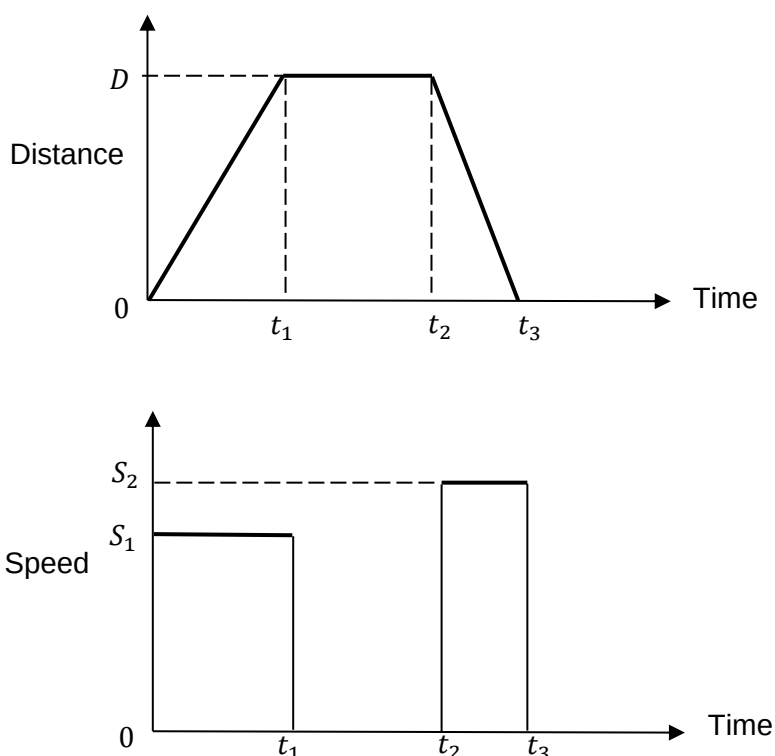
- Rewrite the equation of the non-linear graph with y in terms of x when necessary.
- Draw the graph of the equation above by using a table of x and y values that corresponds to the equation.
- When solving for $y = 0$ graphically, read off the x -coordinates of the point(s) of intersection(s) of the graph with the x -axis.
- When estimating the gradient of the curve at any specific point, draw a tangent to the curve at that point and find the gradient of the tangent.

4. Solving Equations by Graphical Method

- Step 1:** Express the given equation in a manner whereby one side of the equation gives the function of graph while the other side shows a linear expression.
- Step 2:** Draw the graphs of both equations on the same axes.
- Step 3:** The solution will be the x -coordinates of the points of intersection of the two graphs.



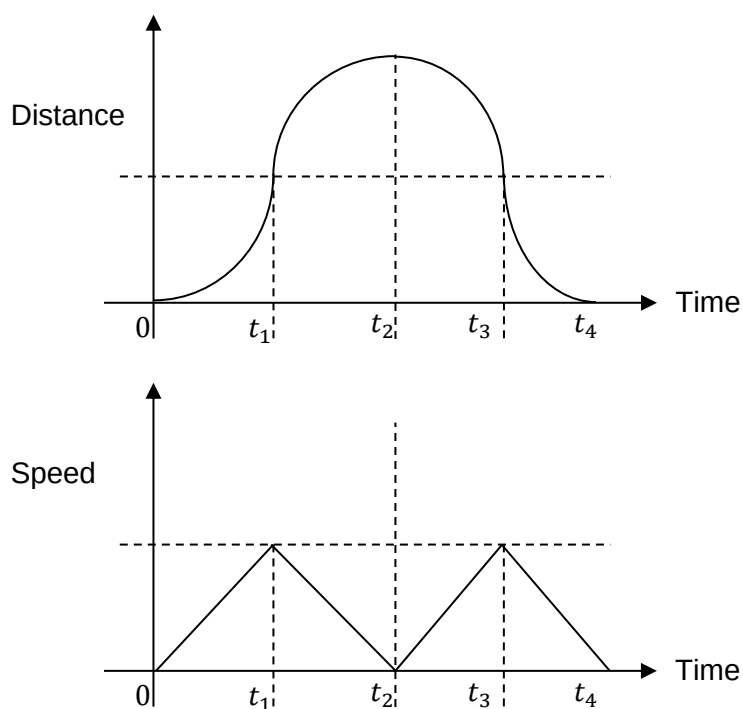
5. Relationship between a distance-time graph and a speed-time graph



- The gradient of a distance-time graph represents the speed of a moving object.
- Distance travelled by the object = Area under the speed-time graph
- Acceleration of the moving object = Gradient in the speed-time graph
- In Figure 1, the object is moving with constant speed at 0 to t_1 and t_2 to t_3 . It is at rest from t_1 to t_2 .
- Figure 2 shows its corresponding speed at the time intervals.
- At $t = 0$, the object is **at the starting point** where Distance and Speed is **zero**.
- From 0 to t_1 , the object is moving towards D where Distance is **increasing** while the Speed is **uniform/ constant** (Acceleration = 0).
- From t_1 to t_2 , the object is **at rest** at D . Therefore, the Distance **remains the same** and the Speed is 0 .
- From t_2 to t_3 , the object is **moving back to the starting point** where Distance is **decreasing** and Speed is **uniform/ constant** (Acceleration = 0).
- At t_3 , object is back at the starting point where Distance and Speed is **zero**.



6. Acceleration in distance-time graph and speed-time graph



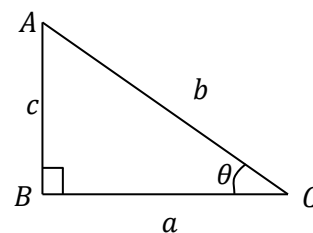
- From the diagrams above,
 - the object is travelling at constant acceleration from 0 to t_1 and t_2 to t_3 .
 - The object is travelling at constant deceleration from t_1 to t_2 and t_3 to t_4 .



Topic 6: Further Trigonometry

1. Pythagoras' Theorem (Recap)

- For right-angle triangle ABC , $a^2 + c^2 = b^2$, where b is the hypotenuse.

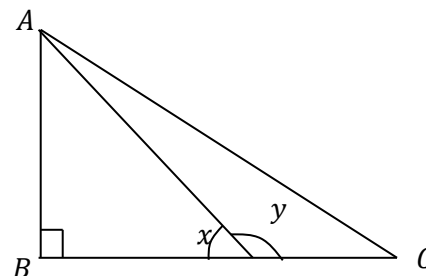


2. Trigonometric Ratios of a right-angle triangle (Recap)

- $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{c}{b}$ [SOH]
- $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{b}$ [CAH]
- $\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{c}{a}$ [TOA]

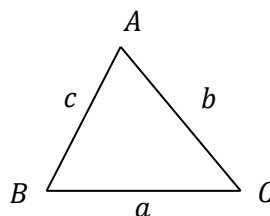
3. Trigonometric Ratios of Acute and Obtuse Angles

- As $x + y = 180^\circ$,
 - $\sin y = \sin(180^\circ - x) = \sin x$
 - $\cos y = \cos(180^\circ - x) = -\cos x$
 - $\tan y = \tan(180^\circ - x) = -\tan x$



4. Area of Triangle

- Area of $\triangle ABC$ is given by:
 - $\frac{1}{2}ab \sin C$
 - $\frac{1}{2}bc \sin A$
 - $\frac{1}{2}ac \sin B$



5. Sine Rule

- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or
- $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Can be used when
 - Given two angles and a side
 - Two sides and an angle opposite a given side
- Ambiguous case:** When a set of given information using Sine Rule gives two possible sets of solutions. It occurs when
 - Length of two sides and a non-included angle are given, **and**
 - The given angle is acute, **and**
 - The length of the side opposite the given angle is less than the other given side.
- No Ambiguous case** when:
 - The given angle is obtuse, **or**
 - The length of side opposite the given angle is greater than the other given side.

6. Cosine Rule

- $a^2 = b^2 + c^2 - 2bc \cos A \leftrightarrow \cos A = \frac{b^2 + c^2 - a^2}{2bc}$
- $b^2 = a^2 + c^2 - 2ac \cos B \leftrightarrow \cos B = \frac{a^2 + c^2 - b^2}{2ac}$
- $c^2 = a^2 + b^2 - 2ab \cos C \leftrightarrow \cos C = \frac{a^2 + b^2 - c^2}{2ab}$
- Can be used when:
 - Given two sides and one included angle
 - Given three sides

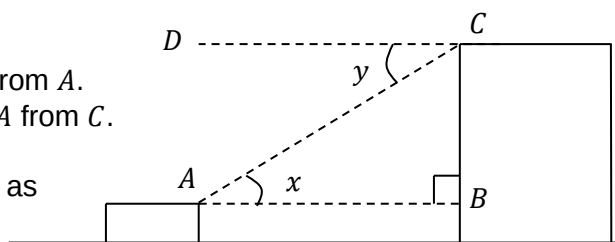


Topic 7: Applications of Trigonometry

1. Angles of Elevation and Depression

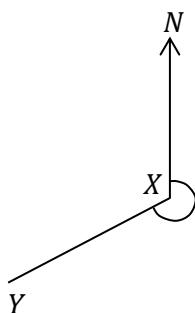
- $\angle BAC$ or $\angle x$ is the **angle of elevation** of C from A .
- $\angle ACD$ or $\angle y$ is the **angle of depression** of A from C .
- Using alternate angles properties, $\angle x = \angle y$.

Angle of elevation of C from A is the same as **angle of depression** of A from C .



2. Bearings

- Use to describe a **precise position** and **direction**.
- A measure from the **North** in **clockwise** direction.
- Ranges from 0° to 360° .
- Written as a **3-digit number** in degrees if it is an integer (for example, 045° , 190° , 005°) and rounded off to 1 decimal place when necessary (for example, 029.9° , 130.4° , 209.5°).



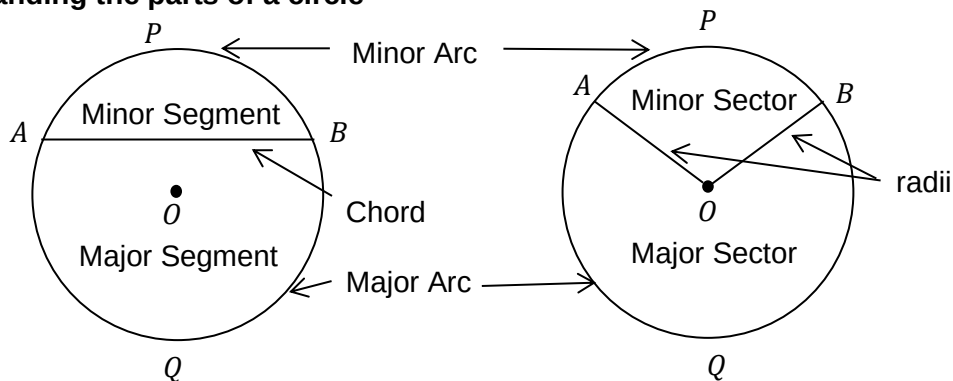
- Finding the bearings from Point Y from Point X :
 - Draw the north line at Point X .
 - Join Point X to Point Y .
 - Measure the angle from the north line at Point X to the line XY in a clockwise direction.

3. **Shortest Distance** from a particular point to a line is the **perpendicular distance** from the point to the line.



Topic 8: Arc Length, Sector Area and Radian Measure

1. Understanding the parts of a circle



- **Chord**, represented by line AB , connects any two points on the circumference of a circle. **Diameter** is the longest chord.
- **Arc** is a part of the circumference of a circle.
 - **Minor arc** is represented by arc APB .
 - **Major arc** is represented by arc AQB .
- **Segment** is the region that is enclosed by a chord and an arc of a circle. A chord divides a circle into two segments.
 - **Minor segment** is represented by the region enclosed by arc APB and chord AB .
 - **Major segment** is represented by the region enclosed by arc AQB and chord AB .
- **Sector** is a part of a circle that is enclosed by two radii and an arc of a circle.
 - **Minor sector** is represented by the region enclosed by $AOBP$.
 - **Major sector** is represented by the region enclosed by $AOBQ$.

2. Length of Arc and Area of Sector (in degrees)

- Arc length $= \frac{x^\circ}{360^\circ} \times \text{Circumference}$
 $= \frac{x^\circ}{360^\circ} \times 2\pi r$
- Sector area $= \frac{x^\circ}{360^\circ} \times \text{Area of circle}$
 $= \frac{x^\circ}{360^\circ} \times \pi r^2$

where x° is the angle subtended by the arc at the center of the circle of radius r .

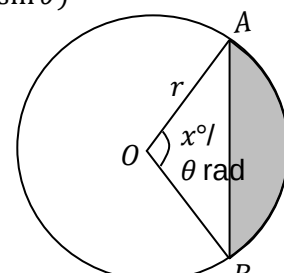
3. Radian Measure

- Angles can be measured in **degrees** or **radians**
- Relationship between degrees and radians: $180^\circ = \pi \text{ rad}$
- Length of arc and area of sector (in radians)
 - Arc length, $s = r\theta$
 - Sector area $= \frac{1}{2}r^2\theta$

where θ is the angle, in radians, subtended by the arc at the centre of the circle of radius r .

4. Area of Segment (Shaded region/ Minor Segment)

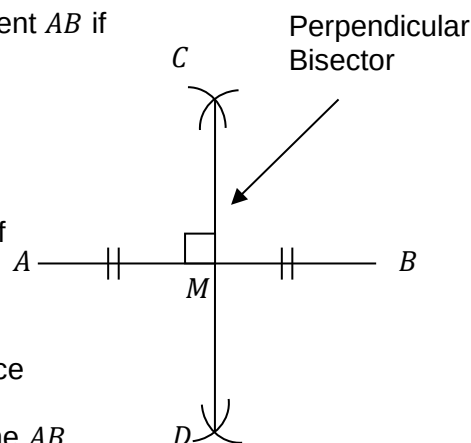
- **Area of Minor Segment (degrees)** $= \frac{x^\circ}{360^\circ} \times \pi r^2 - \frac{1}{2} \times r^2 \times \sin x^\circ$
- **Area of Minor Segment (radians)** $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin \theta = \frac{1}{2}r^2(\theta - \sin \theta)$
- Area of Major Segment (unshaded region)
 $= \text{Area of circle} - \text{area of minor segment}$
 $= \text{Area of major sector} + \text{area of triangle}$



Topic 9: Congruence and Similarity Tests

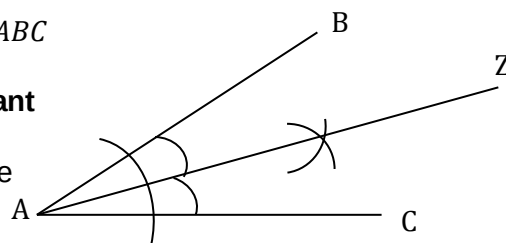
1. Perpendicular Bisector

- A line CD is the **perpendicular bisector** of a line segment AB if
 - CD is perpendicular to AB
 - CD passes through the midpoint, M , of AB
- Any point on the perpendicular bisector CD will be **equidistant** from the points A and B .
- **Constructing a Perpendicular Bisector**
 - Place the compass on point A and adjust the arms of the pair of compasses so that the distance between ends is more than half the length of line AB .
 - Draw an arc that is around point C and D
 - Without adjusting the arc length of the compass, place it on point B . Draw another arc of the same radius to cut the first arc that is drawn above and below the line AB .
 - Label the points of the intersection of the 2 arcs as C and D and mark the right angle at M and the equal lengths of AM and BM .



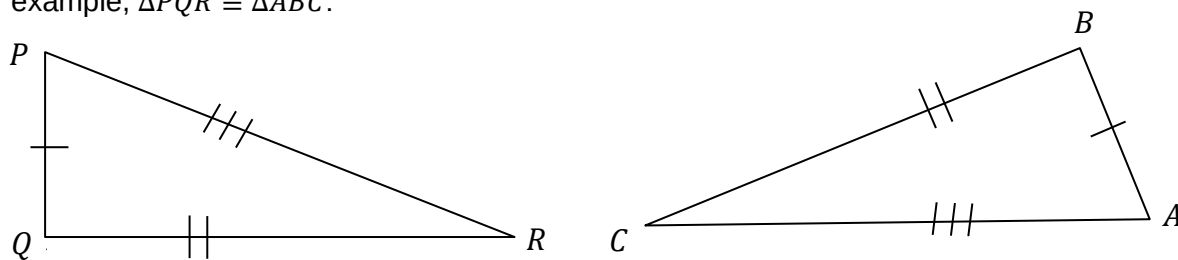
2. Angle Bisector

- If line AZ is the angle bisector of $\angle ABC$, it divides $\angle ABC$ into 2 equal angles, whereby $\angle BAZ = \angle CAZ$.
- Any point on the angle bisector AZ will be **equidistant** from the lines AB and AC .
- A point on the angle bisector is the shortest distance from AB when it is perpendicular to the line AB .
- **Constructing an Angle Bisector**
 - Step 1: Place the compass on point A and draw an arc that crosses both sides of the angle.
 - Step 2: Place the compass on the arc that intersects the line AB and draw an arc of suitable radius (may be different from the step 1) between the two sides of the angle.
 - Step 3: Without adjusting the radius/ the arc length of the compass, place it on the first arc that intersects line AC . Draw another arc that cuts the arc drawn in step 2.
 - Step 4: Draw a straight line from the intersection of the 2 arcs drawn from step 2 and 3 to point A . Mark the equal angle.



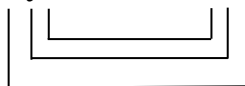
3. Congruent Figures

- Two figures are **congruent** when they have exactly the **same size and shape**, noted by the symbol " $\equiv$ ", representing 'is congruent to'.
- If two figures are congruent,
 - all their corresponding **angles** are **equal** (same shape), **and**
 - all their corresponding **sides** are **equal** (same size)
- For example, $\triangle PQR \equiv \triangle ABC$:



- $\angle P = \angle A$, $\angle Q = \angle B$ and $\angle R = \angle C$, (all corresponding angles are equal)
- $PQ = AB$, $QR = BC$ and $PR = AC$. (all corresponding sides are equal)
- Note that the symbol " $\equiv$ " expresses that the way the two polygons are named have to be in a concise and precise manner.

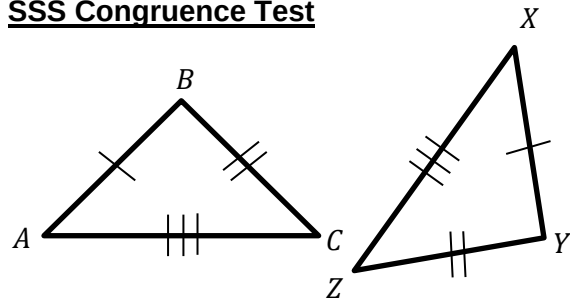
This means that we write $\triangle PQR \equiv \triangle ABC$ in a specific manner, whereby the **order**



of the vertices of one triangle **must correspond** to the other of the vertices of the other triangle.

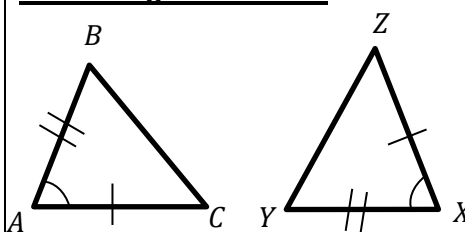
• Congruence Tests

SSS Congruence Test



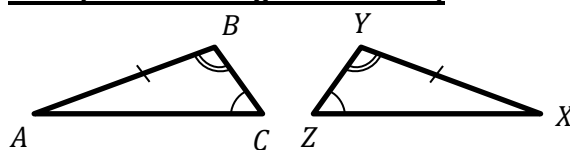
If $AB = XY$, $BC = YZ$ and $AC = XZ$,
then $\triangle ABC \equiv \triangle XYZ$.

SAS Congruence Test



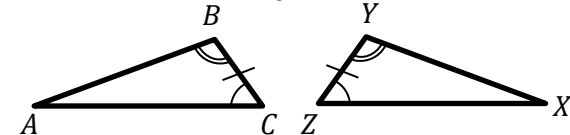
If $AC = XZ$, $AB = XY$ and $\angle BAC = \angle YXZ$,
then $\triangle ABC \equiv \triangle XYZ$.

AAS (or ASA Congruence Test)



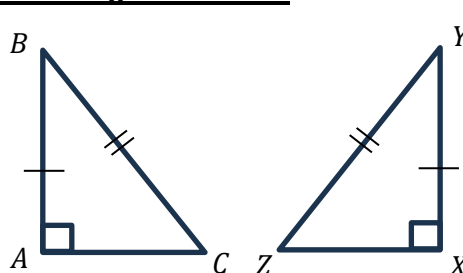
If $AB = XY$, $\angle ABC = \angle XYZ$ and
 $\angle BCA = \angle YZX$, then $\triangle ABC \equiv \triangle XYZ$.

or



If $BC = YZ$, $\angle ABC = \angle XYZ$ and
 $\angle BCA = \angle YZX$, then $\triangle ABC \equiv \triangle XYZ$.

RHS Congruence Test

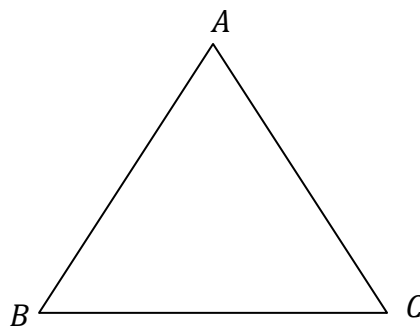
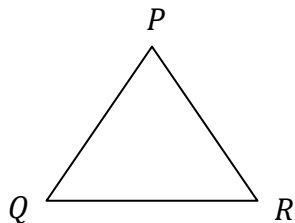


If the hypotenuse side ($BC = YZ$) and one
of the short side ($AB = XY$) of the
right-angled triangles are equal,
then $\triangle ABC \equiv \triangle XYZ$.



4. Similar Figures

- Two figures are **similar** when they have exactly the same shape but do not need to be of the same size. If two similar figures have the same size, they are **congruent**.
- If two figures are similar,
 - all their corresponding **angles** are **equal**, and
 - all the **ratios** of the lengths of their corresponding **sides** are **equal**.
- For example, $\triangle PQR$ is similar to $\triangle ABC$:



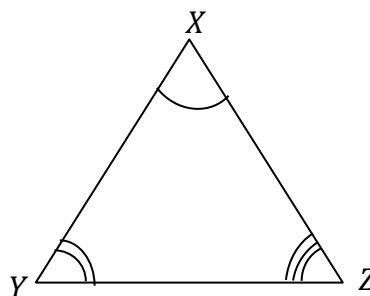
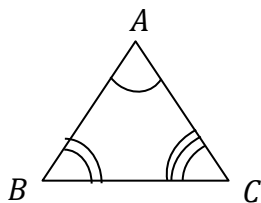
➤ $\angle P = \angle A$, $\angle Q = \angle B$ and $\angle R = \angle C$,

(all corresponding angles are equal)

➤ $\frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC}$

(all the ratios of their corresponding sides are equal)

- Similarity Tests**



- **AA Similarity Test**

Two pairs of corresponding angles are equal.

$\angle ABC = \angle XYZ$ and $\angle BAC = \angle YXZ$

- **SSS Similarity Test**

All corresponding sides are proportional.

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$

- **SAS Similarity Test**

Two pairs of corresponding sides are proportional and the corresponding **included angles** are equal.

$$\frac{AB}{XY} = \frac{BC}{YZ} \text{ and } \angle ABC = \angle XYZ$$



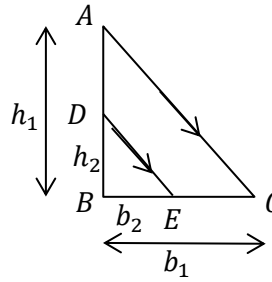
Topic 10: Area and Volume of Similar Figures and Solids

1. Ratios of Area of Similar Figures, $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

- Ratios of Areas of Similar Triangles**

➤ $\triangle ABC$ is similar to $\triangle DBE$

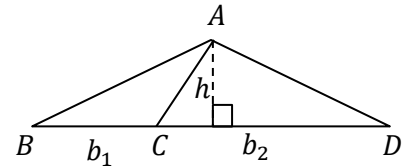
➤ $\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DBE} = \left(\frac{h_1}{h_2}\right)^2 = \left(\frac{b_1}{b_2}\right)^2$



- Ratios of Areas of Triangles with Common Heights**

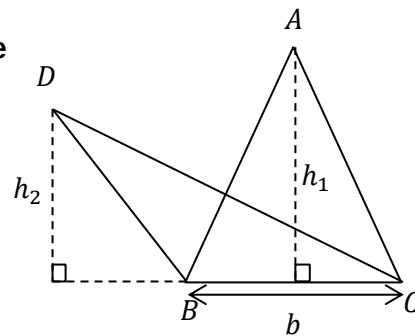
➤ $\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle ACD} = \frac{\frac{1}{2} \times b_1 \times h}{\frac{1}{2} \times b_2 \times h} = \frac{b_1}{b_2}$

➤ $\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle ABD} = \frac{\frac{1}{2} \times b_1 \times h}{\frac{1}{2} \times (b_1 + b_2) \times h} = \frac{b_1}{b_1 + b_2}$



- Ratios of Areas of Triangles with Common Base**

➤ $\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle BCD} = \frac{\frac{1}{2} \times b \times h_1}{\frac{1}{2} \times b \times h_2} = \frac{h_1}{h_2}$



2. Mass and Volume of Similar Figures

- Volume of Similar Figures, $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$**

- Mass of Similar Figures, $\frac{m_1}{m_2} = \frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$, if they have the same density.**

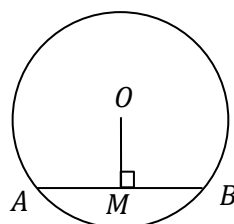
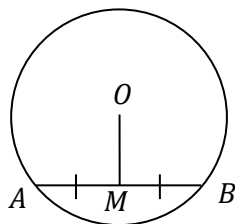
➤ Note: Mass = Volume $\times$ Density



Topic 11: Geometrical Properties of Circles

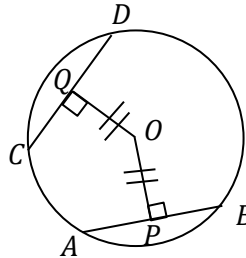
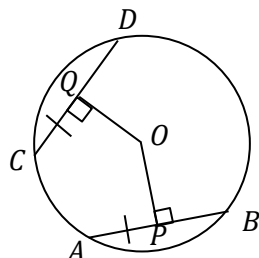
1. Symmetric Properties of Circles

• Perpendicular Bisector of a Chord ($\perp$ Bisector of Chord)



- If a line OM passes through the centre of a circle and **bisects the chord** AB , then the line is **perpendicular to the chord**, $OM \perp AB$.
- If a line OM passes through the centre of a circle and is **perpendicular to a chord** AB , then the **line bisects the chord**, $AM = MB$.
- If a line OM bisects the chord AB of a circle and is perpendicular to the chord AB , the line **passes through the centre** of the circle, where the **perpendicular bisector** of a chord will pass through the centre of the circle.

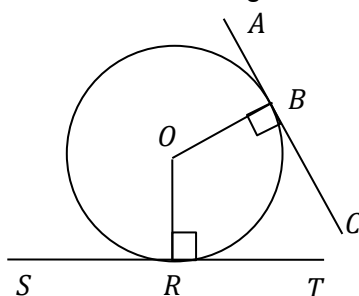
• Equal Chords



- **Equal chords** ($AB = CD$) of a circle are **equidistant from the centre of the circle**, where $OP = OQ$.
- Chords that are **equidistant from the centre of the circle**, $OP = OQ$, are **equal in length**, where $AB = CD$.

• Tangent Perpendicular to Radius (tangent $\perp$ radius)

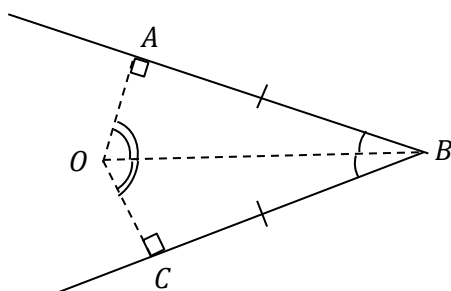
Tangent to a circle is a straight line that meets the circle only at one point.



- Tangents to a circle are **perpendicular** to the radii of the circle at the points of contact.
- $\angle ORT = \angle ORS = 90^\circ$
- $\angle OBC = \angle OBA = 90^\circ$



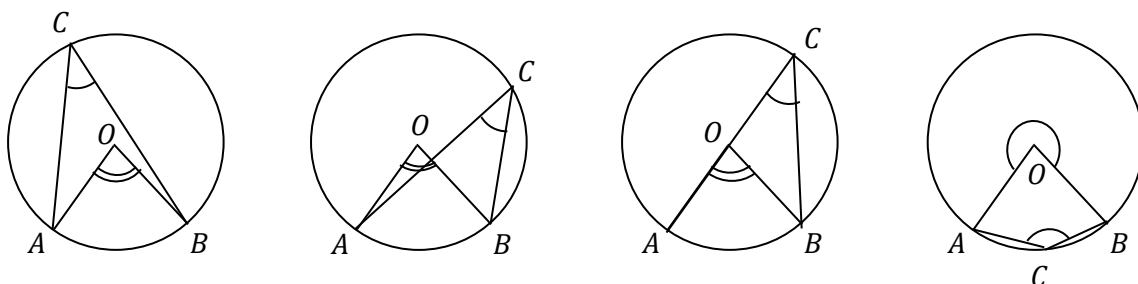
- **Tangents from External Point**



- Tangents from an external point are **equal in length**, $AB = CB$.
- Line from the centre of a circle to an external point **bisects** the angle between the two tangents from the external point, $\angle ABO = \angle CBO$.
- Line from the centre of a circle to an external point **bisects** the angle between the radii OA and OC , $\angle AOB = \angle COB$.
- **Note:** $\triangle OAB \cong \triangle OCB$.

2. Angle Properties of Circles

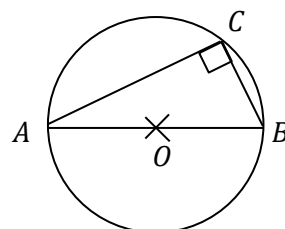
- **Angle at Centre ($\angle$ at centre = $2\angle$ at circumference)**



An angle at the centre of a circle is twice that of any angle at the circumference subtended by the same arc.

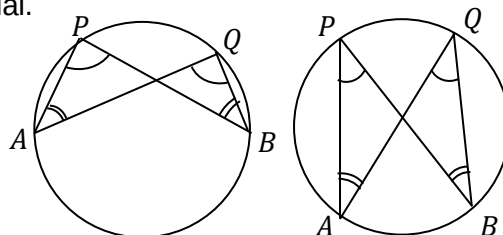
- **Angle in Semicircle (rt. $\angle$ in semicircle)**

- An angle in a semicircle is always a right angle.
- $\angle ACB = 90^\circ$



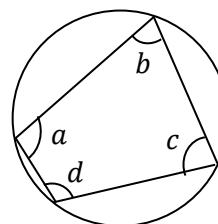
- **Angles in Same Segment ($\angle$ s in same segment)**

- Angles in the same segment of a circle are equal.
- $\angle APB = \angle AQB$
- $\angle PAQ = \angle PBQ$



- **Angles in Opposite Segments ($\angle$ s in Opp. Segments)**

- Angles in opposite segments are supplementary.
- $\angle a + \angle c = 180^\circ$
- $\angle b + \angle d = 180^\circ$



Secondary 4 – Emaths

Topic 1: Set Notation and Language

1. A **set** is a **collection of well-defined and distinct objects**, such as numbers, letters, symbols, etc.
2. The **objects** in a set are called the **members** or **elements** of the set.
3. Ways of describing a set:
 - **Listing all the elements** in a set in **set notation**.
For example, $S = \{a, b, c\}$.
 - **Describing a set in words**.
For example, S is a set of prime numbers.
 - **Describing the elements in a set in set notation**.
For example, $S = \{x : x \text{ is a natural number such that } 1 \leq x < 5\}$.
4. **Set Language and Notation:**
 - $\in$ represents "... is an element of ..."
 - $\notin$ represents "... is not an element of ..."
 - **Number of elements in a set** A is represented by $n(A)$.
 - **Universal set** which contains all the sets is represented by ε .
 - **Null set** or **Empty set** is represented by $\emptyset$ or $\{ \}$ which contains no elements.
 - **Equal set** is represented by $A = B$ when the two sets, A and B have the **same elements**.
 - A is a **proper subset of** B is represented by $A \subset B$, if **every element of** A **is also an element of** B , and $A \neq B$. If $A \subset B$ and $B \subset C$, then $A \subset C$.
 - **Complement of set** A is represented by A' whereby it **contains all elements in ε except those in** A .
 - **Intersection of sets** A and B is represented by $A \cap B$, where it refers to the set of **common elements find in both** A **and** B **only**.
 - **Union of sets** A and B is represented by $A \cup B$, where it refers to the set of elements which belongs to set A or set B or both.
5. **Understanding of Venn Diagrams:** Pictorial representation of relationships involving sets.
6. Note: The set $\{\emptyset\}$ is not an empty set as it contains one element, $\emptyset$. Therefore, the set $\{\emptyset\} \neq \{ \}$ or $\emptyset$.



Topic 2: Statistical Data Analysis

1. Measures of Central Tendency (Recap)

- **Mean: Average** of all the values, obtained by dividing the sum of all values by the number of values.
 - For a frequency distribution where f = frequency, x = value,

$$\text{Mean, } \bar{x} = \frac{\text{Sum of Data}}{\text{Frequency of Data}} = \frac{\sum fx}{\sum f}$$

- For grouped data, the mid-value of each class is used to represent the data value in each group.
- **Median: Middle value** that divides the number of data into two halves. 50% of the observations is below the median and 50% of the observations is above the median. (Note: Always arrange the data in ascending order.)
 - If the total number of data values is odd, the median occurs at exactly the middle term, which is $\left(\frac{n+1}{2}\right)^{\text{th}}$ term.
 - If the total number of data is even, the median is the average of the two middle values, which is the average of $\left(\frac{n}{2}\right)^{\text{th}}$ term and $\left(\frac{n+2}{2}\right)^{\text{th}}$ term.
- **Mode:** Value that occurs **most frequently** in a set of values.
 - If all the data values have the same frequency, the data set has no mode.
 - If a data set has two modes, it is known as a bi-modal distribution.
 - For grouped data, the class interval with the highest frequency is known as the modal class.

2. Standard Deviation: Use to measure the spread of data from the mean.

- Useful in comparing the spread of two separate data sets that have approximately the same mean.
 - The **larger the standard deviation**, the **more wide-spread** the values are from its mean – **less consistency** of the data values relative to the mean.
 - The **smaller the standard deviation**, the **less wide-spread** the values are from its mean – **more consistency** of the data values relative to the mean.
- Standard deviation (SD) is given as:

$$SD = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

3. Median, Quartiles, Percentiles, Range and Interquartile Range

- **Lower Quartile (Q_1):** 25th percentile or the middle value of the lower half of the range.
- **Median (Q_2):** 50th percentile
- **Upper Quartile (Q_3):** 75th percentile or the middle value of the upper half of the range.
- **Range:** Largest value – Smallest Value
- **Interquartile range ($Q_3 - Q_1$):** Indicates how the numbers in a data set are spread around the median. It shows the **degree of variation** the data values are. It is the range of the **middle 50%** of the data.
 - **Small Interquartile Range** indicates that the data is **cluster closely around the median**.
 - **Large Interquartile Range** indicates that the data is **spread across a wider range of values around the median**.
- **P_x :** x^{th} percentile



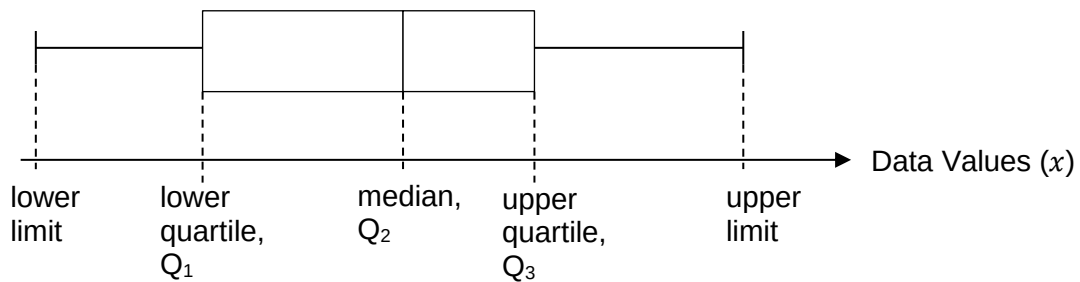
4. **Cumulative Frequency Table and Curve**

- **Cumulative Frequency Distribution**

- **Cumulative Frequency Curve:** Curve that shows the cumulative frequency of a set of data which is obtained from summing up all the total frequencies in previous smaller data values up to that point.
- Plotted with y -axis being cumulative frequency against the ungrouped or grouped data values in the x -axis.
- For grouped data, the curve is drawn by plotting the cumulative frequency against the upper boundary of the class.

5. **Box-and-Whisker Plot:** Simplified diagram that illustrates key data such as **median**, **quartiles** (lower quartile and upper quartile) and the **range** (lower limit and upper limit) for quick comparison of two sets of data.

- **Range** = Upper limit – Lower limit



Topic 3: Probability of Combined Events

1. Terms used in Probability

- **Experiment:** Process with a result/ an outcome that is determined by chance or probability.
- **Outcomes:** Possible results of an experiment.
- **Sample space:** A collection of all possible outcomes of a probability experiment.
- **An event:** Subset of the sample space which results in one or more outcomes of an experiment satisfying a given condition.

2. Probability of an Event

- **Probability, $P(E) = \frac{\text{Number of favorable outcomes for event E}}{\text{Total number of all possible outcomes}}$**
- If E is a **certain event**, $P(E) = 1$.
- If E is an **impossible event**, $P(E) = 0$.
- If E is **any event**, $0 \leq P(E) \leq 1$, and
- E' is the event that E is **not happening**, then $P(E') = 1 - P(E)$.

3. Possibility Diagrams and Tree Diagrams (for simple combined events)

- **Possibility diagrams**
 - Representation of outcomes by points on a grid or using a table to represent the outcomes based on a common context.
 - Used only for two events with many outcomes.
- **Tree diagrams**
 - Representation of different outcomes at the end of the branches with their respective probabilities on the branches.
 - Used for more events (≥ 2) with few outcomes (≤ 3).

4. Mutually Exclusive Events (Addition Law of Probability)

- Mutually Exclusive Events: Events cannot occur simultaneously or at the same time in one occurrence.
- For two mutually exclusive events, A **or** B to occur (no common outcomes in A and B):

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$$

- For three mutually exclusive events, A **or** B **or** C to occur:

$$P(A \text{ or } B \text{ or } C) = P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

5. Independent Events (Multiplication Law of Probability)

- Independent events: Occurrence of event A does not affect that of event B (with replacement)
- For two independent events, A **and** B to occur:

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$

- For three independent events, A **and** B **and** C to occur:

$$P(A, B \text{ and } C) = P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$



6. Dependent Events (Multiplication involving Conditional Probabilities)

- Dependent events: Occurrence of second event will depend on the outcome of the first event (without replacement).
- For two dependent events, A occurs first **and** B occurs next:

$$P(\text{A occurs first and B occurs next}) = P(A) \times P(\text{B occurs after A occurs})$$

- For three dependent events, A occurs, followed by B, followed by C:

$$P(\text{A occurs, followed by B, followed by C}) = P(A) \times P(\text{B occurs after A occurs}) \times P(\text{C occurs after A occurs first followed by B occurs next})$$

- Tree diagrams are commonly used.

7. The minimum value of the probability is always 0 and the maximum value of the probability is always 1. Therefore, $0 \leq P(E) \leq 1$.



Topic 4: Matrices

1. **Definition of Matrix:** A rectangular array of numbers that is arranged in rows and columns.

For example: $\begin{pmatrix} 21 & 32 & 23 & 54 \\ 10 & 11 & 12 & 13 \\ 32 & 28 & 56 & 78 \end{pmatrix}$ is a matrix with 3 rows (horizontal) and 4 columns (vertical)

- The order of the above matrix is **3 by 4**, which is also known as **3×4 matrix**.
- In general, a **$m \times n$ matrix** has **m rows** and **n columns**.
- The **numbers** inside a matrix are called the **elements** of the matrix.

2. **Always** write the number of rows first when stating the order of matrix.

3. **Types of Special Matrices** (Plural of Matrix)

- **Row Matrix:** Matrix with only a single row.
➤ Examples: $(2 \ 4)$, $(5 \ -6 \ 8)$
- **Column Matrix:** Matrix with only a single column.
➤ Examples: $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 5 \\ -8 \end{pmatrix}$
- **Square Matrix:** Matrix that has equal number of rows and columns.
➤ Having order of $n \times n$
➤ Examples: A 2×2 square matrix of $\begin{pmatrix} 2 & 5 \\ 3 & 6 \end{pmatrix}$, and a 3×3 square matrix of $\begin{pmatrix} 1 & -3 & 2 \\ -1 & 4 & 5 \\ 9 & 3 & 6 \end{pmatrix}$.
- **Null/ Zero Matrix:** Matrix with all its elements being zero.
➤ Examples: $(0 \ 0)$, $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

4. **Equal Matrices:** Two matrices are **equal** when they have the **same order** and their **corresponding elements are equal**.

- Example: Given that $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 \\ -1 & 5 \end{pmatrix}$, if $A = B$, then $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ -1 & 5 \end{pmatrix}$.
Therefore, $a = 3$, $b = 2$, $c = -1$, $d = 5$.

5. **Addition and Subtraction of Matrices**

- Matrices must have the **same order** for addition/ subtraction of matrices to take place.
- Addition/ Subtraction of matrices can be obtained by adding the **corresponding elements** of the matrices.
- In general, $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \pm \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} a \pm w & b \pm x \\ c \pm y & d \pm z \end{pmatrix}$

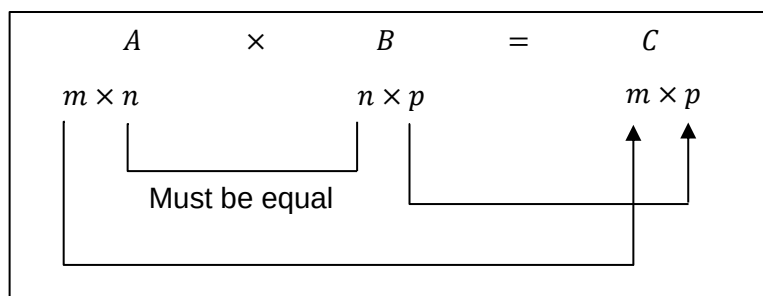
6. **Multiplication of a Matrix by a Scalar/ Real Number**

- In general, when a matrix is multiplied by a scalar k , every element in the matrix is multiplied by k .
- Example: $k \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$



7. Multiplication of Two Matrices

- In general, for multiplication of two matrices A and B , the product AB is only possible if the number of columns of A = number of rows of B .



- If A is a $m \times n$ matrix and B is a $n \times p$ matrix, the result of the product will be a $m \times p$ matrix.
- Example: If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$, then

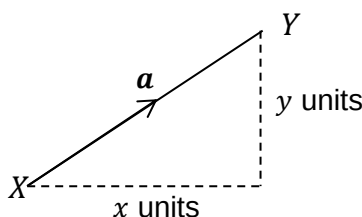
$$AB = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}$$

- The multiplication of matrices is **not commutative**, where $AB \neq BA$.
- The multiplication of matrices is **associative**, where $A(BC) = (AB)C$.



Topic 5: Vectors

- Definition of Vectors:** Quantity that has both **magnitude** and **direction**.
- Representation of Vectors:** It can be represented by a **directed line segment** where the **direction** of the line segment is that of the vector and the **length** of the line segment represents the **magnitude** of that vector.



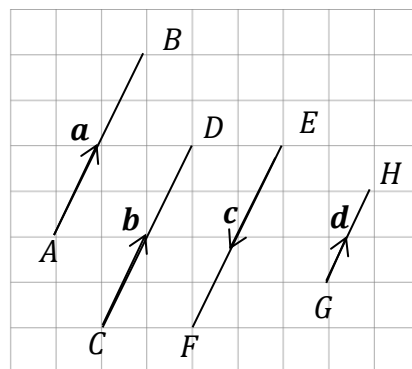
- The vector above is denoted by $\overrightarrow{XY}$, $\mathbf{a}$ and $\begin{pmatrix} x \\ y \end{pmatrix}$. As we cannot bold the letter “ $\mathbf{a}$ ” in writing, we use $\mathbf{a}$.
- Point X is the **initial/starting** point and Point Y is the **ending/terminal** point.
- The magnitude of XY is denoted by $|\overrightarrow{XY}|$ or $|\mathbf{a}|$ or $\left| \begin{pmatrix} x \\ y \end{pmatrix} \right|$.
- $\begin{pmatrix} x \\ y \end{pmatrix}$ is a **column vector** where x represents the x units in x –direction and y represents the y units in y –direction. They are called the components of the vector $\begin{pmatrix} x \\ y \end{pmatrix}$ where x is the x –component and y is the y –components.
 - x –direction is the horizontal direction whereby the position moves to the right in positive units and left in negative units.
 - y –direction is the vertical direction whereby the position moves up in positive units and down in negative units.
- In general, the magnitude of a column vector $\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ is $|\mathbf{a}| = \sqrt{x^2 + y^2}$ by using Pythagoras’ Theorem. Therefore,
 - Magnitude of a horizontal vector $\mathbf{b} = \begin{pmatrix} x \\ 0 \end{pmatrix}$ is $|\mathbf{b}| = x$
 - Magnitude of a vertical vector $\mathbf{c} = \begin{pmatrix} 0 \\ y \end{pmatrix}$ is $|\mathbf{c}| = y$

3. Equal, Negative and Zero Vectors

- Equal vectors** are vectors with the **same magnitude** and **direction**.
 - For example, $\overrightarrow{AB} = \overrightarrow{CD}$ or $\mathbf{a} = \mathbf{b}$.
- Negative vectors** are vectors that have the **same magnitude** but in **opposite directions**.
 - For example, $\overrightarrow{AB} = -\overrightarrow{EF}$ or $\mathbf{a} = -\mathbf{c}$

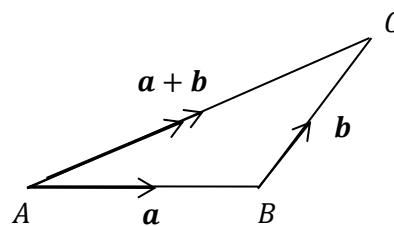
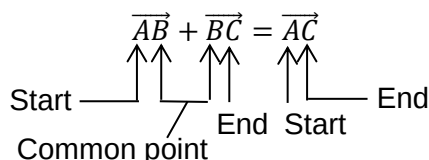
Note: $|\overrightarrow{AB}| = |\overrightarrow{EF}|$

- Zero/ Null vectors** are vectors whose magnitude is 0. It takes place when two vectors are equal in magnitude but in opposite direction are added, where the resultant vector is a zero vector, $\mathbf{0}$.
 - For example, $\overrightarrow{AB} + \overrightarrow{EF} = \mathbf{0}$ or $\mathbf{a} + \mathbf{c} = \mathbf{0}$.
- Note: $\mathbf{a}$ and $\mathbf{d}$ have different magnitudes but same direction. It will be covered in **Scalar Multiples of a Vector**



4. Addition of Vectors

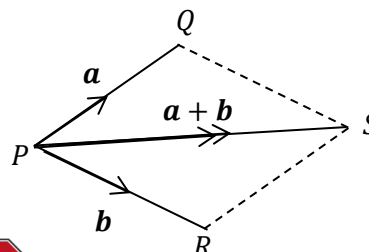
• Triangle Law of Vector Addition:



- In Triangle Law of Vector Addition, the ending point of the first vector must be the starting point of the second vector. In the example above, there is a common point B.

• Parallelogram Law of Vector Addition:

- As $\overrightarrow{QS} = \overrightarrow{PR} = \mathbf{b}$, $\overrightarrow{PQ} + \overrightarrow{PR} = \overrightarrow{PS}$
- In Parallelogram Law of Vector Addition, both vectors $(\overrightarrow{PQ}, \overrightarrow{PR})$, and the resultant vector $(\overrightarrow{PS})$, starts from the same point, P.



- If $\mathbf{a} = \begin{pmatrix} p \\ q \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} r \\ s \end{pmatrix}$, then, in column vectors,

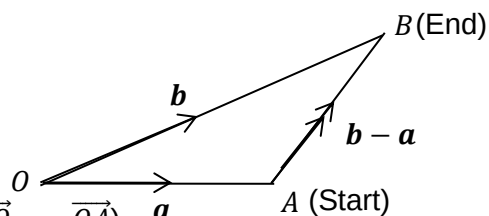
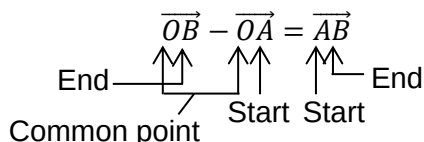
$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p + r \\ q + s \end{pmatrix}.$$



Note that $|\mathbf{a} + \mathbf{b}| \neq |\mathbf{a}| + |\mathbf{b}|$.

5. Vector Subtraction

• Triangle Law of Vector Subtraction:



- $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ (using addition of vector)
 $= -\overrightarrow{OA} + \overrightarrow{OB}$ (using negative vector where $\overrightarrow{AO} = -\overrightarrow{OA}$)
 $= \overrightarrow{OB} - \overrightarrow{OA}$

- In Triangle Law of Vector Subtraction, both vectors $(\overrightarrow{OB}, \overrightarrow{OA})$ must start from the same point.



To determine the direction in vector subtraction, use “end minus start”

- If $\mathbf{a} = \begin{pmatrix} p \\ q \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} r \\ s \end{pmatrix}$, then, in column vectors,

$$\mathbf{a} - \mathbf{b} = \begin{pmatrix} p \\ q \end{pmatrix} - \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p - r \\ q - s \end{pmatrix}.$$



Note that $|\mathbf{a} - \mathbf{b}| \neq |\mathbf{a}| - |\mathbf{b}|$.

6. Scalar Multiples of a Vector and Parallel Vectors

- If $\mathbf{a}$ and $\mathbf{b}$ are two **non-zero, parallel** vectors, and $\mathbf{a}$ has a magnitude $|k|$ times of $\mathbf{b}$, then $\mathbf{a} = k\mathbf{b}$, where k is a scalar and $k \neq 0$.
- If k is positive, $\mathbf{a}$ and $\mathbf{b}$ are in the same direction.
 If k is negative, $\mathbf{a}$ and $\mathbf{b}$ are in the opposite direction.
- In general, if $\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$, then $k\mathbf{a} = \begin{pmatrix} kx \\ ky \end{pmatrix}$, and $|k\mathbf{a}| = |k||\mathbf{a}|$ for any real number of k .
- Points A, B and C are **collinear** if $\overrightarrow{AB} = k\overrightarrow{BC}$, $\overrightarrow{AB} = m\overrightarrow{AC}$ or $\overrightarrow{BC} = n\overrightarrow{AC}$.

7. Expressing a Vector in Terms of Two Other Vectors

- For **non-parallel** vectors $\mathbf{a}$ and $\mathbf{b}$, and p, q, r and s are scalars,
 - If $p\mathbf{a} = q\mathbf{b}$, then $p = 0$ and $q = 0$.
 - If $r\mathbf{a} + s\mathbf{b} = p\mathbf{a} + q\mathbf{b}$, then $r = p$ and $s = q$.
- In general, $\overrightarrow{AB}$ can be expressed uniquely in terms of two other **non-zero** and **non-parallel** vectors, $\mathbf{u}$ and $\mathbf{v}$, that lies on the same plane.
 - For example: $\overrightarrow{AB} = m\mathbf{u} + n\mathbf{v}$, where m and n are scalars.



8. Position Vector and Coordinates of a Point on a Cartesian Plane

- **Position vector** of a point must have a fixed starting point/ reference point, where it is usually the origin O on a Cartesian plane.
- A directed line segment $\overrightarrow{OP}$ indicates the position of a point P that has taken with reference to vectors with the origin O as the initial point.
- In general, the position vector of Point $P(x, y)$ is $\overrightarrow{OP} = \begin{pmatrix} x \\ y \end{pmatrix}$.
- A vector $\overrightarrow{PQ}$ on a Cartesian plane can be expressed in terms of position vectors.
 - For example: $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ (Note: "end minus start" – vector subtraction)

9. Translation of a Point using Column Vector

- Vector $\overrightarrow{PQ}$ can be regarded as movement from P to Q , which is a **translation** from P to Q . $\overrightarrow{PQ}$ becomes a **translation vector** which describe the movement.
- A translation can be represented as a column vector $\begin{pmatrix} h \\ k \end{pmatrix}$. This refers to a movement of h units along the x –axis and k units along the y –axis.
 - For example, Point $P(a, b)$, which can be expressed as $\overrightarrow{OP} = \begin{pmatrix} a \\ b \end{pmatrix}$, undergoes a translation $T = \begin{pmatrix} h \\ k \end{pmatrix}$ to Point Q , then Q can be found by $\overrightarrow{OQ} = \overrightarrow{OP} + \begin{pmatrix} h \\ k \end{pmatrix}$.
Therefore, $\overrightarrow{OQ} = \begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} a + h \\ b + k \end{pmatrix}$.



Secondary 3 – Amaths

Topic 1: Non-Linear Simultaneous Equations

1. Linear and Non-linear Simultaneous Equations

- In general, **n equations** are needed to solve for **n unknowns**.
- Methods to solve simultaneous equation:
 - **Drawing graphs**: Coordinate of the point of intersection(s).
 - **Elimination**: Generally used to solve **linear** simultaneous equation.
 - **Substitution**: Generally used to solve **non-linear** simultaneous equation.

2. Linear Simultaneous Equations involving 2 variables (Recap)

- A unique solution: Two lines intersect at 1 point.
- Infinitely many solutions: Equations for the 2 lines are identical.
- No solution: Two lines are parallel to each other (Same gradient, different y -intercept).

3. Non-linear Simultaneous Equations involving 2 variables

- Unique solution(s): Two equations intersect at distinct point(s).
 - Line tangents to the curve: Intersect at only 1 point.
- Infinitely many solutions: Two identical equations.
- No solution: Two equations that will never intersect each other.



Topic 2: Quadratic Functions, Equations and Inequalities

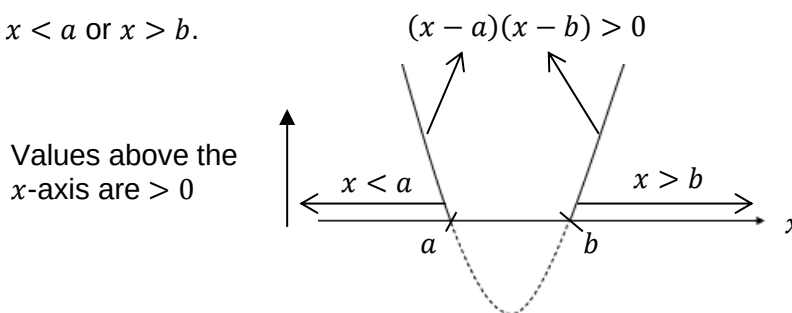
- Completing the Square Form:** Expressing $ax^2 + bx + c$ in completed square form of $a(x + h)^2 + k$.
 - Factorise out the coefficient of x^2 , a , as a common factor.
 - Add and subtract the square of half of the coefficient of x , $\left(\frac{b}{2a}\right)^2$.
 - Express the variable terms as a perfect square.
- Maximum and Minimum Values** for quadratic function $ax^2 + bx + c$ which can be expressed as $a(x + h)^2 + k$.
 - The sign of the value a , determines the nature of the turning point, $(-h, k)$.
 - When $a > 0$, the quadratic function has a minimum point.
 - When $a < 0$, the quadratic function has a maximum point.
- Sketching of Quadratic Graphs** using completed square form.
 - Determine the shape of the graph from a .
 - $a > 0$: Smiley-face graph
 - $a < 0$: Sad-face graph
 - Using the completed square form to obtain the coordinates of maximum or minimum point.
 - Substitute $x = 0$ to find the y -intercept.
 - Substitute $y = 0$ to find the x -intercept.
 - Condition for quadratic curve to lie completely above or below x -axis (no x -intercept).
 - If $a > 0$ and the minimum value, k , is positive, then the curve lies completely above the x -axis.
 - If $a < 0$ and the maximum value, k , is negative, then the curve lies completely below the x -axis.



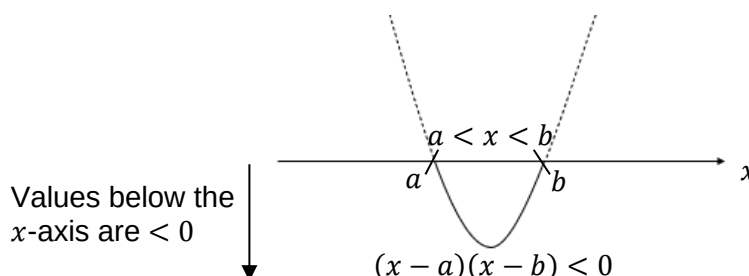
Remember to label all **axes intercepts**, **turning point**, **equation of curve** and the **axes** in your graph.

4. Quadratic Inequalities

- If $(x - a)(x - b) > 0$, then $x < a$ or $x > b$.



- If $(x - a)(x - b) < 0$, then $a < x < b$.



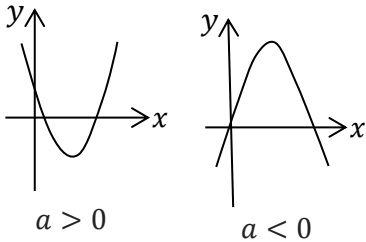
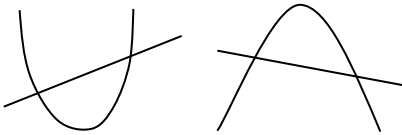
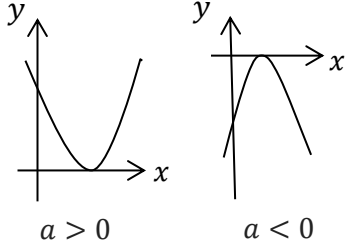
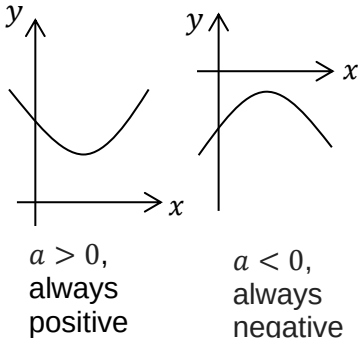
5. Quadratic Formula

- **Roots of a quadratic equation** $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- **Nature of roots** depends on the sign of $(b^2 - 4ac)$, which is called the **discriminant** of the equation. It determines the number of intersection points.

6. Nature of Roots for Quadratic functions using Discriminant $(b^2 - 4ac)$.

$b^2 - 4ac$	Nature of roots	Intersection of quadratic curve with x -axis	Intersection of quadratic curve with a straight line
> 0	2 real and distinct roots	Curve cuts the x -axis at 2 distinct points. 	Line intersects the curve at two distinct points. 
$= 0$	Real and equal roots	Curve touches the x -axis. 	Line is tangent to the curve. 
< 0	No real roots	Curve lies entirely above or entirely below x -axis. 	Line does not intersect the curve. 



When the question states that the quadratic function has real roots, we do not know if the roots are distinct or equal. Therefore, its discriminant, $b^2 - 4ac \geq 0$.



Topic 3: Surds

1. **Definition: Surd** is an irrational root of a real number that is written using a radical sign " $\sqrt{\quad}$ ". It has infinite number of non-recurring decimals. E.g. $\sqrt{5}$, $\sqrt[3]{3}$, etc.

2. Operations on Surds

- **Law of Surds** (Multiplication and division of surds) – Similar to Law of Indices.

➤ $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

➤ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

- **Addition and Subtraction of Surds** (Like surds can be added or subtracted).

➤ $x\sqrt{a} + y\sqrt{a} = (x + y)\sqrt{a}$

➤ $x\sqrt{a} - y\sqrt{a} = (x - y)\sqrt{a}$



Common Misconception (to take note)

- $\sqrt{a} + \sqrt{b} \neq \sqrt{a + b}$
- $\sqrt{a} - \sqrt{b} \neq \sqrt{a - b}$

3. Conjugate Surds

- $(a + b\sqrt{c})$ and $(a - b\sqrt{c})$ are conjugate surds.
- The product of a pair of conjugate surds is always a rational number.
 - For example: $(a + b\sqrt{c})(a - b\sqrt{c}) = a^2 - b^2c$

4. **Rationalising the Denominator:** A technique to rewrite a surd such that the denominator does not contain a surd term. This is to simplify the fraction and make the denominator a rational number.

- **By multiplying both numerator and denominator by the surd itself.**

➤ $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$

- **By multiplying both numerator and denominator by the conjugate surds.**

➤ $\frac{1}{\sqrt{a} + \sqrt{b}} = \frac{1}{\sqrt{a} + \sqrt{b}} \times \frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{\sqrt{a} - \sqrt{b}}{a - b}$

5. Solving Equations involving Surds

- **Methods**

- By squaring both sides of the equations
- By comparing both sides of the equations – Comparing the coefficients of surds

- **Note:** Always substitute the solution back to the original equation to ensure that the solution fits into the original equation as squaring both sides of the equations will introduce another solution that might not fit into the original equation.

6. Note: **Never** square root negative numbers.



Topic 4: Polynomials, Cubic Equations and Partial Fractions

- Polynomial** in x is a mathematical expression of a sum of terms, where each term is in the form of ax^n and a is a constant and n is a non-negative integer.
 - Usually denoted as $f(x)$.
 - General form: $a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$, where a_n, a_{n-1} and a_1 are coefficients and a_0 is the constant term.
 - The degree/ order of $f(x)$ is given by the highest power of x .
 - The value of $f(x)$ at $x = c$ is $f(c)$.
- Identities:** Equations in which the expressions on the left-hand side (LHS) is equal to the expressions on the right-hand side (RHS).
 - Methods of finding the unknown constants**
 - By substituting suitable values of x
 - By equating/ comparing coefficients
- Division Algorithm for Polynomials**
 - Long Division of Polynomials**
 - For example, divide $x^3 + 4x - 3$ by $x - 1$

Divisor	$x - 1$	Quotient
	$ \begin{array}{r} x^3 \quad +0x^2 \quad +4x \quad -3 \\ -(x^3 \quad -x^2) \\ \hline x^2 \quad +4x \\ -(x^2 \quad -x) \\ \hline 5x \quad -3 \\ -(5x \quad -5) \\ \hline 2 \end{array} $	Dividend
	Remainder	

- As the dividend does not have the term in x^2 , we need to put $+0x^2$ as our quotient might have that term.
 - Always leave the dividend in descending powers of x , starting from the highest degree of x to the constant term (x^0). Never leave out any degree of x despite the coefficient of the term is 0.
 - Similar to long division of positive integers, whereby they are arranged in its respective columns of place values, in long division of polynomials, arranged the terms in its respective degree of x .
 - The order of the remainder is always at least one degree less than that of the divisor and the process of long division stopped when the degree of the remainder is less than the degree of the divisor.
- If a polynomial $f(x)$ is divided by another polynomial $D(x)$,
 - $f(x) = D(x) \times Q(x) + R(x)$ whereby

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

And the degree of $R(x) < \text{the degree of } D(x)$.

- Remainder Theorem** states that if a polynomial $f(x)$ is divided by a linear divisor, $ax + b$, the remainder is $f\left(-\frac{b}{a}\right)$.
- Factor Theorem** states that if a polynomial $f(x)$ is divided by a linear divisor, $ax + b$, and that $f\left(-\frac{b}{a}\right) = 0$, then $ax + b$ is a factor of $f(x)$, where the remainder is 0.



6. Cubic expressions, Equations and Identities

- **Cubic expression** is in the form of $ax^3 + bx^2 + cx + d$
- **Cubic equation** has **exactly 3 roots**:
 - 3 distinct real roots,
 - 2 equal real roots and 1 distinct real root,
 - 3 equal real roots,
 - 1 real root and 2 non-real roots.
- **Cubic identities**
 - **Sum of cubes** ($a^3 + b^3$)

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Diagram illustrating the signs in the sum of cubes formula:

- From a^3 to a : Same sign
- From a^3 to $-ab$: Opposite sign
- From b^3 to b^2 : Always Positive

- **Difference of cubes** ($a^3 - b^3$)

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Diagram illustrating the signs in the difference of cubes formula:

- From a^3 to a : Same sign
- From a^3 to $-b$: Opposite sign
- From $-b^3$ to b^2 : Always Positive



Use **SOAP** to factorise both sum of cubes and difference of cubes.

- **Methods of factorising cubic expressions**
 - Trial and error
 - Long division
 - Comparing coefficients

7. Proper and Improper Algebraic Fractions

- **Algebraic Fractions** is the ratio of two polynomials of the form $\frac{P(x)}{D(x)}$, where $P(x)$ and $D(x)$ are polynomials in x .
- **Proper Fraction**: When the degree of $P(x)$ is less than the degree of $D(x)$.
 - For example, $\frac{2x^2}{3x^3+x+1}$.
- **Improper Fraction**: When the degree of $P(x)$ is more than or equal to the degree of $D(x)$.
 - For example, $\frac{x^2}{x^2+1}$, $\frac{2x^3+x^2}{4x^2+1}$.
- From an improper fraction $\frac{P(x)}{D(x)}$, we can obtain $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$ by using **long division**, where $Q(x)$ is a polynomial and $\frac{R(x)}{D(x)}$ is a proper algebraic fraction.



8. Converting Algebraic Fractions into Partial Fractions

- **Step 1:** Determine if the algebraic fraction is proper or improper. If $\frac{P(x)}{D(x)}$ is improper, perform long division to express it as a sum of a polynomial and a proper algebraic fraction.
- **Step 2:** Ensure that the denominator is completely factorised.
- **Step 3:** Express the proper algebraic fraction in partial fractions according to the cases below.

Case	Denominator of the fraction	Proper Algebraic Fraction	Partial Fractions
1	Distinct Linear Factors	$\frac{px + q}{(ax + b)(cx + d)}$	$\frac{A}{ax + b} + \frac{B}{cx + d}$
2	Repeated Linear Factors	$\frac{px + q}{(ax + b)^2}$	$\frac{A}{ax + b} + \frac{B}{(ax + b)^2}$
3	Quadratic factor that cannot be factorised	$\frac{px^2 + qx + r}{(ax + b)(x^2 + c^2)}$	$\frac{A}{ax + b} + \frac{Bx + C}{x^2 + c^2}$

- **Step 4:** Solve the unknown constants by substituting suitable values of x and/or comparing/equating coefficients of like terms, and/or using the Cover-Up Rule.



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Topic 5: Binomial Theorem and its Applications

1. Binomial Expansion of $(1 + b)^n$

- In general, $(1 + b)^n = \binom{n}{0} + \binom{n}{1}b + \binom{n}{2}b^2 + \dots + \binom{n}{n}b^n$, where n is a non-negative integer.

2. Binomial Expansion of $(a + b)^n$

- $(a + b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$
- Formula for $\binom{n}{r}$:
 - $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$
 - $n!$ is read as n factorial where $n! = n \times (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1$, where n is a positive integer and r is an integer such that $0 < r \leq n$.
- Number of terms** in the expansion of $(a + b)^n$ is $n + 1$.

3. General Term

- $(r + 1)^{\text{th}}$ term, $T_{r+1} = \binom{n}{r}a^{n-r}b^r$, where r is an integer such that $0 < r \leq n$.

4. Term Independent of x is referred to as the constant term, where the **power of x is zero**.

5. Some useful notations:

- $\binom{n}{0} = 1$
- $\binom{n}{1} = n$
- $\binom{n}{2} = \frac{n(n-1)}{2!}$
- $\binom{n}{3} = \frac{n(n-1)(n-2)}{3!}$
- $\binom{n}{n} = 1$



One way to remember is to look at the **number at the bottom**. The number at the bottom shows the **number of factors** there are in the numerator. The numerator will always start with n and multiplied by 1 less than its preceding number.



Topic 6: Exponential and Logarithmic Functions

1. Recap on Indices (Secondary 3 EMaths)

- x^4 is read as 'x to the power 4' where **base** is x and **index/ exponent** is 4.
- **Laws of Indices**
 - Law 1: $a^m \times a^n = a^{m+n}$
 - Law 2: $a^m \div a^n = a^{m-n}$
 - Law 3: $a^n \times b^n = (ab)^n$
 - Law 4: $a^n \div b^n = \left(\frac{a}{b}\right)^n$, where $b \neq 0$
 - Law 5: $(a^m)^n = a^{m \times n} = a^{mn}$
 - Law 6: $a^0 = 1$, where $a \neq 0$
 - Law 7: $a^{-m} = \frac{1}{a^m}$, where $a \neq 0$ and m is a positive integer
 - Law 8: $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a \geq 0$ and n is a positive integer
 - Law 9: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$, where $a \geq 0$ and m and n are positive integers

2. Logarithms

- For $a > 0$ and $a \neq 1$, $y = a^x$ (in exponential form) is equivalent to $x = \log_a y$ (in logarithmic form), where a is the base and x is the index.
- For $x = \log_a y$ to be defined, a and y must be positive and $a \neq 1$.
- In logarithm form, **index** is always **by itself** and **base** must be **subscript**.
- **Common logarithms** are logarithms with a base of 10 where by **lg** x is used to represent $\log_{10} x$.



Note: Most calculators use **log** x to represent $\log_{10} x$.

- **Natural logarithm** is logarithms with a base of e , represented by $\log_e x$. The short form for $\log_e x$ is $\ln x$ (pronounced as lawn x).

3. Special Properties of Logarithms

- If $a > 0$ and $a \neq 1$, then
 - $\log_a a = 1$, for example $\lg 10 = 1$ and $\ln e = 1$
 - $\log_a 1 = 0$, for example $\lg 1 = 0$ and $\ln 1 = 0$

4. Laws of Logarithms

- **Product Law of Logarithms**
 - $\log_a xy = \log_a x + \log_a y$
- **Quotient Law of Logarithms**
 - $\log_a \frac{x}{y} = \log_a x - \log_a y$
- **Power Law of Logarithms**
 - $\log_a x^r = r \log_a x$

5. Change of Base Formula

- In general, $\log_a b = \frac{\log_c b}{\log_c a}$, where a, b and $c > 0$ and $a, c \neq 1$
- Special Case: $\log_a b = \frac{\log_b b}{\log_b a} = \frac{1}{\log_b a}$, where $a, b > 0$ and $a, b \neq 1$

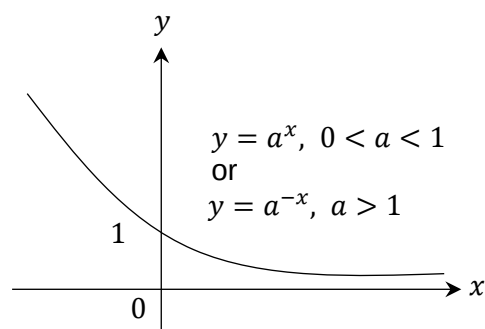
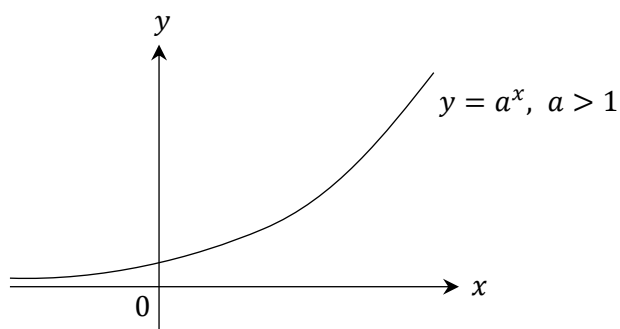
6. Equality of Logarithms

- If $\log_a p = \log_a q$, then $p = q$, where $a, p, q > 0$ and $a \neq 1$



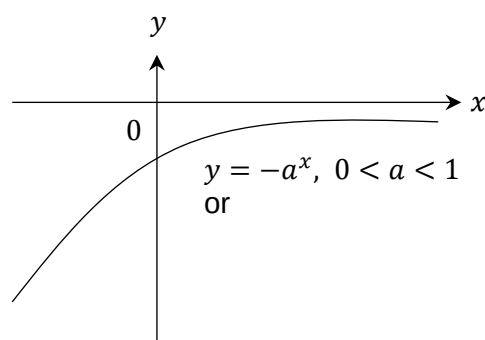
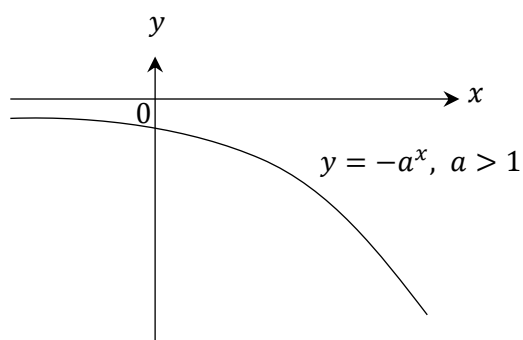
7. Graphs of Exponential Functions

- Graphs of $y = a^x$



The graphs must pass through $(0, 1)$ as $a^0 = 1$.

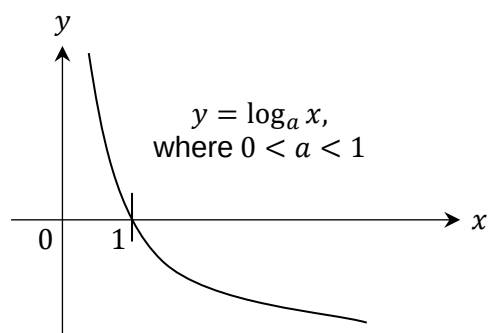
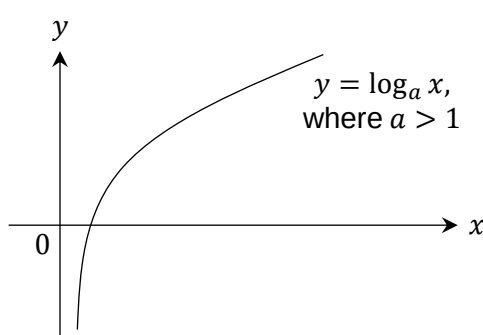
- Graphs of $y = -a^x$



The graphs must pass through $(0, -1)$ as $-a^0 = -1$.

8. Graphs of Logarithmic Functions

- Graphs of $y = \log_a x$



The graphs must pass through the point $(1, 0)$ as $\log_a 1 = 0$.



Topic 7: Coordinate Geometry

1. Coordinate Geometry of a Straight Line, $y = mx + c$

- Distance between 2 points

➤ Length of $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ or $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- Gradient of Line/ Collinear Points

➤ Gradient of line AB , $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1} = \tan \theta$

➤ If $m > 0$, $0^\circ < \theta < 90^\circ$, then the straight-line graph is upward-sloping.

➤ If $m < 0$, $90^\circ < \theta < 180^\circ$, then the straight-line graph is downward-sloping.

➤ If $m = 0$, $\theta = 0^\circ$, $y = y$ -intercept, then the straight-line graph is a horizontal line.

➤ If m is undefined, $\theta = 90^\circ$, $x = x$ -intercept, then the straight-line graph is a vertical line.

➤ If points A, B and M are collinear, then gradient of AB = gradient of AM = gradient of BM .

- Midpoint of 2 points

➤ Midpoint of $AB = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

- Parallel and Perpendicular Lines: for line l_1 has gradient m_1 and line l_2 has gradient m_2

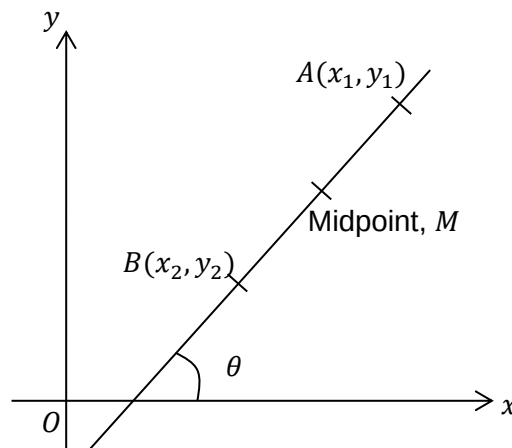
➤ l_1 is parallel to l_2 when $m_1 = m_2$

➤ l_1 is perpendicular to l_2 when $m_1 m_2 = -1$

- Equation of a Straight Line

➤ Gradient form: $y = mx + c$, where m is the gradient and c is the y -intercept.

➤ When the line passes through point $A(x_1, y_1)$, equation of straight line: $y - y_1 = m(x - x_1)$



2. Area of Triangle, Quadrilateral and Polygon

• If $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ are in an **anticlockwise direction**, then

➤ Area of Triangle $ABC = \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$
 $= \frac{1}{2} (x_1 y_2 + x_2 y_3 + x_3 y_1 - x_2 y_1 - x_3 y_2 - x_1 y_3)$

• If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$ are in an **anticlockwise direction**, then

➤ Area of Quadrilateral $ABCD = \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix}$
 $= \frac{1}{2} (x_1 y_2 + x_2 y_3 + x_3 y_4 + x_4 y_1 - x_2 y_1 - x_3 y_2 - x_4 y_3 - x_1 y_4)$

• If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3) \dots N(x_n, y_n)$ are in an **anticlockwise direction** that forms a polygon, then

➤ Area of polygon $= \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & \dots & x_n & x_1 \\ y_1 & y_2 & y_3 & \dots & y_n & y_1 \end{vmatrix}$
 $= \frac{1}{2} (x_1 y_2 + x_2 y_3 + \dots + x_n y_1 - x_2 y_1 - x_3 y_2 - \dots - x_1 y_n)$

• Vertices must be in **cyclic** and **anticlockwise** order. If vertices are in **clockwise direction**, the value obtained will be **negative**.

3. Equation of Circle

• **Standard Form:** $(x - a)^2 + (y - b)^2 = r^2$, where (a, b) is the centre and r is the radius.

• **General Form:** $x^2 + y^2 + 2gx + 2fy + c = 0$, where $(-g, -f)$ is the centre and $\sqrt{g^2 + f^2 - c}$ is the radius.

• To determine if point P lies on the circumference, inside or outside of the circle with centre C and radius of r units,

➤ If $CP = r$, P lies on the circumference of the circle

➤ If $CP < r$, P lies inside the circle

➤ If $CP > r$, P lies outside the circle



Topic 8: Linear Law

1. Linear Law

- **Linear relationship** between the variables x and y are given by the equation $y = mx + c$, where m is the gradient and c is the y -intercept.
 - The graph of values of y is plotted against their respective values of x , forming a straight-line graph.
- **Linear Law** is used to reduce the non-linear functions to the linear form, $Y = mX + c$.
 - To convert non-linear equations involving x and y into linear form, express the non-linear equation in the form of $Y = mX + c$ whereby Y and X are expressions in x and/or y .
 - The variable X and Y can only contain the original variables of x and/or y . It cannot contain the original unknown constants such as a and b .
 - The gradient, m and y -intercept, c , can only contain original unknown constants such as a , and/or b and cannot contain the original variables x and/or y .
 - For example,
Non-linear function: $xy = \frac{a}{x} + bx$
To convert to linear function, express the non-linear function into: $x^2y = bx^2 + a$
Linear function: $(x^2y) = b(x^2) + a$, where $Y = x^2y$ and $X = x^2$.

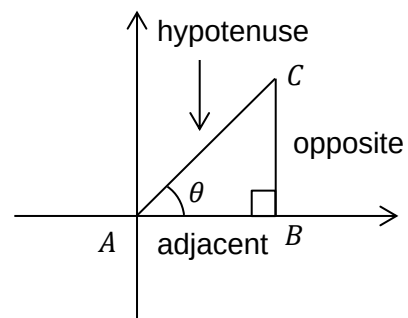


Topic 9: Trigonometric Functions and Graphs

1. Basic Trigonometric Ratios

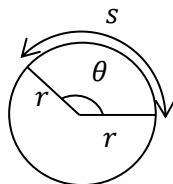
$$\left. \begin{aligned} \sin \theta &= \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AC} & [\text{SOH}] \\ \cos \theta &= \frac{\text{adj}}{\text{hyp}} = \frac{AB}{AC} & [\text{CAH}] \\ \tan \theta &= \frac{\text{opp}}{\text{adj}} = \frac{BC}{AB} & [\text{TOA}] \end{aligned} \right\} \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\begin{aligned} \csc \theta &= \frac{1}{\sin \theta} = \frac{AC}{BC} \\ \sec \theta &= \frac{1}{\cos \theta} = \frac{AC}{AB} \\ \cot \theta &= \frac{1}{\tan \theta} = \frac{AB}{BC} \end{aligned}$$



2. Radian Measure

- θ (in radians) $= \frac{s}{r}$
- $\pi \text{ rad} = 180^\circ$



3. Trigonometric Ratios of Special Angles

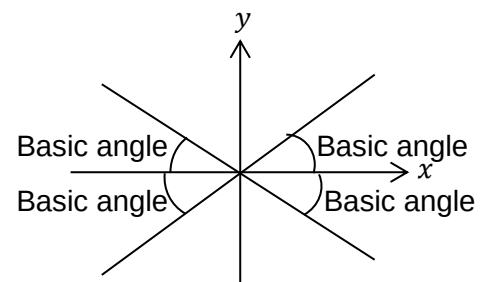
θ	0°	$30^\circ = \frac{\pi}{6}$	$45^\circ = \frac{\pi}{4}$	$60^\circ = \frac{\pi}{3}$	$90^\circ = \frac{\pi}{2}$	$180^\circ = \pi$	$270^\circ = \frac{3\pi}{2}$	$360^\circ = 2\pi$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\tan \theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undefined	0	Undefined	0

4. Basic Angle/ Reference Angle (α)

- An acute angle between a rotating radius about the origin and the x -axis.
 - If θ is in the 1st quadrant, then $\theta = \alpha$.
 - If θ is in the 2nd quadrant, then $\theta = 180^\circ - \alpha$.
 - If θ is in the 3rd quadrant, then $\theta = 180^\circ + \alpha$.
 - If θ is in the 4th quadrant, then $\theta = 360^\circ - \alpha$.

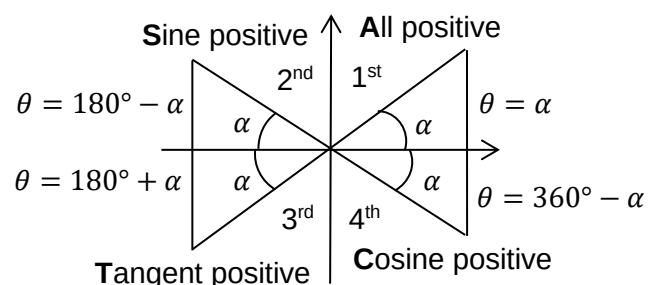


$0^\circ, 90^\circ, 180^\circ, 270^\circ$ and 360° **do not** lie in any quadrant and **do not** have any basic angle.



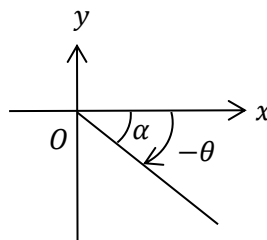
5. Signs of Trigonometric Ratios

- Use **ASTC** to determine the signs of $\sin \theta$, $\cos \theta$ and $\tan \theta$ if $\theta \neq 0^\circ, 90^\circ, 180^\circ, 270^\circ$ and 360° .



6. Signs of Trigonometric Ratios in 2nd, 3rd and 4th quadrant

- **Second quadrant ($\theta = 180^\circ - \alpha$)**
 - $\sin \theta = \sin(180^\circ - \alpha) = \sin \alpha$
 - $\cos \theta = \cos(180^\circ - \alpha) = -\cos \alpha$
 - $\tan \theta = \tan(180^\circ - \alpha) = -\tan \alpha$
- **Third quadrant ($\theta = 180^\circ + \alpha$)**
 - $\sin \theta = \sin(180^\circ + \alpha) = -\sin \alpha$
 - $\cos \theta = \cos(180^\circ + \alpha) = -\cos \alpha$
 - $\tan \theta = \tan(180^\circ + \alpha) = \tan \alpha$
- **Fourth quadrant ($\theta = 360^\circ - \alpha$)**
 - $\sin \theta = \sin(360^\circ - \alpha) = -\sin \alpha$
 - $\cos \theta = \cos(360^\circ - \alpha) = \cos \alpha$
 - $\tan \theta = \tan(360^\circ - \alpha) = -\tan \alpha$



7. Negative Angle ($-\theta$)

- $\sin(-\theta) = -\sin \theta$
- $\cos(-\theta) = \cos \theta$
- $\tan(-\theta) = -\tan \theta$

8. Complementary Angle Formulae

- $\sin(90^\circ - \theta) = \cos \theta$
- $\cos(90^\circ - \theta) = \sin \theta$
- $\tan(90^\circ - \theta) = \frac{1}{\tan \theta} = \cot \theta$

9. Supplementary Angle Formulae

- $\sin(180^\circ - \theta) = \sin \theta$
- $\cos(180^\circ - \theta) = -\cos \theta$
- $\tan(180^\circ - \theta) = -\tan \theta$

10. Graphs of Trigonometric Functions

$y = a \sin bx + c$	$y = a \cos bx + c$	$y = a \tan bx + c$
Amplitude = a	Amplitude = a	—
Period = $\frac{360^\circ}{b}$ or $\frac{2\pi}{b}$	Period = $\frac{360^\circ}{b}$ or $\frac{2\pi}{b}$	Period = $\frac{180^\circ}{b}$ or $\frac{\pi}{b}$
$c - a \leq y \leq c + a$	$c - a \leq y \leq c + a$	—



Topic 10: Trigonometric Equations and Identities

1. Principal Values of $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$

- $-90^\circ \leq \sin^{-1} x \leq 90^\circ$ or $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$, where $-1 \leq x \leq 1$
- $0^\circ \leq \cos^{-1} x \leq 180^\circ$ or $0 \leq \cos^{-1} x \leq \pi$, where $-1 \leq x \leq 1$
- $-90^\circ < \tan^{-1} x < 90^\circ$ or $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$, where x is any real value.
- Calculator will always give the principal value of the respective inverse trigonometric functions.

2. Basic Trigonometric Identities

- $\sin^2 \theta + \cos^2 \theta = 1$
- $\sec^2 \theta = 1 + \tan^2 \theta$
- $\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$

3. Addition Formulae/ Compound Angle Formulae

- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$



$$\sin(A + B) \neq \sin A + \sin B$$

4. Double Angle Formulae

- $\sin 2A = 2 \sin A \cos A$
- $\cos 2A = \cos^2 A - \sin^2 A$
 $= 2 \cos^2 A - 1$
 $= 1 - 2 \sin^2 A$
- $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$



- $\sin 2A \neq 2 \sin A$
- $\cos 2A \neq 2 \cos A$
- $\tan 2A \neq 2 \tan A$

5. Half Angle Formulae (By replacing A with $\frac{A}{2}$ in the Double Angle Formulae)

- $\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2}$
- $\cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2}$
 $= 2 \cos^2 \frac{A}{2} - 1$
 $= 1 - 2 \sin^2 \frac{A}{2}$
- $\tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}}$

6. Proving of Identities

- **Strategy 1:** Always start with simplifying the expression on the side of the identity that is **more complicated** as there is more room to manipulate using trigonometric formulae.
- **Strategy 2:** Observe and think of which definition or known identity that can be applied at each step.
- **Strategy 3:** Work from both sides of the equation when stuck halfway, but present the final solution from one side of the identity to the other side.

7. R-Formulae

- $a \sin \theta \pm b \cos \theta = R \sin(\theta \pm \alpha)$
- $a \cos \theta \pm b \sin \theta = R \cos(\theta \mp \alpha)$ } $R = \sqrt{a^2 + b^2}$, $\tan \alpha = \frac{b}{a}$
- For the expression $R \sin(\theta \pm \alpha)$ or $R \cos(\theta \mp \alpha)$,
 - Maximum value = R , when $\sin(\theta \pm \alpha) = 1$ or $\cos(\theta \mp \alpha) = 1$
 - Minimum value = R , when $\sin(\theta \pm \alpha) = -1$ or $\cos(\theta \mp \alpha) = -1$

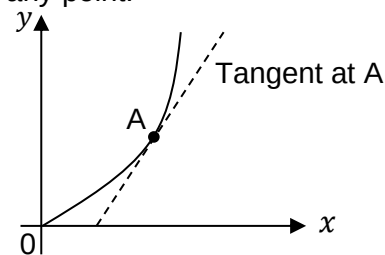


Secondary 4 – Amaths

Topic 1: Gradients, Derivatives and Differentiation Techniques

1. **Differentiation** from first principles is the **gradient of a curve** at any point.

- **Gradient of curve**, $y = f(x)$ at point A
= Gradient of tangent to the curve at A
= Value of $\frac{dy}{dx}$ at A
- **Gradient function** $\frac{dy}{dx}$ is also known as:
 - **Derivative of y with respect to x**
 - **Differentiate y with respect to x**
 - Can also be written as $f'(x)$ [f prime x] for the curve $y = f(x)$



Differentiation is a **more accurate method** of finding a gradient of a curve at any point as compared to gradient of a tangent.

Gradient of a tangent is only an **estimate** due to the inaccuracy of plotting the points on a curve and drawing a tangent.

2. **Differentiation of Power Functions**

- $\frac{d}{dx}(x^n) = nx^{n-1}$, where n is a real number
- $\frac{d}{dx}(ax^n) = anx^{n-1}$
- $\frac{d}{dx}(k) = 0$

Bring down the power by multiplying it and subtract 1 from the current power to get the new power.



3. **Five Rules of Differentiation**

- **Rule 1: Scalar Multiple Rule**
 - $\frac{d}{dx}[kf(x)] = k \frac{d}{dx}[f(x)]$
- **Rule 2: Addition/Subtraction Rule**
 - $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$
- **Rule 3: Chain Rule** (Derivative of a composite function)
 - $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
- **Rule 4: Product Rule**
 - $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
- **Rule 5: Quotient Rule**
 - $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

You can differentiate **each term** separately, but you cannot differentiate **each factor** separately.

First function $\times$ derivative of second function + second function $\times$ derivative of first function

Denominator $\times$ derivative of numerator – numerator $\times$ derivative of denominator; divided by the square of denominator

4. **Higher Derivatives**

- **First Derivative** (differentiating y with respect to x): $\frac{dy}{dx}$ or $f'(x)$
- **Second Derivative** (differentiating $\frac{dy}{dx}$ with respect to x): $\frac{d^2y}{dx^2}$ or $f''(x)$

5. **Increasing/ Decreasing Functions**

- If y is an **increasing function** (y increases as x increases), $\frac{dy}{dx} > 0$; gradient is positive.
- If y is a **decreasing function** (y decreases as x increases), $\frac{dy}{dx} < 0$; gradient is negative.



Topic 2: Application of Differentiation

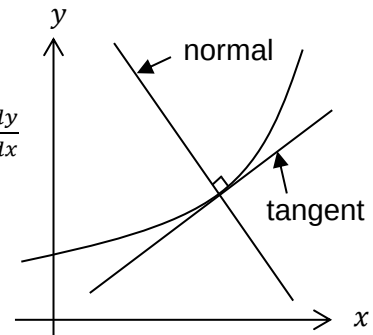
1. Equations of Tangents and Normals

- Equation of curve, $y = f(x)$
- Tangent to a Curve**

➤ **Gradient of Tangent** to the curve (at any point x), $m = \frac{dy}{dx}$



- Equation of straight line, $y = mx + c$
- Gradient of straight line (tangent) = m
- Gradient of curve = $\frac{dy}{dx}$



➤ **Equation of Tangent** at (x_1, y_1) , $y - y_1 = m(x - x_1)$

- Normal to a curve** is the line **perpendicular** to the tangent.

➤ **Gradient of Normal** to the curve = $-\frac{1}{m}$

➤ **Equation of Normal** at (x_1, y_1) , $y - y_1 = -\frac{1}{m}(x - x_1)$

Using $y = mx + c$,
at (x_1, y_1) , $c = y_1 - mx_1$.



Product of the Gradients of 2 perpendicular lines, $m_1 m_2 = -1$

2. Rate of Change

- $\frac{dy}{dx}$ is the rate of change of y with respect to x .
- $\frac{dy}{dt}$ is the rate of change of y with respect to time, t .
- $\frac{dx}{dt}$ is the rate of change of x with respect to time, t .
- A positive rate of change is an increase in the magnitude of the quantity involved as the time increases.
- A negative rate of change is a decrease in the magnitude of the quantity involved as the time increases.



Take note that when the question says that the rate of change is decreasing, the value of the rate of change must be negative.

3. Connected Rate of Change

- When the function, $y = f(x)$, both varies with another variable, time (t), the rate of change with respect to t are related by the **Chain Rule**.
- $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$
- $\frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$, where $\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$



When a straight line is **parallel** to the tangent of a curve, the **gradient of straight line** is **equal** to the **gradient of tangent to the curve**.

4. Stationary Points

- If a point (x_0, y_0) is a stationary point of the curve $y = f(x)$, then $\frac{dy}{dx} = 0$ when $x = x_0$. (i.e. the gradient of the tangent at $x = x_0$ is zero.)
- A stationary point can be a **maximum point**, a **minimum point**, or a **point of inflexion**.



5. Determining the Nature of the Stationary Points

- **First Derivative Test:** By using $\frac{dy}{dx}$.
 - **Maximum Point:** The gradient $\frac{dy}{dx}$ changes from a positive value to a negative value as x increases through the stationary point.
 - **Minimum Point:** The gradient $\frac{dy}{dx}$ changes from a negative value to a positive value as x increases through the stationary point.
 - **Stationary Point of Inflexion:** The gradient $\frac{dy}{dx}$ remains positive/ negative as x increases through the stationary point.

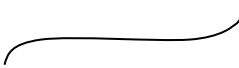
Maximum Point

x	x^-	x_0	x^+
$\frac{dy}{dx}$	> 0	0	< 0
Sketch of tangent			
Stationary Point			

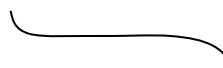
Minimum Point

x	x^-	x_0	x^+
$\frac{dy}{dx}$	< 0	0	> 0
Sketch of tangent			
Stationary Point			

Point of Inflexion

x	x^-	x_0	x^+
$\frac{dy}{dx}$	> 0	0	> 0
Sketch of tangent			
Stationary Point			

Point of Inflexion

x	x^-	x_0	x^+
$\frac{dy}{dx}$	< 0	0	< 0
Sketch of tangent			
Stationary Point			

- **Second Derivative Test:** By using $\frac{d^2y}{dx^2}$.
 - If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$ at $x = x_1$, the stationary point at x_1 is a maximum point.
 - If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$ at $x = x_1$, the stationary point at x_1 is a minimum point.
 - If $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ at $x = x_1$, the stationary point at x_1 can be a maximum point, minimum point or a point of inflexion. Use the **First Derivative Test** to determine the nature of the stationary point. Second Derivative Test **cannot** determine point of inflexion.



Topic 3: Differentiation of Trigonometric, Exponential and Logarithmic Functions

1. Differentiation of Trigonometric Functions

- Always ensure that the calculator is in **radian mode**
- Derivatives of $\sin x$, $\cos x$ and $\tan x$:**
 - $\frac{d}{dx}(\sin x) = \cos x$
 - $\frac{d}{dx}(\cos x) = -\sin x$
 - $\frac{d}{dx}(\tan x) = \sec^2 x$, where x is in radians.
- Derivatives of $\sin f(x)$, $\cos f(x)$ and $\tan f(x)$:**
 - $\frac{d}{dx}[\sin f(x)] = f'(x) \cos f(x)$
 - $\frac{d}{dx}[\cos f(x)] = -f'(x) \sin f(x)$
 - $\frac{d}{dx}[\tan f(x)] = f'(x) \sec^2 f(x)$, where x is in radians.
- Derivatives of $\sin^n f(x)$, $\cos^n f(x)$ and $\tan^n f(x)$:**
 - $\frac{d}{dx}[\sin^n f(x)] = n [\sin^{n-1} f(x)] \times \frac{d}{dx}[\sin f(x)]$
 - $\frac{d}{dx}[\cos^n f(x)] = n [\cos^{n-1} f(x)] \times \frac{d}{dx}[\cos f(x)]$
 - $\frac{d}{dx}[\tan^n f(x)] = n [\tan^{n-1} f(x)] \times \frac{d}{dx}[\tan f(x)]$

} Using Chain Rule

2. Differentiation of Exponential Functions

- Derivative of e^x :**
 - $\frac{d}{dx}[e^x] = e^x$
- Derivative of e^{ax+b} :**
 - $\frac{d}{dx}[e^{ax+b}] = a e^{ax+b}$
- In general, $\frac{d}{dx}[e^{f(x)}] = e^{f(x)} \times f'(x)$

3. Differentiation of Logarithmic Functions

- Derivative of $\ln x$:**
 - $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- Derivative of $\ln(ax + b)$:**
 - $\frac{d}{dx}[\ln(ax + b)] = \frac{a}{ax+b}$
- In general, $\frac{d}{dx}[\ln f(x)] = \frac{f'(x)}{f(x)}$
- Always try to make use of the laws of logarithms to simplify logarithmic expressions before finding the derivatives.



Topic 4: Integration

1. Integration as Reverse of Differentiation (Antiderivative)

- Differentiation is the process of obtaining the derivative $\frac{dy}{dx}$ from y .

Integration is the **reverse** process of obtaining y from $\frac{dy}{dx}$.

- If $\frac{d}{dx}[F(x)] = f(x)$, then $\int f(x) dx = F(x) + c$, where c is an **arbitrary constant**.
- $\int f(x) dx$ is called the **indefinite integral** of $f(x)$ with respect to x .

2. Two Rules of Integration

- **Rule 1: Scalar Multiple Rule of Integration**
 - $\int kf(x)dx = k \int f(x)dx$
- **Rule 2: Additional/Subtraction Rule of Integration**
 - $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

3. Integration of power functions

- $\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$, where $n \neq -1$ and c is an arbitrary constant.
- $\int (ax + b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)(a)} + c$, where $n \neq -1$, $a \neq 0$ and c is an arbitrary constant.
 - **Note:** This formula only applies for linear factor, $(ax + b)$.

4. Integration of Trigonometric Functions of $\sin x$, $\cos x$ and $\sec^2 x$

- $\int \sin x dx = -\cos x + c$
- $\int \cos x dx = \sin x + c$
- $\int \sec^2 x dx = \tan x + c$,

where x is in **radians** and c is an **arbitrary constant**.

5. Integration of Trigonometric Functions of $\sin(ax + b)$, $\cos(ax + b)$ and $\sec^2(ax + b)$

- $\int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + c$
- $\int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + c$
- $\int \sec^2(ax + b) dx = \frac{1}{a} \tan(ax + b) + c$

where x is in **radians**, a and b are constants, and c is an **arbitrary constant**.

6. Methods of Integrating Trigonometric Functions:

- Using trigonometric identities
 - For example, $1 + \tan^2 x = \sec^2 x$
- Using double angle formulae
 - For example, $\cos 2x = 2 \cos^2 x - 1$

7. Integration of Exponential Functions

- $\int e^x dx = e^x + c$
- $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$

where a and b are constants, and c is an arbitrary constant.

8. Integration of functions of the form $\frac{1}{x}$ and $\frac{1}{ax+b}$

- $\int \frac{1}{x} dx = \ln x + c$, where $x > 0$
- $\int \frac{1}{ax+b} dx = \frac{1}{a} \ln(ax + b) + c$, where $ax + b > 0$

where a and b are constants, and c is an arbitrary constant.

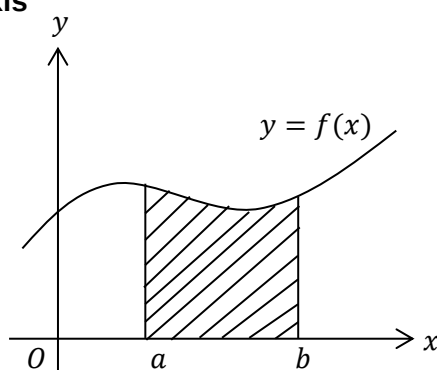


Topic 5: Applications of Integration

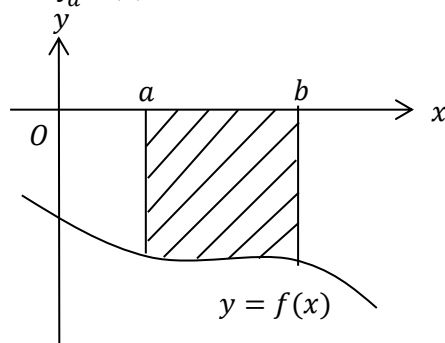
1. Definite Integrals

- If $\int f(x) dx = F(x) + c$, then $\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$
- $\int_a^b f(x) dx = -\int_b^a f(x) dx$
- $\int_a^a f(x) dx = 0$
- $\int_a^b hf(x) dx = h \int_a^b f(x) dx$
- $\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$
- $\int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$

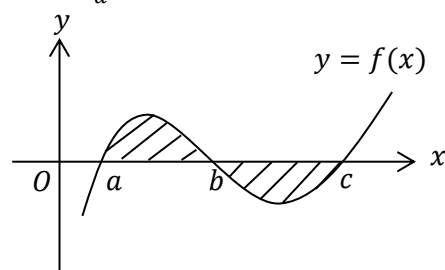
2. Area bounded by x -axis



- For region above the x -axis, area bounded by the curve $y = f(x)$, the x -axis and lines $x = a$ and $x = b$ is $\int_a^b f(x) dx$.



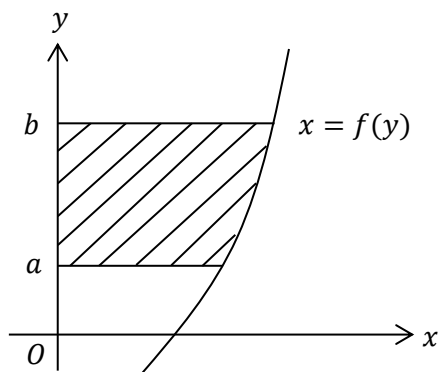
- For region below the x -axis, area bounded by the curve $y = f(x)$, the x -axis and lines $x = a$ and $x = b$ is $-\int_a^b f(x) dx$.



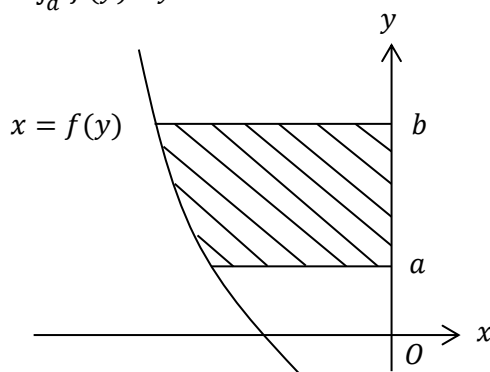
- For region enclosed above and below the x -axis, area bounded by the curve $y = f(x)$ and the x -axis is $\int_a^b f(x) dx - \int_b^c f(x) dx$.



3. Area bounded by the y -axis

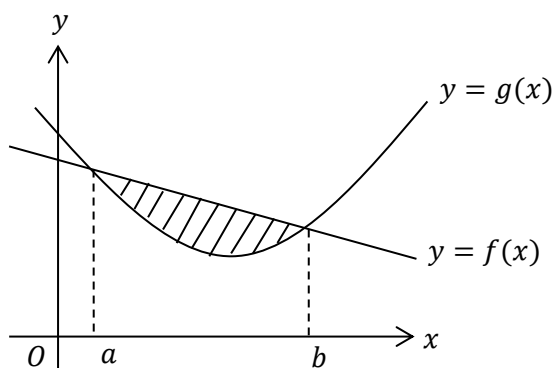


- For region on the right of y -axis, area bounded by the curve $x = f(y)$, the y -axis and lines $y = a$ and $y = b$ is $\int_a^b f(y) dy$.



- For region on the left of y -axis, area bounded by the curve $x = f(y)$, the y -axis and the lines $y = a$ and $y = b$ is $-\int_a^b f(y) dy$.

4. Area bounded by a Curve and a Line

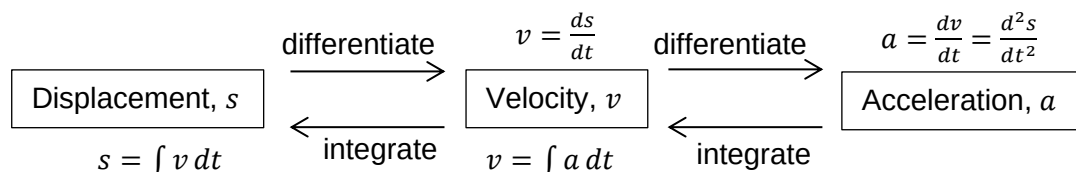


- For region bounded by a curve and a line as shown above, area bounded by the curve $y = g(x)$ and the line $y = f(x)$ is $\int_a^b [f(x) - g(x)] dx$.



Topic 6: Kinematics

1. **Displacement, s** , is the change of position of a particle with respect to the fixed point O at an instant. It has both **magnitude** (distance from the initial position to the final position) and **direction** (direction from the initial position to the final position). It can be positive, zero or negative.
 - If $s > 0$, the particle is in the positive s –axis
 - If $s = 0$, the particle is at the fixed point O
 - If $s < 0$, the particle is in the negative s –axis
2. **Velocity, v** , is the rate of change of displacement with respect to time. It is given by $\frac{ds}{dt}$ and can be seen as speed with direction. It can be positive, zero or negative.
 - If $v > 0$, the particle is moving in the direction of positive displacement (positive s –axis)
 - If $v = 0$, the particle is **instantaneously at rest**
 - If $v < 0$, the particle is moving in the direction of negative displacement (negative s –axis)
3. **Acceleration, a** , is the rate of change of velocity with respect to time. It is given by $\frac{dv}{dt}$ and its direction is dependent on the velocity of the particle (increases/ decreases) and the direction of the moving particle (positive/ negative displacement).
 - If $a > 0$, the velocity increases with respect to time, and the particle is accelerating
 - If $a < 0$, the velocity decreases with respect to time, and the particle is decelerating
4. **Relationship between Displacement, Velocity and Acceleration**



5. **Common Terms in Kinematics**
 - Initial: Time, $t = 0$
 - At rest/ stationary: Velocity, $v = 0$
 - Particle at fixed point: Displacement, $s = 0$
 - Maximum/ minimum displacement: Velocity, $v = 0$
 - Maximum/ minimum velocity: Acceleration, $a = 0$
6. **Average Speed** = $\frac{\text{Total Distance travelled}}{\text{Total time taken}}$
7. **Distance travelled** is different from **displacement** as it does not have direction. To find distance travelled in the first n seconds,
 - **Step 1:** Find the value(s) of t when $v = 0$.
 - **Step 2:** Use each of the value(s) of t found in Step 1 to find s .
 - **Step 3:** Find the value of s when $t = 0$ and $t = n$.
 - **Step 4:** Draw the path of the particle to visualise the movements of the particle using the displacements found to find the total distance travelled by the particle.



Topic 7: Proof in Plane Geometry

1. Basic Proofs in Plane Geometry (RECAP)

- Adjacent angles on a straight line are supplementary.
- Vertically opposite angles are equal.
- Corresponding angles are equal.
- Alternate angles are equal.
- Interior angles between parallel lines are supplementary.
- Sum of angles in a triangle is 180° .
- Exterior angle of a triangle.
- Base angles of isosceles triangle.
- Angles of equilateral triangle.
- Perpendicular bisector
- Angle bisector

2. Proofs Using Congruent and Similar Triangles

• Tests of Congruent Triangles

- SSS
- SAS
- AAS or ASA
- RHS

• Tests of Similar Triangles

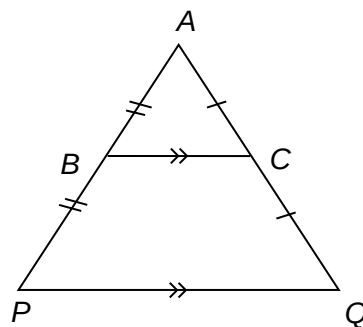
- AA Similarity test
- SSS Similarity test
- SAS Similarity test

3. Properties of Circles

- Angle at centre is twice of the angle subtended by the same arc.
- Right angle in a semicircle.
- Angles in the same segment are equal.
- Angles in the opposite segment are supplementary.
- Line drawn from the centre perpendicular to chord bisects the chord.
- Equal chords are equidistant from the centre of the circle.
- Tangent is perpendicular to radius drawn to the point of contact.
- Tangents from external point are equal.

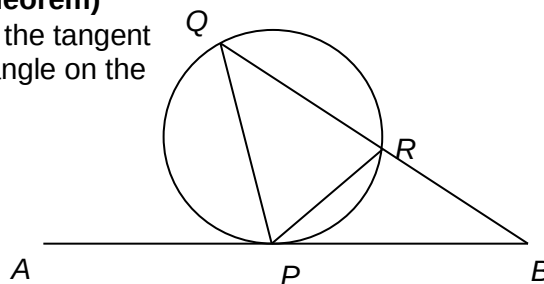
4. Midpoint Theorem for Triangles

- If B and C are midpoints of AP and AQ respectively, then $BC \parallel PQ$ and $BC = \frac{1}{2}PQ$.
- $\triangle ABC$ is similar to $\triangle APQ$



5. Tangent-Chord Theorem (Alternate Segment Theorem)

- Angle between a tangent and a chord meeting the tangent at the point of contact is equal to the inscribed angle on the opposite side of the chord.
 - $\angle BPR = \angle PQR$
 - $\angle QPA = \angle QRP$



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