Class	Register Number	Name

南洋女子中学校

NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2018
Secondary Three

INTEGRATED MATHEMATICS 1

2 hours

Thursday **4 October 2018** **0845 – 1045**

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Answer questions 1 – 11 before attempting the Bonus Question.
2. Write your answers and working on the separate writing paper provided.
3. Write your name, register number and class on each separate sheet of paper that you use and fasten the separate sheets together with the string provided. Do not staple your answer sheets together.
4. Omission of essential steps will result in loss of marks.
5. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

1. The number of marks is given in brackets [ ] at the end of each question or part question.
2. The total number of marks for this paper is 80.
3. The use of an approved scientific calculator is expected, where appropriate.
4. You are reminded of the need for clear presentation in your answers.

Mathematical Formulae

1. TRIGONOMETRY

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

2. MENSURATION

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

1. Boyle's Law states that at constant temperature for a fixed mass, the absolute pressure, P, is inversely proportional to the volume, V, of a gas.
A sample of a gas has a volume of $0.005\ m^3$ at a pressure of 7600 Pascal (Pa).

(i) Find an equation connecting P and V. [2]

(ii) What will the volume of the gas be if the pressure is increased to 18600 Pa? [1]

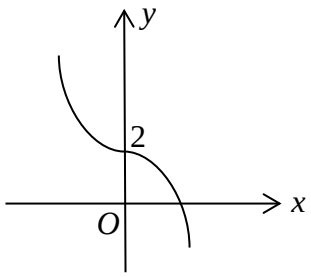
(iii) Sketch a graph to represent the relationship between P and V. [1]

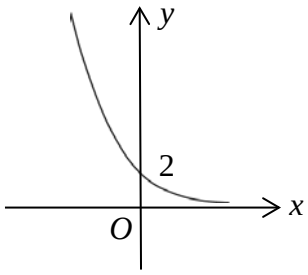
2. **(a)** Sketch each of the graphs of $y = ax^n$ separately, where

(i) $a < 0, n = 2,$ **(ii)** $a > 0, n = -2.$ [2]

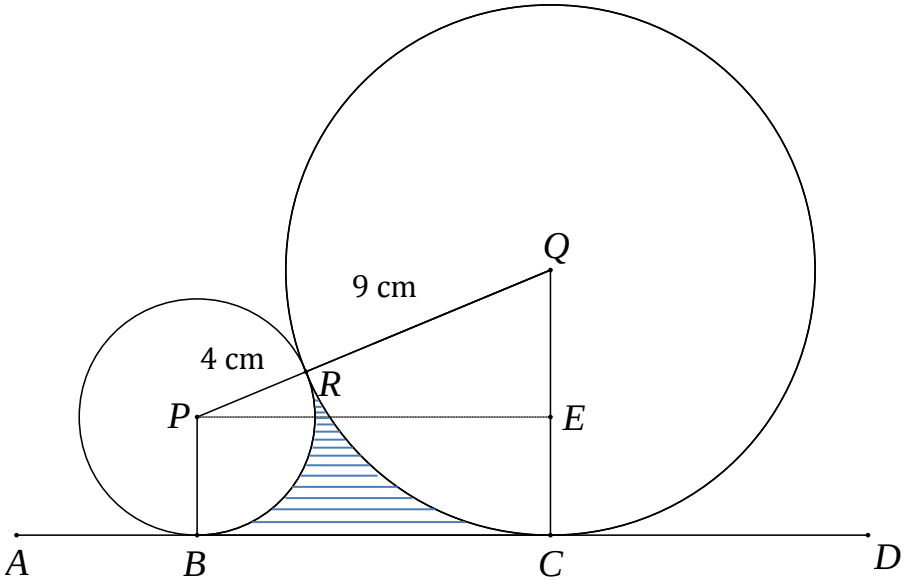
(b) Write down a possible equation for each of the following graphs.

(i) **(ii)** [2]





3. A small circle of centre P, radius 4 cm, touches a large circle of centre Q, radius 9 cm, at the point R. The circles are lying on a straight line $ABCD$. The point E is the foot of the perpendicular from P to QC. Find the perimeter of the shaded region. [5]



4. Variables x and y are known to be related by an equation of the form $xy = ax^3 + bx^2$, where a and b are constants and $x \neq 0$.

When the graph of $\frac{y}{x^2}$ against $\frac{1}{x}$ is plotted, a straight line passing through the points (0.1, 0.9) and (0.5, 1.5) is formed.
Estimate

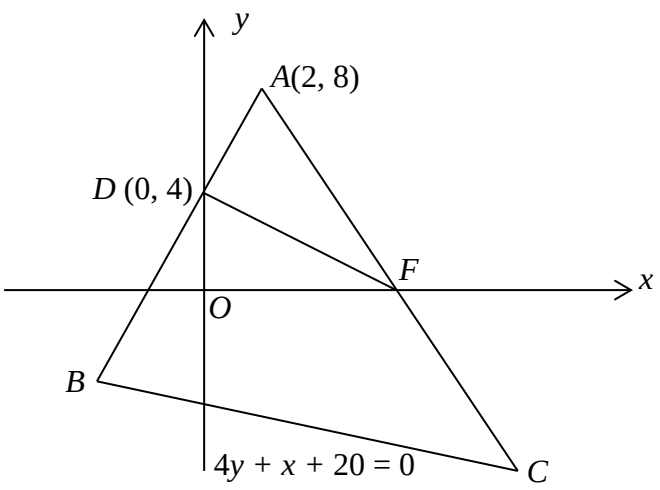
(i) the value of a and of b, [3]

(ii) the value(s) of x when $y = 2$. [3]

If a new line is formed when $\frac{y}{x}$ is plotted against x,

(iii) find the intercept on the x-axis. [2]

5.



The diagram shows a triangle ABC where A has coordinates (2, 8) and AC is parallel to $5y + 8x + 4 = 0$.
AB meets the y-axis at D (0, 4) and AC meets the x-axis at F.

(i) Find the equation of the line AC. [2]

(ii) Determine if DF is perpendicular to AB. [3]

(iii) Given that $AB:BD = 3:2$, find the coordinates of B. [2]

(iv) If the equation of BC is $4y + x + 20 = 0$, find the coordinates of C. [3]

(v) Find the area of quadrilateral $BCFD$. [2]

6. An outbreak of a contagious disease has hit a local secondary school.
The number of lower and upper secondary boys and girls in the school and the percentage of lower and upper secondary students who are currently healthy, ill or carriers of the disease are given in the table below.

	Boys	Girls	Healthy	Ill	Carriers
Lower Secondary	120	160	25%	30%	45%
Upper Secondary	140	120	35%	25%	40%

(i) **P** is a 2×2 matrix representing the number of boys and girls in the lower and upper secondary sections respectively. **Q** is a 2×3 matrix representing the percentage of lower and upper secondary students who are currently healthy, ill or carriers of the disease. Evaluate **PQ** and describe what the elements of matrix **PQ** represent. [2]

(ii) As a result of the outbreak, the school sent all the students for a check-up. The students received treatment if they were ill or carriers. The check-up and/or treatment for students who are healthy, ill and carriers cost \$10, \$25 and \$15 respectively. Write down a 3×1 matrix **R** such that the product **PQR** gives the total medical cost the school incurred for the boys and for the girls respectively. Find the total medical cost the school incurred for the boys and for the girls respectively. [3]

(iii) Given that the School Health Services decided to give the female students a discount of 30% of the medical cost, **using matrix multiplication**, find the total medical costs incurred by the school for all the students as a result of the discount. [2]

7. At a cross-country run, the first-aid post F is located 650 m north of an ambulance A. A milo van M is located 450 m away from the first-aid post such that the bearing of M from A is 319°.

(i) Find the two possible values of angle AMF. [3]

(ii) A drone D is hovering at a point vertically above the ambulance. A student on duty at the first-aid post spotted the drone at an angle of elevation of 5°. She walked south towards the ambulance and reached a point N. The angle of elevation of the drone from the student at N is 10°. Find the distance the student walked towards N. [3]

8. **Answer the whole of this question on the INSERT provided.**

The graph of $y = \frac{1}{5}x^3 + \frac{2}{x}$ is partially shown on the INSERT.

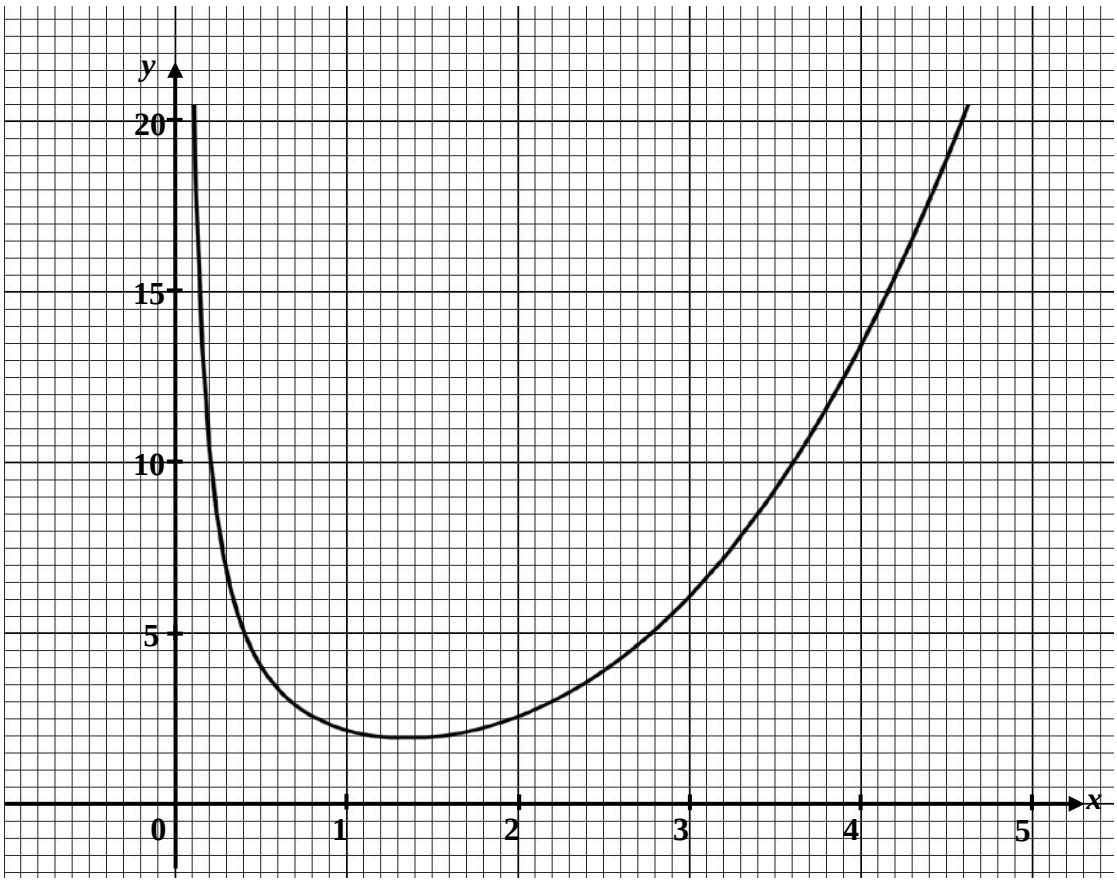
(a) By drawing a tangent, find the gradient of the graph at the point where $x = 0.5$. [3]

(b) Find the x-coordinate of a point on the curve at which the gradient is 5. [2]

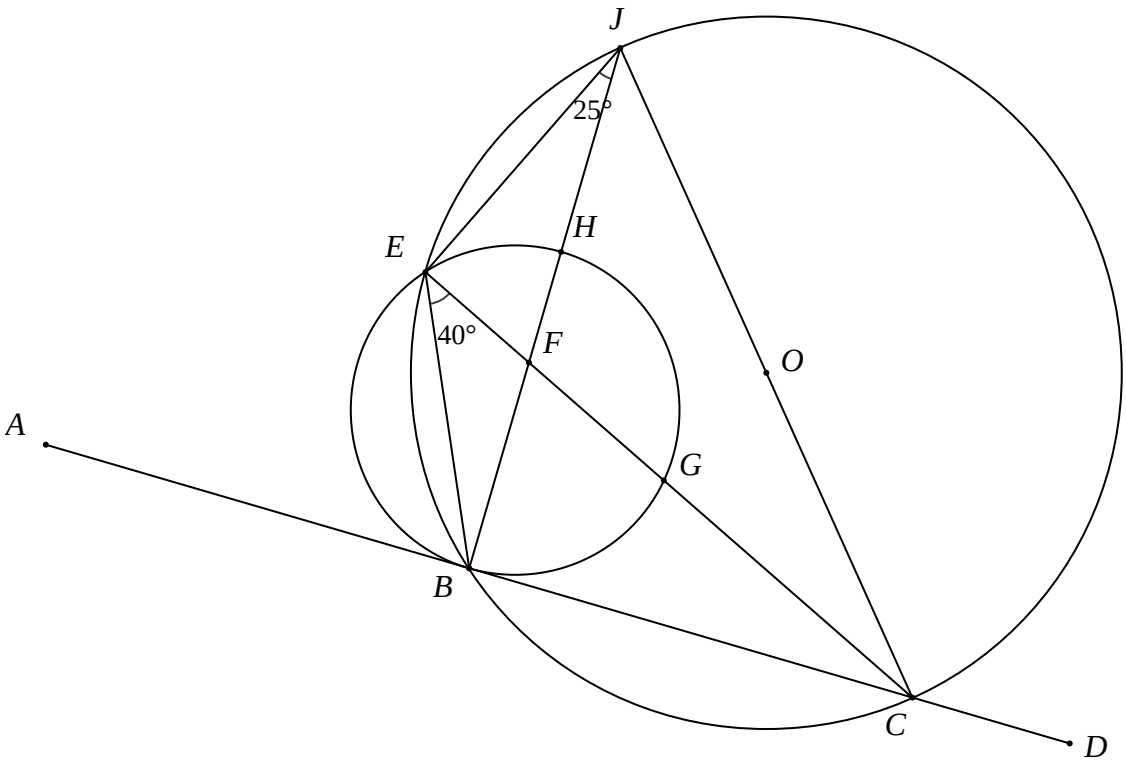
(c) By drawing suitable straight lines on the graph, find

(i) the range of values of x for which $\frac{1}{5}x^3 + \frac{2}{x} \leq 6 - x$, [2]

(ii) the solutions to the equation $x^4 - 5x^2 - 15x + 10 = 0$. [3]



9.



ABCD is a tangent to the smaller circle *BGHE*.

The circles *BCJE* and *BGHE* intersect at *B* and *E*.
JC is the diameter of the larger circle *BCJE*, with centre *O*.
EFGC and *JHFB* are straight lines that intersect at F.
It is given that angle *BEC* = 40° and angle *EJB* = 25°.

(a) Find

(i) angle *BJC*, [1]

(ii) angle *EBA*, [1]

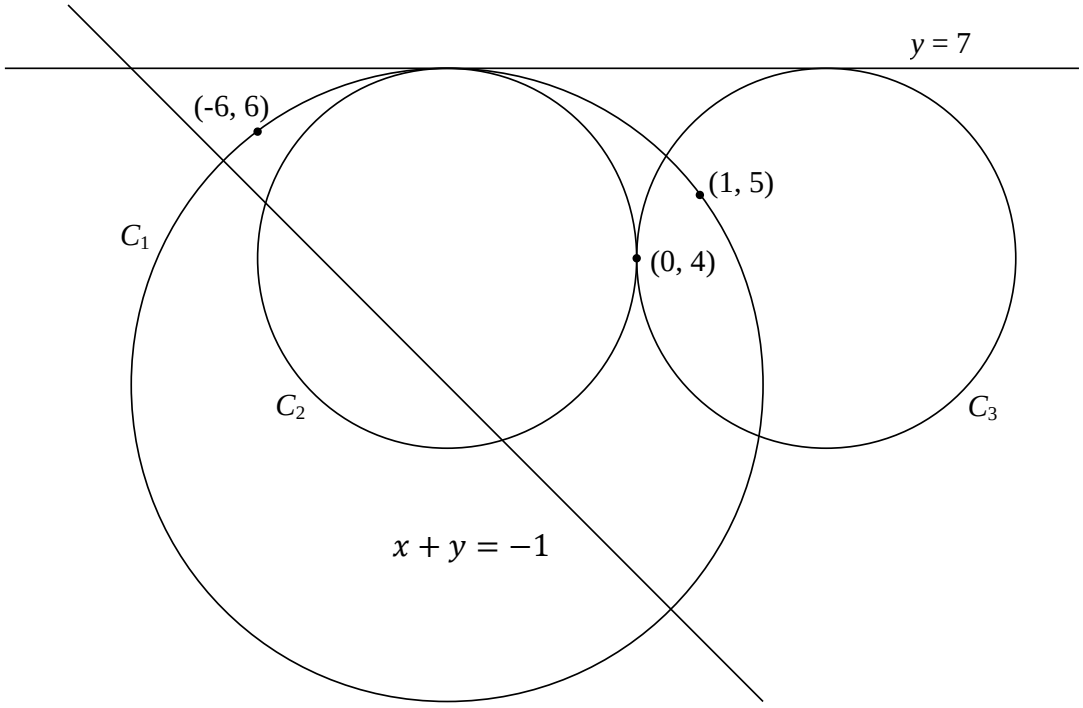
(iii) angle *JCE*, [2]

(iv) angle *BFC*. [1]

(b) State the geometrical property used in **(a)(i)**. [1]

(c) Given that minor arc *JE* = 3 cm, find the length of minor arc *EB*, explaining your reasons clearly. [2]

10. The diagram below shows circle C_1 which passes through the points (1, 5) and $(-6, 6)$ and has its centre lying on the line $x + y = -1$.



(i) Show that the centre of circle C_1 is $(-3, 2)$. [4]

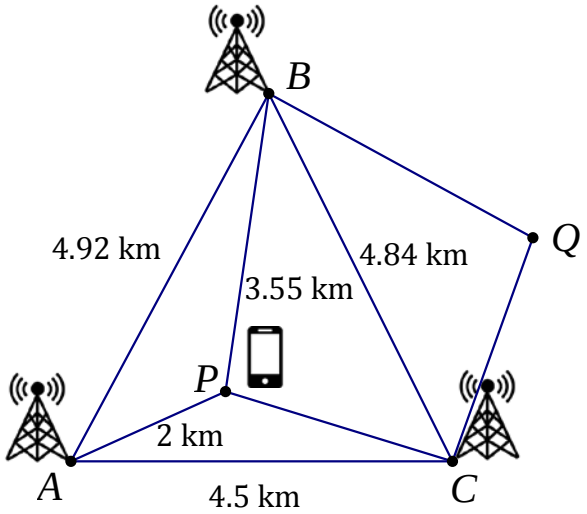
(ii) Hence, find the equation of circle C_1. [2]

The line $y = 7$ is a tangent to circles C_1, C_2 and C_3.
Circles C_2 and C_3 have a radius of 3 and touch the y-axis at (0, 4).

(iii) Write down the coordinates of the centre of C_2 and of C_3. [2]

(iv) Explain why it is possible to draw a circle C_4 passing through the centres of the three circles. Find the centre of C_4. [2]

11.



Three cell towers *A*, *B* and *C* are located such that *AB* = 4.92 km, *AC* = 4.5 km and *BC* = 4.84 km. A cell phone is located at *P*. Once the phone signal is picked up by each of the three cell towers, cell tower triangulation is used to determine its exact location. The overlap of the three signals, gives an estimation of the location of the cell phone. The phone is estimated to be at a distance of 2 km from *A* and 3.55 km from *B*.

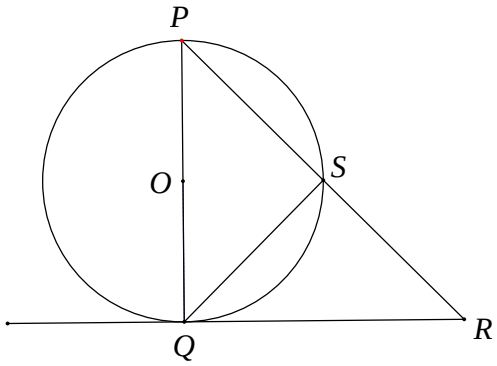
Calculate

(i) angle *ABC*, [2]

(ii) the distance travelled by the owner of the cell phone as he moves along the straight road from *P* to *Q*, given that *PBQC* is a kite. [4]

12. Bonus

QR is a tangent to the circle at centre O. PQ is the diameter of the circle and PSR is a straight line. Prove that $PR^2 = PQ^2 + PR \times SR$. [2]



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