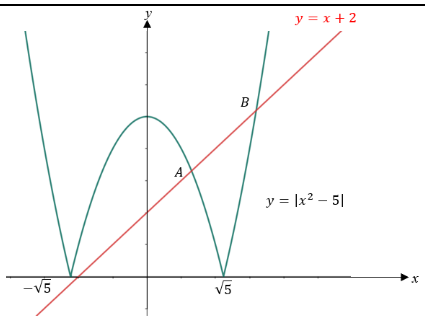
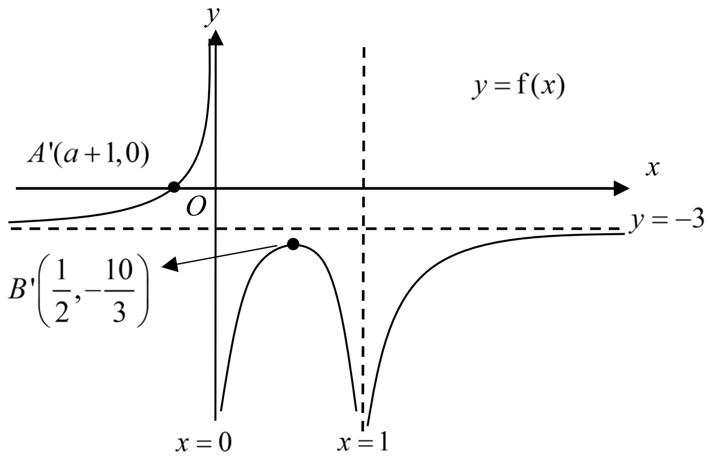
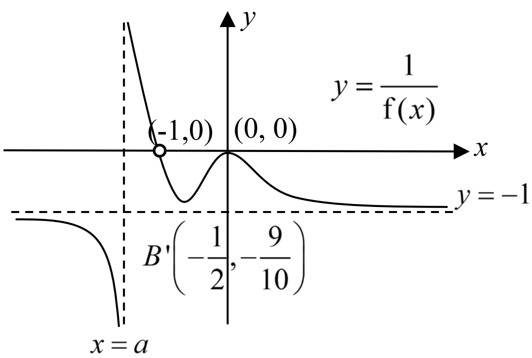
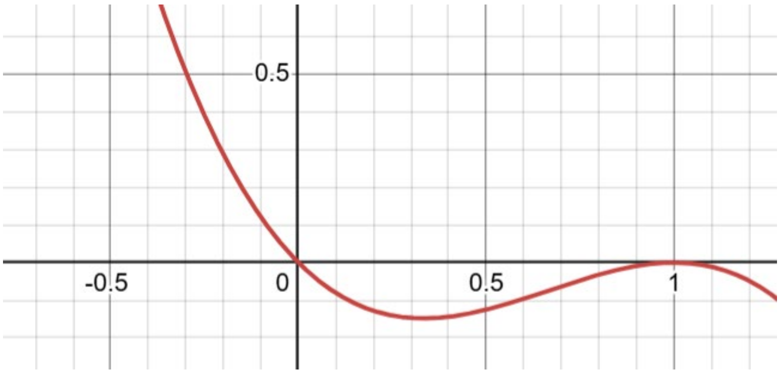
EJC H2 Mathematics 2025 Promo Solutions

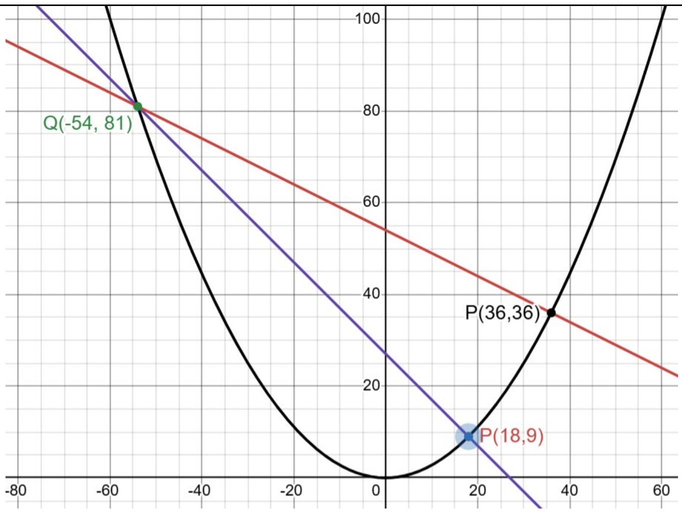
1	**Solution**
	Let x, y and z be the respective price per kilogram for pumpkins, carrots and broccoli respectively. $1.20x+2.50y+1.25z=21.86$ $1.50x+2.70y+1.35z=25.20$ $1.30x+2.60y+1.10z=21.90$ $\therefore x=7.80, y=1.90, z=6.20$ Amount that Denise paid $=2.30\times 7.80+3.10\times 1.90+2.10\times 6.20=\$36.85$

2	**Solution**
	
	The points of intersection: Point A: $-\left(x^2-5\right)=x+2 \Rightarrow x^2+x-3=0$ $$x=\frac{-1\pm\sqrt{1-4(1)(-3)}}{2}$$ $$x=\frac{-1+\sqrt{13}}{2} \text{ or } x=\frac{-1-\sqrt{13}}{2} \text{ (reject since } x>0\text{)}$$ Point B: $x^2-5=x+2 \Rightarrow x^2-x-7=0$ $$x=\frac{1+\sqrt{29}}{2} \text{ or } x=\frac{1-\sqrt{29}}{2} \text{ (reject since } x>0\text{)}$$ From the graph: $x<\frac{-1+\sqrt{13}}{2}$ or $x>\frac{1+\sqrt{29}}{2}$

3	**Solution**
(a)	We can deduce that vectors **a** and **b** are parallel or either vector **a** or **b** is the zero vector.
(b)	$\mathbf{n} = \frac{1}{\sqrt{14}}\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ or $-\frac{1}{\sqrt{14}}\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$
(c)	Let the acute angle between $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and the y-axis be θ. $\cos\theta = \frac{\left\lvert \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\rvert}{\sqrt{14}\sqrt{1}} = \frac{1}{\sqrt{14}}$

4	**Solution**
(a)	
(b)	
(c)	The point of intersection of the transformed curve with the x-axis is the point that was transformed from point $A(a,0)$. $(a,0) \xrightarrow{P} (-a,0) \xrightarrow{Q} (-ka,0)$

5	**Solution**
(a)	If the sequence converges, let L be the limit. Then as $n \to \infty$, $x_n \to L$ and $x_{n+1} \to L$. So we have $L = L^2(2-L) \Rightarrow L^3 - 2L^2 + L = 0 \Rightarrow L = 0$ or 1.
(bi)	$x_1 = 0.5,\ x_2 = 0.375,\ x_3 = 0.2285,\ x_4 = 0.0925, \ldots$ As $n \to \infty$, $x_n \to 0$
(bii)	$x_1 = 1.5,\ x_2 = 1.1429,\ x_3 = 1.1074,\ x_4 = 1.0946, \ldots$ As $n \to \infty$, $x_n \to 1$
(biii)	$x_1 = -1,\ x_2 = 3,\ x_3 = -9,\ x_4 = 891, \ldots$ Sequence diverges **OR** does not converge **OR** Sequence oscillates between positive and negative terms with increasing magnitude
(c)	Observe that $x_{n+1} - x_n = 2x_n^2 - x_n^3 - x_n$, so $y = x_{n+1} - x_n$.  From the graph of $y = 2x^2 - x^3 - x$, when $0 < x_n < 1$, $y < 0$. So $x_{n+1} - x_n < 0$ (shown). Now, $x_{n+1} - x_n < 0 \Rightarrow x_{n+1} < x_n$, i.e. $x_1 > x_2 \ldots > x_n > x_{n+1} > \ldots$ So the (convergent) sequence in **b(i)** is (strictly) decreasing.

6	**Solution**
(a)	$x=6t,\ y=t^2$ $x=6t \Rightarrow \dfrac{dx}{dt}=6$ and $y=t^2 \Rightarrow \dfrac{dy}{dt}=2t$, so $\dfrac{dy}{dx}=\dfrac{dy}{dt}\div\dfrac{dx}{dt}=\dfrac{2t}{6}=\dfrac{t}{3}$. At the point $P(6p,\ p^2)$, $t=p$, so $\dfrac{dy}{dx}=\dfrac{p}{3}$ So gradient of normal $=-\dfrac{3}{p}$ The equation of the normal at the point $P(6p,\ p^2)$ is $y-p^2=-\dfrac{3}{p}(x-6p)$ $yp-p^3=-3x+18p$ $p^3+(18-y)p-3x=0$ (shown)
(b)	Since $Q(-54,\ 81)$ lies on the normal, $p^3+(18-81)p-3(-54)=0$ $p^3-63p+162=0$ From the GC $p=-9,\ 3,\ 6$ Since $p=-9$ gives the point Q, the possible values of p are 3 and 6.
(c)	

7	**Solution**

Let S be the external surface area of the toy.

$$S = \pi r^2 + 2\pi rh + \frac{1}{2}\left(4\pi r^2\right) = 3\pi r^2 + 2\pi rh$$

Let V be the volume of the toy.

$$V = \pi r^2 h - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \pi r^2 h - \frac{2}{3}\pi r^3$$

Since $V = k$, $k = \pi r^2 h - \frac{2}{3}\pi r^3 \quad \Rightarrow \quad h = \frac{1}{\pi r^2}\left(k + \frac{2}{3}\pi r^3\right)$

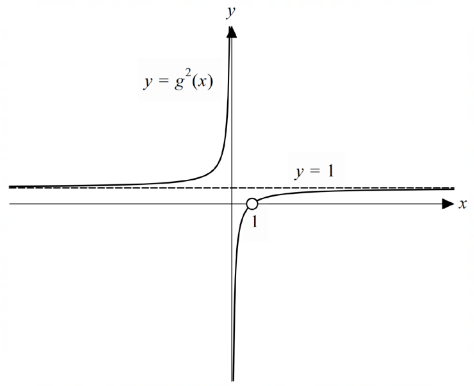
Sub $h = \frac{1}{\pi r^2}\left(k + \frac{2}{3}\pi r^3\right)$ into S,

$$\begin{aligned} S &= 3\pi r^2 + 2\pi r\left[\frac{1}{\pi r^2}\left(k + \frac{2}{3}\pi r^3\right)\right] \\ &= 3\pi r^2 + \frac{2}{r}\left(k + \frac{2}{3}\pi r^3\right) \\ &= 3\pi r^2 + \frac{2k}{r} + \frac{4}{3}\pi r^2 = \frac{13}{3}\pi r^2 + \frac{2k}{r} \end{aligned}$$

For S to be minimum,

$$\begin{aligned} \frac{\mathrm{d}S}{\mathrm{d}r} &= 0 \\ \frac{26}{3}\pi r - \frac{2k}{r^2} &= 0 \\ \frac{26\pi r}{3} &= \frac{2k}{r^2} \\ r^3 &= \frac{6k}{26\pi} \\ r &= \left(\frac{3k}{13\pi}\right)^{\frac{1}{3}} \end{aligned}$$

$$\begin{aligned} h &= \frac{1}{\pi\left(\frac{3k}{13\pi}\right)^{\frac{2}{3}}}\left(k + \frac{2}{3}\pi\left(\frac{3k}{13\pi}\right)\right) \\ &= \frac{1}{\pi}\left(\frac{3k}{13\pi}\right)^{-\frac{2}{3}}\left(k + \frac{2k}{13}\right) \\ &= \frac{1}{\pi}\left(\frac{3k}{13\pi}\right)^{-\frac{2}{3}}\left(\frac{15k}{13}\right) \\ &= 5\left(\frac{3k}{13\pi}\right)^{-\frac{2}{3}}\left(\frac{3k}{13\pi}\right) \\ &= 5\left(\frac{3k}{13\pi}\right)^{\frac{1}{3}} \end{aligned}$$

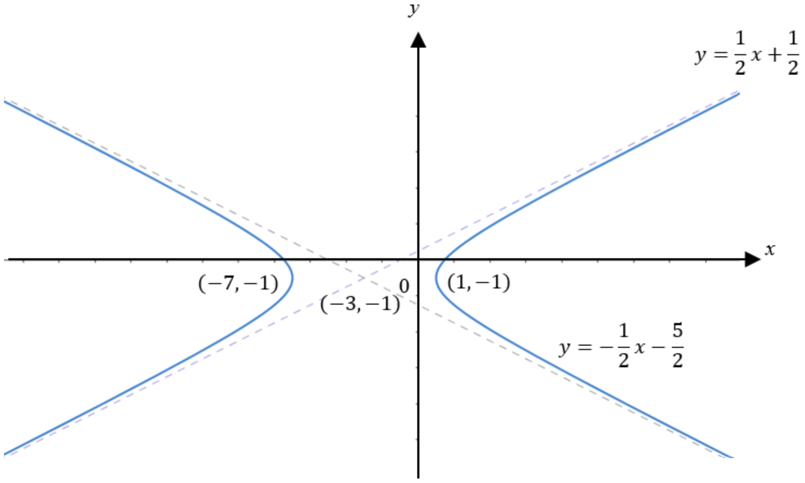
8	**Solution**
(a)	$R_f = \mathbb{R}\setminus\{0\}$ while $D_f = \mathbb{R}\setminus\{1\}$ so $R_f \not\subseteq D_f$. Hence f^2 does not exist. <u>Alternatively</u>: $f(0) = 1 \in R_f$ but $1 \notin D_f$. So $R_f \not\subseteq D_f$. Hence f^2 does not exist.
(b)	$g^2(x) = g\left(\frac{1}{1-x}\right) = \frac{1}{1-\frac{1}{1-x}} = \frac{1-x}{(1-x)-1} = \frac{x-1}{x}$ $\left(\text{or } 1-\frac{1}{x}\right)$ $D_{g^2} = D_g = \mathbb{R}\setminus\{0,1\}$ or $(-\infty, 0)\cup(0,1)\cup(1,\infty)$ So $g^2 : x \mapsto 1-\frac{1}{x},\quad x\in\mathbb{R},\, x\neq 0,1$ By considering the graph of $y = 1-\frac{1}{x}$ for $x \neq 0,1$, $R_{g^2} = \mathbb{R}\setminus\{0,1\}$ or $(-\infty,0)\cup(0,1)\cup(1,\infty)$.  <u>Alternative for finding range through mapping:</u> $\underbrace{\mathbb{R}\setminus\{0,1\}}_{D_{g^2}=D_g} \xrightarrow{g} \underbrace{\mathbb{R}\setminus\{0,1\}}_{R_g} \xrightarrow{g} \underbrace{\mathbb{R}\setminus\{0,1\}}_{R_{g^2}}$
(c)	<u>Method 1:</u> $g^3 = gg^2$ $g^3(x) = g\left(1-\frac{1}{x}\right) = \frac{1}{1-\left(1-\frac{1}{x}\right)} = \frac{1}{\left(\frac{1}{x}\right)} = x$ <u>Method 2:</u> $g^3 = g^2g$ $g^3(x) = g^2\left(\frac{1}{1-x}\right) = 1-\frac{1}{\left(\frac{1}{1-x}\right)} = 1-(1-x) = x$ Since $g^3(x) = x$, $g^{3k}(x) = x$ for all integers *k*. We have $2025 = 3\times 675$, hence $g^{2025}(x) = x$. So $g^{2025}(x) + g(x) = 0$ is equivalent to $x + \frac{1}{1-x} = 0$ $\Rightarrow x^2 - x - 1 = 0$ Solving using G.C., $x = -0.618$ or 1.62. <u>Alternatively:</u> $x^2 - x - 1 = 0 \quad\Rightarrow\quad x = \frac{-(-1)\pm\sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1}{2}\pm\frac{\sqrt{5}}{2}$

9	**Solution**

(a)

$$(x+3)^2-4(y+1)^2=16 \Rightarrow \frac{(x+3)^2}{4^2}-\frac{(y+1)^2}{2^2}=1$$

Equations of oblique asymptotes: $y+1=\pm\frac{1}{2}(x+3) \quad \Rightarrow \quad y=\frac{x}{2}+\frac{1}{2},\ y=-\frac{x}{2}-\frac{5}{2}$



(b)

$(x+3)^2-4(y+1)^2=16 ----(1)$

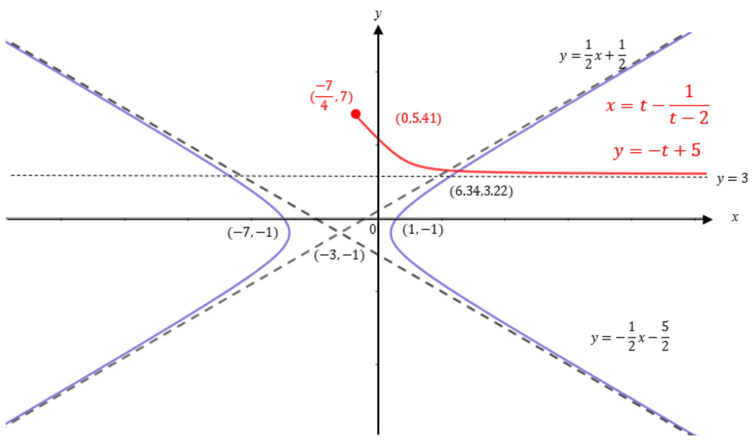
$x=t-\frac{1}{t-2}$ and $y=-t+5$ ------(2)

Sub (2) into (1): $\left(t-\frac{1}{t-2}+3\right)^2-4(-t+6)^2-16=0$

From GC, $t=1.7806, 2.0720$ (rej), 3.6944 (rej), 14.453 (rej), since $-2\le t<2$

The coordinates of the point of intersection $(6.34, 3.22)$

(c)



y-intercept: $\quad x=0 \Rightarrow t-\frac{1}{t-2}=0 \Rightarrow t^2-2t-1=0$

$\Rightarrow t=-0.41421$ or 2.4142 (rej $\because -2\le t<2$)

$\Rightarrow y=5.41$ (3s.f.)

Asymptote: $-2\le t<2 \Rightarrow 3<5-t\le 7$

i.e. $3<y\le 7 \Rightarrow y=3$ is the horizontal asymptote

10	**Solution**
(a)	$y=\tan\left[\ln(1+2x)\right]$ <u>Method 1 – direct differentiation</u> $\frac{\mathrm{d}y}{\mathrm{d}x}=\sec^2\left[\ln(1+2x)\right]\times\frac{1}{1+2x}\times 2=\frac{2\sec^2\left[\ln(1+2x)\right]}{1+2x}=\frac{2(1+y^2)}{1+2x}$ $(1+2x)\frac{\mathrm{d}y}{\mathrm{d}x}=2(1+y^2)$ (shown) <u>Method 2 – arctan first</u> $\tan^{-1}y=\ln(1+2x)$ Diff w.r.t. x, $\frac{1}{1+y^2}\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{1+2x}$ $(1+2x)\frac{\mathrm{d}y}{\mathrm{d}x}=2(1+y^2)$ (shown)
(b)	When $x=0$, $y=0$ (So we need at least term in x^3 for 1st 3 non-zero terms) Diff **(a)** implicitly w.r.t. x, $(1+2x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\frac{\mathrm{d}y}{\mathrm{d}x}=4y\frac{\mathrm{d}y}{\mathrm{d}x}$ ---(*) Diff (*) implicitly w.r.t. x, $(1+2x)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=4\left(y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}y}{\mathrm{d}x}\right)\Rightarrow(1+2x)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+(4-4y)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=4\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ When $x=0$, $\frac{\mathrm{d}y}{\mathrm{d}x}=2$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-4$, $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=32$ So $y=0+2x+\frac{-4}{2!}x^2+\frac{32}{3!}x^3+...=2x-2x^2+\frac{16}{3}x^3+...$
(c)	$2x(1+ax)^b=2x\left(1+b(ax)+\frac{b(b-1)}{2!}(ax)^2+...\right)=2x+2abx^2+a^2b(b-1)x^3+...$ From part **(b)**, $ab=-1$ ---(1) and $a^2b^2-a^2b=\frac{16}{3}$ ---(2) Sub (1) into (2): $(-1)^2-a(-1)=\frac{16}{3}\Rightarrow a=\frac{13}{3}$ Sub into (1): $\frac{13}{3}(b)=-1\Rightarrow b=-\frac{3}{13}$ $\therefore a=\frac{13}{3}, b=-\frac{3}{13}$.

11	**Solution**
(a)	$\int \frac{e^{\sqrt{x-1}}}{\sqrt{x-1}} \, dx = 2\int \frac{1}{2}(x-1)^{-\frac{1}{2}} e^{\sqrt{x-1}} \, dx = 2e^{\sqrt{x-1}} + c,$ where c is an arbitrary constant
(b)	$\int \frac{1}{4x^2+4x+17} \, dx = \int \frac{1}{(2x+1)^2+16} \, dx = \frac{1}{2}\int \frac{2}{(2x+1)^2+4^2} \, dx$ $= \frac{1}{2} \cdot \frac{1}{4} \tan^{-1} \frac{2x+1}{4} + c$ $= \frac{1}{8} \tan^{-1} \frac{2x+1}{4} + c$, where c is an arbitrary constant <u>Alternative: factorise 4 first</u> $\int \frac{1}{4x^2+4x+17} \, dx = \frac{1}{4}\int \frac{1}{x^2+x+\frac{17}{4}} \, dx = \frac{1}{4}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+2^2} \, dx$ $= \frac{1}{4}\left[\frac{1}{2}\tan^{-1}\frac{x+\frac{1}{2}}{2}\right] + c$ $= \frac{1}{8} \tan^{-1} \frac{2x+1}{4} + c$, where c is an arbitrary constant
(c)	$u = \sqrt{2x+1} \Rightarrow u^2 = 2x+1 \Rightarrow \frac{dx}{du} = u$ (or $\frac{du}{dx} = \frac{1}{2}(2x+1)^{-\frac{1}{2}}(2) = \frac{1}{u}$) Sub into integral: $\int x\sqrt{2x+1} \, dx = \int \frac{u^2-1}{2}(u) \, u \, du = \frac{1}{2}\int u^4 - u^2 \, du$ $= \frac{1}{2}\left(\frac{u^5}{5} - \frac{u^3}{3}\right) + c = \frac{(2x+1)^{\frac{5}{2}}}{10} - \frac{(2x+1)^{\frac{3}{2}}}{6} + c$, where c is an arbitrary constant

12	**Solution**

(a)

$l_2 : \mathbf{r} = \begin{pmatrix} 7 \\ 2 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix}, \mu \in \mathbb{R}.$

$l_1 \perp l_2$ implies that $\begin{pmatrix} b \\ 2 \\ 1 \end{pmatrix} . \begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix} = 0 \Rightarrow 3b + 2 - 5 = 0 \Rightarrow b = 1$

Since the two lines intersect, $\begin{pmatrix} a \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix}$ for some $\lambda, \mu \in \mathbb{R}$.

So we have $a + \lambda = 7 + 3\mu ----(1)$

$-3 + 2\lambda = 2 + \mu ---(2)$

$1 + \lambda = -2 - 5\mu ---(3)$

Solving (2) and (3), $\lambda = 2, \mu = -1$.

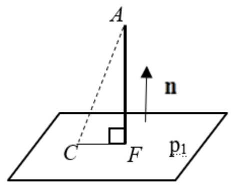
Substitute into (1): $a = 2$

Thus $\overrightarrow{OA} = \begin{pmatrix} 7 \\ 2 \\ -2 \end{pmatrix} + (-1) \begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$

Coordinates of point A are $(4, 1, 3)$.

(b)

Let $C(-2,0,0)$ be a point on the plane p_1.



Then the shortest distance from A to p_1 is the length of projection of $\overrightarrow{CA}$ onto $\mathbf{n}$.

So shortest distance from A to $p_1 = \dfrac{\left| \overrightarrow{CA} . \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right|}{\sqrt{5}} = \dfrac{\left| \begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix} . \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right|}{\sqrt{5}} = \dfrac{15}{\sqrt{5}} = 3\sqrt{5}$

<u>Alternative method: find foot of perpendicular first (not recommended in this case as *F* is not requested)</u>

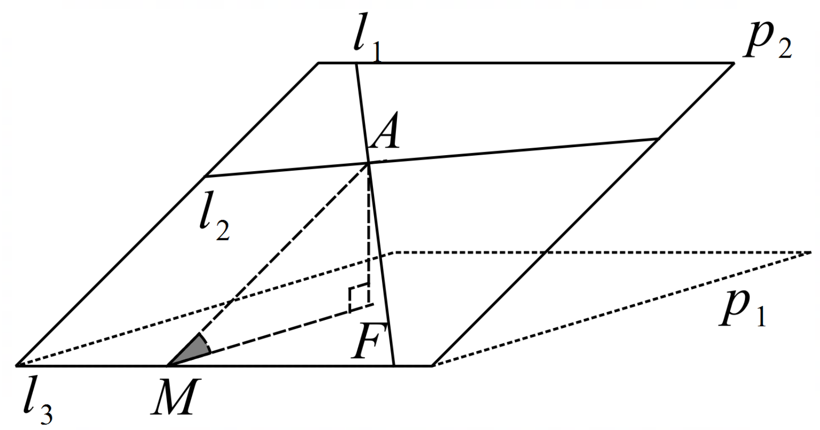
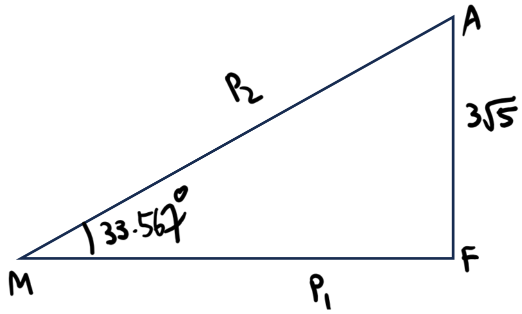
Let F be the foot of perpendicular from A to the plane p_1.

F lies on the line AF, so $\overrightarrow{OF} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ for some $t \in \mathbb{R}$.

F lies on p_1, so $\overrightarrow{OF} . \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \left[\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right] . \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = -4 \qquad \Rightarrow 5t + 11 = -4 \Rightarrow t = -3$

$\overrightarrow{AF} = \overrightarrow{OF} - \overrightarrow{OA} = \left[\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} - 3 \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right] - \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} = -3 \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

So $\left| \overrightarrow{AF} \right| = 3\sqrt{5}$.

(c)	Let θ be the acute angle between the planes p_1 and p_2. $\cos\theta = \dfrac{\left\lvert \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -11 \\ 8 \\ -5 \end{pmatrix} \right\rvert}{\sqrt{5}\sqrt{210}}$ $= \dfrac{27}{\sqrt{5}\sqrt{210}}$ $\theta = 33.567^0 \approx 33.6^0$
(d)	<u>Hence method</u>   Cross-section view of the planes p_1 and p_2 - line of intersection appears as a point *M*. Required distance is the length *AM*. $\sin\theta = \dfrac{AF}{AM} \quad \Rightarrow \quad AM = \dfrac{3\sqrt{5}}{\sin 33.567^\circ} = 12.1$ (to 3 sf)

13	**Solution**
(a)	The yearly decrease in effective range

Year (N)	Yearly decrease in effective range (u_N)
1	5.6
2	5.6 + 0.75
3	5.6 + 2(0.75)
…	…

i.e. Decrease in effective range is in A.P with $a = 5.6$; $d = 0.75$

$u_7 = 5.6 + 6(0.75) = 10.1$ (Shown)

(b)	[Effective range = Initial range − **total** decrease in effective range]

effective range $< 0.75 \times 460 = 345$

$\Rightarrow 460 - S_n < 345$

$\Rightarrow S_n > 115$

$\Rightarrow \frac{n}{2}\left[2(5.6)+(n-1)(0.75)\right] > 115$

n	S_n
11	$102.85 < 115$
12	$116.7 > 115$

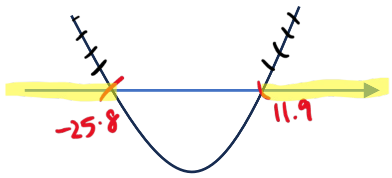
A battery replacement is recommended in year 12 of car ownership.

Alternative Method to solve Inequality

$\frac{n}{2}\left[2(5.6)+(n-1)(0.75)\right] > 115$

$5.6n + 0.375n^2 - 0.375n - 115 > 0$

$0.375n^2 + 5.225n - 115 > 0$



Thus, $n < -25.8$ *or* $n > 11.9$.

Since n is a positive integer (n refers to number of years), least $n = 12$.

(c)

Mth	Loan amt at start of mth	Loan amt at end of mth
1	$150000(1.002)$	$150000(1.002) - x$
2	$\left[150000(1.002) - x\right](1.002)$ $= 150000(1.002)^2 - x(1.002)$	$150000(1.002)^2 - x(1.002) - x$
3	$150000(1.002)^3 - x(1.002)^2 - x(1.002)$	$150000(1.002)^3 - x(1.002)^2 - x(1.002) - x$
…		
n	$150000(1.002)^n - x(1.002)^{n-1} - ... - x(1.002)$	$150000(1.002)^n - x(1.002)^{n-1} - ... - x(1.002) - x$

Outstanding loan amt at end of n months:

$$150000(1.002)^n - x\left[(1.002)^{n-1} + ... + (1.002) + 1\right] = 150000(1.002)^n - \frac{x\left(1.002^n - 1\right)}{1.002 - 1}$$

$$= 150000(1.002)^n - 500x\left(1.002^n - 1\right) \quad \text{(Shown)}$$

(d) (i)

$150\,000\left(1.002^{60}\right) - 500x\left(1.002^{60} - 1\right) = 0$

Solving, $x = 2655.50$.

(ii)

Total interest paid $= (2655.50 \times 60) - 150000 = 9330$