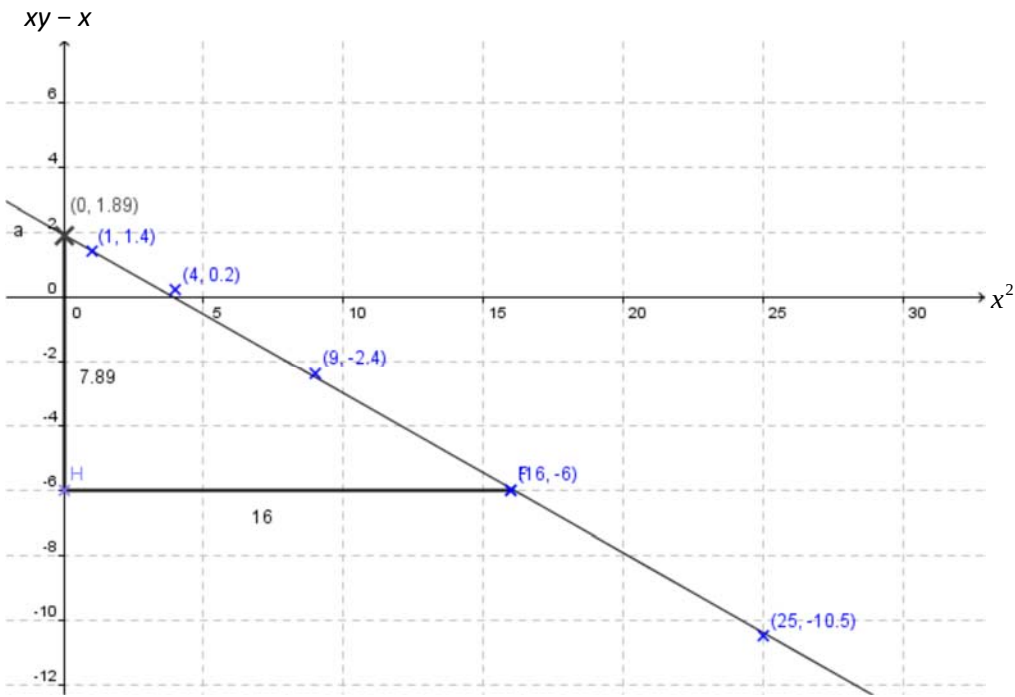


Topic: Linear Law

Question 1 [2008 EOY, Q4]	marks
Two variables x and y are such that when $\lg y$ is drawn against $\lg x$, a straight line of gradient -3 is obtained, passing through the point $(-2, 9)$. (i) Express y in terms of x. <u>Solution:</u> $\lg y - 9 = -3(\lg x + 2)$ $\Rightarrow \lg y = -3\lg x + 3 \Rightarrow \lg y = \lg x^{-3} + \lg 10^3 \Rightarrow \lg y = \lg \left(\frac{10}{x} \right)^3$ Hence $y = \left(\frac{10}{x} \right)^3$	[4]
(ii) Find the value of x when $y = -1$. <u>Solution:</u> (a) When $y = -1$, $\left(\frac{10}{x} \right)^3 = -1 \Rightarrow \frac{10}{x} = -1 \Rightarrow x = -10$	[2]

Question 2 [2009 EOY, Q15]		marks																									
Answer the whole of this question on a sheet of graph paper. The table shows experimental values of two variables, x and y, which are known to be related by the equation $y - 1 = px + \frac{q}{x}$. <table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>2.4</td><td>1.1</td><td>0.2</td><td>-0.5</td><td>-1.1</td></tr></table> (i) Choosing a suitable scale, plot $(xy - x)$ against x^2 and draw a straight line. Solution: <table><tr><td>$xy - x$</td><td>1.4</td><td>0.2</td><td>-2.4</td><td>-6</td><td>-10.5</td></tr><tr><td>x^2</td><td>1</td><td>4</td><td>9</td><td>16</td><td>25</td></tr></table> 			1	2	3	4	5	y	2.4	1.1	0.2	-0.5	-1.1	$xy - x$	1.4	0.2	-2.4	-6	-10.5	x^2	1	4	9	16	25	[5]	
	1	2	3	4	5																						
y	2.4	1.1	0.2	-0.5	-1.1																						
$xy - x$	1.4	0.2	-2.4	-6	-10.5																						
x^2	1	4	9	16	25																						
(ii) Use your graph to estimate the values of p and q. Solution: $x(y - 1) = px^2 + q$ Fr graph, $q = 1.89$, $p = -0.493$																											
(iii) Explain how you would use your graph to determine if the set of experimental values $x = 10$ and $y = -4$ are related by the equation $y - 1 = px + \frac{q}{x}$. Solution: $xy - x = -50$, $x^2 = 100$. Extend the line. If it passes through $(100, -50)$, then the values are related by the equation $y - 1 = px + \frac{q}{x}$.		[1]																									

Question 3 [2010 EOY, Q8]

marks

The table below shows experimental readings of y for given values of x such that $xy = ay + b\sqrt{x}$ where a and b are real constants.

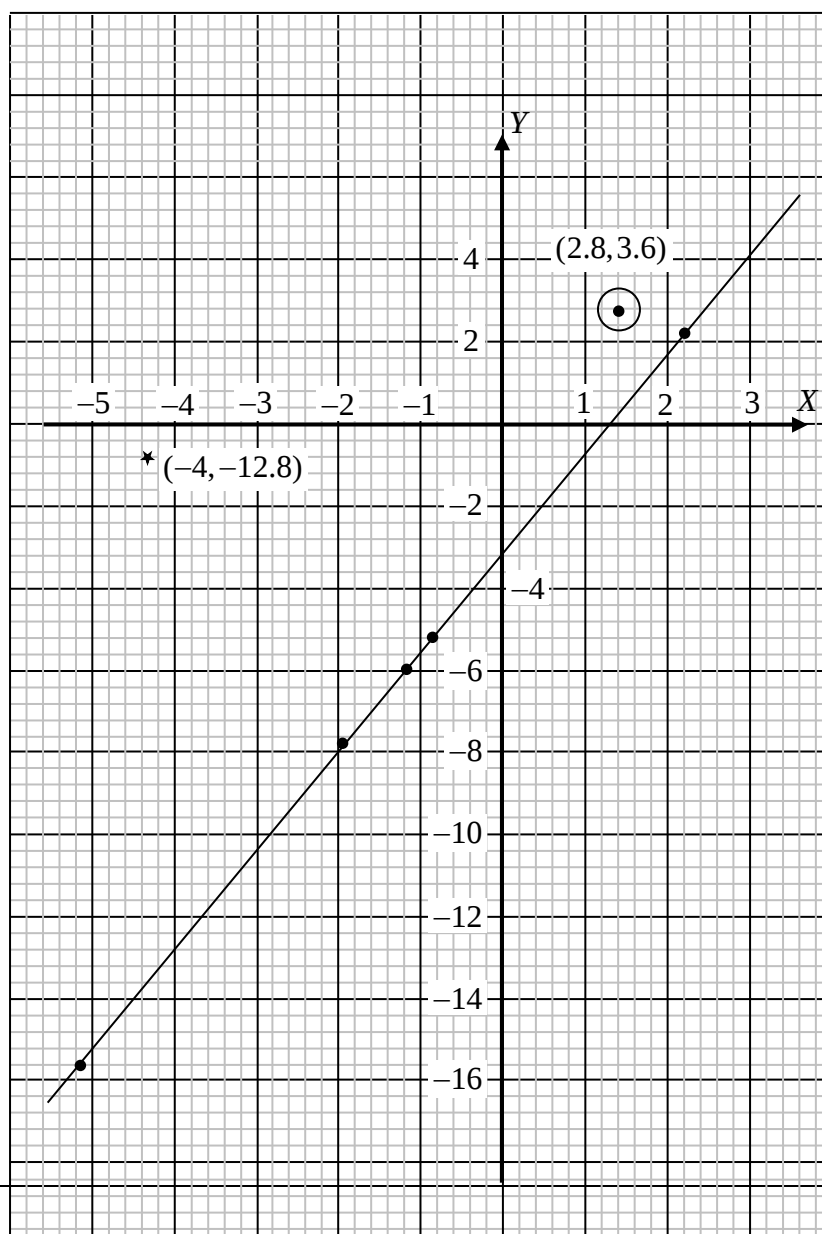
x	0	1	2	3	4	5	6
y	0	2.21	1.96	-8.95	-3.88	-2.67	-2.11

By drawing a straight line graph of $\sqrt{x}y$ against $\frac{y}{\sqrt{x}}$, identify the incorrect reading,

Solution: $xy = ay + b\sqrt{x} \Rightarrow \sqrt{x}y = a\frac{y}{\sqrt{x}} + b$ so let $X = \frac{y}{\sqrt{x}}$ and $Y = \sqrt{x}y$

X	2.21	1.39	-5.17	-1.94	-1.19	-0.86
Y	2.21	2.77	-15.5	-7.76	-5.97	-5.17

From graph, the incorrect reading is 1.96



[4]

(ii) estimate the values of a and b. <u>Solution:</u> From the graph, $a \approx \frac{3.6 - (-12.8)}{2.8 - (-4)}$ $= 2.4$ (1 d.p.) $b \approx -3.2$ (1 d.p.)	[2]												
Question 4 [2012 EOY, Q12] The table shows experimental values of two variables, x and y, which are known to be related by the equation $\sqrt{y}(x^b) = a$, where a and b are constants. <table><tr><td>x</td><td>1.0</td><td>1.5</td><td>2.0</td><td>2.5</td><td>3.0</td></tr><tr><td>y</td><td>7.3</td><td>3.5</td><td>2.0</td><td>1.3</td><td>0.9</td></tr></table> (i) Express this equation in a form suitable for drawing a straight line graph. <u>Solution:</u> $\sqrt{y}(x^b) = a \Rightarrow \frac{1}{2} \lg y + b \lg x = \lg a \Rightarrow \lg y = -2b \lg x + 2 \lg a$	x	1.0	1.5	2.0	2.5	3.0	y	7.3	3.5	2.0	1.3	0.9	marks [2]
x	1.0	1.5	2.0	2.5	3.0								
y	7.3	3.5	2.0	1.3	0.9								

(ii)

Using a suitable scale, plot a straight line graph of $\lg y$ against $\lg x$.

Use your graph to estimate the value of a and of b .

[6]

Solution:

$\lg x$	0		0.18	0.30	0.40	0.48
$\lg y$	0.86		0.54	0.30	0.11	-0.05

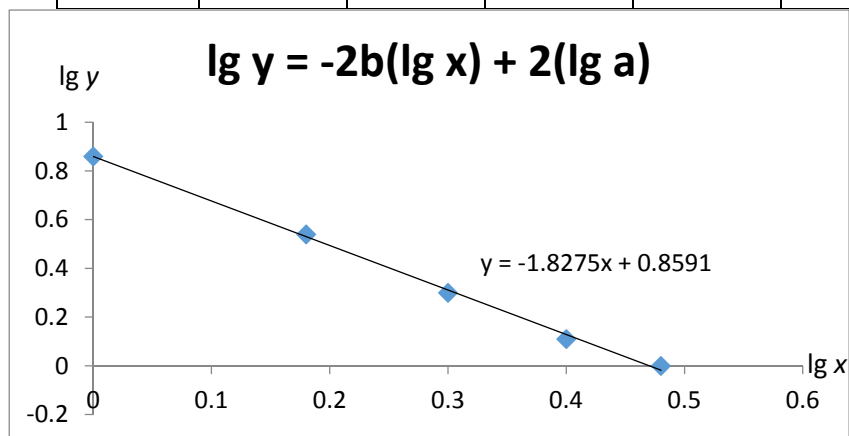


Table of values

Label axes

Plot points correctly

Best fit line

From graph,

$$2\lg a \approx 0.86$$

$$a = 2.69$$

$$-2b \approx \frac{0.86 - 0}{0 - 0.47}$$

$$b = 0.915$$

Question 5 [2013 EOY, Q15]

marks

The table shows the experimental values of two variables x and y .

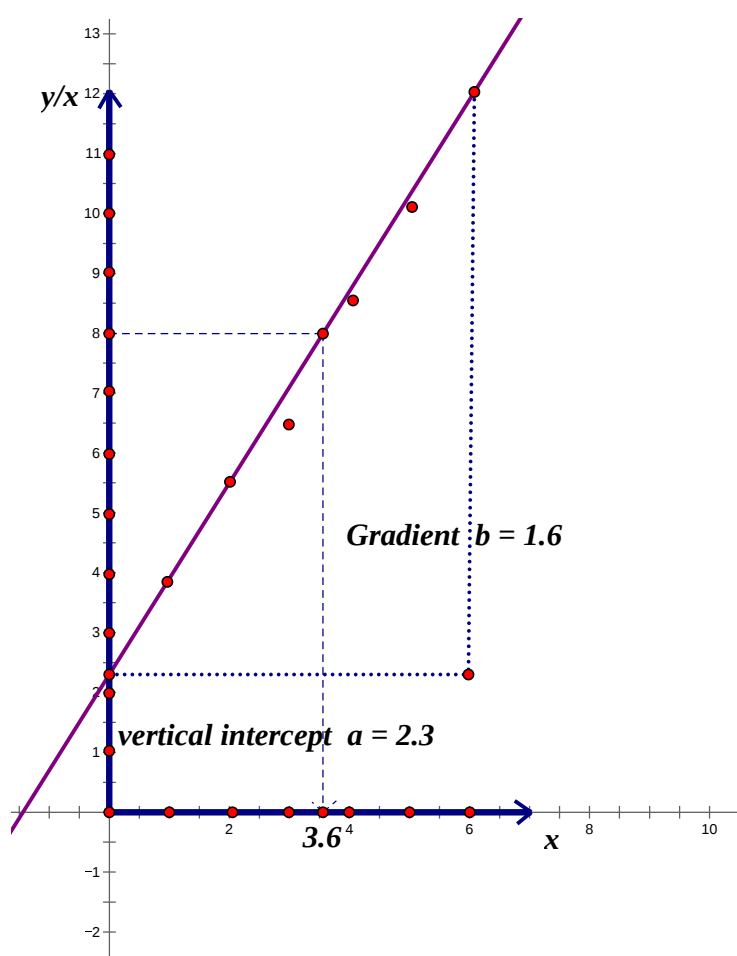
x	1	2	3	4	5
y	3.8	11.2	20.5	33.2	50.6
$\frac{y}{x}$	3.8	5.6		8.3	

It is known that x and y are related by the equation $y = ax + bx^2$

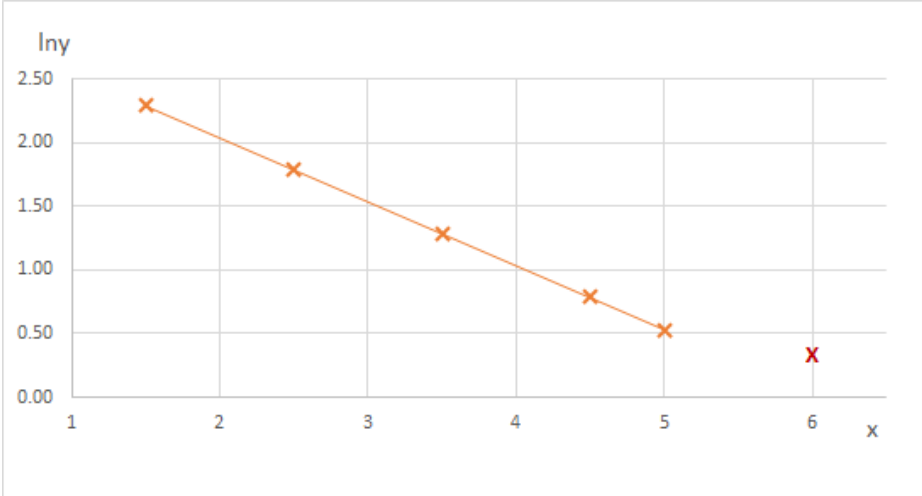
Plot $\frac{y}{x}$ against x and draw a suitable straight line.

Solution:

x	1	2	3	4	5
y	3.8	11.2	20.5	33.2	50.6
$\frac{y}{x}$	3.8	5.6	6.8	8.3	10.1



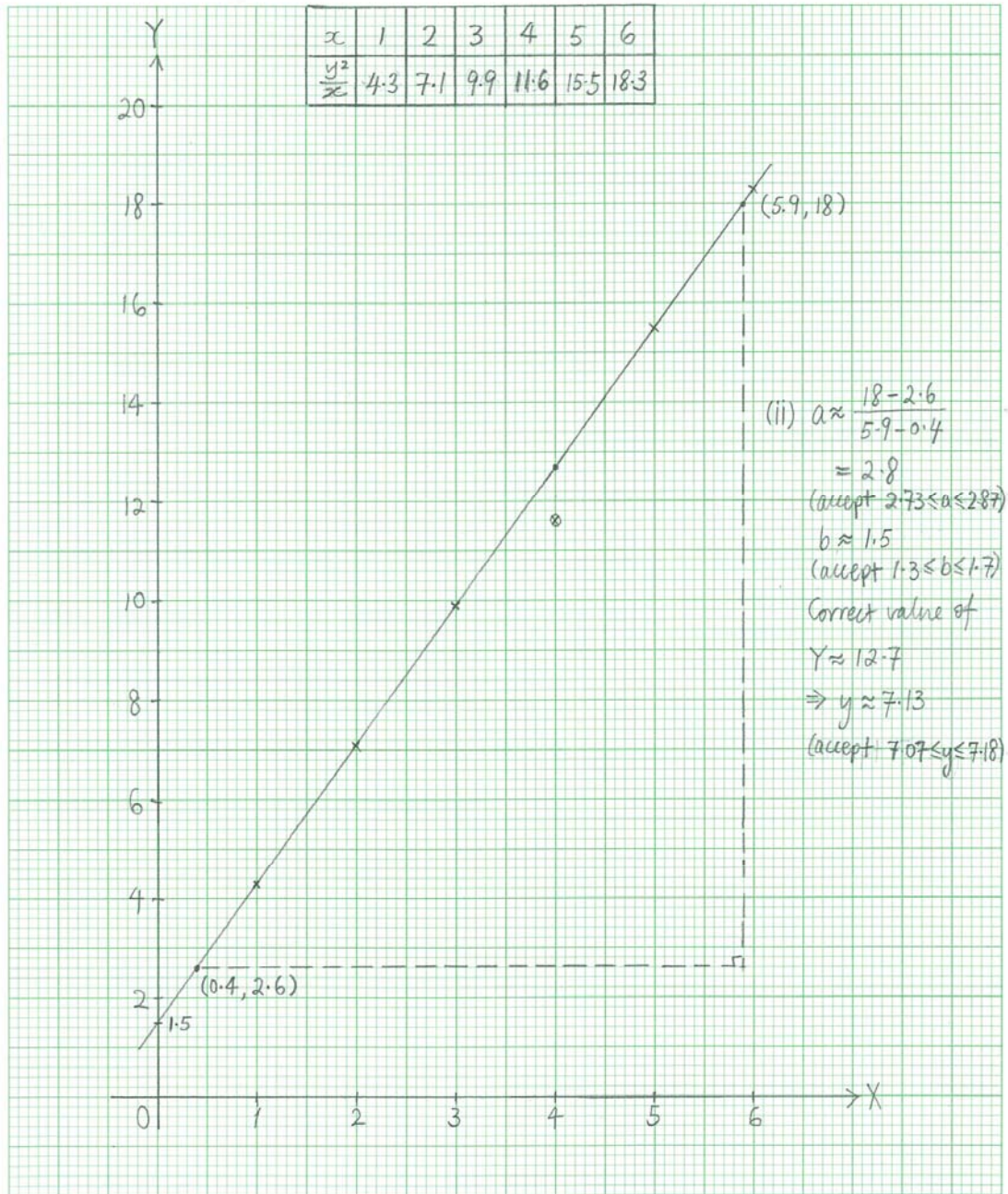
(a) Use the graph to estimate the value of a and of b, <u>Solution:</u> (a) Plot $\frac{y}{x}$ against x. From the diagram, The gradient $b = 1.6$ (± 0.2) and the vertical intercept is $a = 2.3$. (± 0.1)	[4]
(ii) the value of x when $y = 8x$. <u>Solution:</u> When $y = 8x \Rightarrow \frac{y}{x} = 8$. From the diagram, the value of x is 3.6 (± 0.1)	[1]
(b) The diagram shows the straight line graph of xy against x^2 passing through the points $A(0, 6)$ and $B(8, 4)$. Express y in terms of x. The figure shows a Cartesian coordinate system. The vertical axis is labeled 'xy' and the horizontal axis is labeled 'x^2'. A straight line is drawn through two points: A(0, 6) on the vertical axis and B(8, 4) on the line. The origin is labeled 'O'. <u>Solution:</u> xy – intercept is 6 gradient $\frac{6-4}{0-8} = -\frac{1}{4}$ equation of straight line is of the form “$y = mx + c$” i.e. $xy = mx^2 + c$ $xy = -\frac{1}{4}x^2 + 6$ Hence $y = -\frac{1}{4}x + \frac{6}{x}$ or $-\frac{x^2 + 24}{4x}$	[3]

Question 6 [2014 EOY, Q10]	marks																																			
Answer the whole of this question on a sheet of graph paper. The table shows experimental values of two variables x and y. It is known that x and y are related by an equation $y = ae^{-bx}$, and that one of these values of y given in the table is subject to an abnormally large error. <table><tr><td>x</td><td>1.5</td><td>2.5</td><td>3.5</td><td>4.5</td><td>5.0</td><td>6.0</td></tr><tr><td>y</td><td>9.9</td><td>6.0</td><td>3.6</td><td>2.2</td><td>1.7</td><td>1.4</td></tr></table> (i) Draw a straight line graph of $\ln y$ against x, using a scale of 2 cm to 1 unit on the x-axis and 1 cm to 0.2 units on the $\ln y$-axis. Solution: $y = ae^{-bx} \Rightarrow \ln y = \ln a - bx$ <table><tr><td>x</td><td>1.5</td><td>2.5</td><td>3.5</td><td>4.5</td><td>5.0</td><td>6.0</td></tr><tr><td>y</td><td>9.9</td><td>6.0</td><td>3.6</td><td>2.2</td><td>1.7</td><td>1.4</td></tr><tr><td>$\ln y$</td><td>2.29</td><td>1.79</td><td>1.28</td><td>0.79</td><td>0.53</td><td>0.34</td></tr></table> 	x	1.5	2.5	3.5	4.5	5.0	6.0	y	9.9	6.0	3.6	2.2	1.7	1.4	x	1.5	2.5	3.5	4.5	5.0	6.0	y	9.9	6.0	3.6	2.2	1.7	1.4	$\ln y$	2.29	1.79	1.28	0.79	0.53	0.34	[4]
x	1.5	2.5	3.5	4.5	5.0	6.0																														
y	9.9	6.0	3.6	2.2	1.7	1.4																														
x	1.5	2.5	3.5	4.5	5.0	6.0																														
y	9.9	6.0	3.6	2.2	1.7	1.4																														
$\ln y$	2.29	1.79	1.28	0.79	0.53	0.34																														
(ii) Use your graph to estimate the value of a and of b. Solution: $\ln a \approx 3.04$ $\Rightarrow a \approx 20.9$ (accept 20.0 to 21.8) $b \approx 0.500$ (accept 0.47 to 0.53)	[3]																																			

(iii)															
Identify the abnormal reading of y and estimate its correct value.	[2]														
<u>Solution:</u> The abnormal reading is 1.4. [B1] When $x = 6.0$, $\ln y$ is estimated to be 0.04. Hence, the correct value of $y \approx e^{0.04} \approx 1.04$. (accept 1.00 to 1.04)															
Question 7 [2015 EOY, Q8]	marks														
Answer the whole of this question on a sheet of graph paper. Two variables x and y are related such that $y^2 = ax^2 + bx$, where a and b are unknown constants. In an experiment, values of y are found for several values of x and the values are tabulated as shown below. It is also found that one particular value of y is incorrectly recorded. <table border="1"><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>y</td><td>2.07</td><td>3.77</td><td>5.45</td><td>6.81</td><td>8.81</td><td>10.48</td></tr></table>	x	1	2	3	4	5	6	y	2.07	3.77	5.45	6.81	8.81	10.48	
x	1	2	3	4	5	6									
y	2.07	3.77	5.45	6.81	8.81	10.48									
(i) Using a scale of 2 cm to 1 unit for the horizontal axis and 1 cm to 1 unit for the vertical axes, plot $\frac{y^2}{x}$ against x and draw a straight line graph, and use it to identify the incorrect value of y recorded.	[4]														
<u>Solution:</u>															

Name..... Subject

C.T Group/Class Serial No. Date



Incorrect value of y recorded is 6.81.

(ii)

Use your graph to estimate, to one decimal place, the values of a and b , and hence estimate, to two decimal places, the correct value of y in (i).

[4]

Solution:

The gradient represents a , so $a \approx 2.8$ (accept $2.73 \leq a \leq 2.87$)

The vertical intercept represents b , so $b \approx 1.5$ (accept $1.3 \leq b \leq 1.7$)

$$\frac{y^2}{x} \approx 12.7 \Rightarrow y \approx 7.13 \text{ (accept } 7.07 \leq y \leq 7.18 \text{)}$$