Tutorial 4: Functions

Section A (Basic Questions)

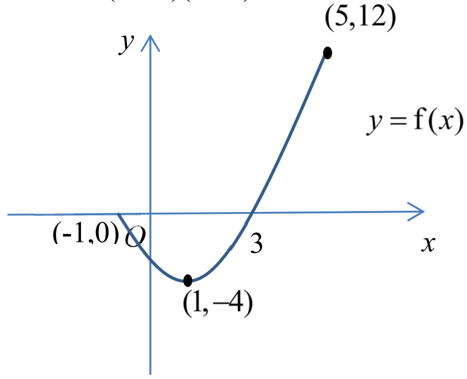
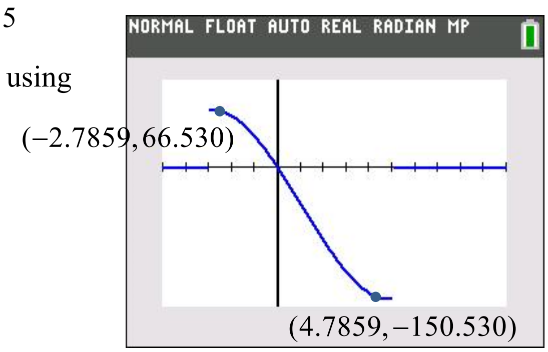
1 Find the range of the following functions:

(a) $\mathrm{f}: x \mapsto x^2 - 2x - 3, \quad x \in \mathbb{R},\ -1 \le x \le 5,$

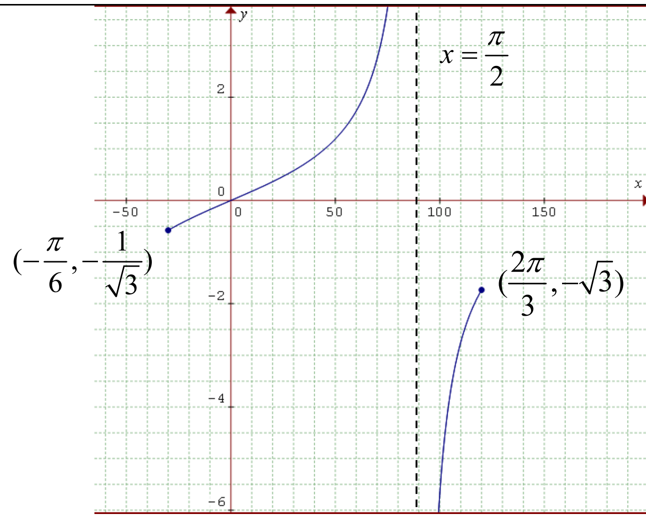
(b) $\mathrm{g}: x \mapsto x(x+5)(x-8),\ x \in \mathbb{R},\ -3 \le x \le 5,$

(c) $\mathrm{h}: x \mapsto \tan x, \quad x \in \mathbb{R}, -\frac{\pi}{6} \le x \le \frac{2\pi}{3}, x \ne \frac{\pi}{2}$

[(a) $[-4,12]$ **(b)** $[-151,66.5]$ **(c)** $\left(-\infty,-\sqrt{3}\right] \cup \left[-\frac{1}{\sqrt{3}},\infty\right)$**]**

(a)	$\mathrm{f}: x \mapsto x^2 - 2x - 3, \quad x \in \mathbb{R},\ -1 \le x \le 5$ $y = x^2 - 2x - 3 = (x-3)(x+1)$  Min turning point is at (1,-4) When $x = 5$, $\mathrm{f}(5) = 12$ $R_{\mathrm{f}} = [-4,12]$
1(b)	$\mathrm{g}: x \mapsto x(x+5)(x-8),\ x \in \mathbb{R},\ -3 \le x \le 5$ Find the maximum and minimum point using GC and $R_g = [-151,66.5]$ 

(c)



$\mathrm{h}: x \mapsto \tan x, \quad x \in \mathbb{R}, -\frac{\pi}{6} \le x \le \frac{2\pi}{3}, x \ne \frac{\pi}{2}$

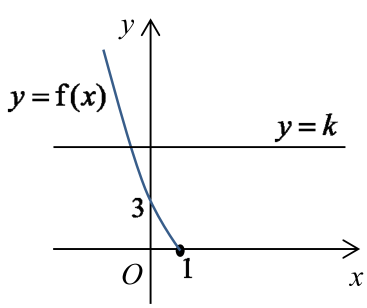
$\mathrm{R_h} = \left(-\infty, -\sqrt{3}\right] \cup \left[-\frac{1}{\sqrt{3}}, \infty\right)$

2 For each of the following functions, sketch the graph and determine if the inverse function exists. If it does, find it in a similar form.

(a) $\mathrm{f}(x) = x^2 - 4x + 3, \ x \in \mathbb{R}, \ x \le 1,$ **(b)** $\mathrm{g}(x) = x^{\frac{2}{3}}, \quad x \in \mathbb{R},$

(c) $\mathrm{h}(x) = \ln(2x-1), \ x \in \mathbb{R}, \ x > \frac{1}{2}$.

(a) $\mathrm{f}(x) = x^2 - 4x + 3 = (x-1)(x-3), \quad x \in \mathbb{R}, \ x \le 1$



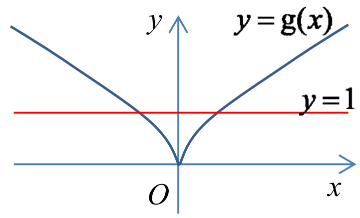
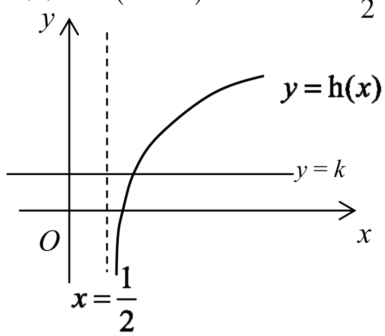
Since every horizontal line $y = k, \quad k \in [0, \infty)$ cuts the graph of f at one and only one point, f is a one-one function and hence the inverse function of f exists.

Let $y = x^2 - 4x + 3 = (x-2)^2 - 1$

Then $(x-2)^2 = y + 1$

$x = 2 \pm \sqrt{y+1}$

Since $x \le 1$, $x = 2 - \sqrt{y+1}$ $\quad \therefore \mathrm{f}^{-1}(x) = 2 - \sqrt{x+1}, \ x \ge 0$.

(b)	$\mathrm{g}(x)=x^{\frac{2}{3}},\quad x\in\mathbb{R}$ Note : g is an even function. Gradient at $x=0$ is undefined.  Since $\mathrm{g}(-8)=\mathrm{g}(8)=4$, g is not a one-one function. Therefore, inverse function of g does not exist. OR: Since the horizontal line $y=1$ cuts the graph of g at more than one point, g is not a one-one function and therefore the inverse function of g does not exist.
(c)	$\mathrm{h}(x)=\ln(2x-1),\ x\in\mathbb{R},\ x>\frac{1}{2}$  Since every horizontal line $y=k,\ k\in\mathbb{R}$ cuts the graph of h at one and only one point, h is a one-one function and hence the inverse function of h exists. Let $y=\ln(2x-1)$. Then $\mathrm{e}^{y}=2x-1$ $x=\frac{1}{2}(\mathrm{e}^{y}+1)\ \therefore \mathrm{h}^{-1}(x)=\frac{1}{2}(\mathrm{e}^{x}+1)\quad,\quad x\in\mathbb{R}$

3 The function f is defined by $\mathrm{f}:x\mapsto 2-(x+1)^2,\ x\in\mathbb{R},\ x\le k$.

Determine the largest value of k for which the function f^{-1} exists.

[largest value of $k=-1$]

In order for f^{-1} to exist, f needs to be a 1-1 function.   The function f has a turning point at $(-1,2)$ and therefore $k\le -1$.   Therefore the largest value of k is -1.