Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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MA303.2.E1 – Linear Law Exercise

1. Additional Mathematics 360 (2nd Edition) Exercise 9.1
   - 1.1 Question 10
   - 1.2 Question 13
   - 1.3 Question 15

2. Additional Mathematics 360 (2nd Edition) Exercise 9.2
   - 2.1 Question 11
   - 2.2 Question 13

B-

1.1 The varibles x and y are related by the equation $y=\frac{ax}{x-b}$, where a and b are constants. When y is plotted against $\frac{y}{x}$, a straight line graph is obtained. The gradient of the line is 2 and the y-intercept is -1.

(i) Find the value of a and of b.

$y=\frac{y}{x}\cdot 2-1$

$=\frac{2y}{x}-1$

$=\frac{2y-x}{x}$

$xy=2y-x$ —— ①

$y=\frac{ax}{x-b}$

$xy-by=ax$

$xy=ax+by$ —— ②

① = ②

$\therefore 2y-x=by+ax$

$\therefore b=2$

$a=-1$

(ii) Find the values of x for which $y=3x$

$y=-\frac{x}{x-2}$

$3x=-\frac{x}{x-2}$

$3x^2-6x=-x$

$3x^2-5x=0$

$x=0$ or $\frac{5}{3}$

1.2 The variables x and y are related such that, when $\frac{y}{x^2}$ is plotted against $\frac{1}{x}$, a straight line is obtained. The line passes through the points (3, 10) and (9, 28).

(i) Express y in terms of x.

Gradient of line $= \frac{28-10}{9-3}$

$= 3$

By gradient-point form,

$Y - 10 = 3(X-3)$

$Y = 3X + 1$

$\frac{y}{x^2} = \frac{3}{x} + 1$

$\frac{y}{x^2} = \frac{3+x}{x}$

$xy = 3x^2 + x^3$

$y = x^2 + 3x$

(ii) Find the values of y when $x = \frac{1}{\sqrt{2}}$.

Sub $x = \frac{1}{\sqrt{2}}$ into $x^2 + 3x = y$

$\therefore y = \left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{2} + \frac{3}{\sqrt{2}}$

~~= 2.62 (3s.f.)~~

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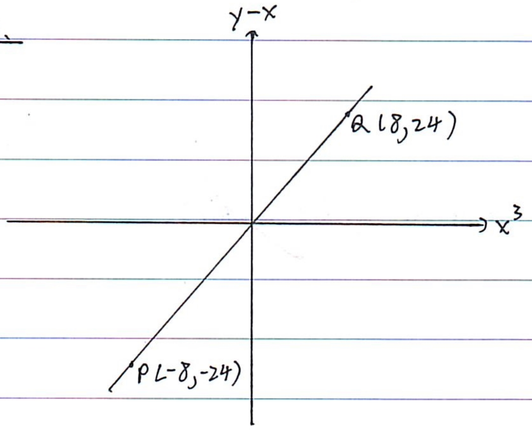
1.3 The diagram shows that the straight line graph of $(y-x)$ against x^3. The line passes through the points $P(-8, 24)$ and $Q(8,24)$.



(i) Express y in terms of x.

$\text{Gradient} = \frac{24-(-24)}{8-(-8)}$

$= 3$

By gradient-point form,

$Y - 24 = 3(X-8)$

$Y = 3X$

$y - x = 3x^3$

$y = 3x^3 + x$

(ii) Find the value of y when $x=-1$.

Sub $x=-1$ into $y=3x^3+x$

$\therefore y = 3(-1)^3 - 1$

$y = -4$

(iii) The point $R(m,n)$, where $m+2n=18$ and $n^2-m=17$, lies on this line. Find the values of x and y corresponding to the point R.

$m+2n = 18$ —— ①

$n^2 - m = 17$ —— ②

Let ① + ②

$\therefore n^2+2n = 35$

$n^2+2n-35=0$

$n = 5$ or -7

When $n = 5$,

$5^2 - m = 17$

$m = 8$

When $n = -7$

$(-7)^2 - m = 17$

$m = 32$ (reject as both n and m must be positive or negative together)

$\therefore (m, n) = (8, 5)$

$x^3 = 8$

$x = 2$

$y - x = 5$

$y - 2 = 5$

$y = 7$

~~R: (2, 7)~~ X R(m, n) not R(x, y)

R: (8, 5)

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2.1 Radio waves. The table shows the frequencies (kHz) of a radio station at various lengths. It is known that a relationship of the form $f = k\lambda^n$ exists between f and λ, where k and n are constants.

f (kHz)	1190	647	469	250	200
λ (m)	252	464	640	1200	1500
lg f	3.08	2.81	2.67	2.40	2.30
lg λ	2.40	2.67	2.81	3.08	3.18

} move table on graph paper.

(i) Plot lg f against lg λ and draw a straight line graph

(ii) Use your graph to estimate the value of k and of n.

$f = k\lambda^n$

$\lg f = \lg (k\lambda^n)$

$\lg f = \lg k + n \lg \lambda$

$\lg k = 5.47$

$k \approx 295000$ (3sf.) sf ?

$n = \text{gradient of line} = \frac{5.25 - 2.00}{0.47 - 3.47}$ ≈

$= -1.08$ (3sf.)

|| poor choice of points

$n \approx -1.1$

(iii) By drawing a suitable line on your graph, solve the equation $k\lambda^n = 10^{-2.5n}$

$k\lambda^n = 10^{-2.5n}$

$\lg (k\lambda^n) = -2.5n$

$\lg k + n \lg \lambda = -2.5n$ —— ①

Sub $n = -1.1$ into ①

$5.47 - 1.1 \lg \lambda = 2.75$

$-1.1 \lg \lambda = 2.72$

$\lg \lambda = \left| -\frac{136}{55} \right|$

$\lambda = 297$ (3sf.)

? are you using the graph?

?? what concept?

$f = 10^{-2.5n}$

$\lg f = -2.5n$

$\lg f = 2.75$

From graph, $\lg \lambda = 2.725$

$\therefore \lambda = 531$ (3s.f.)

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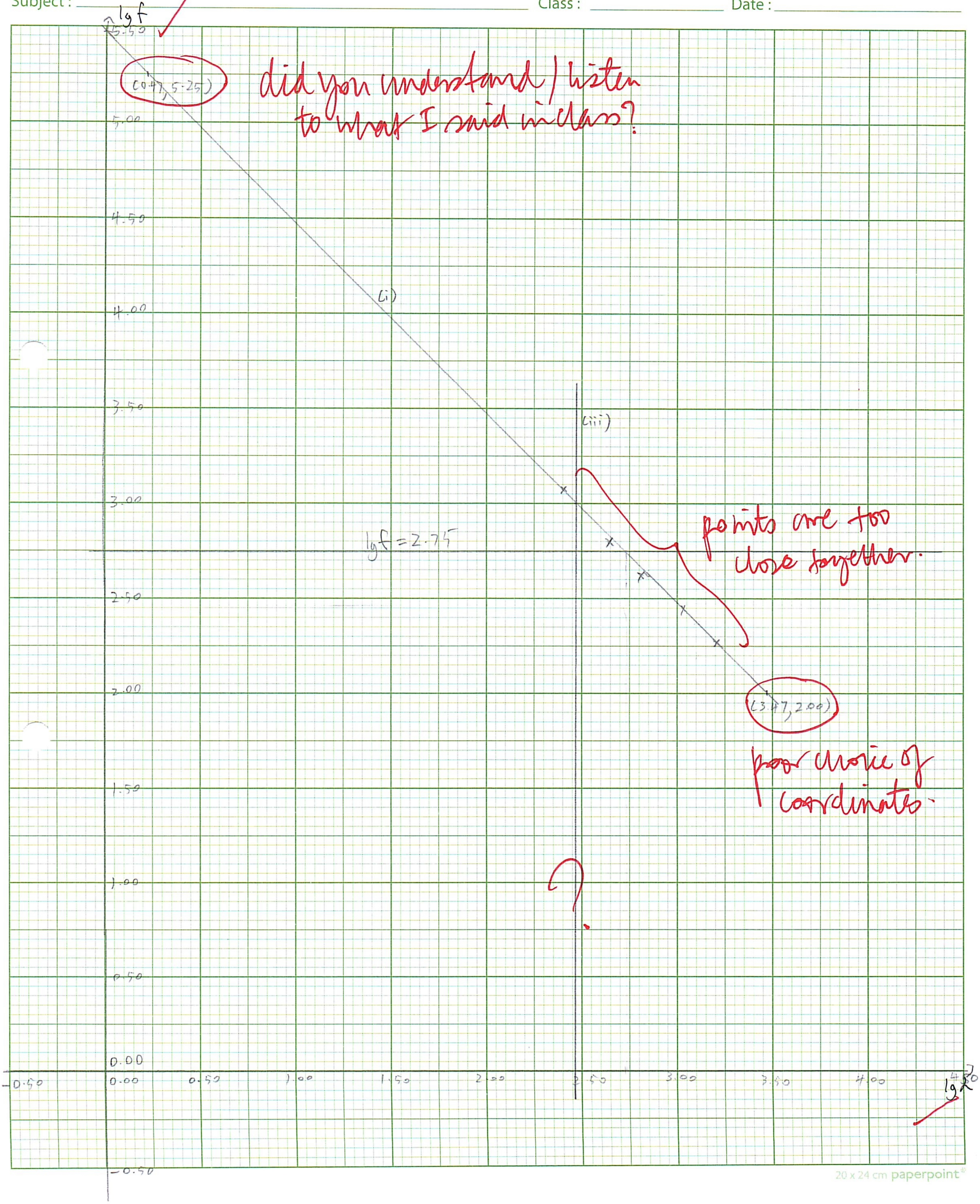

20 x 24 cm paperpoint

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2.2 The table shows experimental values of two variables, x and y.

x	1	2	3	4	5
y	-1.25	-2.29	-3.3	-4.31	-5.31
$\frac{y}{x+y}$	5.00	7.90	11.0	13.9	17.1
x	1	2	3	4	5

} show on graph paper.

It is known that x and y are related by the equation $y = ax^2 + bx + axy + by$, where a and b are constants.

(i) Draw the graph of $\frac{y}{x+y}$ against x.

(ii) Use your graph to estimate the values of a and b.

$\frac{y}{x+y} = mx + c$

$y = mx^2 + mxy + cx + cy$

$= ax^2 + bx + axy + by$

Comparing coefficients,

$a = k$ and $b = c$.

$\therefore a \approx \frac{16.4 - 2.05}{4.7 - 0.1}$ $\quad a \approx \frac{11.4 - 2.2}{4.7 - 0.1}$

≈ 3.1 (2s.f.) $\quad \approx 3.1$ (2sf)

$b \approx 1.9$

(iii) Explain how you use your graph to solve the value of x in the equation $(5-a)(x^2+xy) = b(x+y)$.

Sub $a = 3.1$ and $b = 1.9$ into the equation. $\quad \frac{y}{x+y} = 5x$

$\therefore 1.9(x^2+xy) = 1.9(x+y)$? $\quad$ x-coordinate of the intersection point is the value of x.

$1.9x^2 + 1.9xy - 1.9x - 1.9y = 0$? $\quad$ wrong understanding.

Comparing coefficients, gradient of straight is 1.9 and y-intercept is -1.9.

Draw the line and find the x-intercept to solve for x.

Substitute in the x-value and solve for y.

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