1 If $6x^3+17x^2-26x+18\equiv(Ax-1)(3x-2)(x+B)+C$, find the value of the constants A, B and C. [3]

2 A circular disc has a volume of $5\,\text{cm}^3$. If the radius of its circular cross-section is $(\sqrt{3}+1)\text{cm}$, find the thickness of the disc in terms of π. Leave your answer as an exact value, in the form $a+b\sqrt{3}$. [3]

3 Given that $\cos 220° = -p$, find in terms of p,

(i) $\cos 40°$, [1]

(ii) $\tan 140°$, [2]

(iii) $\sin(-320°)$. [1]

4 The quadratic equation $x^2+2x+k=0$ has roots α and β. Find the quadratic equation with roots $\dfrac{2\alpha}{\beta+2}$ and $\dfrac{2\beta}{\alpha+2}$. [5]

5 Sketch the graph of $y=\ln(1-x)$, labelling the asymptote(s) and intercept(s) clearly.

Find the equation of the straight line that needs to be added onto the same axes so as to solve the equation $e^{2x}=e\,(1-x)$. **It is not necessary to draw the line on your sketch.** [4]

6 The line $y=mx+1$ does not intersect the curve $y=x^2+2x+5$. However, it intersects a second curve $y=mx^2-3$ at two distinct points. Find the range of possible values of m. [6]

7 The curve $y = a\sin bx$ has a maximum value of 4, a minimum value of -4 and a period of $\frac{2\pi}{3}$.

(i) State the values of a and b. [2]

(ii) Hence, sketch the graph of $y = a\sin bx$ for $0 \le x \le \pi$, indicating clearly all intercepts. [2]

8 **Answer the whole of this question on the INSERT provided.**

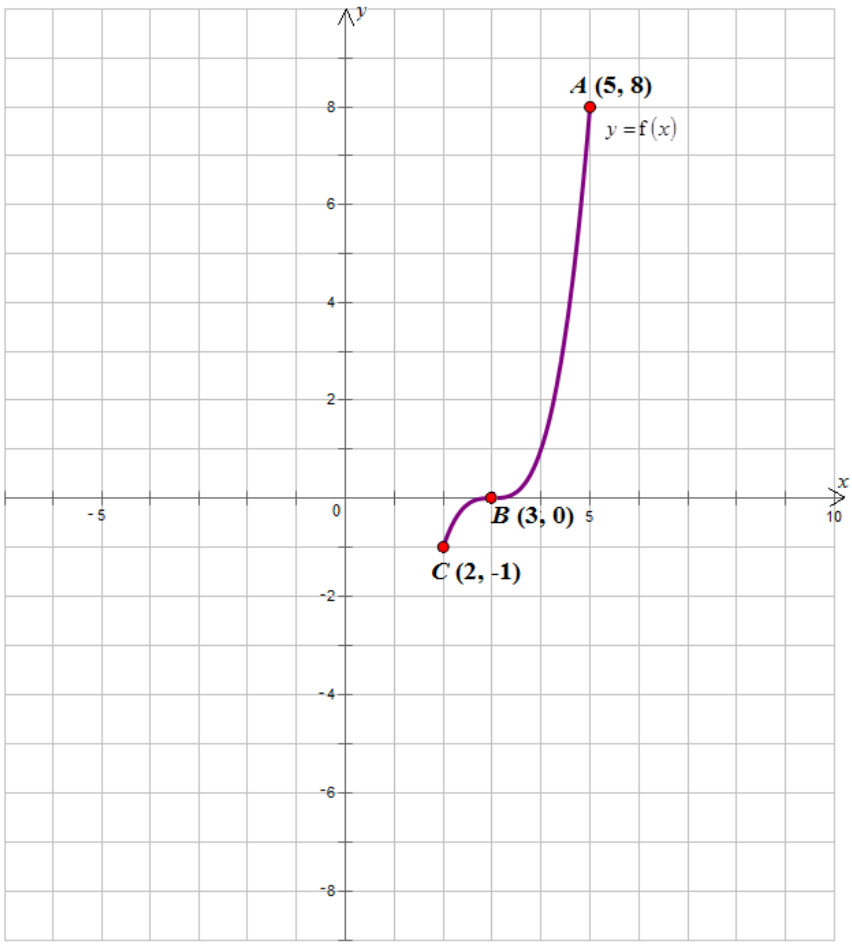
The graph of $y = \mathrm{f}(x)$ is given in the diagram below. The graph passes through the points $A(5, 8)$, $B(3, 0)$ and $C(2, -1)$. On the same axes, sketch the graph of

(a) $y = \mathrm{f}(x-3)$, [2]

(b) $y = -\frac{1}{2}\mathrm{f}(x)$, [2]

(c) $y = \mathrm{f}^{-1}(x)$, [2]

showing clearly the coordinates of the image of each of the points *A*, *B* and *C*.



9 **(a)** The function f is defined by $\mathrm{f}: x \mapsto 2x^2 - 12x + 33$, for $1 \le x \le 6$.

(i) By using completing the square method, express $2x^2 - 12x + 33$ in the form $a(x+b)^2 + c$. [3]

(ii) Find the range of f. [2]

(iii) Does f have an inverse? Explain your answer. [1]

(iv) The function g is defined by $\mathrm{g}: x \mapsto 2x^2 - 12x + 33$. Write down a possible domain for g so that g has an inverse. [1]

(b) If a function h is defined as $\mathrm{h}: x \mapsto \dfrac{x}{2x-3}$ for $x \neq \dfrac{3}{2}$, find, in similar form, an expression for h^{-1}, stating the restriction on its domain. [3]

10 **(a)** Given that $x = \log_2 a$ and $y = \log_2 b$, express $\log_2 \dfrac{\sqrt{b^3}}{8a^2}$ in terms of *x* and *y*.

[3]

(b) Solve the equation $2^{2x+1} = 3(2^x) + 2$. [4]

(c) Solve the equation $\log_2 (5x+1)^2 - \log_{(5x+1)} 2 = 1$. [5]

11 **(a)** The function $\mathrm{f}(x) = 6x^3 + ax^2 + bx - 2$ where *a* and *b* are constants, is exactly divisible by $x-1$ and leaves a remainder of -36 when divided by $x+2$. Find the values of *a* and *b*. [3]

(b) **(i)** Solve the equation $x^3 - 4x^2 - 4x + 16 = 0$. [4]

(ii) Hence, solve the equation $|x|^3 + 4x^2 + 16 = 8x^2 + 4|x|$. [2]

(c) A quadratic function $\mathrm{f}(x)$ is such that $\mathrm{f}(2) = 3$, $\mathrm{f}(0) = 1$ and $\mathrm{f}(1) = 0$. Find the expression for $\mathrm{f}(x)$. [3]

12 **(a)** Prove that $\operatorname{cosec}^4 x - \cot^4 x \equiv 2\operatorname{cosec}^2 x - 1$. [3]

(b) Solve the equation $2\cos^2 x = 2 + \sin x$, for the interval $0 \le x \le 2\pi$, leaving your answer(s) as exact value(s). [4]

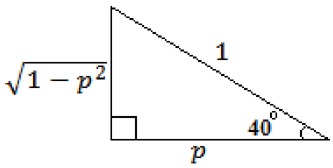
(c) Solve the equation $6\tan 2A + 1 = \cot 2A$, for the interval $0 \le A \le 360^\circ$. [4]

<u>Bonus question</u>

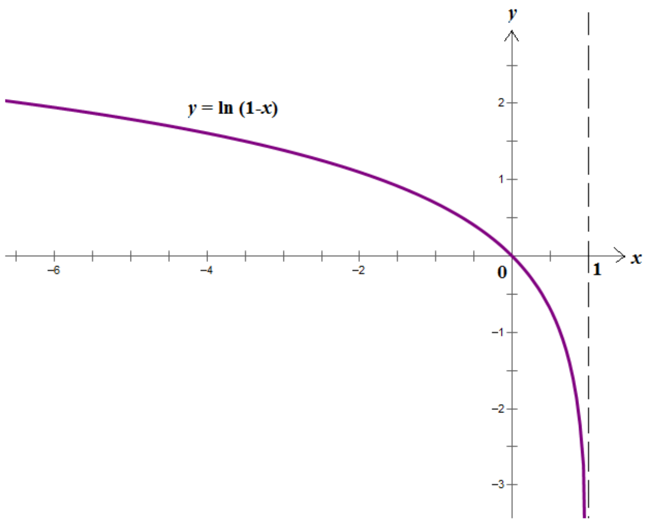
13 Prove that $\tan\{\sin^{-1}[\cos(\tan^{-1} k)]\} = \dfrac{1}{k}$, where k is a positive constant. [3]

- End of Paper -

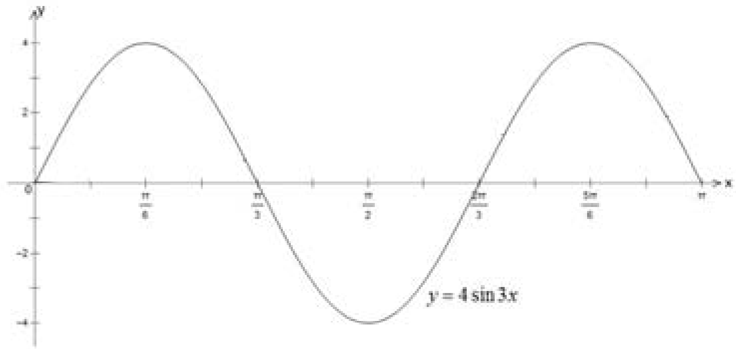
IM2 EOY Marking Scheme

1	$A = 2$ $B = 4$ $C = 10$	
2	$Volume\ of\ circular\ disc = \pi r^2 h$ $5 = \pi(\sqrt{3}+1)^2 h$ $h = \frac{5}{\pi(4+2\sqrt{3})}$ $h = \frac{5}{\pi(4+2\sqrt{3})} \times \frac{4-2\sqrt{3}}{4-2\sqrt{3}}$ $h = \frac{20-10\sqrt{3}}{4\pi}$ $h = \frac{5}{2\pi}(2-\sqrt{3})\ or\ \frac{10-5\sqrt{3}}{2\pi}$ cm OR $5 = \pi(\sqrt{3}+1)^2 h$ $h = \frac{5}{\pi(\sqrt{3}+1)^2}$ $h = \frac{5}{\pi(\sqrt{3}+1)^2} \times \frac{(\sqrt{3}-1)^2}{(\sqrt{3}-1)^2}$ $h = \frac{5}{\pi} - \frac{5}{2\pi}\sqrt{3}$ cm	
3i	$\cos 40° = p$	
3ii	 $\tan 140° = -\tan 40° = -\frac{\sqrt{1-p^2}}{p}$	
3iii	$\sin(-320°) = \sin 40° = \sqrt{1-p^2}$	

4	$\alpha + \beta = -2, \alpha\beta = k$	

	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=4-2k$ Sum of roots: $\frac{2\alpha}{\beta+2}+\frac{2\beta}{\alpha+2}=\frac{2(\alpha^2+\beta^2)+4(\alpha+\beta)}{2(\alpha+\beta)+4+\alpha\beta}$ $=\frac{2(4-2k)+4(-2)}{2(-2)+4+k}$ $=-4$ Product of roots: $\left(\frac{2\alpha}{\beta+2}\right)\left(\frac{2\beta}{\alpha+2}\right)=\frac{4\alpha\beta}{2(\alpha+\beta)+4+\alpha\beta}$ $=\frac{4k}{2(-2)+4+k}$ $=4$ So the equation is $x^2+4x+4=0$	
5	 $e^{2x}=e(1-x)$ $e^{2x-1}=1-x$ $2x-1=\ln(1-x)$ $\therefore$ draw $y=2x-1$	

6	If $y = mx + 1$ does not intersect $y = x^2 + 2x + 5$, then $x^2 + x(2 - m) + 4 = 0$ has no real roots, and hence $$(2-m)^2 - 4(1)(4) < 0$$ $$m^2 - 4m - 12 < 0$$ $$(m+2)(m-6) < 0$$ $$-2 < m < 6$$ If $y = mx + 1$ intersects $y = mx^2 - 3$ at two distinct points, then $mx^2 - mx - 4 = 0$ has 2 roots, and hence, $$m^2 - 4(m)(-4) > 0$$ $$m(m+16) > 0$$ $$m < -16 \text{ or } m > 0$$ Combining the two inequalities, obtain the range of possible values of m: $$0 < m < 6$$	

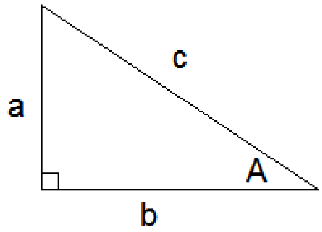
7i	$a = 4$ $b = 3$	
7ii	 	

9ai	$2x^2 - 12x + 33$ $= 2(x^2 - 6x) + 33 \quad or \quad = 2\left(x^2 - 6x + \frac{33}{2}\right)$ $= 2(x-3)^2 - 18 + 33$ $= 2(x-3)^2 + 15$	
9aii	$f(1) = 23$ $f(6) = 33$ $15 \leq \mathrm{f}(x) \leq 33$	
9aiii	f **does no**t have an inverse. It is not a **one-to-one** function.	
9aiv	Any acceptable answer Eg. $x \leq 3$ or $x \geq 3$	
9b	$Let\ y = \frac{x}{2x-3}$ $2xy - 3y = x$ $x(2y-1) = 3y$ $x = \frac{3y}{2y-1}$ Hence, $\mathrm{h}^{-1}: x \mapsto \frac{3x}{2x-1}, x \neq \frac{1}{2}$	

10a	$$\log_2 \frac{\sqrt{b^3}}{8a^2} = \frac{3}{2}[\log_2 b] - [\log_2 8 + 2\log_2 a]$$ $$= \frac{3}{2}(y) - (3 + 2x)$$ $$= -2x + \frac{3}{2}y - 3$$ OR $$\log_2 \frac{\sqrt{b^3}}{8a^2} = \log_2 b^{\frac{3}{2}} - [\log_2 8 + \log_2 a^2]$$ $$= \frac{3}{2}\log_2 b - [\log_2 2^3 + 2\log_2 a]$$ $$= \frac{3}{2}y - 3 - 2x$$	
10b	$$2^{2x+1} = 3(2^x) + 2$$ $$2(2^x)^2 - 3(2^x) - 2 = 0$$ Let $2^x = y$, hence $$2y^2 - 3y - 2 = 0$$ $$(2y+1)(y-2) = 0$$ $$2^x = 2 \Rightarrow x = 1$$ $$2(2^x) = -1\ (NA)$$	
10c	$$\log_2(5x+1)^2 - \log_{(5x+1)} 2 = 1$$ $$\log_2(5x+1)^2 - \frac{\log_2 2}{\log_2(5x+1)} = 1$$ $$2[\log_2(5x+1)]^2 - 1 = \log_2(5x+1)$$ Let $y = \log_2(5x+1)$, hence $$(2y+1)(y-1) = 0$$ $$\log_2(5x+1) = -\frac{1}{2} \Rightarrow x = \frac{1}{5}\left(\frac{\sqrt{2}}{2} - 1\right)\ or\ -0.0586$$ or $$\log_2(5x+1) = 1 \Rightarrow x = \frac{1}{5}\ or\ 0.2$$	
11a	$\mathrm{f}(1) = 0$, hence, $6 + a + b - 2 = 0$ $$a + b = -4$$ $\mathrm{f}(-2) = -36$, hence, $-48 + 4a - 2b - 2 = -36$ $$2a - b = 7$$ Solving simultaneously, obtain	

	$a = 1$ $b = -5$	
11bi	$\mathrm{f}(2) = 0$, hence $(x-2)$ is a factor. $x^3 - 4x^2 - 4x + 16 = 0$ $(x-2)(x^2 - 2x - 8) = 0$ $(x-2)(x-4)(x+2) = 0$ Hence, $x = 2, -2, \text{or } 4$	
11bii	$\lvert x\rvert^3 - 4\lvert x\rvert^2 - 4\lvert x\rvert + 16 = 0$ $\lvert x\rvert = 2, -2\ (rej), \text{or } 4$ $x = 2, -2, 4 \text{ or } -4$	
11c	Consider the fact that $\mathrm{f}(1) = 0$, and that $\mathrm{f}(x)$ is a quadratic. This allows us to deduce that $\mathrm{f}(x) = (ax+b)(x-1)$ Since $\mathrm{f}(2) = 3$, $2a + b = 3$ Since $\mathrm{f}(0) = 1$, $-b = 1$ Solving simultaneously, we have $a = 2$, $b = -1$ Hence, $f(x) = (2x-1)(x-1)$, OR Let $\mathrm{f}(x) = ax^2 + bx + c$ $\mathrm{f}(0) = 1 \Rightarrow \mathrm{c} = 1$ Since $\mathrm{f}(2) = 3$, $4a + 2b + 1 = 3$ Since $\mathrm{f}(1) = 0$, $a + b + 1 = 0$ Solving simultaneously, we have $a = 2$, $b = -3$ Hence, $f(x) = 2x^2 - 3x + 1$.	

12a	$LHS = \operatorname{cosec}^4 x - \cot^4 x$ $= (\operatorname{cosec}^2 x - \cot^2 x)(\operatorname{cosec}^2 x + \cot^2 x)$ $= \operatorname{cosec}^2 x + \cot^2 x$ $= \operatorname{cosec}^2 x + \operatorname{cosec}^2 x - 1$ $= 2\operatorname{cosec}^2 x - 1 = RHS$ $\therefore LHS \equiv RHS$	
12b	$2\cos^2 x = 2 + \sin x$ $2\sin^2 x + \sin x = 0$ $\sin x(2\sin x + 1) = 0$ Hence, either $\sin x = 0 \quad \Rightarrow \quad x = 0, \pi, 2\pi$ or $\sin x = -\frac{1}{2} \quad \Rightarrow \quad x = \frac{7\pi}{6}, \frac{11\pi}{6}$	
12c	$6\tan 2A + 1 = \cot 2A$ $6\tan^2 2A + \tan 2A - 1 = 0$ $(3\tan 2A - 1)(2\tan 2A + 1) = 0$ Since $0 \le A \le 360^\circ$, $0 \le 2A \le 720^\circ$ Hence, $\tan 2A = \frac{1}{3}$ Basic reference angle $\alpha = \tan^{-1}\frac{1}{3} = 18.435^\circ$ $2A = 18.435^\circ$ or 198.435° or 378.435° or 558.435° $(3dp)$ $A = 9.2^\circ$ or 99.2° or 189.2° or 279.2° $(1dp)$ OR $\tan 2A = -\frac{1}{2}$ Basic reference angle $\alpha = \tan^{-1}\frac{1}{2} = 26.565^\circ$ $2A = 153.435^\circ$ or 333.435° or 513.435° or 693.435° $(3dp)$ $A = 76.7^\circ$ or 166.7° or 256.7° or 346.7° $(1dp)$	

Bonus Qns 13	 Consider the triangle Observe that $\sin^{-1}\cos A = \frac{\pi}{2} - A$, and $\tan\left(\frac{\pi}{2} - A\right) = \cot A$. Hence, letting $A = \tan^{-1} k$, we have $\tan\{\sin^{-1}[\cos(\tan^{-1} k)]\}$ $= \tan\left\{\frac{\pi}{2} - \tan^{-1} k\right\}$ $= \cot\{\tan^{-1} k\}$ $= \frac{1}{\tan\{\tan^{-1} k\}}$ $= \frac{1}{k}$	