Name:	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR 2022
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4049/02**
Paper 2 **30 August 2022**

Candidates answer on the Question Paper. **2 hours 15 mins**
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your name and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **90**.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10
Marks										

Table of Penalties		Qn. No.		
Presentation	–1			
Significant Figures/ Units	–1		Parent's/Guardian's Signature	90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-r)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

1 Show that $x=-1$ is a solution of the equation $4x^3-7x-3=0$ and hence solve the equation completely. [5]

Let $f(x)=4x^3-7x-3$

$f(-1)=-4+7-3=0$ [M1 – apply factor thm]

Hence, by the factor Theorem, $x=-1$ is a solution of $4x^3-7x-3=0$. [A1]

$\therefore f(x)=(x+1)(ax^2+bx+c)$

Comparing coefficient of x^3 : $a=4$

Comparing constant term : $c=-3$

Comparing coefficient of x^2 : $b=-4$ [M1, A1]

Hence $(x+1)(4x^2-4x-3)=0$

$(x+1)(2x+1)(2x-3)=0$

$x=-1, -\frac{1}{2}$ or $\frac{3}{2}$ [A1]

2 Show that the equation $e^{x+1} - 6e^{-x} = 2e - 3$ has only one solution and find its exact value. [5]

$e^{x+1} - 6e^{-x} = 2e - 3$

Let $y = e^x$

$e.y - \frac{6}{y} = 2e - 3$ [M1, M1 – apply laws of indices]

$e.y^2 - 6 = (2e - 3)y$

$e.y^2 + (3 - 2e)y - 6 = 0$ [A1 – form correct quad eqn]

$(ey + 3)(y - 2) = 0$ [A1 – correct factorization]

$\therefore e^x = 2$ or $e^x = -\frac{3}{e}$ (NA)

$x = \ln 2$

Hence $x = \ln 2$ is the only solution. [A1]

3(a) Given that $y = \dfrac{2x}{\sqrt{3x+1}}$, show that $\dfrac{dy}{dx} = \dfrac{3x+2}{\sqrt{(3x+1)^3}}$, [3]

$$\frac{dy}{dx} = \frac{\sqrt{3x+1}.(2)-(2x).\frac{3}{2\sqrt{3x+1}}}{(3x+1)}$$ [M1, M1 – Quotient rule and chain rule]

$$= \frac{\frac{2(3x+1)-3x}{\sqrt{3x+1}}}{(3x+1)}$$ [M1 – simplifying numerator]

$$= \frac{3x+2}{\sqrt{(3x+1)^3}}$$

3(b) Hence find the value of $\int_1^5 \dfrac{6x+3}{\sqrt{(3x+1)^3}}\,dx$. [5]

$$\int_1^5 \frac{6x+3}{\sqrt{(3x+1)^3}}\,dx = \int_1^5 \frac{3x+2+3x+1}{\sqrt{(3x+1)^3}}\,dx$$

$$= \int_1^5 \frac{3x+2}{\sqrt{(3x+1)^3}}\,dx + \int_1^5 \frac{3x+1}{\sqrt{(3x+1)^3}}\,dx$$ [M1]

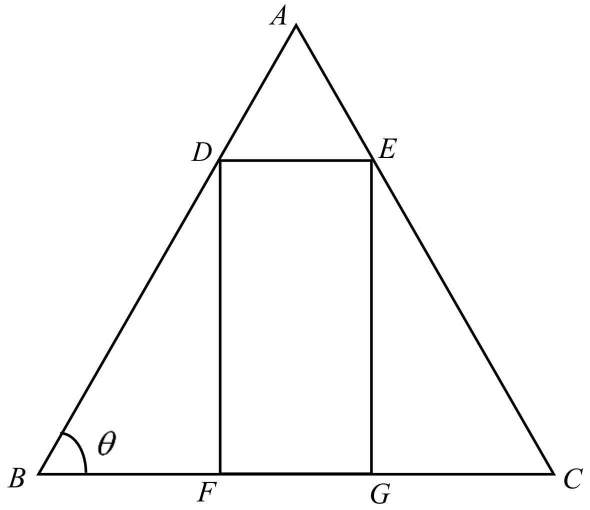
$$= \left[\frac{2x}{\sqrt{3x+1}}\right]_1^5 + \int_1^5 (3x+1)^{-\frac{1}{2}}\,dx$$ [M1, M1]

$$= \left(\frac{10}{4}-\frac{2}{2}\right) + \left[\frac{(3x+1)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)(3)}\right]_1^5$$ [M1]

$$= 1\frac{1}{2} + \frac{2}{3}(4-2)$$

$$= 2\frac{5}{6}$$ [A1]

4



In the diagram, *DFGE* is a rectangle and triangle *ABC* is an isosceles with $AB = AC = 5$ m.

ADB and *AEC* are straight lines. *DE* is parallel to *BC* and $AB = 4AD$. Angle $ABC = \theta$ where $0° < \theta < 90°$.

(a) Show that the perimeter, *P* metres, of rectangle *DFGE* is given by $P = \frac{5}{2}(2\cos\theta + 3\sin\theta)$.

[3]

By similar triangles, $\angle ABC = \angle ADE = \theta$

$BD = \frac{3}{4}AB = \frac{15}{4}$ m, $\therefore DF = \frac{15}{4}\sin\theta$ m. [B1]

$BC = 2(5\cos\theta)$ m, $\therefore DE = \frac{1}{4}BC = \frac{5}{2}\cos\theta$ m. [B1]

$$P = 2(DE + DF)$$

$$= 2\left(\frac{5}{2}\cos\theta + \frac{15}{4}\sin\theta\right)$$

$$= \frac{5}{2}(2\cos\theta + 3\sin\theta) \quad \text{[A1]}$$

(b) Express P in the form $\frac{5}{2}R\cos(\theta-\alpha)$, where $R>0$ and write down the largest possible value of P. [3]

$R=\sqrt{2^2+3^2}=\sqrt{13}$ [M1]

$\alpha=\tan^{-1}\left(\frac{3}{2}\right)=56.310°$ [M1]

$P=\frac{5}{2}\left(\sqrt{13}\cos(\theta-56.3°)\right)$ [–1 if final answer not given]

Largest possible P $=\frac{5\sqrt{13}}{2}$ [A1]

(c) Find the value of θ for which $P=6$. [3]

$\frac{5}{2}\left(\sqrt{13}\cos(\theta-56.310°)\right)=6$

$\cos(\theta-56.310°)=\frac{2.4}{\sqrt{13}}$ [M1]

Basic angle = $\cos^{-1}\frac{2.4}{\sqrt{13}}=48.269°$ [M1]

$\theta-56.310°=-48.269°$

$\theta=8.041°\approx 8.0°$ [A1]

5(a) A student is asked to solve the quadratic inequality $(2x-1)(x-3)<0$.

The first step of her solution is: $2x-1<0$ and $x-3<0$.

State whether this step is correct and support your answer with a clear explanation. [2]

No. [A1]

Since $(2x-1)(x-3)<0$, $(2x-1)$ and $(x-3)$ cannot be negative at the same time, as $(-\text{ve})(-\text{ve})=(+\text{ve})$. [A1]

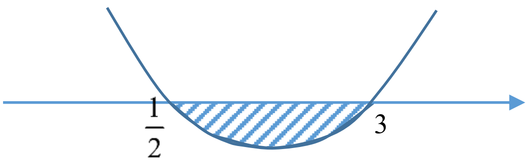
(b) The equation of a curve is given by $y=2x^2-6x+7$.

(i) Find the set of values of x for which the curve lies below the line $y=x+4$. [3]

$2x^2-6x+7<x+4$ [M1]

$2x^2-7x+3<0$

$(2x-1)(x-3)<0$ [M1]



Hence $\frac{1}{2}<x<3$ [A1]

(ii) The line $y = 2x + k$ is tangent to the curve at the point Q, where k is a constant. Find the value of the constant k. [3]

$2x^2 - 6x + 7 = 2x + k$ [M1]

$2x^2 - 8x + (7 - k) = 0$

Since line is tangent to curve, $b^2 - 4ac = 0$.

$(-8)^2 - 4(2)(7 - k) = 0$ [M1]

$k = -1$ [A1]

[Turn over

6(a) Write down and simplify the second and fourth term of the expansion $\left(x^3 - \frac{2}{x^2}\right)^n$. [4]

$T_2 = \binom{n}{1}\left(x^3\right)^{n-1}\left(-\frac{2}{x^2}\right)$ [M1]

$= -2nx^{3n-5}$ [A1]

$T_4 = \binom{n}{3}\left(x^3\right)^{n-3}\left(-\frac{2}{x^2}\right)^3$ [M1]

$= -\frac{4n(n-1)(n-2)}{3}x^{3n-15}$ [A1]

(b) Given that coefficient of the fourth term to the coefficient of the second term is 20 : 1, find the value of n. [3]

$\dfrac{\frac{-4n(n-1)(n-2)}{3}}{-2n} = \frac{20}{1}$ [M1]

$n^2 - 3n + 2 = 30$

$n^2 - 3n - 28 = 0$

$(n-7)(n+4) = 0$ [M1]

$\therefore n = 7$ or $n = -4$ (rejected) [A1]

(c) Using the value of n found in part **(b)** and without expanding $\left(x^3 - \frac{2}{x^2}\right)^n$, explain why there is no constant term in the expansion. [3]

$$T_{r+1} = \binom{7}{r}\left(x^3\right)^{7-r}\left(-\frac{2}{x^2}\right)^r \quad \text{[M1]}$$

$$= \binom{7}{r}(-2)^r x^{21-5r}$$

For constant term $x^{21-5r} = x^0$

i.e. $21 - 5r = 0$ [M1]

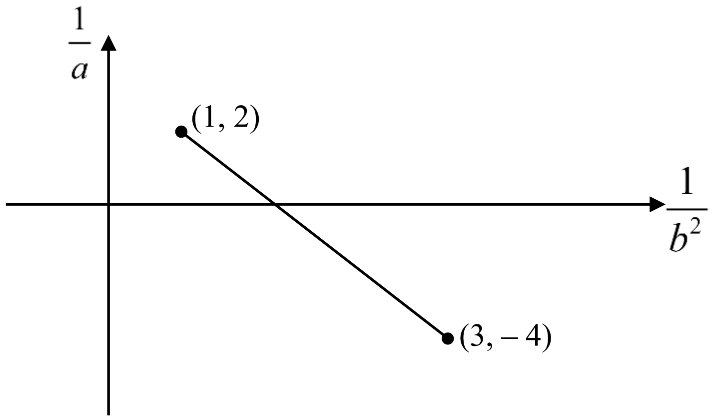
or $r = \frac{21}{5}$

Since r is not a positive integer, there is no constant term in the expansion. [A1]

[Turn over

7(a) The diagram shows part of a straight line obtained by plotting $\frac{1}{a}$ against $\frac{1}{b^2}$.

Obtain an equation relating a and b, expressing a in terms of b. [4]



Gradient $= \frac{2-(-4)}{1-3} = -3$ [B1]

Hence $\frac{1}{a} = -3\left(\frac{1}{b^2}\right) + c$, where c is the vertical intercept

i.e $\quad 2 = -3(1) + c \Rightarrow c = 5$ [B1]

Therefore, straight line equation is $\frac{1}{a} = -3\left(\frac{1}{b^2}\right) + 5$

$$\frac{1}{a} = \frac{-3+5b^2}{b^2} \quad \text{[M1]}$$

$$a = \frac{b^2}{5b^2-3} \quad \text{[A1]}$$

(b) It is given that variable x and y are related by an equation in the form $y = m^{x+c}$, where $m > 1$ and m and c are constants.

(i) Explain why plotting the graph of y against x would not result in a straight line graph. [1]

$y = m^{x+c}$ is not of the form $Y = mX + c$, where m is the gradient, c is the vertical intercept and X and Y are variables. [B1]

[except the explanation that the given equation is an exponential equation]

(ii) Express $y = m^{x+c}$ in a form suitable for drawing a straight line graph and state clearly how the values of m and c can be obtained. [4]

$\lg y = \lg m^{x+c}$ (accept ln) [M1]

$\lg y = (x+c)\lg m$

$\lg y = x\lg m + c\lg m$ [A1]

$\lg m = \text{gradient}$

$m = 10^{\text{gradient}}$ [A1]

$c\lg m = \lg y - \text{intercept}$

$c = \dfrac{\lg y - \text{intercept}}{\text{gradient}}$ [A1] $\lg y - \text{intercept}$ is vertical intercept

[Turn over

8 A particle moves in a straight line such that its velocity, v m/s is given by $v = t^2 - 10t + 9$, where t is the time in seconds after passing a fixed point O.

(a) Calculate the values of t when the particle is instantaneously at rest. [2]

When particle is instantaneously at rest, velocity = 0.

i.e. $t^2 - 10t + 9 = 0$ [M1]

$(t-1)(t-9) = 0$

$t = 1$ or $t = 9$ [A1]

(b) Calculate the stationary value of the velocity and explain whether this velocity is a maximum or a minimum. [4]

$\frac{dv}{dt} = 2t - 10$ [B1]

$\frac{d^2v}{dt^2} = 2 > 0$ [B1]

For stationary velocity, $\frac{dv}{dt} = 0$

i.e. $2t - 10 = 0$ [M1]

$t = 5$

Therefore, stationary value of velocity = 25 – 50 + 9 = –16 m/s and it is a minimum value. [A1]

(c) Calculate the velocity of the particle when it first returns to O. [6]

$s = \int t^2 - 10t + 9 \text{ d}t$ [M1]

$= \frac{t^3}{3} - 5t^2 + 9t + d$, where d is an arbitrary constant

When $t = 0$, $s = 0$, hence $d = 0$.

` Hence, $s = \frac{t^3}{3} - 5t^2 + 9t$ [A1, no mark awarded if there is no attempt to find d]

When particle returns to O, $s = 0$.

i.e. $t\left(\frac{t^2}{3} - 5t + 9\right) = 0$ [M1]

$t = 0$ or 2.0917 or 12.908 [A1]

Particle first returns to O at 2.0917 s. [A1]

Corresponding velocity = –7.5418 $\approx$ –7.54 m/s (correct to 3sf) [A1]

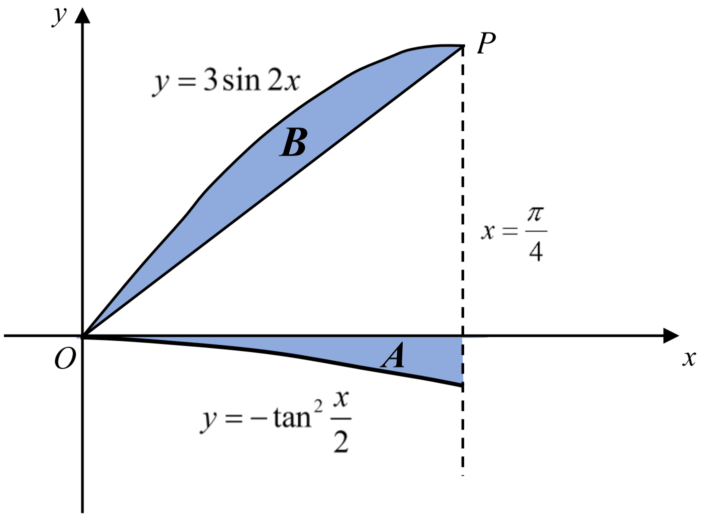
(d) Find the total distance travelled in the first 3 seconds. [2]

When $t = 1$, $s = 4\frac{1}{3}$ [M1]

When $t = 3$, $s = -9$

Total distance travelled $= 2 \times 4\frac{1}{3} + 9 = 17\frac{2}{3}$ m [A1]

9 The diagram below, not drawn to scale, shows part of the graphs of $y = 3\sin 2x$ and $y = -\tan^2 \frac{x}{2}$ for $0 \le x \le \frac{\pi}{4}$ and OP is a straight line.



(a) Find the area of shaded region A, correct to 2 significant figures. [5]

Area of region A = $-\int_0^{\frac{\pi}{4}} -\tan^2 \frac{x}{2} \, dx$ [M1]

$= \int_0^{\frac{\pi}{4}} \sec^2 \frac{x}{2} - 1 \, dx$ [M1]

$= \left[\frac{\tan\left(\frac{x}{2}\right)}{\frac{1}{2}} - x \right]_0^{\frac{\pi}{4}}$ [B1]

$= \left(2\tan\left(\frac{\pi}{8}\right) - \frac{\pi}{4} \right) - 0$ [M1]

≈ 0.043 sq units [A1]

(b) Calculate the exact area of the shaded region B. [7]

When $x = \frac{\pi}{4}$, $y = 3\sin\frac{\pi}{2} = 3$

Hence gradient of OP = $\dfrac{3-0}{\frac{\pi}{4}-0} = \dfrac{12}{\pi}$

Equation of OP is $y = \dfrac{12}{\pi}x$ [M1, A1]

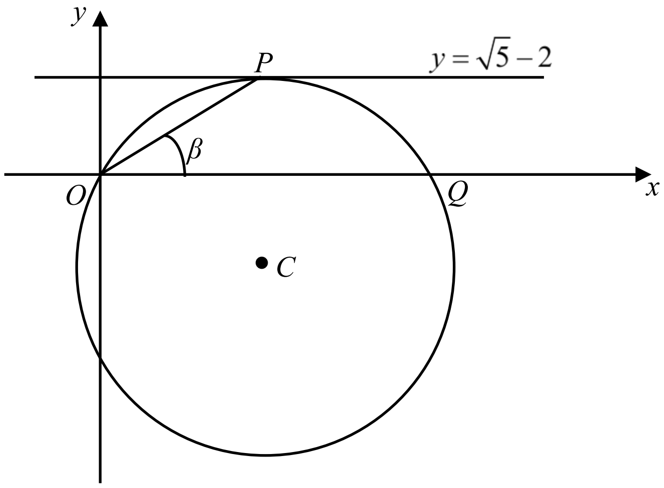
Area of region B = $\int_0^{\frac{\pi}{4}} 3\sin 2x - \dfrac{12}{\pi}x \, dx$ [M1]

$= \left[\dfrac{-3\cos 2x}{2} - \dfrac{12}{\pi}\left(\dfrac{x^2}{2}\right)\right]_0^{\frac{\pi}{4}}$ [B1, B1]

$= \left(0 - \dfrac{12}{\pi}\left(\dfrac{\pi^2}{32}\right)\right) - \left(-\dfrac{3}{2} - 0\right)$ [M1]

$= \left(\dfrac{3}{2} - \dfrac{3\pi}{8}\right)$ sq units [A1]

10



In the diagram, the line $y=\sqrt{5}-2$ is tangent to the circle C_1 with centre C at the point P. The circle passes through the origin O and the point Q on the x-axis. The line OP makes an angle β with the x-axis, where $\tan\beta=\sqrt{5}-2$.

(a) Find the exact coordinates of points P, Q and C. [5]

Let the foot of the perpendicular from P to the x-axis be R.

Since $\tan\beta=\dfrac{PR}{OR}=\dfrac{\sqrt{5}-2}{OR}$, $OR=1$ [A1]

Hence coordinates of $P=\left(1,\ \sqrt{5}-2\right)$ [A1]

Since $OR=RQ$,

Coordinates of $Q=(2,\ 0)$ [A1]

Let the coordinates of C be $(1, y)$.

Since $OC=PC$,

$\sqrt{(1-0)^2+(y-0)^2}=\sqrt{(1-1)^2+\left(y-\left(\sqrt{5}-2\right)\right)^2}$ [M1]

$1+y^2=y^2-2\left(\sqrt{5}-2\right)y+\left(\sqrt{5}-2\right)^2$

$2\left(\sqrt{5}-2\right)y=\left(5-4\sqrt{5}+4\right)-1$

$y=\dfrac{4\left(2-\sqrt{5}\right)}{2\left(\sqrt{5}-2\right)}=-2$ [A1]

Hence, coordinates of $C=(1,-2)$ [–1 if not given]

(b) Find the equation of circle C_1. [2]

Radius of circle $C_1 = \sqrt{(1-0)^2+(-2-0)^2} = \sqrt{5}$ [M1]

Therefore equation of circle C_1 is :

$(x-1)^2+(y+2)^2=5$. [A1]

(c) Circle C_2 is the reflection of circle C_1 about the line $y=\sqrt{5}-2$. Find the equation of circle C_2. [3]

Radius of circle $C_2 = \sqrt{5}$ [M1]

Centre of circle $C_2 = \left(1,\ \sqrt{5}+\left(\sqrt{5}-2\right)\right)=\left(1,\ 2\sqrt{5}-2\right)$ [M1]

Equation of circle C_2 is :

$(x-1)^2+\left(y-2\sqrt{5}+2\right)^2=5$ [A1]

End of Paper