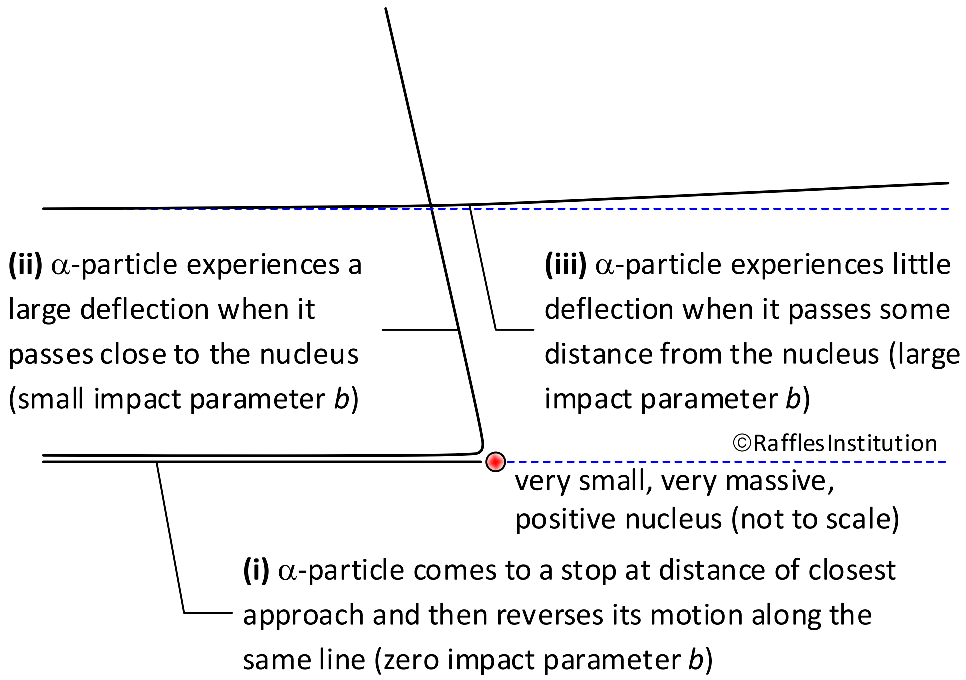
Tutorial 20 Nuclear Physics Suggested Solutions

DISCUSSION QUESTIONS

Nuclear Reactions

D1 **(a)**



(b)

Observation	Conclusion
Most α-particles experienced **little to no deflection**	Results as **expected** in Thomson's plum-pudding model of atom.
A small fraction of α-particles experienced **large deflection**; a few α-particles are deflected at angles $\sim 170^\circ$.	Large deflection possible only if the nucleus is **massive** (99.99% mass of atom). The very small fraction indicates that the nucleus is **very small** ($\sim 10^{-14} - 10^{-15}$ m)

D2 **(a)** Neutrons are **neutral** and cannot be measured directly using electric or magnetic mass spectrometer.

Note

Experimentally, we can determine the energy or speed of a charged particle using either electric or magnetic mass spectrometer (for low energy) by measuring the amount of deflection, provided the mass of the particle is known.

For high energy charged particles, an electromagnetic calorimeter is more appropriate. When a charged particle enters the calorimeter, it initiates a particle showers, i.e., the original charge particle transfers all its energy to other particles (via electromagnetic interaction). The energy of the particle shower is then deposited within the calorimeter and measured.

https://www.desy.de/~garutti/LECTURES/ParticleDetectorSS12/L10_Calorimetry.pdf

(b) Maximum recoil speed of the target nucleus occurs when the neutron collides **head-on** with the target nucleus.

(c) Consider the head-on collision between a neutron and the target nucleus.

before collision after collision

Principle of conservation of momentum:

$$m_n u_n = m_n v_n + mv \qquad ❶$$

Since the collision is elastic,

$$u_n = v - v_n \quad \Rightarrow \quad v_n = v - u_n \qquad ❷$$

Substituting ❷ into ❶,

$$m_n u_n = m_n\left(v - u_n\right) + mv$$

$$\Rightarrow \quad u_n = \frac{m_n v + mv}{2m_n}$$

Since the target nuclei has mass m_1 and m_2 with recoil speed v_1 and v_2, respectively,

$$u_n = \frac{m_n v_1 + m_1 v_1}{2m_n} = \frac{m_n v_2 + m_2 v_2}{2m_n}$$

$$\Rightarrow \quad m_n = \frac{m_1 v_1 - m_2 v_2}{v_2 - v_1}$$

(d) The data for the collision are as follow:

$$m_{\mathrm{p}} = 1.00\ u\ ;\ v_{\mathrm{p}} = 3.30\times10^{7}\ \mathrm{m\ s^{-1}}$$

$$m_{\mathrm{N}} = 14.0\ u\ ;\ v_{\mathrm{N}} = 4.70\times10^{6}\ \mathrm{m\ s^{-1}}$$

Substituting the date into the equation

$$m_n = \frac{1.00\ u\times3.30\times10^{7} - 14.00\ u\times4.70\times10^{6}}{4.70\times10^{6} - 3.30\times10^{7}}$$

$$= \frac{-3.28\times10^{7}\ u}{-2.83\times10^{7}} = 1.16\ u$$

D3 Nuclear equation:

$${}^{2}_{1}\mathrm{H} + {}^{6}_{3}\mathrm{Li} \rightarrow 2\,{}^{4}_{2}\mathrm{He}$$

Decrease in mass

$$\Delta m = 2.013553\ u + 6.013476\ u - 2\times4.001505\ u$$

$$= 0.024019\ u$$

Since the total momentum of the system is zero, the α-particles must have equal and opposite momentum and hence have the same energy.

Energy of each α-particle

$$E = \tfrac{1}{2}\Delta mc^{2} = \tfrac{1}{2}\times0.024019\times1.66\times10^{-27}\times\left(3.00\times10^{8}\right)^{2}$$

$$= 1.79\times10^{-12}\ \mathrm{J}$$

D4 **(a)** Nuclear equation:

$${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n}$$

(b) Nuclear fission

(c) Energy released

$$E = \mathrm{B.E.}_{\mathrm{products}} - \mathrm{B.E.}_{\mathrm{reactants}}$$

$$= 141\times8.5 + 92\times8.5 - 235\times7.6 = 194.5 \approx 195\ \mathrm{MeV}$$

(d) Kinetic energies of products (${}^{141}_{56}\mathrm{Ba}$, ${}^{92}_{36}\mathrm{Kr}$ and neutrons) and gamma radiation.

D5 **(a)** Decrease in mass

$$\Delta m = 2m_{\text{Deuterium-atom}} - m_{\text{Helium-atom}}$$
$$= 2 \times 2.014102\ u - 4.002603\ u = 0.025601\ u$$

(b) Number of deuterium atoms

$$N = \frac{m}{M_R} \times N_A = \frac{1.0}{2.014102} \times 6.02 \times 10^{23} = 2.99 \times 10^{23}$$

Number of nuclear fusion reaction is 1.494×10^{23}.

Energy released from 1.0 g of deuterium

$$E = 1.494 \times 10^{23} \times 0.025601 \times 1.66 \times 10^{-27} \times \left(3.00 \times 10^{8}\right)^{2}$$
$$= 5.72 \times 10^{11}\ \text{J}$$

Note: Answer given is different from above as it was calculated using approximated mass of each nucleus.

(c) Mass of deuterium needed

$$m = \frac{100 \times 10^{6} \times 24 \times 60 \times 60}{5.72 \times 10^{11}} \times 1.0 \div 0.40 = 37.8\ \text{g}$$

Radioactivity

D6 **(a)** **(i)** A **spontaneous** decay is one of which the **probability of decay** of a nucleus is **unaffected by any external factors** such as temperature, pressure or chemical composition.

(ii) **Random** means that it is **impossible to predict** **when a nucleus will decay**. This results in fluctuations in the count rate measured by a detector.

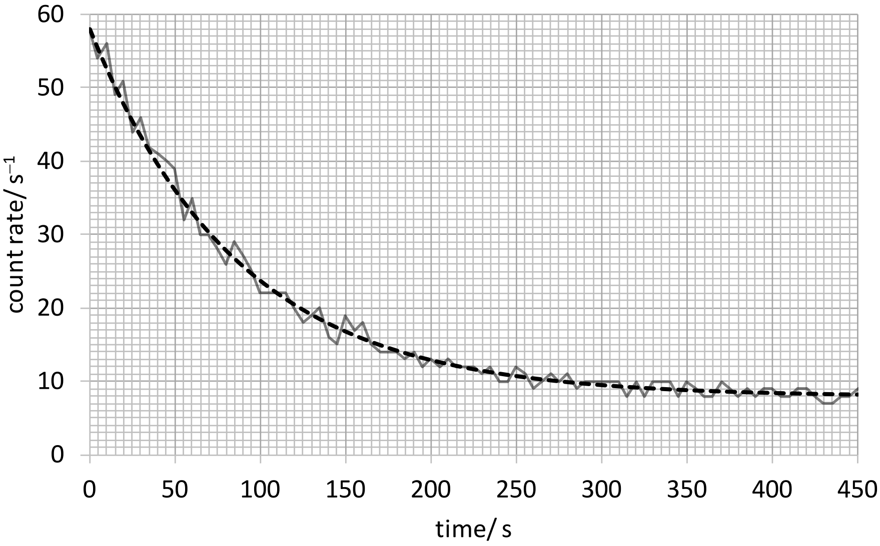
(b) Since cobalt-60 nucleus decays into a different nucleus (nickel-60) and no nuclei escape the box, the number of nuclei in a box remains constant.

Correct definition

The half-life of a radioactive nuclide is the **time taken** for the **number of undecayed nuclei** to decay **to half** its original number.

D7 **(a)** **(i)** The **fluctuating count rate** shows that radioactive decay is random.

To minimize uncertainty in the determination of half-life, we must draw a **line of best fit** through these data points.



(ii) The relatively constant count rate (subject to random fluctuations) after $t = 400$ s is the background count rate. After a sufficiently long time, the count rate due to the radioactive source becomes **negligible**.

Background count rate must be **subtracted** from the total count rate to obtain the true count rate due to the radioactive source.

Note

Background count rate is usually determined in by measuring the count rate with a radiation detector in the **absence of a radioactive source**.

(b) The background count rate is approximately 8.0 Bq.

Using the equation $A = A_0 e^{-\lambda t} = A_0 e^{-t\ln 2/t_{1/2}}$ and rearranging,

$$t_{1/2} = -\frac{t\ln 2}{\ln(A/A_0)}$$

We read off two points $(0.0, 58.0)$ and $(275.0, 10.0)$ from the line of best fit. Accounting for background count rate, we have $(0.0, 50.0)$ and $(275.0, 2.0)$. So

$$t_{1/2} = -\frac{275.0 \times \ln 2}{\ln(2.0/50.0)} = 59.2 \text{ s}$$

OR

Actual count rate of source $= 58 - 8 = 50$ s^{-1}

For 1 half life, count rate of source should drop to 25 s^{-1}.

This should display on the graph as $= 25 + 8 = 33\ \text{s}^{-1}$

Time taken to reach $33\ \text{s}^{-1} = 60\ \text{s}$

D8 **(a)** **(i)** Decay constant is the **probability per unit time** of the decay of a nucleus.

(ii) Decay constant

$$\lambda = \frac{\ln 2}{5.3} = 0.131\ \text{yr}^{-1} = 4.14 \times 10^{-9}\ \ \text{s}^{-1}$$

(b) **(i)** Number of nuclei in 2.0 mg of ${}^{60}_{27}\text{Co}$

$$N_0 = \frac{m}{M_R} \times N_A = \frac{2.0 \times 10^{-3}}{60} \times 6.02 \times 10^{23} = 2.01 \times 10^{19}$$

(ii) Initial activity of ${}^{60}_{27}\text{Co}$

$$A_0 = \lambda N_0 = 4.14 \times 10^{-9} \times 2.01 \times 10^{19} = 8.32 \times 10^{10}\ \text{Bq}$$

(iii) Initial power output from 2.0 mg of ${}^{60}_{27}\text{Co}$

$$P_0 = A_0 E = 8.32 \times 10^{10} \times 2.62 = 2.18 \times 10^{11}\ \text{MeV s}^{-1} = 3.49 \times 10^{-2}\ \text{W}$$

(iv) Activity after 21.2 years

$$A = A_0 \left(\tfrac{1}{2}\right)^{21.2/5.3} = 5.20 \times 10^{9}\ \text{Bq}$$

Power output from 2.0 mg of ${}^{60}_{27}\text{Co}$ after 21.2 years

$$P = AE = 5.20 \times 10^{9} \times 2.62 = 1.36 \times 10^{10}\ \text{MeV s}^{-1} = 2.18 \times 10^{-3}\ \text{W}$$

(c) Most of the γ rays are not absorbed by internal organs as γ rays have **low ionizing power** OR γ rays are highly penetrative.

(d) **1.** The decay of cobalt-60 generates **radioactive wastes** which must be properly **stored for a long time in secure facilities**.

Linear accelerator do not generate radioactive waste.

2. The radiation from cobalt-60 is emitted in all directions and is thus difficult to focus them on the tumor. The X-rays from accelerators can be focused more easily.

From wiki

The **radiation dose** in photon radiation therapy (gamma rays or x-rays) is measured in grays and varies depending on the type and stage of cancer being treated.

Radiation dose is defined as the **energy absorbed per unit mass** of the irradiated matter.

One gray (Gy) is one **joule of radiation energy per kilogram** of matter.

For curative cases, the typical dose for a solid epithelial tumour ranges from 60 to 80 Gy, while lymphomas are treated with 20 to 40 Gy.

D9 **(a)** **(i)** The half-life $t_{1/2}$ of a radioactive nuclide is the **time taken** for the **number of undecayed nuclei** to be **reduced to half** its original number.

(ii) Half-life of ${}^{40}_{19}\mathrm{K}$

$$t_{1/2} = \frac{\ln 2}{5.33 \times 10^{-10}} = 1.30 \times 10^{9} \text{ yr}$$

(iii) Since $A = A_0 \left(\tfrac{1}{2}\right)^n$, rearranging,

$$n = \frac{\ln\left(A/A_0\right)}{\ln\left(\tfrac{1}{2}\right)} = \frac{\ln\left(2.4/38.4\right)}{\ln\left(\tfrac{1}{2}\right)} = 4.0$$

Age of rock

$$t = nt_{1/2} = 5.20 \times 10^{9} \text{ yr}$$

(b) **(i)** Number of ${}^{40}_{19}\mathrm{K}$ nuclei

$$N = \frac{A}{\lambda} = \frac{2.4}{5.33 \times 10^{-10}} \times 24 \times 60 \times 60 \times 365 = 1.42 \times 10^{17}$$

(ii) 1. No change. The half life is not affected by the number of nuclei of the sample.

2. No change.
Radioactive decay is **spontaneous**, i.e., rate of decay is not affected by external factors such as temperature, pressure or chemical composition. Hence half-life of ${}^{40}_{19}\mathrm{K}$ remains the **same** at higher temperature.