

Name of Candidate: _____ () Class: _____ Calculator Model:



BUKIT PANJANG GOVERNMENT HIGH SCHOOL
PRELIMINARY EXAMINATION 2023
SECONDARY 4/5
GCE 'O' LEVEL SYLLABUS

MATHEMATICS

Paper 1

4052 / 01

Date: 21 August, 2023

Duration: 2 h 15 mins

Time: 0750 – 1005 h

Additional Materials: NIL

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

Setter: Mr Sim Meng Hoe

[Turn over]

This paper consists of 22 printed pages.

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

- 1 Calculate $\sqrt[3]{\frac{2.13 \times 81.23}{3.524} \times 3.2^3}$, giving your answer correct to 4 significant figures.

Answer [1]

- 2 At a warehouse sale, all prices are reduced by 15%.
The price of a set of ear pods during the sale is \$221.

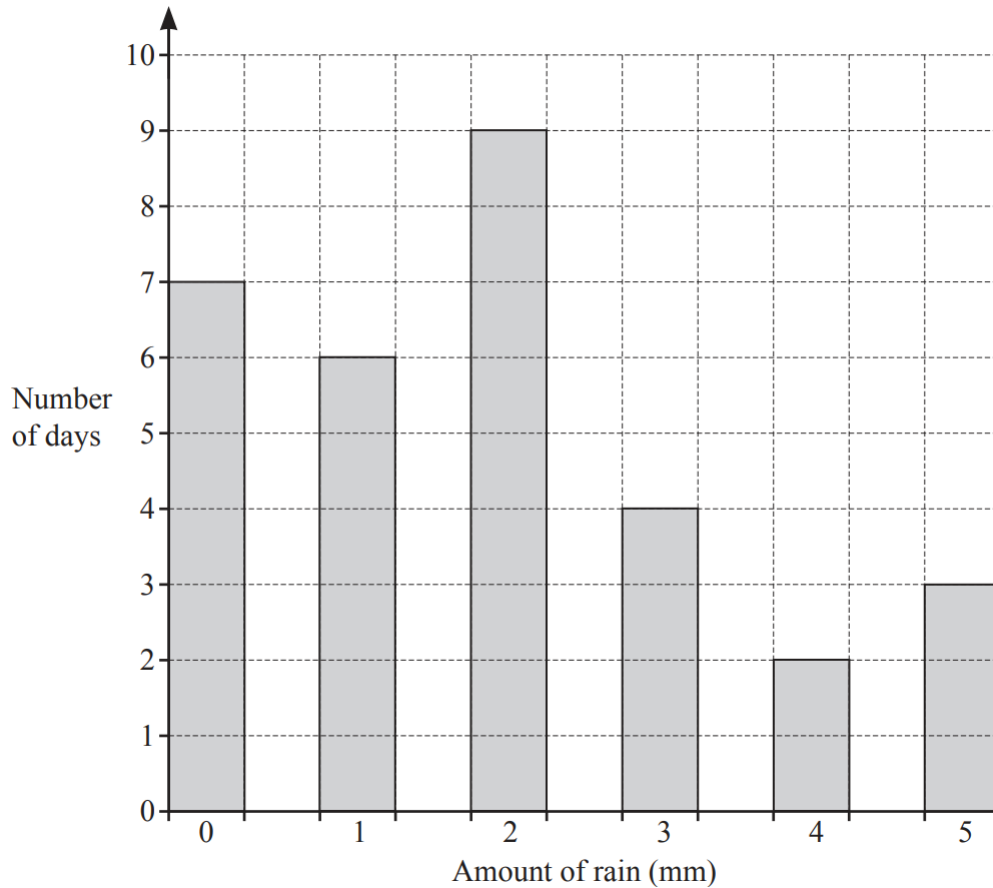
(i) Find its original price.

Answer \$ [1]

- (ii) If the salesman still earns a profit of 10.5% during the sale, find the percentage profit he earns from selling the pair of ear pods if it is not on sale.

Answer % [3]

- 3 Ethan measures the amount of rain, in millimetres (mm), each day for 31 days. The bar chart shows his results.



- (a) Write down the median amount of rain.

Answer mm [1]

- (b) Find the mean amount of rain per day.

Answer mm [2]

- (c) Ethan picks one of these days at random. Find the probability that, on that day, the amount of rain was 3 mm or more.

Answer [1]

- 4(a) These are the first four terms in a sequence.

9 5 1 -3

Find an expression, in term of n , for the n th term of the sequence.

Answer [1]

- (b) The n th term of another sequence is given by $T_n = \frac{3n+4}{4n+3}$.

Determine if it is possible to have a term in the sequence to be greater than 1.

Explain your answer.

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..... [2]

- 5 A drawer contains 40 balls. 15 balls are black, 5 balls are grey and 20 balls are white. Samuel chooses one ball from the drawer at random.

(a) Find the probability that the ball is not black.

Answer [1]

Samuel adds a few more grey balls. The probability of choosing a grey ball is now $\frac{2}{7}$.

(b) Find the number of grey balls he added.

Answer balls [2]

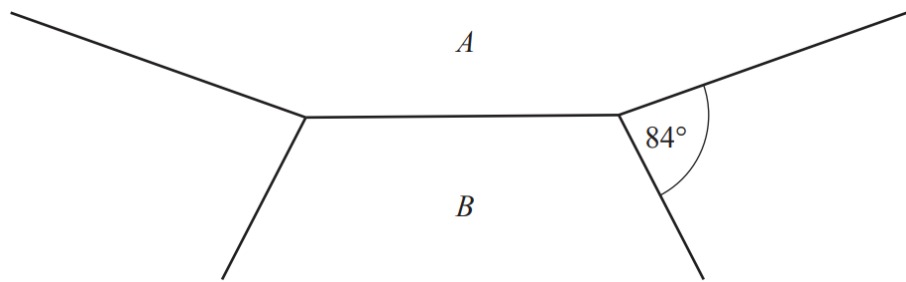
- 6 Solve the simultaneous equations.

$$7x + 2y = 8$$

$$2x - 3y = 13$$

Answer $x = \dots\dots\dots, y = \dots\dots\dots$ [3]

7



The diagram shows part of a polygon A and part of polygon B .
 A is a regular polygon with n sides.
 B is a regular hexagon.
Find the value of n .

Answer $n = \dots\dots\dots$ [4]

- 8 Mary needs to pack 210 oranges, 252 apples and 294 pears into identical fruit baskets.
- (i) What is the largest possible number of fruit baskets that can be packed?

Answer fruit baskets [1]

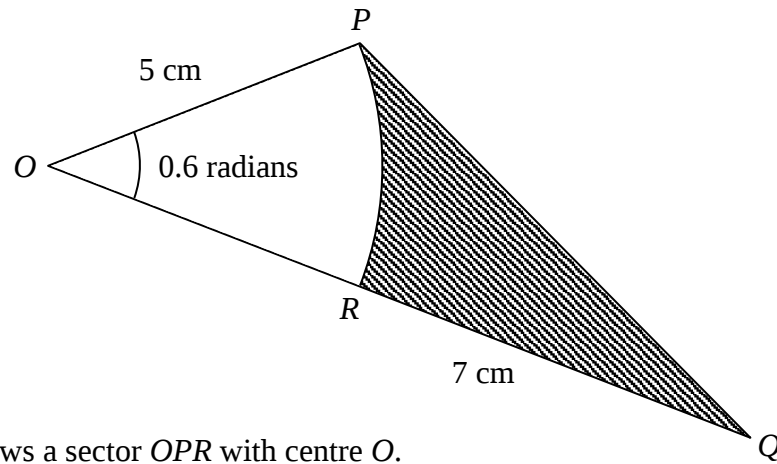
- (ii) State the number of each fruit in a fruit basket.

Answer oranges

..... apples

..... pears [1]

9



The diagram shows a sector OPR with centre O .

ORQ is a straight line.

Angle $POQ = 0.6$ radians.

$OP = 5$ cm and $RQ = 7$ cm.

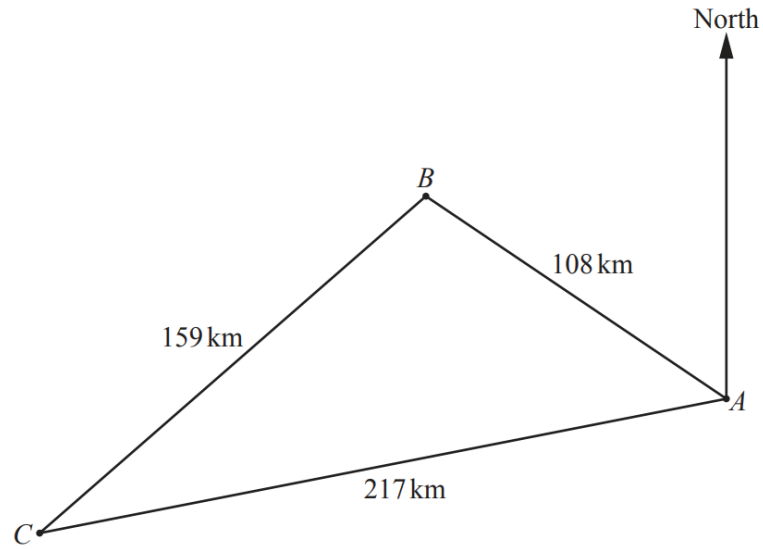
- (i) Calculate the perimeter of the shaded region PQR .

Answer cm [3]

- (ii) Calculate the area of the shaded region PQR .

Answer cm² [2]

10



A , B and C are three ports.

- (a) Show that angle $ABC = 107.2^\circ$ correct to 1 decimal place. [2]

- (b) The bearing of B from A is 305° .
Find the bearing of C from A .

Answer $^\circ$ [2]

- 11 Expand and simplify $7x - 2(3x - 2)^2$.

Answer [2]

- 12 The length of one of the diagonals of a rhombus of side 13 cm is 24 cm. Find the area of the rhombus.

Answer [3]

- 13 p varies inversely as the square root of q .
When $q = 9$, $p = 12$.
Find p when $q = 16$.

Answer $p = \dots\dots\dots$ [3]

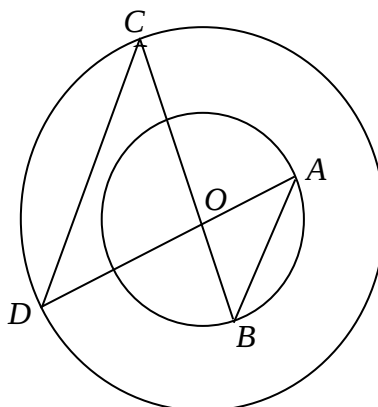
- 14 Andrea and Beatrice each have a savings account.
The ratio of Andrea's savings to Beatrice's savings is 2 : 3.
They each spend \$50 from their savings. Andrea then gives Beatrice \$20 from her savings. The new ratio of Andrea's savings to Beatrice's savings is 5 : 9.
Find the amount of money Andrea now has in her account.

Answer \$ $\dots\dots\dots$ [4]

- 15 Solve the inequalities $-4 \leq \frac{2x+6}{2} < \frac{11+2x}{3}$.

Answer [3]

- 16 In the diagram, O is the centre of two concentric circles. A and B lie on the circumference of the smaller circle. C and D lie on the circumference of the larger circle. AD and BC intersect at O .



Prove that

- (a) $\triangle AOC$ and $\triangle BOD$ are congruent

.....

 [2]

- (b) $\triangle ADB$ and $\triangle BCA$ are congruent.

.....

 [2]

- 17 $PQRS$ is a trapezium where P is the point $(-3, 2)$, Q is the point $(5, 8)$ and R is the point $(3, 2)$. PQ is parallel to RS .

(a) Find the equation of the line RS .

Answer [2]

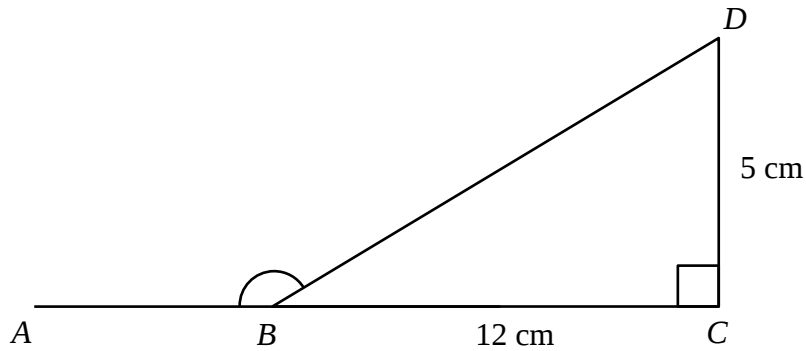
(b)(i) Find the length of PQ .

Answer units [2]

(ii) Hence find the perpendicular distance from R to PQ .

Answer units [2]

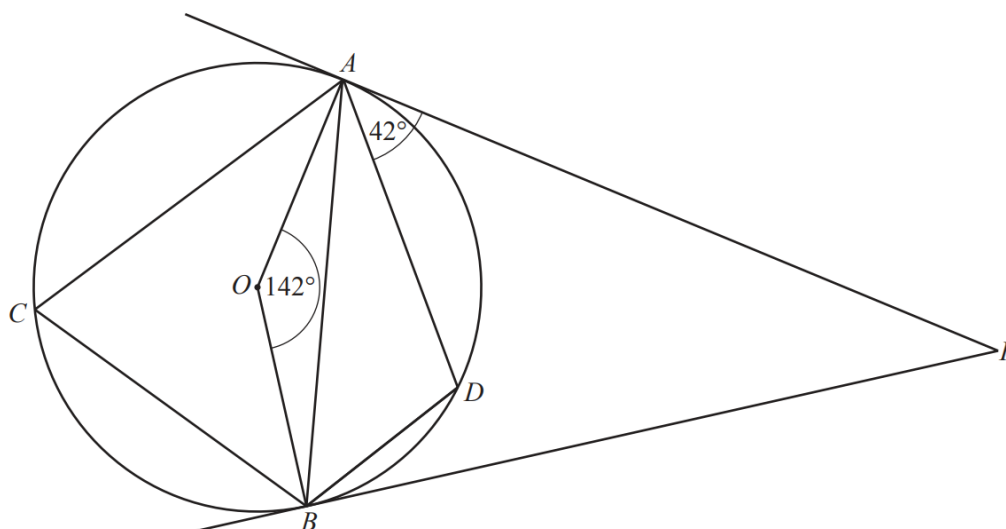
- 18 In the diagram, points A , B and C lie on a straight line. $BC = 12$ cm, $DC = 5$ cm and angle $BCD = 90^\circ$. Find the value of $\cos \angle ABD$.



Answer $\cos \angle ABD = \dots\dots\dots$ [2]

- 19 Write as a single fraction in its simplest form $\frac{1}{9x^2 - 1} - \frac{2}{1 - 3x}$.

Answer $\dots\dots\dots$ [3]



A, D, B and C lie on a circle, centre O .

AP is a tangent to the circle at A and BP is a tangent to the circle at B .

Angle $AOB = 142^\circ$ and angle $DAP = 42^\circ$.

(a) Find the value of

(i) angle ACB ,

Answer $^\circ$ [1]

(ii) angle ADB ,

Answer $^\circ$ [1]

(b) Is OB parallel to AD ? Explain.

.....

.....

.....

..... [2]

- 21 The admission tickets to the Singapore Zoo are \$50 for an adult, \$36 for a child and \$20 for a senior citizen. On a particular Tuesday, there were 212 adults, 251 children and 15 senior citizens who visited the Singapore Zoo and on a particular Wednesday, there were 231 adults, 266 children and 12 senior citizens who visited the Singapore Zoo.

The number of visitors on the particular Tuesday and Wednesday can be represented by the matrix $\mathbf{V} = \begin{pmatrix} 212 & 251 & 15 \\ 231 & 266 & 12 \end{pmatrix}$.

- (i) Write a 3×1 matrix, $\mathbf{P}$, to represent the price of the admission tickets.

Answer $\mathbf{P} = \dots\dots\dots$ [1]

- (ii) Find the matrix $\mathbf{T} = \mathbf{VP}$.

Answer $\dots\dots\dots$ [2]

- (iii) Explain what does each of the elements represents.

.....

 [1]

- (iv) Find the total amount collected from the sales of the tickets for the 2 days.

Answer \$ $\dots\dots\dots$ [1]

- 22 Rearrange this equation to make x the subject.

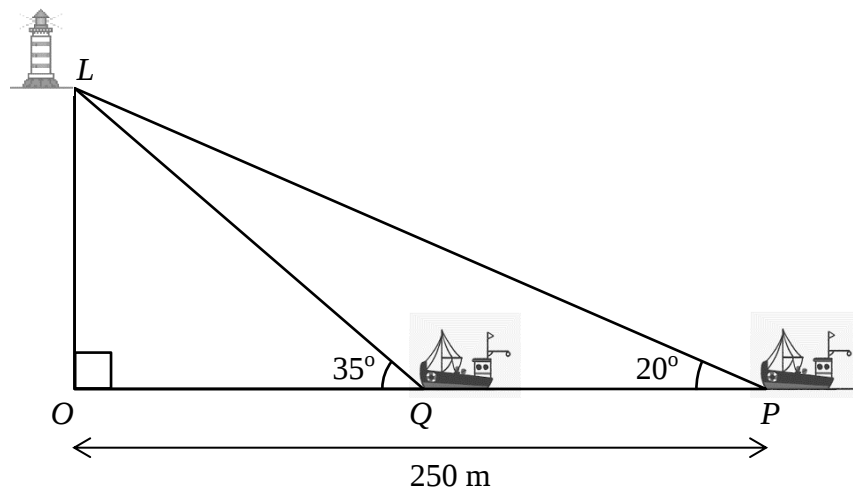
$$\frac{a}{2x-3} = \frac{b}{5x}$$

Answer $x = \dots\dots\dots$ [3]

- 23 Given that $2 \times 16^{2x} = 8^3$, find the value of x .

Answer $x = \dots\dots\dots$ [3]

- 24 The angle of elevation of the base of a lighthouse, L , from two fishing boat P and Q are 20° and 35° respectively. Given that the fishing boat P is 250 m from the lighthouse, find the distance between the two fishing boats.



Answer m [3]

- 25 100 newborn babies are weighed and their masses are recorded in the table below.

Mass (x kg)	Frequency
$2.8 < x \leq 3.0$	6
$3.0 < x \leq 3.2$	21
$3.2 < x \leq 3.4$	47
$3.4 < x \leq 3.6$	22
$3.6 < x \leq 3.8$	4

- (a) Calculate an estimate for
 (i) the mean mass of these new born babies,

Answer kg [2]

- (ii) the standard deviation of the mass of these newborn babies.

Answer kg [2]

- (b) If the weighing machine has an error of weighing an extra 0.2 kg, comment on how this error affects the standard deviation of the mass of these newborn babies.

.....

 [1]

$$26(a) \quad \xi = \left\{ 0, -1, 3\pi, 3, \frac{2}{11}, 7 \right\}$$

$$A = \{x : x \text{ is a rational number}\}$$

$$B = \{x : x \text{ is an integer}\}$$

$$C = \{x : x \text{ is a positive number}\}$$

(i) Use one of the symbols below to fill in the blank in the answer space.

= $\subset$ $\not\subset$ $\in$ $\notin$

Answer $\{-1, 3, 7\}$ B [1]

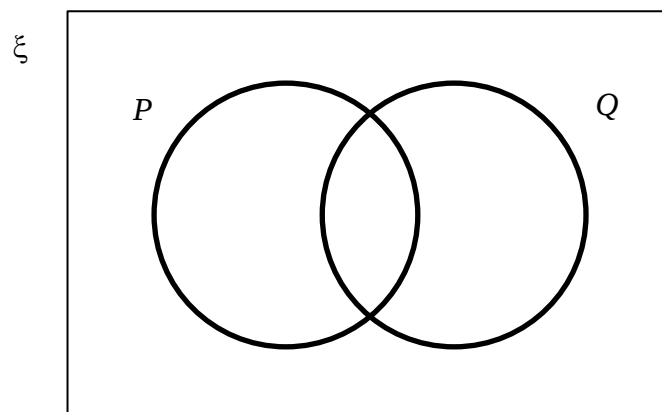
(ii) List the elements of $A \cap C$.

Answer $A \cap C =$ [1]

(iii) Write down, in set notation, in terms of A , B and/or C the set that represents rational numbers that are **not** integers.

Answer [1]

(b) In the diagram, shade the region representing $P' \cap Q$.



[1]

Solutions to Preliminary Examinations 2023 Mathematics 4052

Qn No.	Solution
1	11.72
2(i)	$\begin{aligned}\text{Original sale price} &= \frac{100}{85} \times \$221 \\ &= \$260\end{aligned}$
2(ii)	$\begin{aligned}\text{Cost price of a set of ear pods} &= \frac{100}{110.5} \times \$221 \\ &= \$200\end{aligned}$ $\begin{aligned}\text{Profit without sale} &= \$260 - \$200 \\ &= \$60\end{aligned}$ $\begin{aligned}\text{Percentage profit} &= \frac{\$60}{\$200} \times 100\% \\ &= 30\%\end{aligned}$
3(a)	$\text{Middle position} = \frac{31+1}{2} = 16$ Median = 2 mm
3(b)	$\begin{aligned}\text{Mean} &= \frac{7(0) + 6(1) + 9(2) + 4(3) + 2(4) + 3(5)}{31} \\ &= \frac{59}{31} \\ &= 1.90 \text{ mm}\end{aligned}$
3(c)	Number of days with rain of 3 mm or more = $4 + 2 + 3 = 9$ $P(\text{the day chosen rain 3 mm or more}) = \frac{9}{31}$

Qn No.	Solution
4(a)	$9 - 4(n - 1)$ $= 9 - 4n + 4$ $= 13 - 4n$
4(b)	If a term is greater than 1, $\frac{3n + 4}{4n + 3} > 1$ $3n + 4 > 4n + 3$ $3n - 4n > 3 - 4$ $-n > -1$ $n < 1$ Since $n < 1$ and n has to be an integer greater than or equal to 1 for T_n, There is no such term in the sequence that is greater than 1.
5(a)	P(the ball chosen is not black) $= 1 - \text{P(the ball chosen is black)}$ $= 1 - \frac{15}{40}$ $= \frac{25}{40}$ $= \frac{5}{8}$
5(b)	Let the number of grey balls added be x. $\frac{5 + x}{40 + x} = \frac{2}{7}$ $7(5 + x) = 2(40 + x)$ $35 + 7x = 80 + 2x$ $7x - 2x = 80 - 35$ $5x = 45$ $x = 9$

Qn No.	Solution
9(i)	Arc length $PR = 5 \times 0.6 = 3$ cm $PQ^2 = 5^2 + 12^2 - 2(5)(12)\cos 0.6$ $PQ = \sqrt{5^2 + 12^2 - 2(5)(12)\cos 0.6} \text{ cm}$ Perimeter of $PQR = 7 + 3 + \sqrt{5^2 + 12^2 - 2(5)(12)\cos 0.6}$ $= 18.4$ cm (to 3 s.f.)
9(ii)	Area of triangle $OPQ = \frac{1}{2} \times 5 \times 12 \times \sin 0.6$ $= 16.939 \text{ cm}^2$ Area of sector $= \frac{1}{2} \times 5^2 \times 0.6 \text{ cm}^2$ $= 7.5 \text{ cm}^2$ Area of shaded region $= 16.939 - 7.5 \text{ cm}^2$ $= 9.44 \text{ cm}^2$ (to 3 s.f.) }
10(a)	Using cosine rule $\cos \angle ABC = \frac{159^2 + 108^2 - 217^2}{2(159)(108)}$ $\angle ABC = \cos^{-1} \left(\frac{159^2 + 108^2 - 217^2}{2(159)(108)} \right)$ $= 107.179^\circ$ $= 107.2^\circ$ (to 1 d.p.) (Shown!)
10(b)	Using sine rule, $\frac{\sin \angle BAC}{159} = \frac{\sin 107.179^\circ}{217}$ $\sin \angle BAC = \frac{159 \sin 107.179^\circ}{217}$ $\angle BAC = \sin^{-1} \left(\frac{159 \sin 107.179^\circ}{217} \right)$ $= 44.429^\circ$ Bearing of C from $A = 305^\circ - 44.429^\circ = 260.6^\circ$ (to 1 d.p.)
11	$7x - 2(3x - 2)^2$ $= 7x - 2(9x^2 - 12x + 4)$ $= 7x - 18x^2 + 24x - 8$ $= -18x^2 + 31x - 8$

Qn No.	Solution
12	The diagonals of a rhombus intersect perpendicularly. Let the length of the other diagonal be $2x$ cm. By Pythagoras' Theorem, $x^2 + 12 = 13^2$ $x^2 = 13^2 - 12^2$ $x = \sqrt{13^2 - 12^2} = 5$ Hence area of rhombus $= 2 \times \frac{1}{2} \times 24 \times 5 \text{ cm}^2$ $= 120 \text{ cm}^2$
13	$p\sqrt{q} = k$, where k is a constant $q = 9$ when $p = 12$ $12\sqrt{9} = k$ $k = 12(3) = 36$ Hence, $p = \frac{36}{\sqrt{q}}$ When $q = 16$, $p = \frac{36}{\sqrt{16}} = 9$

Qn No.	Solution
14	Let Andrea's original savings be \$x. Beatrice's original savings is $\\$ \frac{3}{2}x$. $\frac{x-50-20}{\frac{3}{2}x-50+20} = \frac{5}{9}$ $9(x-70) = 5\left(\frac{3}{2}x-30\right)$ $9x-630 = \frac{15}{2}x-150$ $9x - \frac{15}{2}x = 630-150$ $\frac{3}{2}x = 480$ $x = \frac{2}{3} \times 480 = 320$ Amount of money Andres now has = \$320 - \$50 - \$20 = \$ 250
15	$-4 \leq \frac{2x+6}{2} < \frac{11+2x}{3}$ $-4 \leq \frac{2x+6}{2} \quad \text{and} \quad \frac{2x+6}{2} < \frac{11+2x}{3}$ $-8 \leq 2x+6 \quad \text{and} \quad 3(2x+6) < 2(11+2x)$ $2x \geq -8-6 \quad \text{and} \quad 6x+18 < 22+4x$ $2x \geq -14 \quad \text{and} \quad 6x-4x < 22-18$ $x \geq -7 \quad \text{and} \quad 2x < 4$ $\text{and} \quad x < 2$ $\therefore -7 \leq x < 2$
16(a)	$\left. \begin{array}{l} \angle AOC = \angle BOD \text{ (vert. opp. } \angle) \\ OC = OD \text{ (radii of a circle)} \\ OA = OB \text{ (radii of a circle)} \end{array} \right\}$ $\therefore$ by SAS congruency test, $\triangle AOC$ and $\triangle BOD$ are congruent.
16(b)	$\left. \begin{array}{l} BC = AD \text{ (Sum of radii of circle)} \\ AB = BA \text{ (common length)} \end{array} \right\}$ From (a), $\triangle AOC$ is congruent to $\triangle BOD$. Hence, $AC = BD$. By SSS congruency test, $\triangle ADB$ and $\triangle BCA$ are congruent

Qn No.	Solution
17(a)	Gradient of $RS = \frac{8-2}{5-(-3)}$ $= \frac{3}{4}$ Equation of RS : $2 = \frac{3}{4}(3) + c$ $c = 2 - \frac{9}{4} = -\frac{1}{4}$ Hence, the equation of RS is $y = \frac{3}{4}x - \frac{1}{4}$
17(b)(i)	$PQ = \sqrt{(-3-5)^2 + (2-8)^2}$ $= \sqrt{64+36}$ $= 10 \text{ units}$
17(b)(ii)	Area of triangle $PQR = \frac{1}{2} \times (8-2) \times (3+3)$ $= 18 \text{ units}^2$ Let the perpendicular distance of R to PQ be h. $\frac{1}{2} \times 10 \times h = 18$ $h = \frac{18 \times 2}{10} = 3.6 \text{ units}$
18	By Pythagoras' Theorem, $BD^2 = 5^2 + 12^2$ $BD = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$ $\cos \angle ABD = -\cos \angle DBC = -\frac{12}{13}$

Qn No.	Solution
19	$\frac{1}{9x^2-1} - \frac{2}{1-3x}$ $= \frac{1}{(3x+1)(3x-1)} - \frac{2}{1-3x}$ $= \frac{1}{(3x+1)(3x-1)} + \frac{2}{(3x-1)}$ $= \frac{1+2(3x+1)}{(3x+1)(3x-1)}$ $= \frac{1+6x+2}{(3x+1)(3x-1)}$ $= \frac{6x+3}{(3x+1)(3x-1)}$
20(a)(i)	$\angle ACB = \frac{142^\circ}{2} \text{ (}\angle \text{ at centre = twice } \angle \text{ at circumference)}$ $= 71^\circ$
20(a)(ii)	$\angle ADB = 180^\circ - 71^\circ \text{ (}\angle \text{s in opp. segment)}$ $= 109^\circ$
20(b)	$\angle OAD = 90^\circ - 42^\circ \text{ (tan } \perp \text{ radius)}$ $= 48^\circ$ $\angle BOA + \angle OAD = 142^\circ + 48^\circ$ $= 190^\circ$ Since $\angle BOA + \angle OAD \neq 180^\circ$, OB is not parallel to AD

Qn No.	Solution
21(i)	$\mathbf{P} = \begin{pmatrix} 50 \\ 36 \\ 20 \end{pmatrix}$
21(ii)	$\mathbf{T} = \begin{pmatrix} 212 & 251 & 15 \\ 231 & 266 & 12 \end{pmatrix} \begin{pmatrix} 50 \\ 36 \\ 20 \end{pmatrix}$ $= \begin{pmatrix} 212(50) + 251(36) + 15(20) \\ 231(50) + 266(36) + 12(20) \end{pmatrix}$ $= \begin{pmatrix} 19936 \\ 21366 \end{pmatrix}$
21(iii)	Each element represents the total amount collected from the sales of admission tickets for Tuesday and Wednesday respectively.
21(iv)	Total amount collected for the 2 days $= \$19\,936 + \$21\,366$ $= \$41\,302$
22	$\frac{a}{2x-3} = \frac{b}{5x}$ $5ax = b(2x-3)$ $5ax = 2bx - 3b$ $5ax - 2bx = -3b$ $x(5a - 2b) = -3b$ $x = -\frac{3b}{(5a-2b)} \quad \text{or} \quad x = \frac{3b}{(2b-5a)}$
23	$2 \times 16^{2x} = 8^3$ $2 \times (2^4)^{2x} = (2^3)^3$ $2 \times 2^{8x} = 2^9$ $2^{1+8x} = 2^9$ Comparing indices, $1 + 8x = 9$ $8x = 9 - 1$ $8x = 8$ $x = 1$

Qn No.	Solution
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24	$\tan 20^\circ = \frac{OL}{250}$ $OL = 250 \tan 20^\circ = 90.993 \text{ m (to 5 s.f.)}$ $\tan 35^\circ = \frac{OL}{OQ}$ $\tan 35^\circ = \frac{250 \tan 20^\circ}{OQ}$ $OQ = \frac{250 \tan 20^\circ}{\tan 35^\circ} = 129.95 \text{ m (to 5 s.f.)}$ $\begin{aligned} \text{Distance between two fishing boat} &= 250 - \frac{250 \tan 20^\circ}{\tan 35^\circ} \text{ m} \\ &= 120 \text{ m (to 3 s.f.)} \end{aligned}$
25(a)(i)	$\begin{aligned} \text{Mean} &= \frac{6(2.9) + 21(3.1) + 47(3.3) + 22(3.5) + 4(3.7)}{100} \text{ kg} \\ &= 3.294 \text{ kg} \end{aligned}$
25(a)(ii)	$\begin{aligned} \frac{\Sigma fx^2}{\Sigma f} &= \frac{6(2.9)^2 + 21(3.1)^2 + 47(3.3)^2 + 22(3.5)^2 + 4(3.7)^2}{100} \\ &= \frac{1088.36}{100} \end{aligned}$ $\begin{aligned} \text{Standard deviation} &= \sqrt{\frac{1088.36}{100} - (3.294)^2} \\ &= 0.182 \text{ kg (to 3 s.f.)} \end{aligned}$
25(b)	As all the mass will be reduced by the same amount, the standard deviation remains the same / is unaffected.

Qn No.	Solution
26(a)(i)	$\subset$
26(a)(ii)	$\left\{ 3, \frac{2}{11}, 7 \right\}$
26(a)(iii)	$A \cap B'$
26(b)	ξ 