RAFFLES INSTITUTION
2025 Year 5 H2 Mathematics Timed Practice Paper
Questions and Solutions with comments

General Comments

- A small number of students wrote the wrong Exam Index Number on their scripts. For example, 260227 was mistakenly written as 262027, which belongs to another actual student.
  👉 Please **double-check your Index Number** before writing it down in future assessments.

- A significant number of scripts had solutions written outside the designated boxes for each part of the question.
  📌 Please note that for the actual A-level exams, your scripts will be digitally scanned, and any writing outside the boxes may not be captured.
  👉 Always **keep your work within the provided boundaries**.

- A small number of students wrote their solutions for one question in the space of another question.
  👉 **Do not do this**—there may be **different markers assigned to different questions**, and your answer may be **missed** if it is in the wrong place.

- A significant number of scripts that used Additional Page to write solutions did not indicate on the original allocated solution space to inform the marker to refer to the Additional Page.
  👉 If you use an Additional Page, always **write "See Additional Page"** clearly in the allocated space, or the marker may **miss your continued work**.

- Most scripts were well presented, with clear and logical mathematical arguments. 👍
  However, in some cases:
  - the handwriting was difficult to read, or
  - the writing was too small, making it illegible, or
  - the ink was too light, making answers illegible

  ✏️ **Reminder:** Markers can only award marks for work they can read. Please ensure your handwriting is **neat and legible**, and that you write at a **reasonable size**.

Solutions

1 Given that the curve $y = ax + b|x|$ passes through $(-1, -4)$ and intersects another curve $y = cx - 3$ at $x = -3$ and $x = 3$, find the values of *a*, *b* and *c*. [4]

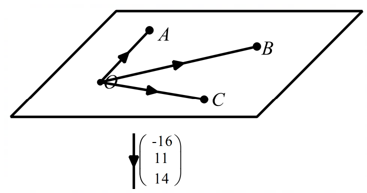
[4]	Substituting $(-1, -4)$ into $y = ax + b\|x\|$, $-4 = -a + b \quad \Rightarrow \quad a - b = 4 \quad ----- (1)$ At points of intersection, $ax + b\|x\| = cx - 3$ When $x = -3$, $-3a + 3b = -3c - 3$ $\Rightarrow \quad a - b - c = 1 ----- (2)$ When $x = 3$, $3a + 3b = 3c - 3$ $\Rightarrow \quad a + b - c = -1 ----- (3)$ Using GC, $a = 3$, $b = -1$, $c = 3$	Most students were able to solve this question and arrive at the correct answer. However, a significant number chose to solve the simultaneous equations by hand, and some made careless mistakes in the process.

2 Referred to an origin O, the position vectors of three points A, B and C are $3\mathbf{i}-2\mathbf{j}+5\mathbf{k}$, $\mathbf{i}+4\mathbf{j}-2\mathbf{k}$ and $\alpha\mathbf{i}+2\mathbf{j}+4\mathbf{k}$ respectively.

(a) Determine the value of α for which points O, A, B and C are coplanar. [2]

(b) Given that $\overrightarrow{OD}=\overrightarrow{OA}+\lambda\overrightarrow{OB}$, find the value of λ for which $\overrightarrow{OD}$ is perpendicular to the y – axis. [2]

(c) Find the exact area of triangle OAD. [2]

(a) **[2]**	**<u>Method 1</u>** A vector normal to plane $OABC$ $=\overrightarrow{OA}\times\overrightarrow{OB}$ $=\begin{pmatrix}3\\-2\\5\end{pmatrix}\times\begin{pmatrix}1\\4\\-2\end{pmatrix}=\begin{pmatrix}-16\\11\\14\end{pmatrix}$  So, $\overrightarrow{OC}\bullet\begin{pmatrix}-16\\11\\14\end{pmatrix}=\begin{pmatrix}\alpha\\2\\4\end{pmatrix}\bullet\begin{pmatrix}-16\\11\\14\end{pmatrix}=0$ $-16\alpha+22+56=0$ $\alpha=\dfrac{39}{8}$ **<u>Method 2</u>** Given the points are coplanar, $\begin{pmatrix}\alpha\\2\\4\end{pmatrix}=s\begin{pmatrix}3\\-2\\5\end{pmatrix}+t\begin{pmatrix}1\\4\\-2\end{pmatrix}$, where s and t are constants. $\alpha=3s+t$ --- (1) $2=-2s+4t$ --- (2) $4=5s-2t$ --- (3) Using GC to solve the above 3 eqns, we get $\alpha=\dfrac{39}{8},\ s=\dfrac{5}{4},\ t=\dfrac{9}{8}.$	Some students misunderstood the term “coplanar,” mistaking it for “collinear,” which led to incorrect interpretations in their answers. Note that when we say “four points are coplanar”, it means that there is a plane that the four points can lie on.
(b) **[2]**	$\overrightarrow{OD}=\overrightarrow{OA}+\lambda\overrightarrow{OB}=\begin{pmatrix}3\\-2\\5\end{pmatrix}+\lambda\begin{pmatrix}1\\4\\-2\end{pmatrix}$ A direction vector of the y-axis is $\begin{pmatrix}0\\1\\0\end{pmatrix}$. If $\overrightarrow{OD}$ is perpendicular to the y – axis,	This part is well done.

	$\overrightarrow{OD}\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}=0$ $\begin{pmatrix}3+\lambda\\-2+4\lambda\\5-2\lambda\end{pmatrix}\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}=0$ $-2+4\lambda=0$ $\lambda=\frac{1}{2}$	
(c) **[2]**	$\overrightarrow{OD}=\overrightarrow{OA}+\frac{1}{2}\overrightarrow{OB}=\begin{pmatrix}3\\-2\\5\end{pmatrix}+\frac{1}{2}\begin{pmatrix}1\\4\\-2\end{pmatrix}=\begin{pmatrix}3.5\\0\\4\end{pmatrix}$ Area of ΔOAD $=\frac{1}{2}\left\|\overrightarrow{OA}\times\overrightarrow{OD}\right\|$ $=\frac{1}{2}\left\|\begin{pmatrix}3\\-2\\5\end{pmatrix}\times\begin{pmatrix}3.5\\0\\4\end{pmatrix}\right\|=\frac{1}{2}\left\|\begin{pmatrix}-8\\5.5\\7\end{pmatrix}\right\|$ $=\frac{1}{2}\sqrt{64+30.25+49}$ $=\frac{1}{4}\sqrt{573}$	The part is well done

3 The function f is defined by

$$\mathrm{f} : x \mapsto (3-x)(x+1), \text{ where } x \in \mathbb{R},\ x \le 1.$$

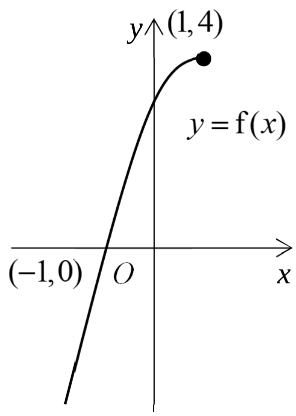
(a) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

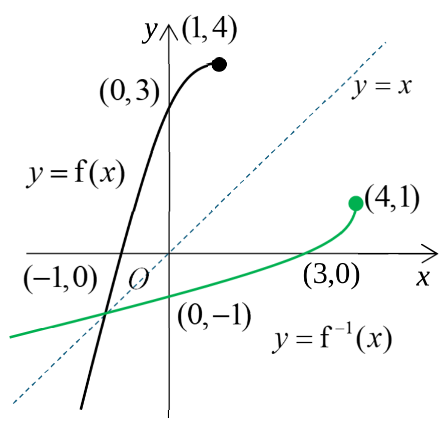
(b) Sketch on the same diagram the graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$, giving the coordinates of any points where the curves cross the axes. [3]

The function g is defined by

$$\mathrm{g} : x \mapsto \mathrm{e}^{-x}, \text{ where } x \in \mathbb{R},\ x \ge 0.$$

(c) Show that the composite function fg exists and state the range of fg. [2]

<table>
<tr>
<td>(a)
[4]</td>
<td>$y=(3-x)(x+1) \quad , \ x \le 1$
$=-(x^2-2x-3)$
$=-\left[(x-1)^2-4\right]$
$=4-(x-1)^2$
$4-y=(x-1)^2$
$x=1\pm\sqrt{4-y}$
Since $x \le 1$, $x=1-\sqrt{4-y}$
$\therefore \mathrm{f}^{-1}(x)=1-\sqrt{4-x}$
From the graph of f, $\mathrm{R_f}=(-\infty,4]$
Domain of $\mathrm{f}^{-1}=\mathrm{R_f}=(-\infty,4]$

</td>
<td>A few students did not recall that if $a^2=b$, then $a=\pm\sqrt{b}$.
So, they wrote $x-1=\sqrt{4-y}$.

Similar Qn
• Tut 4 Sect A Qn 2(a), Sect B Qn 3(ii)</td>
</tr>
<tr>
<td>(b)
[3]</td>
<td>
</td>
<td>Recall that
• the graphs of f and f^{-1} are symmetrical about $y=x$.
• the x- and y-scale should be the same
• the question ask for the intercepts in coordinates-form
• it is important to write down the coordinates of the endpoints on the graphs, so that one will know where does the graph starts from/ends at.

Similar Qn
• Tut 4 Sect B Qn 5(iii)</td>
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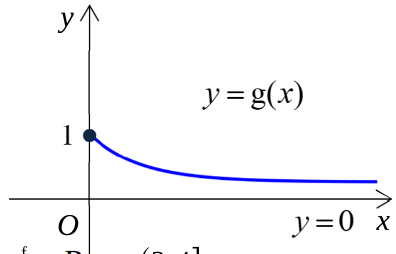
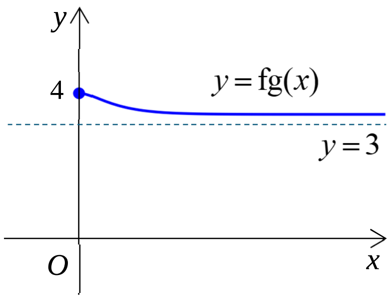
</table>

(c) **[2]**	For fg to exist, $R_g \subseteq D_f$. $R_g = (0,1]$, $D_f = (-\infty,1]$ Since $R_g \subseteq D_f$, fg exists. Using the graphs of f and g,  $D_g = [0,\infty) \xrightarrow{g} R_g = (0,1] \xrightarrow{f} R_{fg} = (3,4]$ **Alternative Method to find R_{fg}:** Consider the graph of fg, $fg(x) = (3-e^{-x})(e^{-x}+1)$, $x \geq 0$ $R_{fg} = (3,4]$  	Most students are able to recall the condition for composite function to exist. **Similar Qn** • Tut 4 Sect B Qn 2, Qn 6(ii)

4 It is given that $2+\mathrm{i}$ is a root of the equation

$$2z^4+az^3+bz^2+az-15=0\ .$$

(a) Find the values of the real numbers a and b, and without the use of a calculator, find the other roots of the equation. [6]

(b) Deduce the roots of the equation

$$2w^4+a\mathrm{i}w^3-bw^2-a\mathrm{i}w-15=0. \qquad [2]$$

<table>
<tr>
<td>(a)
[6]</td>
<td>

Method 1

Since all coefficients of the equation are real and $2+\mathrm{i}$ is a root, then $2-\mathrm{i}$ is also a root.

The quadratic factor of the equation is

$$\begin{aligned}\left[z-(2+\mathrm{i})\right]\left[z-(2-\mathrm{i})\right]&=\left[(z-2)-\mathrm{i}\right]\left[(z-2)+\mathrm{i}\right]\\&=(z-2)^2-(-1)\\&=z^2-4z+5\end{aligned}$$

Let $2z^4+az^3+bz^2+az-15$

$$\begin{aligned}&=\left(z^2-4z+5\right)\left(2z^2+cz-3\right)\\&=2z^4+(c-8)z^3+(7-4c)z^2+(5c+12)z-15\end{aligned}$$

Comparing coefficients of z^3 and z ,

$$\begin{aligned}c-8=a&=5c+12\\4c&=-20\\\therefore\quad c&=-5\end{aligned}$$

Thus $a=-5-8=-13$ and $b=7-4(-5)=27$

$$\begin{aligned}2z^4-13z^3+27z^2-13z-15&=0\\\left(z^2-4z+5\right)\left(2z^2-5z-3\right)&=0\\\left[z-(2+\mathrm{i})\right]\left[z-(2-\mathrm{i})\right](2z+1)(z-3)&=0\end{aligned}$$

Hence the other roots are $2-\mathrm{i},\ -\dfrac{1}{2}$ and $3.$

Method 2

$$\begin{aligned}2(2+\mathrm{i})^4+a(2+\mathrm{i})^3+b(2+\mathrm{i})^2+a(2+\mathrm{i})-15&=0\\2(-7+24\mathrm{i})+a(2+11\mathrm{i})+b(3+4\mathrm{i})+a(2+\mathrm{i})-15&=0\\-29+4a+3b+48\mathrm{i}+12a\mathrm{i}+4b\mathrm{i}&=0\end{aligned}$$

Comparing real and imaginary parts,

Real part: $-29+4a+3b=0$ ... (1)

Imaginary part: $12+3a+b=0\ \Rightarrow\ b=-12-3a$ ... (2)

Substitute (2) into (1), $-29+4a+3(-12-3a)=0$

$$\begin{aligned}-5a&=65\\a&=-13\end{aligned}$$

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Most students used the first method and able to get the answer.

As the question says that "without the use of a calculator, find the other roots of the equation", one should factorise $2z^2-5z-3$ to obtain the roots $-\frac{1}{2}$ and $3,$ otherwise it will be treated as using calculator to obtain the roots.

Similar Qn

- Tut 5 Sect B Qn 6

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	Substitute $a=-13$ into (2), $b=-12-3\left(-13\right)=27$ $2z^4-13z^3+27z^2-13z-15=0$ Since all coefficients of the equation are real and $2+\mathrm{i}$ is a root, then $2-\mathrm{i}$ is also a root. $\left[z-\left(2+\mathrm{i}\right)\right]\left[z-\left(2-\mathrm{i}\right)\right]=\left[\left(z-2\right)-\mathrm{i}\right]\left[\left(z-2\right)+\mathrm{i}\right]$ $=\left(z-2\right)^2-\left(-1\right)$ $=z^2-4z+5$ $2z^4-13z^3+27z^2-13z-15=0$ $\left(z^2-4z+5\right)\left(2z^2-5z-3\right)=0$ $\left[z-\left(2+\mathrm{i}\right)\right]\left[z-\left(2+\mathrm{i}\right)\right]\left(2z+1\right)\left(z-3\right)=0$ Hence the other roots are $2-\mathrm{i},\ -\frac{1}{2}$ and 3.	
(b) **[2]**	Replace z with $-\mathrm{i}w$ (or Replace w with $\mathrm{i}z$), $2z^4+az^3+bz^2+az-15=0$ $2\left(-\mathrm{i}w\right)^4+a\left(-\mathrm{i}w\right)^3+b\left(-\mathrm{i}w\right)^2+a\left(-\mathrm{i}w\right)-15=0$ $2w^4+a\mathrm{i}w^3-bw^2-a\mathrm{i}w-15=0$ $\therefore\ -\mathrm{i}w=\ 2+\mathrm{i}\ ,\ 2-\mathrm{i}\ ,\ -\frac{1}{2}\ ,\ 3$ $w\ =\ \frac{2+\mathrm{i}}{-\mathrm{i}}\ ,\ \frac{2-\mathrm{i}}{-\mathrm{i}}\ ,\ \frac{1}{2\mathrm{i}}\ ,\ \frac{3}{-\mathrm{i}}$ Hence $w=-1+2\mathrm{i}\ ,\ 1+2\mathrm{i}\ ,\ -\frac{1}{2}\mathrm{i}\ ,\ 3\mathrm{i}$	

5 The curve C has equation $x^2+3xy-y^2+4=0$.

(a) Find the exact x-coordinates of the stationary points of C. [4]

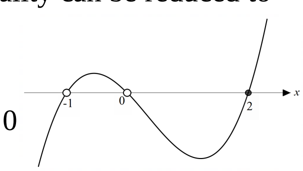
(b) For the stationary point with $x<0$, determine whether it is a maximum or minimum. [4]

(a) **[4]**	$x^2+3xy-y^2+4=0$ --- (1) Differentiate with respect to x: $$2x+3x\frac{\mathrm{d}y}{\mathrm{d}x}+3y-2y\frac{\mathrm{d}y}{\mathrm{d}x}=0 \text{ --- (2)}$$ For stationary points, let $\frac{\mathrm{d}y}{\mathrm{d}x}=0$. $$2x+3y=0 \Rightarrow y=-\frac{2}{3}x$$ Sub $y=-\frac{2}{3}x$ into (1): $$x^2+3x\left(-\frac{2x}{3}\right)-\left(-\frac{2x}{3}\right)^2+4=0$$ $$-\frac{13}{9}x^2+4=0$$ $$x=\pm\frac{6}{\sqrt{13}}$$	This part is well done. **Similar Qn** • Tut 6B Sect B Qn 2
(b) **[4]**	Differentiate (2) with respect to x. $$2+3x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+3\frac{\mathrm{d}y}{\mathrm{d}x}+3\frac{\mathrm{d}y}{\mathrm{d}x}-2y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=0 \quad \cdots(*)$$ When $x=-\frac{6}{\sqrt{13}}$, we get • $y=\frac{4}{\sqrt{13}}$ from $y=-\frac{2}{3}x$, • $\frac{\mathrm{d}y}{\mathrm{d}x}=0$. Sub into $(*)$, $$2+3\left(-\frac{6}{\sqrt{13}}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2\left(\frac{4}{\sqrt{13}}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$$ $$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{\sqrt{13}}>0$$ Thus, $x=-\frac{6}{\sqrt{13}}$ gives the minimum point.	A significant number of scripts use $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2x+3y}{2y-3x}$ and then differentiate again using quotient rule or product rule, which makes the expression very complicated. Many careless mistakes are made. The skill of doing implicit differentiation is important and makes the working much neater.

6 **(a)** Express x^3-8 in the form $(x-2)(ax^2+bx+c)$, where a, b and c are constants to be determined. [1]

(b) Without using a calculator, solve the inequality $\dfrac{x^2}{x+1} \ge \dfrac{8}{x(x+1)}$. [4]

(c) Hence, solve $\dfrac{(q+2)^2}{\lvert q+2\rvert+1} \ge \dfrac{8}{\lvert q+2\rvert(\lvert q+2\rvert+1)}$ [3]

(a) **[1]**	$x^3-8=(x-2)(ax^2+bx+c)$ By observation, $a=1$, $c=4$. Compare coefficient of x: $0=4-2b$ $b=2$ $\therefore x^3-8=(x-2)(x^2+2x+4)$ $a=1,\ b=2,\ c=4$	According to the O-level Math syllabus, students should know the result $a^3-b^3=(a-b)(a^2+ab+b^2)$ and able to just write down the answer for this part.
(b) **[4]**	$\dfrac{x^2}{x+1} \ge \dfrac{8}{x(x+1)}$ $\dfrac{x^2}{x+1}-\dfrac{8}{x(x+1)} \ge 0$ $\dfrac{x^3-8}{x(x+1)} \ge 0$ $\dfrac{(x-2)(x^2+2x+4)}{x(x+1)} \ge 0$ $\dfrac{(x-2)\left[(x+1)^2+3\right]}{x(x+1)} \ge 0$ But $x^2+2x+4=(x+1)^2+3 \ge 3>0$ for all real values of x, hence the inequality can be reduced to $\dfrac{x-2}{x(x+1)} \ge 0$  $x(x+1)(x-2) \ge 0,\ x \ne -1,\ 0$ $-1<x<0$ or $x \ge 2$	**Similar Qn** • Tut 3 Sect A Qn 2(b), Sect B Qn 2
(c) **[3]**	$\dfrac{(q+2)^2}{\lvert q+2\rvert+1} \ge \dfrac{8}{\lvert q+2\rvert(\lvert q+2\rvert+1)}$ $\dfrac{\lvert q+2\rvert^2}{\lvert q+2\rvert+1} \ge \dfrac{8}{\lvert q+2\rvert(\lvert q+2\rvert+1)}$ Replace x by $\lvert q+2\rvert$: $-1<\lvert q+2\rvert<0$ or $\lvert q+2\rvert \ge 2$ No solution $\quad q+2 \le -2$ or $q+2 \ge 2$ $q \le -4$ or $q \ge 0$ The solution is $q \le -4$ or $q \ge 0$	

7 A particle moves on the Cartesian plane in such a way that at time t, its position is given by the parametric equations $x = t\cos t,\ y = t\sin t$. The particle starts moving from the origin.

(a) Find the equation of the normal to the path of the particle at $t = \frac{\pi}{2}$, leaving your answer in exact form. [5]

(b) It is given that the instantaneous speed of the particle at time t is defined as $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$.

(i) Find an expression for $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ in terms of t, giving your answer in its simplest form. [2]

(ii) Find the exact instantaneous speed of the particle when it crosses the positive x-axis for the first time. [2]

(iii) Explain if the particle will ever come to rest. [1]

(a) **[5]**	$\frac{dx}{dt} = -t\sin t + \cos t,\ \frac{dy}{dt} = t\cos t + \sin t$ So, $\frac{dy}{dx} = \frac{t\cos t + \sin t}{-t\sin t + \cos t}$ The coordinates of particle at $t = \frac{\pi}{2}$ is $\left(0, \frac{\pi}{2}\right)$. Gradient of the normal is $-\left.\frac{dx}{dy}\right\|_{t=\frac{\pi}{2}} = -\frac{-\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2}}{\frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}} = \frac{\pi}{2}$ Equation of normal is $y - \frac{\pi}{2} = \frac{\pi}{2}(x - 0) \Rightarrow y = \frac{\pi}{2}x + \frac{\pi}{2}$.	This part is well done
(b) **(i)** **[2]**	$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ $= \sqrt{(-t\sin t + \cos t)^2 + (t\cos t + \sin t)^2}$ $= \sqrt{t^2\sin^2 t - 2t\sin t\cos t + \cos^2 t + t^2\cos^2 t + 2t\sin t\cos t + \sin^2 t}$ $= \sqrt{t^2\sin^2 t + t^2\cos^2 t + \sin^2 t + \cos^2 t} = \sqrt{t^2(\sin^2 t + \cos^2 t) + (\sin^2 t + \cos^2 t)}$ $= \sqrt{t^2 + 1}$	This part is well done
(ii) **[2]**	When it crosses the x-axis, $y = 0 \Rightarrow t\sin t = 0$ $t = 0, \pi, 2\pi, 3\pi, \ldots$ When $t = 0$, $x = 0$; When $t = \pi$, $x = -\pi < 0$ When $t = 2\pi$, $x = 2\pi > 0$ The curve first crosses the positive x-axis at $t = 2\pi$. When $t = 2\pi$, speed is $\sqrt{4\pi^2 + 1}$.	A significant number of students did not check the required condition that the particle is at **positive** x-axis.
(iii) **[1]**	Speed at time t is $\sqrt{t^2 + 1} \geq 1 > 0$ for all t. Hence it will never come to rest.	

8 A home interior designer is looking to design a pull-down wardrobe lift. To test his concept, the designer built a small-scale model of it, in which points (x, y, z) are defined relative to a corner of the model at $O(0, 0, 0)$. The rods used in the model can be modelled by equations of straight lines.

The main rod can be modelled by the line l_1 with equation $\mathbf{r} = \begin{pmatrix} 5 \\ -5 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \lambda \in \mathbb{R}$.

Another rod, line l_2, passes through the points $A(5, -5, 8)$ and $B(8, -4, 3)$.

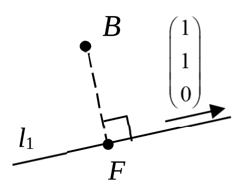
(a) A supporting wire connects B to a point F on l_1 such that $\overrightarrow{BF}$ is perpendicular to l_1. Find the position vector of F. [3]

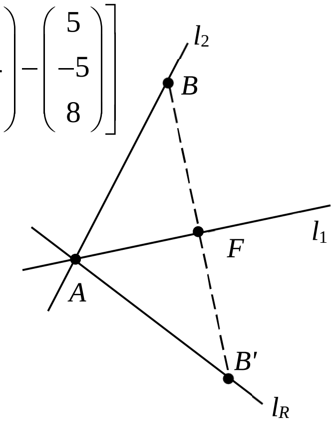
(b) A retractable rod, line l_R, is placed such that it can be modelled by reflecting l_2 about l_1. Find a vector equation of l_R. [3]

The panels of the small-scale model can be modelled by equations of planes. The main panel, plane p_1, contains the main rod l_1 and an attachment point $C(0, 0, 6)$.

(c) Show that p_1 can be represented by the cartesian equation $-x + y + 5z = 30$. [3]

(d) A decorative plank, plane p_2, has equation $p_2 : -3x + \alpha y + 15z = \beta$, where $\alpha, \beta \in \mathbb{R}$. If the distance between p_1 and p_2 is exactly $3\sqrt{3}$ units, find the possible values of α and β. [3]

(a) **[3]**	$\overrightarrow{OA} = \begin{pmatrix} 5 \\ -5 \\ 8 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 8 \\ -4 \\ 3 \end{pmatrix}$  Since F lies on l_1, $\overrightarrow{OF} = \begin{pmatrix} 5 \\ -5 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ for some $\lambda \in \mathbb{R}$ $\overrightarrow{BF} = \begin{pmatrix} 5 \\ -5 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 8 \\ -4 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ To find point F, $\left[\begin{pmatrix} -3 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right] \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0$ $-4 + 2\lambda = 0$ $\lambda = 2$ $\overrightarrow{OF} = \begin{pmatrix} 5 \\ -5 \\ 8 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \\ 8 \end{pmatrix}$	This part is well done. **Similar Questions** • Tut 2B Sect B Qn 1(b)(i), 2(ii), 3(ii)

(b) **[3]**	**Method 1: Ratio theorem (w.r.t. point *A*)** $\overrightarrow{AF}=\frac{\overrightarrow{AB}+\overrightarrow{AB'}}{2}$ $\overrightarrow{AB'}=2\overrightarrow{AF}-\overrightarrow{AB}$ $=2\left[\begin{pmatrix}7\\-3\\8\end{pmatrix}-\begin{pmatrix}5\\-5\\8\end{pmatrix}\right]-\left[\begin{pmatrix}8\\-4\\3\end{pmatrix}-\begin{pmatrix}5\\-5\\8\end{pmatrix}\right]$ $=2\begin{pmatrix}2\\2\\0\end{pmatrix}-\begin{pmatrix}3\\1\\-5\end{pmatrix}=\begin{pmatrix}1\\3\\5\end{pmatrix}$  $l_R:\mathbf{r}=\begin{pmatrix}5\\-5\\8\end{pmatrix}+\mu\begin{pmatrix}1\\3\\5\end{pmatrix},\ \mu\in\mathbb{R}$ **Method 2: Ratio theorem (w.r.t. point *O*)** $\overrightarrow{OF}=\frac{\overrightarrow{OB}+\overrightarrow{OB'}}{2}$ $\overrightarrow{OB'}=2\overrightarrow{OF}-\overrightarrow{OB}=2\begin{pmatrix}7\\-3\\8\end{pmatrix}-\begin{pmatrix}8\\-4\\3\end{pmatrix}=\begin{pmatrix}6\\-2\\13\end{pmatrix}$ $\overrightarrow{AB'}=\begin{pmatrix}6\\-2\\13\end{pmatrix}-\begin{pmatrix}5\\-5\\8\end{pmatrix}=\begin{pmatrix}1\\3\\5\end{pmatrix}$ $l_R:\mathbf{r}=\begin{pmatrix}5\\-5\\8\end{pmatrix}+\mu\begin{pmatrix}1\\3\\5\end{pmatrix},\ \mu\in\mathbb{R}$	A few students did not visualise what is given and what needs to be found, and did not use the appropriate method to find the answer. For such question, do visualise. **Similar Question** • Tut 2B Sect B Qn 6(iv)
(c) **[3]**	$\overrightarrow{CA}=\begin{pmatrix}5\\-5\\8\end{pmatrix}-\begin{pmatrix}0\\0\\6\end{pmatrix}=\begin{pmatrix}5\\-5\\2\end{pmatrix}$ or $\overrightarrow{CF}=\begin{pmatrix}7\\-3\\8\end{pmatrix}-\begin{pmatrix}0\\0\\6\end{pmatrix}=\begin{pmatrix}7\\-3\\2\end{pmatrix}$ $n_1=\begin{pmatrix}5\\-5\\2\end{pmatrix}\times\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}-2\\2\\10\end{pmatrix}=2\begin{pmatrix}-1\\1\\5\end{pmatrix}$ $\mathbf{r}\cdot\begin{pmatrix}-1\\1\\5\end{pmatrix}=\begin{pmatrix}0\\0\\6\end{pmatrix}\cdot\begin{pmatrix}-1\\1\\5\end{pmatrix}=30$ $p_1:-x+y+5z=30$ (Shown)	This part is well done

(d)
[3]

$$p_2: -3x + \alpha y + 15z = \beta \quad \Rightarrow \quad \mathbf{r}\cdot\begin{pmatrix}-3\\ \alpha\\ 15\end{pmatrix} = \beta$$

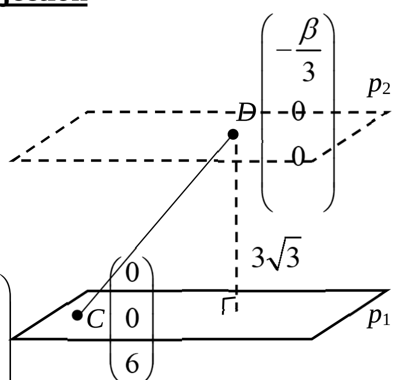
For non-zero distance between 2 planes, p_1 is parallel to p_2. So, they have the same normal vector.

$$\begin{pmatrix}-1\\ 1\\ 5\end{pmatrix} = k\begin{pmatrix}-3\\ \alpha\\ 15\end{pmatrix} \quad \Rightarrow k = \frac{1}{3},\ \alpha = 3$$

<u>**Method 1: Length of Projection**</u>

Let a point on p_2 be D.

Choose $\overrightarrow{OD} = \begin{pmatrix}-\frac{\beta}{3}\\ 0\\ 0\end{pmatrix}$



$$\overrightarrow{CD} = \begin{pmatrix}-\frac{\beta}{3}\\ 0\\ 0\end{pmatrix} - \begin{pmatrix}0\\ 0\\ 6\end{pmatrix} = \begin{pmatrix}-\frac{\beta}{3}\\ 0\\ -6\end{pmatrix}$$

The distance between the 2 planes

=Length of projection of $\overrightarrow{CD}$ onto the normal of the plane

$$\left|\frac{\begin{pmatrix}-\frac{\beta}{3}\\ 0\\ -6\end{pmatrix}\cdot\begin{pmatrix}-1\\ 1\\ 5\end{pmatrix}}{\left|\begin{pmatrix}-1\\ 1\\ 5\end{pmatrix}\right|}\right| = 3\sqrt{3}$$

$$\left|\frac{\beta}{3} - 30\right| = 3\sqrt{3}\left(\sqrt{27}\right)$$

$$\frac{\beta}{3} - 30 = \pm 27$$

$\frac{\beta}{3} = 57$ or $\frac{\beta}{3} = 3$

$\beta = 171$ or $\beta = 9$

	<u>Method 2: Distance between 2 planes</u> $p_2 : \mathbf{r} \cdot \begin{pmatrix} -3 \\ 3 \\ 15 \end{pmatrix} = \beta \Rightarrow \quad p_2 : \mathbf{r} \cdot \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix} = \frac{\beta}{3}$ Since distance between p_1 and p_2 is exactly $3\sqrt{3}$ units, $\left\| \frac{30 - \frac{\beta}{3}}{\sqrt{(-1)^2 + 1^2 + 5^2}} \right\| = 3\sqrt{3}$ $\left\| 30 - \frac{\beta}{3} \right\| = 27$ $\frac{\beta}{3} = 30 \pm 27$ $\therefore \beta = 9 \text{ or } \beta = 171$	Method 2 uses the idea in Example 5 of Chap 2C Lecture Notes (see page 8). In order to use this method, one need to ensure that the normal vector of both planes are the **same**, and <u>not</u> a scalar multiple of each other. So, quite a significant wrote $\left\| \frac{30 - \beta}{\sqrt{(-1)^2 + 1^2 + 5^2}} \right\|$ and not able to get the correct answer. **Similar Qn** • Timed Prac Topical Revision Exercise on Vectors Qn 4(iv)