1		Suggested Solution
(a)	(i)	$\nabla(\mathrm{f}+\mathrm{g})=\frac{\partial}{\partial x}(\mathrm{f}+\mathrm{g})\mathbf{i}+\frac{\partial}{\partial y}(\mathrm{f}+\mathrm{g})\mathbf{j}$ $=\left(\frac{\partial \mathrm{f}}{\partial x}+\frac{\partial \mathrm{g}}{\partial x}\right)\mathbf{i}+\left(\frac{\partial \mathrm{f}}{\partial y}+\frac{\partial \mathrm{g}}{\partial y}\right)\mathbf{j}$ $=\left(\frac{\partial \mathrm{f}}{\partial x}\mathbf{i}+\frac{\partial \mathrm{f}}{\partial y}\mathbf{j}\right)+\left(\frac{\partial \mathrm{g}}{\partial x}\mathbf{i}+\frac{\partial \mathrm{g}}{\partial y}\mathbf{j}\right)$ $=\nabla \mathrm{f}+\nabla \mathrm{g}$
	(ii)	$\nabla(\mathrm{f}\,\mathrm{g})=\frac{\partial}{\partial x}(\mathrm{f}\,\mathrm{g})\mathbf{i}+\frac{\partial}{\partial y}(\mathrm{f}\,\mathrm{g})\mathbf{j}$ $=\left(\mathrm{f}\frac{\partial \mathrm{g}}{\partial x}+\mathrm{g}\frac{\partial \mathrm{f}}{\partial x}\right)\mathbf{i}+\left(\mathrm{f}\frac{\partial \mathrm{g}}{\partial y}+\mathrm{g}\frac{\partial \mathrm{f}}{\partial y}\right)\mathbf{j}$ $=\mathrm{f}\left(\frac{\partial \mathrm{g}}{\partial x}\mathbf{i}+\frac{\partial \mathrm{g}}{\partial y}\mathbf{j}\right)+\mathrm{g}\left(\frac{\partial \mathrm{f}}{\partial x}\mathbf{i}+\frac{\partial \mathrm{f}}{\partial y}\mathbf{j}\right)$ $=\mathrm{f}\,\nabla \mathrm{g}+\mathrm{g}\,\nabla \mathrm{f}$
(b)	(i)	$H=0\Rightarrow 100-\left(x^2+y^2\right)=0$ $\Rightarrow x^2+y^2=100$ The set of points of S which lie on the x-y plane $=\left\{(x,y)\in\mathbb{R}^2 : x^2+y^2=100\right\}$ which is the set of points on the circle with centre $(0,0)$ and radius 10.
	(ii)	The highest point of S is $(0,0,100)$. The directional derivative of H at the highest point of S in the direction of any unit vector $\mathbf{u}$ is $D_{\mathbf{u}}H(0,0)=\nabla H(x,y)\bullet\mathbf{u}$ $=\left(\frac{\partial H(0,0)}{\partial x}\mathbf{i}+\frac{\partial H(0,0)}{\partial y}\mathbf{j}\right)\bullet\mathbf{u}$ $=\left(-2(0)\mathbf{i}-2(0)\mathbf{j}\right)\bullet\mathbf{u}$ $=0$

2	**Suggested Answers**
(a)	$x = \frac{\mathrm{d}y}{\mathrm{d}t}$ Differentiating with respect to t, $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}^2 y}{\mathrm{d}t^2}$ Using chain rule, $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}y}\frac{\mathrm{d}y}{\mathrm{d}t} = x\frac{\mathrm{d}x}{\mathrm{d}y}$ $x\frac{\mathrm{d}x}{\mathrm{d}y} = -ax^2 + by$.
(b)	$u = x^2$ $\frac{\mathrm{d}u}{\mathrm{d}y} = 2x\frac{\mathrm{d}x}{\mathrm{d}y}$ $\frac{1}{2}\frac{\mathrm{d}u}{\mathrm{d}y} = -au + by$ $\frac{\mathrm{d}u}{\mathrm{d}y} + 2au = 2by$ Integrating Factor is $\mathrm{e}^{\int 2a\mathrm{d}y} = \mathrm{e}^{2ay}$ Multiplying throughout by e^{2ay}, $\mathrm{e}^{2ay}\frac{\mathrm{d}u}{\mathrm{d}y} + 2a\mathrm{e}^{2ay}u = 2by\mathrm{e}^{2ay}$ $\frac{\mathrm{d}}{\mathrm{d}y}\left(u\mathrm{e}^{2ay}\right) = 2by\mathrm{e}^{2ay}$ Integrating throughout with respect to y, $u\mathrm{e}^{2ay} = \frac{b}{a}y\mathrm{e}^{2ay} - \int \frac{b}{a}\mathrm{e}^{2ay}\mathrm{d}y$ $= \frac{b}{a}y\mathrm{e}^{2ay} - \frac{b}{2a^2}\mathrm{e}^{2ay} + C$ $u = \frac{b}{a}y - \frac{b}{2a^2} + C\mathrm{e}^{-2ay}$
(c)	$x^2 = 25y - 625 + C\mathrm{e}^{-0.04y}$ $\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = 25y - 625 + C\mathrm{e}^{-0.04y}$ $0 = 2500 - 625 + C\mathrm{e}^{-4}$ $C = -1875\mathrm{e}^4$ $\frac{\mathrm{d}y}{\mathrm{d}t} = \sqrt{25y - 625 - 1875\mathrm{e}^4\mathrm{e}^{-0.04y}}$

3	
(a)	We have $\omega^3=1, \omega\neq 1$. Then $\omega^3=1\Rightarrow \omega^4=\omega, \omega^5=\omega^2$. $1+\omega+\omega^2=\dfrac{1-\omega^3}{1-\omega}=0$ since $\omega^3=1,\omega\neq 1$ $(2-\omega)(2-\omega^2)(2-\omega^4)(2-\omega^5)$ $=(2-\omega)(2-\omega^2)(2-\omega)(2-\omega^2)$ $=\left[(2-\omega)(2-\omega^2)\right]^2$ $=\left[4-2(\omega+\omega^2)+\omega^3\right]^2$ $=\left[4-2(-1)+1\right]^2$ $=49$
(b)	$(r-c^*)z+(r-c)z^*=r^2-\lvert c\rvert^2$ $\Rightarrow -c^*z-cz^*+\lvert c\rvert^2=-rz-rz^*+r^2$ $\Rightarrow zz^*-c^*z-cz^*+\lvert c\rvert^2=zz^*-rz-rz^*+r^2$ $\Rightarrow (z-c)(z^*-c^*)=(z-r)(z^*-r)$ $\Rightarrow \lvert z-c\rvert^2=\lvert z-r\rvert^2$ $\Rightarrow \lvert z-c\rvert=\lvert z-r\rvert$ which is the perpendicular bisector of the line segment joining the points representing the complex number c and the real number r.
	Alternative Solution: Write $z=x+y\mathrm{i}$ and let $c=a+b\mathrm{i}$. $r^2-\lvert c\rvert^2$ $=(r-c^*)z+(r-c)z^*$ $=\left[r-(a-b\mathrm{i})\right](x+y\mathrm{i})+\left[r-(a+b\mathrm{i})\right](x-y\mathrm{i})$ $=(r-a)x-by+(bx+(r-a)y)\mathrm{i}+(r-a)x-by-(bx+(r-a)y)\mathrm{i}$ $=2(r-a)x-2by$ which is the line with axial intercepts $\left(\dfrac{r^2-\lvert c\rvert^2}{2(r-a)},0\right),\left(0,\dfrac{\lvert c\rvert^2-r^2}{2b}\right)$ provided $r\neq a, b\neq 0$ OR a line with gradient $\frac{r-a}{b}$ and y-intercept $\dfrac{\lvert c\rvert^2-r^2}{2b}$.

Qn 4

(a) Area of sector OAB,

$$T=\int_0^{\alpha}\frac{1}{2}r^2\ \mathrm{d}\theta$$

$$=\int_0^{\alpha}2\mathrm{e}^{\frac{2\theta}{k}}\ \mathrm{d}\theta$$

$$=\left[k\mathrm{e}^{\frac{2\theta}{k}}\right]_0^{\alpha}$$

$$=k(\mathrm{e}^{\frac{2\alpha}{k}}-1)$$

$$r=2\mathrm{e}^{\frac{1}{k}\theta}\quad\Rightarrow\quad\frac{\mathrm{d}r}{\mathrm{d}\theta}=\frac{2}{k}\mathrm{e}^{\frac{1}{k}\theta}$$

Length of arc AB,

$$S=\int_0^{\alpha}\sqrt{r^2+\left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\ \mathrm{d}\theta$$

$$=\int_0^{\alpha}\sqrt{4\mathrm{e}^{\frac{2\theta}{k}}+\frac{4}{k^2}\mathrm{e}^{\frac{2\theta}{k}}}\ \mathrm{d}\theta$$

$$=\int_0^{\alpha}\sqrt{\frac{4}{k^2}\mathrm{e}^{\frac{2\theta}{k}}(k^2+1)}\ \mathrm{d}\theta$$

$$=2(k^2+1)^{\frac{1}{2}}\int_0^{\alpha}\frac{1}{k}\mathrm{e}^{\frac{\theta}{k}}\ \mathrm{d}\theta$$

$$=2(k^2+1)^{\frac{1}{2}}\left[\mathrm{e}^{\frac{\theta}{k}}\right]_0^{\alpha}\quad=2(k^2+1)^{\frac{1}{2}}(\mathrm{e}^{\frac{\alpha}{k}}-1)$$

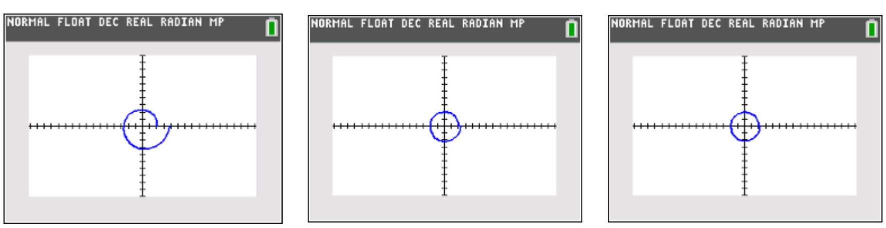
(b) Consider $\dfrac{T}{S}$:

$$\frac{T}{S}=\frac{k(\mathrm{e}^{\frac{2\alpha}{k}}-1)}{2(k^2+1)^{\frac{1}{2}}(\mathrm{e}^{\frac{\alpha}{k}}-1)}=\frac{k}{\sqrt{k^2+1}}(\mathrm{e}^{\frac{\alpha}{k}}+1)$$

As $k\to\infty$, $\dfrac{k}{\sqrt{k^2+1}}\to 1$ and $\mathrm{e}^{\alpha/k}\to 1$, and so $\dfrac{T}{S}\to 1$

$\therefore\quad T\approx S$

(c) As k increases, the curve approaches a circle centred at O with radius 2.



$$\therefore\quad\frac{T}{S}\approx\frac{\pi(2)^2}{2\pi(2)}=1\quad\Rightarrow\quad T\approx S\ \text{(same as above)}$$

5	
(a)	$\mathrm{f}(x,y)=x^2+y^2-4x+(ax-y)^2$ $\mathrm{f}_x=2x-4+2a(ax-y)=(2+2a^2)x-2ay-4$ $\mathrm{f}_y=2y-2(ax-y)=4y-2ax$ $\mathrm{f}_{xx}=2+2a^2$ $\mathrm{f}_{yy}=4$ $\mathrm{f}_{xy}=-2a$
(b)	$\mathrm{f}_x=0\Rightarrow 2x-4+2a(ax-y)=0$ --- (1) $\mathrm{f}_y=0\Rightarrow 2y-2(ax-y)=0$ --- (2) From (2): $y=\frac{1}{2}ax$ Sub $y=\frac{1}{2}ax$ into (1): $2x-4+2a(ax-y)=0$ $2x-4+2a\left(ax-\frac{1}{2}ax\right)=0$ $x-2+a\left(\frac{1}{2}ax\right)=0$ $\left(\frac{1}{2}a^2+1\right)x=2$ $x=\frac{4}{a^2+2}$ $y=\frac{1}{2}a\left(\frac{4}{a^2+2}\right)=\frac{2a}{a^2+2}$ Since $a\in\mathbb{R}$, there is unique solution for x and y value. Hence, surface S has exactly one stationary point. $D=\mathrm{f}_{xx}\mathrm{f}_{yy}-(\mathrm{f}_{xy})^2$ $=(2+2a^2)(4)-(-2a)^2$ $=8+8a^2-4a^2$ $=4a^2+8>0$ for all $a\in\mathbb{R}$. And $\mathrm{f}_{xx}=2+2a^2>0$ for all $a\in\mathbb{R}$. The stationary point is minimum point.
(c)	$\mathrm{f}(x,y)\approx \mathrm{f}(0,1)+\mathrm{f}_x(0,1)(x-0)+\mathrm{f}_y(0,1)(y-1)$ $\mathrm{f}_x(0,1)=2(0)-4+2a(a(0)-1)=-4-2a$ $\mathrm{f}_y(0,1)=2(1)-2(a(0)-1)=4$ $\mathrm{f}(x,y)\approx 2+(-4-2a)(x-0)+4(y-1)$ $2+2h\approx 2+(-4-2a)(k-0)+4(h+1-1)$ $2+2h\approx 2+(-4-2a)k+4h$

	$2h \approx (4+2a)k$ $h \approx (2+a)k$
(d)(i)	At point P, normal vector of the tangent plane is $\begin{pmatrix} -4-2a \\ 4 \\ -1 \end{pmatrix}$. The tangent line at point P which is parallel to $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ also lies in the tangent plane, we have $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -4-2a \\ 4 \\ -1 \end{pmatrix} = 0$ $-4-2a+8-1=0$ $2a=3$ $a=\frac{3}{2}$
(d)(ii)	Since P lies on the plane $\alpha x+\beta y+\gamma z=0$, $\alpha(0)+\beta(1)+\gamma(2)=0$ $\beta=-2\gamma$ --- (1) Since the tangent line also lies in the plane $\alpha x+\beta y+\gamma z=0$, we have $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \cdot = 0$ $\alpha+2\beta+\gamma=0$ --- (2) Substitute (1) into (2), $\alpha+2(-2\gamma)+\gamma=0$ $\alpha=3\gamma$ $(3\gamma)x+(-2\gamma)y+\gamma z=0$ Since $\gamma \neq 0$, the equation of the plane is $3x-2y+z=0$ **Alternative Solution:** Clearly, O lies on the plane $\alpha x+\beta y+\gamma z=0$. Since $P(0,1,2)$ also lies on the plane, $\overrightarrow{OP}=\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ is a direction vector of the plane. Hence $\mathbf{n}=\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$. So cartesian equation of the plane is $3x-2y+z=0$.

6	**Suggested Solution**
(a)	H_0 : there is no association between the students' year of study and their Primary study approach H_1 : there is association between the students' year of study and their Primary study approach Level of significance: 5% Assumption: Assume that the 100 students surveyed constitute a random sample. Test statistic: $\chi^2 = \sum_i \sum_j \frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$ Computation: Under H_0, expected frequencies, $E_{ij} = \frac{r_i \times c_j}{N}$ (N = grand total) are shown below:

Primary study approach	Year 1	Year 3	Total
Study Alone	20.25	24.75	45
Group Study	13.5	16.5	30
Online Resources	11.25	13.75	25
Total	45	55	100

Calculation of contributions to test statistic:

Primary study approach	Year 1	Year 3
Study Alone	1.361	1.113
Group Study	3.130	2.561
Online Resources	0.1389	0.1136

$$\therefore \chi^2 = 8.4175$$

Degrees of freedom: $v = (2-1)(3-1) = 2$

$$p-\text{value} = \mathrm{P}\left(\chi^2 \geq 8.4175\right) = 0.01486$$

Conclusion: Since $p-\text{value} = 0.01486 > 0.01$ $\therefore H_0$ is not rejected at the 1% significance level. Hence there is insufficient evidence to conclude that there is association between the students' year of study and their primary study approach

(b)

The two cells that contribute the most to the chi-square test statistics are:

- Group Study, Year 1
- Group Study, Year 3

These high contributions suggest that more Year 1 students chose Group Study than expected, while fewer Year 3 students did so.

A possible explanation is that Year 1 students may prefer collaborative learning environments to adjust to university life, while Year 3 students, being more experienced and focused on final-year projects, may prefer studying alone or using online resources.

No, this does not contradict the conclusion in part (a). While these cells contribute significantly to the chi-square statistic, the overall test did not provide enough evidence to conclude an association at the 1% significance level. The high contributions highlight specific deviations but do not change the overall result

7	**Suggested Solution**

(a)	Possible conditions are: The average rate of reports being made is constant throughout any 1-hour time interval. Or Each report is independent of any other report. Reason why the condition may not hold: During peak period, there is a higher chance of report being made during the flight arrival period and hence rate of report being made is not constant throughout any time interval.
(b)	$Y \sim P_0(4.5)$ $\mathrm{P}(Y \geq 1) = 1 - \mathrm{P}(Y = 0) = 0.989(3\text{s.f})$
(c)	Let W denotes random variable the number of calls in a t-hour period. $W \sim \mathrm{Po}(4.5t)$ $\mathrm{P}(W \geq 1) = 1 - \mathrm{P}(W = 0) = 1 - \mathrm{e}^{-4.5t} \dfrac{(4.5t)^0}{0!}$ $= 1 - \mathrm{e}^{-4.5t}$ (shown)
(d)	$\mathrm{F}(t) = \mathrm{P}(T \leq t) = \mathrm{P}(W \geq 1)$ $= 1 - \mathrm{e}^{-4.5t}$ $\mathrm{f}(t) = \dfrac{\mathrm{d}}{\mathrm{d}t}\mathrm{F}(t) = \dfrac{\mathrm{d}}{\mathrm{d}t}\left(1 - \mathrm{e}^{-4.5t}\right) = 4.5\mathrm{e}^{-4.5t}$ Therefore, T has an exponential distribution with parameter 4.5
(e)	$\mathrm{P}\left(T > \dfrac{15}{60} \mid T > \dfrac{10}{60}\right) = \mathrm{P}\left(T > \dfrac{5}{60}\right)$ ('lack of memory' property) $= \int_{\frac{1}{12}}^{\infty} 4.5\mathrm{e}^{-4.5t}\,\mathrm{d}t$ $= \left[-\mathrm{e}^{-4.5t}\right]_{\frac{1}{12}}^{\infty}$ $= \mathrm{e}^{-4.5\left(\frac{1}{12}\right)}$ $= 0.687(3\text{s.f})$

Qn 8	Suggested Solution
(a)	$\int_k^{\infty} \frac{k}{(x-1)^3}\,\mathrm{d}x = 1$ $\left[\frac{-k}{2(x-1)^2}\right]_k^{\infty} = 1$ $2k^2 - 5k + 2 = 0$ $k = 2$ or $k = \frac{1}{2}$ (reject) $\mathrm{E}(X-1) = \int_2^{\infty} \frac{2(x-1)}{(x-1)^3}\,\mathrm{d}x$ $\mathrm{E}(X) = 1 + 2\left[-\frac{1}{x-1}\right]_2^{\infty} = 3$
(b)	For $x \geq 2, \mathrm{F}(x) = \mathrm{P}(X \leq x)$ $= \int_2^{x} \frac{2}{(t-1)^3}\,\mathrm{d}t$ $= \left[\frac{2}{-2(t-1)^2}\right]_2^{x}$ $= 1 - \frac{1}{(x-1)^2}$ $\therefore \mathrm{F}(x) = \begin{cases} 0 & x < 2 \\ 1 - \frac{1}{(x-1)^2} & x \geq 2 \end{cases}$ $W = \max(X_1, X_2, X_3)$ $\mathrm{G}(w) = \mathrm{P}(W < w) = \mathrm{P}(X_1 \leq w, X_2 \leq w, X_3 \leq w)$ $= \mathrm{P}(X_1 < w)\mathrm{P}(X_2 < w)\mathrm{P}(X_3 < w)$ $= \left[1 - \frac{1}{(w-1)^2}\right]^3$ $G(w) = \begin{cases} 0 & w < 2 \\ \left(1 - \frac{1}{(w-1)^2}\right)^3 & w \geq 2 \end{cases}$ $\mathrm{P}(W > 4) = 1 - G(4)$ $= 1 - \left(1 - \frac{1}{3^2}\right)^3 = \frac{217}{729}$

(c)	$S \sim \text{Geo}\left(\frac{217}{729}\right)$ $\text{E}(S) = \frac{729}{217}$ $\text{P}(S > \text{E}(S)) = \text{P}(S > \frac{729}{217}) = P\left(S > 3.36\right) = P\left(S > 3\right)$ $= \left(1 - \frac{217}{729}\right)^3 = 0.346$

9	
(a)	Sign Test

Let X be the potassium level in the soil of a randomly chosen farm

Let m be the population median potassium level a randomly chosen farm

$\mathrm{H}_0 : m = 250$ vs $\mathrm{H}_1 : m < 250$

Level of significance: 5 %

Farm	1	2	3	4	5	6	7	8	9	10
Potassium Level (mg/kg)	180	220	250	190	210	271	89	242	199	262
$d = x - 250$	-70	-30	0	-60	-40	21	-161	-8	-51	12
Sign	$-$	$-$	0	$-$	$-$	$+$	$-$	$-$	$-$	$+$

Test statistic:

Let S_+ be the number of farms out of 9 with soil sample having a potassium level more than 250

From the table, $s_- = 8 - 1 = 7$, $s_+ = 2$.

Under H_0, $S_+ \sim \mathrm{B}(9,\ 0.5)$

$p - \text{value} = \mathrm{P}(S_+ \le s_+) = \mathrm{P}(S_+ \le 2) = 0.0898$ (3sf)

Since $p\text{-value} = 0.0898 > 0.05$ we do not reject H_0 and conclude that there is insufficient evidence, at 5% level of significance that the potassium level is below the recommended threshold of 250 mg/kg.

Test Statistic:
Let P and Q be the sum of the ranks corresponding to the positive and negative differences respectively.

Let T be the smaller of P and Q where T is the test statistic
To reject H_0, $T \le 8$ (from MF 27)

We rank all the Farms except 3 and 7, Let the rank of the correct potassium level of Farm 7 be r.

Farm	1	2	3	4	5	6	7	8	9	10
Potassium Level (mg/kg)	180	220	250	190	210	271	x	242	199	262
$d = x - 250$	-70	-30	0	-60	-40	21	$x - 250$	-8	-51	12
Sign	$-$	$-$	0	$-$	$-$	$+$	$+$	$-$	$-$	$+$
Rank of $\lvert d \rvert$	8	4		7	5	3	r	1	6	2

There are only 3 positive ranks in total (Farm 6,7,10)
There are 6 negative ranks in total
Since we have so many negative ranks, $P = T \le 8$

Case 1: $r = 1$
If $r = 1$, then since we cannot have tied rank, all ranks must shift down by 1.
Therefore $P = 1 + 3 + 4 = 8$ (Farm 7 + Farm 10 + Farm 6)

Case 2: $r = 2$
If $r = 2$, then since we cannot have tied rank, all ranks except rank 1 must shift down by 1. Therefore $P = 2 + 3 + 4 = 9 > 8$

	Therefore Farm 7 can only be positively rank 1 so that $P = T \leq 8$. Which means the difference must be less than 8 Let w be the correct potassium level of Farm 7, Therefore $250 < w < 258$
(c)	Mostly likely the conclusion made by Researcher A is more reliable because we do not know whether the underlying distribution is normal.

10	**Solutions**
(a)	We need to assume that the battery life of smartphones follows a normal distribution.
(b)	Let X denote the battery life (in hours) of a randomly chosen smartphone. From GC, $\bar{x} = 10.475$, $s^2 = 0.52304^2$ $\dfrac{\bar{X}}{S/\sqrt{n}} \sim t(7)$ $\bar{x} + t\dfrac{s}{\sqrt{n}} = 11.029$ $t = 2.9958$ $q = \mathrm{P}\left(-2.9958 < \dfrac{\bar{X}}{S/\sqrt{n}} < 2.9958\right)\times 100$ $= 97.993$ $= 98$ (nearest integer)
(c)	Since 11.1 is not in the 98% confidence interval (9.9206, 11.029), there is sufficient evidence to conclude that Aphone's battery life was lower than 11.1 hours at 1% significance level. Since there is already significant evidence at 1% significance level, there will, too, be significant evidence to conclude the same at 5% significant level.
(d)	Let $D =$ Version 3 battery life $-$ Version 2 battery life Let μ_D be population mean of D Test at 5% significance level $\mathrm{H}_0 : \mu_D = 0$ $\mathrm{H}_1 : \mu_D > 0$ Assume that D follows a normal distribution. Under H_0, $T = \dfrac{\bar{D}}{\sqrt{S_D^2/8}} \sim t(7)$ From GC, p-value $= 0.095489$ Since p-value > 0.05, we do not reject H_0, and conclude that there is insufficient evidence at 5% significance level that the software update from Version 2 to Version 3 improved Aphone's battery life.
(e)	Let X_2 and X_3 be battery life of Aphones with Version 2 and Version 3 software respectively, and μ_2 and μ_3 be their corresponding population mean battery life.

	$H_0 : \mu_3 - \mu_2 = 0$ $H_1 : \mu_3 - \mu_2 > 0$ Unbiased estimate of population variance: $s_2^2 = \frac{120}{119}(0.9^2)$ $s_3^2 = \frac{140}{139}(1^2)$ Under H_0, $Z = \frac{\overline{X_3} - \overline{X_2}}{\sqrt{\frac{s_2^2}{120} + \frac{s_3^2}{140}}} \sim N(0,1)$ approximately by Central Limit Theorem. $z\text{-calc} = \frac{10.6 - 10.2}{\sqrt{\frac{s_2^2}{120} + \frac{s_3^2}{140}}} = 3.3805$ $p\text{-value} = 0.000361$ Since $p\text{-value} < 0.05$, we reject H_0 and conclude that there is sufficient evidence at 5% significance level, that the population mean battery life had improved from Version 2 to Version 3.
(f)	In part (d), the computed p-value of 0.095489 was already relatively small even when a small sample size was taken. Thus, the manufacturer should concur with the conclusion in part (e) that the population mean battery life had improved.