Hwa Chong Institution 华侨中学
Mathematics (SMTP)

Name: Wan Run Hao (27) Class: 3I1 Date: 16th Aug

MA304.2.2 – Trigonometric Ratios of Acute Angles

At the end of this unit, you should be able to

- **recall and use the three trigonometric ratios: sine, cosine and tangent and use** $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
- **find trigonometric ratios using the right-angle triangle**
- **derive and use trigonometric ratios of special angles**
- **simplify trigonometric ratios of complementary angles**

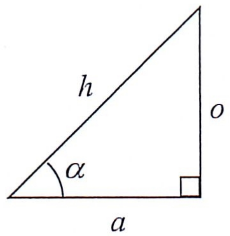
Based on a right-angled triangle, let the basic angle be α.

Then,

$$\sin\alpha = \frac{\text{opposite}}{\text{hypothenuse}} = \frac{o}{h}$$

$$\cos\alpha = \frac{\text{adjcent}}{\text{hypothenuse}} = \frac{a}{h}$$

$$\tan\alpha = \frac{\text{opposite}}{\text{adjcent}} = \frac{o}{a}$$



How do you plan to remember the definitions?

TOH CAH SOH

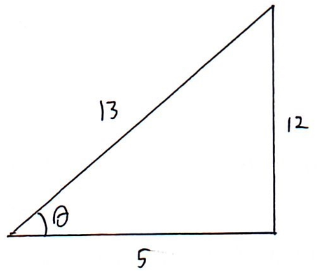
E1. Show that $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$ from the above.

$$\frac{o}{a} = \frac{o}{h} \cdot \frac{h}{a}$$

$$\therefore \frac{o}{a} = \frac{o}{a} \text{ (shown)}$$

E2. Find the value of $\sin\theta$ and $\tan\theta$ given that $\cos\theta = \dfrac{5}{13}$ and θ is acute.

Step 1: Draw the triangle and fill in the information.



Step 2: Find the missing information (which side is unknown?).

Let the unknown side be a.

$a^2 = 13^2 - 5^2$

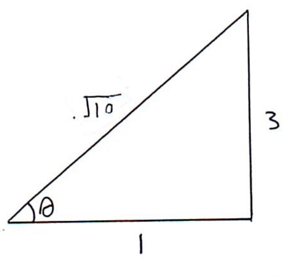
$a = 12$ units

Step 3: Evaluate the trigonometric ratios.

$\sin\theta = \dfrac{12}{13}$

$\tan\theta = \dfrac{12}{5}$

P1. Find the value of $\sin\theta$ and $\cos\theta$ given that $\tan\theta = 3$ and $0 < \theta < \dfrac{\pi}{2}$.



Let the unknown side be h.

$h^2 = 3^2 + 1^2$

$h = \sqrt{10}$

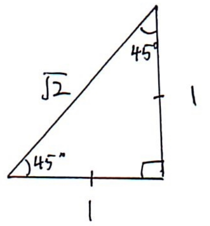
$\sin\theta = \dfrac{3}{\sqrt{10}}$

$\cos\theta = \dfrac{1}{\sqrt{10}}$

Special angles

E3. Draw an isosceles right-angled triangle with base and height equals to 1 unit.

(a) What is the exact length of the hypotenuse?



hypotenuse $= \sqrt{1^2 + 1^2}$ units

$= \sqrt{2}$ units

(b) What is the angle made between the hypotenuse and the base/height? Explain.

45°.

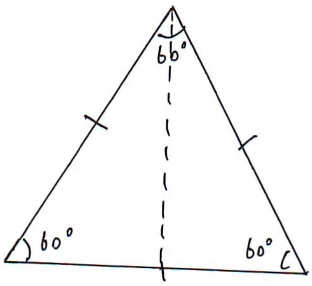
$\frac{(180° - 90°)}{2} = 45°$

(c) Hence or otherwise evaluate the following:

$\sin 45^\circ = \frac{1}{\sqrt{2}}$	$\cos 45^\circ = \frac{1}{\sqrt{2}}$	$\tan 45^\circ = 1$

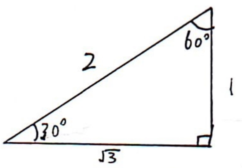
P2. Draw an equilateral triangle with side 2 units.

(a) What is the angle at each vertex?



$60°$

(b) By dividing the triangle into two equal parts, explain how you can derive the trigonometric values of $\sin 30^\circ$, $\cos 30^\circ$, $\tan 30^\circ$, $\sin 60^\circ$, $\cos 60^\circ$, and $\tan 60^\circ$.

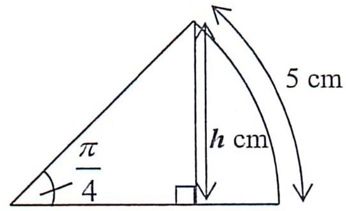


Adjacent $= \sqrt{2^2-1^2}$
$= \sqrt{3}$

$\sin 30^\circ = \frac{1}{2}$
$\cos 30^\circ = \frac{\sqrt{3}}{2}$
$\tan 30^\circ = \frac{1}{\sqrt{3}}$
$\sin 60^\circ = \frac{\sqrt{3}}{2}$
$\cos 60^\circ = \frac{1}{2}$
$\tan 60^\circ = \sqrt{3}$

$\sin 30^\circ = \frac{1}{2}$	$\cos 30^\circ = \frac{\sqrt{3}}{2}$	$\tan 30^\circ = \frac{1}{\sqrt{3}}$
$\sin 60^\circ = \frac{\sqrt{3}}{2}$	$\cos 60^\circ = \frac{1}{2}$	$\tan 60^\circ = \sqrt{3}$

P3. Find the value of h in the diagram on the right.



$$\frac{\pi}{4} = \frac{5}{r}$$

$$r = \frac{20}{\pi}$$

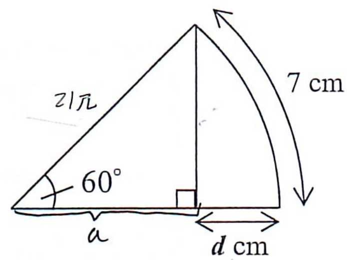
$$\sin\frac{\pi}{4} = \frac{h}{\frac{20}{\pi}}$$

$$h = \frac{20}{\pi}\sin\left(\frac{\pi}{4}\right)$$

$$= \frac{20}{\pi} \times \frac{\sqrt{2}}{2}$$

$$= \frac{10\sqrt{2}}{\pi} \text{ cm}$$

P4. Find the value of d in the diagram on the right.



$$7 = \frac{60^\circ}{360^\circ} \times \pi \times r \times 2$$

$$7 = \frac{1}{3}\pi r$$

$$r = 21\pi$$

$$\cos 60^\circ = \frac{a}{21\pi}$$

$$a = 21\pi \cos 60^\circ$$

$$= \frac{21\pi}{2}$$

Complementary angles

E4. Consider the right-angle triangle again.

(a) Find, in terms of a, b and/or c, the following ratios:

$\sin\theta$, $\sin(90^\circ-\theta)$, $\cos\theta$, $\cos(90^\circ-\theta)$, $\tan\theta$, $\tan(90^\circ-\theta)$

$\sin\theta = \frac{b}{c}$

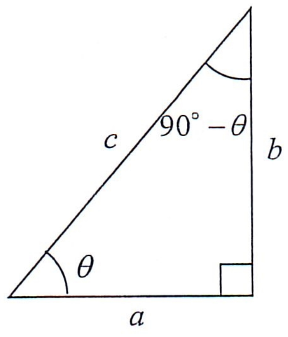
$\sin(90^\circ-\theta) = \frac{a}{c}$

$\cos\theta = \frac{a}{c}$

$\cos(90^\circ-\theta) = \frac{b}{c}$

$\tan\theta = \frac{b}{a}$

$\tan(90^\circ-\theta) = \frac{a}{b}$



(b) Complete the following about the equality between the 6 terms.

$$\boxed{\begin{aligned} \sin(90^\circ-\theta) &= \frac{a}{c} \\ \cos(90^\circ-\theta) &= \frac{b}{c} \\ \tan(90^\circ-\theta) &= \frac{a}{b} \end{aligned}}$$

(c) Given that $\tan A = 4$, find the value of $2\tan(90^\circ - A) - \tan A$.

$2 \times \frac{1}{4} - 4 = -\frac{7}{2}$

E5. Without using a calculator, evaluate

(a) $\dfrac{\sin 45^\circ}{2\cos 30^\circ - \sin 60^\circ}$

$= \dfrac{\frac{1}{\sqrt{2}}}{2 \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}}$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{2}{\sqrt{3}}$

$= \dfrac{2}{\sqrt{6}}$

(b) $\tan\dfrac{\pi}{4} + 2\tan\dfrac{\pi}{6}\tan\dfrac{\pi}{3}$

$= 1 + 2 \times \frac{1}{\sqrt{3}} \times \sqrt{3}$

$= 3$

P5. Without using a calculator, find the value of

(a) $\dfrac{\cos 50^\circ}{\sin 40^\circ} + \sin 90^\circ$

$= \dfrac{\cos(90^\circ - 40^\circ)}{\sin(40^\circ)} + 1$

$= \dfrac{\sin(40^\circ)}{\sin(40^\circ)} + 1$

$= 1 + 1$

$= 2$

(b) $\dfrac{\tan\frac{\pi}{4} + \cos\frac{\pi}{3}}{\tan\frac{2\pi}{5}\tan\frac{\pi}{10} + \sin\frac{\pi}{6}}$

$= \dfrac{1 + \frac{1}{2}}{\tan(\frac{\pi}{2} - \frac{\pi}{10})\tan(\frac{\pi}{10}) + \frac{1}{2}}$

$= \dfrac{\frac{3}{2}}{\cot(\frac{\pi}{10})\tan(\frac{\pi}{10}) + \frac{1}{2}}$

$= \dfrac{\frac{3}{2}}{\frac{1}{\tan(\frac{\pi}{10})} \times \tan(\frac{\pi}{10}) + \frac{1}{2}}$

$= \dfrac{\frac{3}{2}}{1 + \frac{1}{2}}$

$= \dfrac{\frac{3}{2}}{\frac{3}{2}}$

$= 1$