

S3Math13 Tutor

- 1 Solve the simultaneous equations

$$\begin{aligned} 8^{x+3y} &= 4 & \rightarrow 3x+9y &= 2 \\ 2x-3y-1 &= 0. \end{aligned} \quad \begin{cases} x = \frac{5}{9} \\ y = \frac{1}{27} \end{cases} \quad [4]$$

- 2 (a) Simplify $3\sqrt{216} - \sqrt{24} + \frac{4}{\sqrt{6}}$, expressing your answer in the form of

$$k\sqrt{6}, \text{ where } k \text{ is a constant.} \quad \frac{50\sqrt{6}}{3} \quad k = \frac{50}{3} \quad [3]$$

- (b) Solve the equation $8(5^{2x-1}) - 5^x = 2(5^{x+2})$. $x = 1 + \log_5\left(\frac{51}{8}\right)$ [4]
 $\int x = \frac{255}{8} \quad \approx 2.15095$

- 3 (i) Factorise $2x^3 - 128$ completely. $2(x-4)(x^2+4x+16)$ [2]

- (ii) Hence or otherwise, solve the equation $2x^3 - 128 = 4 - x$. [3]
 $x = 4$

- 4 (a) Find the range of values of x for which $(3x-2)(x+6) - 3x > -2$. [3]
 $x < -5 \text{ or } x > \frac{2}{3}$

- (b) Given that the line $2y - b = ax$ is a tangent to the curve $4x^2 + 5y^2 - 20 = 0$,
 show that $b^2 = 5a^2 + 16$. $(16+5a^2)y^2 - 16by + 4b^2 - 20a^2 = 0$ [4]
 $\Delta = 0 \quad b^2 = 5a^2 + 16$

- 5 (a) Solve the equation $x - 5 = 4\sqrt{x}$. $x = 25$ [3]

- (b) Solve the equation $\log_3 x^2 + 2 \log_x 3 = 5$. $x_1 = \sqrt{3} \quad x_2 = 9$ [5]

- 6 (i) On the same axes, sketch the graphs of $y = -x$ and $y = |-2x - 7|$, indicating clearly the x -intercepts and y -intercepts. [4]

- (ii) Hence, or otherwise, solve the inequality $|-2x - 7| \leq -x$. [4]

$$-7 \leq x \leq -\frac{7}{3}$$

[Turn over

- 7 (a) The points $A(3,0)$, $B(9,18)$ and $C(13,10)$ lie on a quadratic curve $y = f(x)$ and B is the maximum point on the curve. Find the equation of the curve in the form $y = ax^2 + bx + c$, where a , b and c are constants. [3]

$$y = \frac{-1}{2}x^2 + 9x + \frac{45}{2}$$

- (b) Peter used 20 metres of fence to enclose two flower beds. One flower bed is in the shape of an equilateral triangle of side x metres. The other is in the shape of a circle of radius r metres.

- (i) Express r in terms of x and π . $r = \frac{1}{2\pi}(20 - 3x)$ [1]

- (ii) Show that the total area, $A \text{ m}^2$ of the two flower beds is given by $A = \frac{\sqrt{3}\pi x^2 + (20 - 3x)^2}{4\pi}$. [3]

- (iii) Hence, by completing the square, find the minimum total area of the two flower beds. [3]

$$A = \frac{1}{4\pi} \left[(\sqrt{3}\pi + 9)x^2 - 120x + 400 \right] \quad \text{min} = c - \frac{b^2}{4a} = 12$$

- 8 $P(-3,3)$ and $T(0,2)$ lie on a circle with centre $Q(-1,4)$.

- (i) Given that PS is the diameter of the circle, find the coordinates of S . $(1, 5)$ [2]

- (ii) Find the equation of the circle with centre Q in the general form. $(x+1)^2 + (y-4)^2 = 5$ [3]

- (iii) $R(2,3)$ lies outside of the circle. By calculating the gradient of RS and

- PS , show that RS is a tangent to the circle in (i). $k_{RS} = \frac{3-5}{2-1} = -2$ [3]

- (iv) Find the area of the quadrilateral $PTRS$. $k_{PS} = \frac{5-3}{1-(-3)} = \frac{1}{2}$ [2]

$$S = \frac{21}{2}$$

$$= \frac{1}{2} \begin{vmatrix} -3 & 0 & 2 & 1 & -3 \\ 3 & 2 & 3 & 5 & 3 \end{vmatrix}$$

- 9 (i) Given that $f(x) = 2e^x$ and $g(x) = 5 - 2e^x$, describe 2 successive transformations that will map the graph of $f(x)$ to the graph of $g(x)$. [2]
- (ii) Sketch the graph of $y = 5 - 2e^x$, indicating clearly the asymptote, the values of the x -intercept and y -intercept. [4]
- (iii) In order to solve the equation $\ln\left(\frac{x+6}{4}\right) = x$, a graph of a suitable straight line is drawn on the same axes as the graph of $y = 5 - 2e^x$. Find the equation of the straight line. $y = -\frac{1}{2}x + 2$ [3]

- 10 (a) Given that $A = \begin{pmatrix} -2 & -3 \\ 4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 \\ -3 & 5 \end{pmatrix}$, find $3A - B$. $\begin{pmatrix} -4 & -10 \\ 15 & 4 \end{pmatrix}$ [2]

- (b) E'Lite shop sells air-conditioning unit, instant water heater and humidifier. The number of air-conditioning unit, instant water heater and humidifier sold in 2020 and 2021 is given in the following table.

Type \ Year	air-conditioning	instant water heater	humidifier
2021	290	240	200
2020	235	362	117

- (i) Write down a 2×3 matrix C which represents the number of air-conditioning unit, instant water heater and humidifier sold in 2021 and 2020 respectively.

$$C = \begin{pmatrix} 290 & 240 & 200 \\ 235 & 362 & 117 \end{pmatrix} \quad [1]$$

- (ii) Given that the profit made from selling one air-conditioning unit is \$240, one instant water heater unit is \$95 and one humidifier unit \$110, write down a column matrix D that represents the profit made from selling each unit.

$$D = \begin{pmatrix} 240 \\ 95 \\ 110 \end{pmatrix} \quad [1]$$

- (iii) Hence, find matrix E where $E = CD$. [2]

- (iv) Explain what matrix E represents. [1]

$$E = \begin{pmatrix} 114,400 \\ 103,660 \end{pmatrix}$$

profit of year

[Turn over]

- 11 The function $f(x) = ax^3 + 7x^2 - bx - 4$ where a and b are constants, is divisible by $x + 2$ and leaves a remainder of 185 when divided by $x - 3$.

(i) Show that $a = 6$ and $b = 12$.

$$\begin{cases} 4a - b = 12 \\ 9a - b = 42 \end{cases}$$

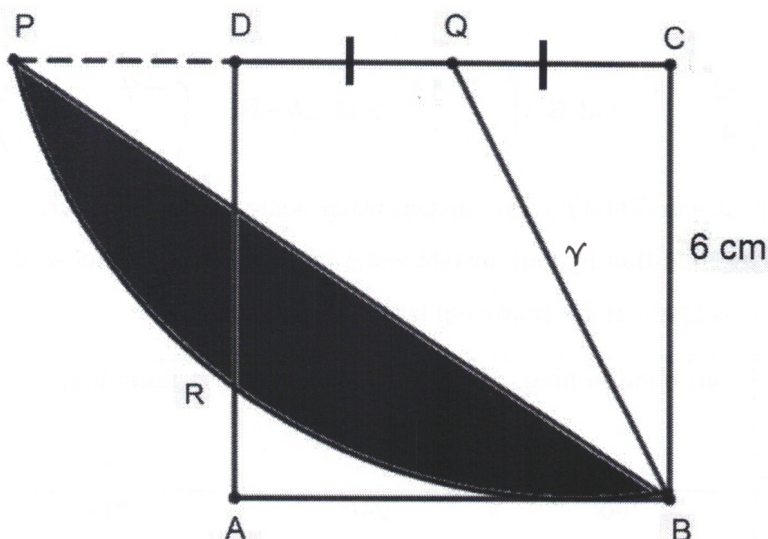
[3]

(ii) Use the value of a and b from (i), by using long division, find the remainder when $f(x)$ is divided by $2x^2 + 3$.

$$\frac{6x^3 - 14x^2 - 21x - \frac{29}{2}}{2x^2 + 3} = -21x - \frac{29}{2}$$

[3]

- 12 $ABCD$ is a square of sides 6 cm. Q is the mid-point of DC .
An arc BRP is drawn with centre Q and P lies on DC produced.



- (i) Show that the $\angle PQB = 2.03$.
- (ii) Calculate the perimeter of $CPRBC$.
- (iii) Calculate the area of the shaded region.

$$\tan \angle BQC = \frac{6}{3} = 2$$

[2]

$$= 29.32$$

[3]

$$\widehat{PRB} = \angle PQB \cdot r \quad r = \sqrt{6^2 + 3^2} = 3\sqrt{5}$$

$$CR = r + 3$$

$$P = \widehat{PRB} + CR + CB$$

$$CB = 6$$

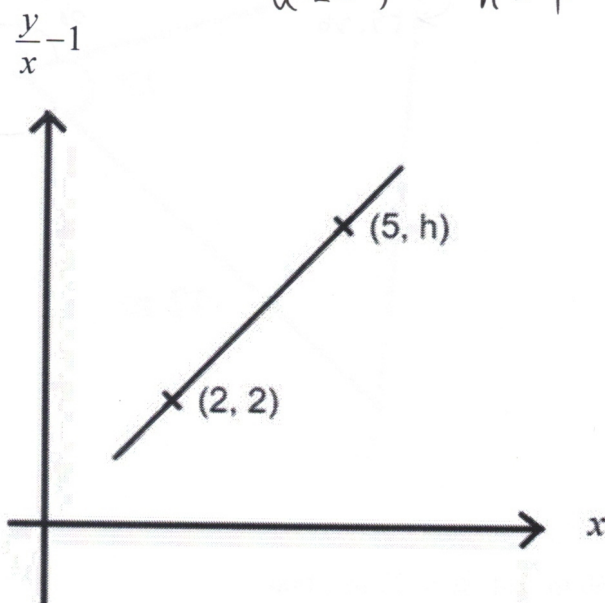
$$S = \frac{1}{2} \angle PQB \cdot r^2 - \frac{1}{2} (6) r = 12.13$$

- 13 The variables x and y are connected by the equation $2x^2 - ax - 3y = 0$, where a is a constant.

(i) Express $\frac{y}{x}$ in terms of x and a . $\frac{y}{x} = \frac{2}{3}x - \frac{a}{3}$ [1]

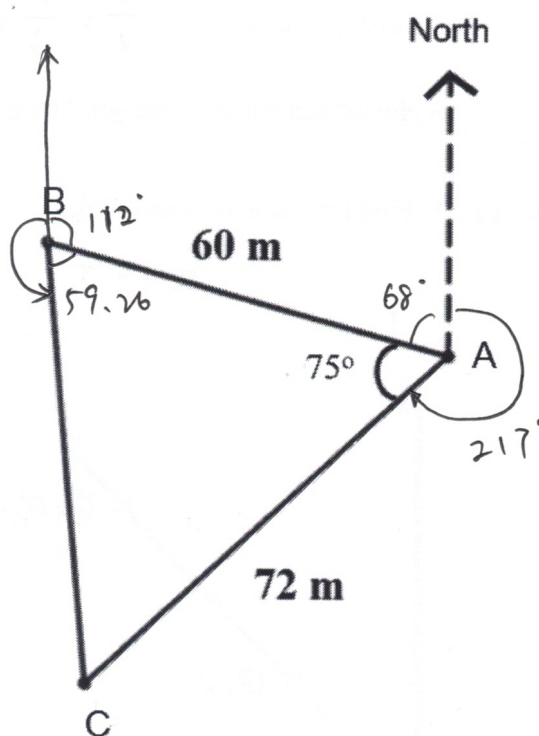
- (ii) When $\left(\frac{y}{x} - 1\right)$ is plotted against x , a straight line is obtained which passes through $(2, 2)$ and $(5, h)$. Find the value of a and of h . [4]

$$a = -5 \quad h = 4$$



[Turn over

- 14 In the diagram, the points A , B and C are on level ground. The bearing of C from A is 217° and $\angle BAC = 75^\circ$.



Given that $AB = 60$ m and $AC = 72$ m, find

- | | | | |
|-------|-------------------------------|----------------|-----|
| (i) | the length BC , | 80.92 | [2] |
| (ii) | the angle ABC , | 59.26° | [2] |
| (iii) | the bearing of C from B . | 188.74° | [2] |