Name	Index Number	Class

WOODGROVE SECONDARY SCHOOL

A COMMUNITY OF FUTURE-READY LEARNERS AND THOUGHTFUL LEADERS

N-LEVEL PRELIMINARY EXAMINATIONS 2025

LEVEL & STREAM	: SECONDARY 4 NORMAL ACADEMIC
SUBJECT (CODE)	: ADDITIONAL MATHEMATICS (4051)
PAPER	: 02
DATE (DAY)	: 01 AUGUST 2025 (FRIDAY)
DURATION	: 1 HOUR 45 MINUTES

MARKING SCHEME

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top o
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or gra
Do not use staples, paper clips, glue or co

Answer **all** the questions
Give non-exact nu ures, or 1 decimal place in the case of angles in d ecified in the question.
The ected, where appropriate.
You a sentation in your answer.

The nu ven in brackets [ ] at the end of each question or part question.

The total number of marks in this paper is 70.

DO NOT TURN OVER THE QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Student's Signature		**Parent's Signature**	
Date		**Date**	

/ **70**

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Setter: Mdm Tan Hwee Lin

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve the simultaneous equations.

$$y = x^2 + 4x - 5$$
$$x - y - 1 = 0$$

[4]

$$y = x^2 + 4x - 5 \cdots (1)$$
$$x - y - 1 = 0 \quad \Rightarrow \quad y = x - 1 \quad \cdots (2)$$

M1

Subst. (2) into (1),

$$x - 1 = x^2 + 4x - 5$$
$$x^2 + 3x - 4 = 0$$
$$(x+4)(x-1) = 0$$

M1

$$x = -4,\ x = 1$$

A2

When $x = -4, y = -5$, and when $x = 1, y = 0$

2 **(a)** State the amplitude and period of the graph of $y = 3\sin 2x + 1$. [2]

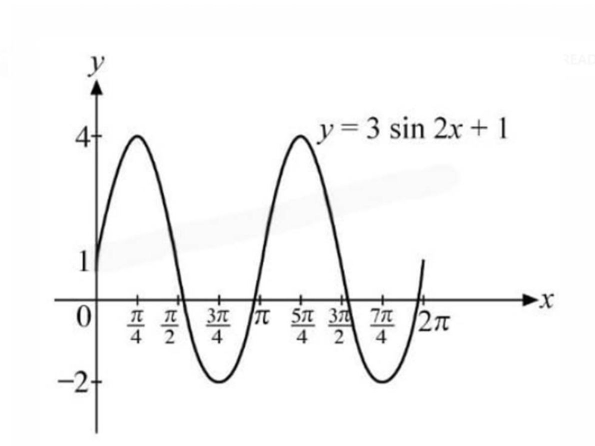
Given that $y = 3\sin 2x + 1$,

amplitude = 3 — B1

period=360°÷2

=180° or π — B1

(b) On the axes below, sketch the graph of $y = 3\sin 2x + 1$ for $0 \leq x \leq 2\pi$. [3]



Correct shape – B1
Correct max and min value of *y* - B1
Correct Period- B1

3 Show that the quadratic expression $x^2 - px + p^2 + 1$ is always positive for all real values of x. [3]

Let $y = x^2 - px + p^2 + 1$

$$\begin{aligned} \text{discriminant, } D &= (-p)^2 - 4(1)\left(p^2+1\right) \\ &= p^2 - 4p^2 - 4 \\ &= -\left(3p^2+4\right) \\ &< 0 \end{aligned}$$

Since <u>discriminant < 0</u> and coefficient of $x^2 > 0$, $x^2 - px + p^2 + 1$ is always positive for all real values of x.

M1
M1
A1

4 A curve is defined by equation $y = 3x^2 + 1$. Find the rate at which y is changing when $x = 2$, given that x is increasing at a rate of 0.5 units per second. [3]

$$\begin{aligned} y &= 3x^2 + 1 \\ \frac{\mathrm{dy}}{\mathrm{dt}} &= 6x \\ \frac{\mathrm{dy}}{\mathrm{dt}} &= \frac{\mathrm{dy}}{\mathrm{dx}} \times \frac{\mathrm{dx}}{\mathrm{dt}} \\ &= 6(2) \times 0.5 \\ &= 6 \text{ cm/s} \end{aligned}$$

M1
M1
A1

5 **(a)** Differentiate $\left(5-x^2\right)^7$ with respect to x. [2]

$$\frac{\mathrm{d}y}{\mathrm{d}x}=7\left(5-x^2\right)^6(-2x) \quad \text{M1}$$
$$=-14x\left(5-x^2\right)^6 \quad \text{A1}$$

(b) Hence, find $\int x\left(5-x^2\right)^6 dx$. [3]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(5-x^2\right)^7=-14x\left(5-x^2\right)^6$$
$$-\frac{1}{14}\frac{\mathrm{d}}{\mathrm{d}x}\left(5-x^2\right)^7=x\left(5-x^2\right)^6 \quad \text{M1}$$
$$\int x\left(5-x^2\right)^6 dx=\int-\frac{1}{14}\frac{\mathrm{d}}{\mathrm{d}x}\left(5-x^2\right)^7 \quad \text{M1}$$
$$=-\frac{1}{14}\left(5-x^2\right)^7+C \quad \text{A1}$$

6 The area and length of a rectangle are 2 cm^2 and $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ cm respectively.

Express, in the form $a+b\sqrt{3}$,

(a) the breadth, [3]

Breadth = Area ÷ Length

$$=2\div\frac{2+\sqrt{3}}{2-\sqrt{3}} \quad \text{M1}$$
$$=2\times\frac{\left(2-\sqrt{3}\right)}{2+\sqrt{3}}$$
$$=\frac{2\left(2-\sqrt{3}\right)\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)} \quad \text{M1}$$
$$=\frac{2\left(4-4\sqrt{3}+3\right)}{2^2-3}$$
$$=2\left(7-4\sqrt{3}\right) \quad \text{A1}$$
$$\therefore \text{breadth}=\left(14-8\sqrt{3}\right)\text{ cm}$$

(b) the perimeter of the rectangle. [3]

Perimeter = 2(length + breadth)

$$= 2\left[\left(\frac{2+\sqrt{3}}{2-\sqrt{3}}\right) + \left(14-8\sqrt{3}\right)\right] \quad \text{M1}$$

$$= 2\left[\left(\frac{2+\sqrt{3}}{2-\sqrt{3}}\right)\left(\frac{2+\sqrt{3}}{2+\sqrt{3}}\right) + \left(14-8\sqrt{3}\right)\right] \quad \text{M1}$$

$$= 2\left[\frac{2^2+4\sqrt{3}+3}{4-3} + 14 - 8\sqrt{3}\right]$$

$$= 2\left(7+4\sqrt{3}+14-8\sqrt{3}\right)$$

$$= 2\left(21-4\sqrt{3}\right)$$

$$= \left(42-8\sqrt{3}\right)\text{ cm} \quad \text{A1}$$

7 **(a)** Factorise $8x^3+125$. [1]

$8x^3+125 = (2x+5)(4x^2-10x+25)$ B1

(b) Hence,

(i) Express 1125 as a product of two integers, [2]

$$8x^3+125 = 1125$$
$$8x^3 = 1000$$
$$x^3 = 125$$
$$x = 5 \quad \text{M1}$$

$$1125 = (2\times5+5)(4\times5^2-10\times5+25)$$
$$= 15\times75 \quad \text{A1}$$

(ii) Solve $8x^3 + 125 = 39(2x+5)$ [4]

$8x^3 + 125 = 39(2x+5)$	
$(2x+5)(4x^2 - 10x + 25) = 39(2x+5)$	M1
$(2x+5)(4x^2 - 10x + 25) - 39(2x+5) = 0$	
$(2x+5)(4x^2 - 10x + 25 - 39) = 0$	
$(2x+5)(4x^2 - 10x - 14) = 0$	
$(2x+5)(2x^2 - 5x - 7) = 0$	M1
$(2x+5)(2x-7)(x+1) = 0$	M1
$x = -\frac{5}{2}, \frac{7}{2} \; or \; -1$	A1

8 The equation of a curve is $y=(x+1)(2x^2-5x-31)$.

(a) Find the coordinates of the stationary points of the curve. [5]

$$y=(x+1)(2x^2-5x-31)$$

$$\frac{dy}{dx}=(2x^2-5x-31)+(x+1)(4x-5)$$ M1

$$=2x^2-5x-31+4x^2-5x+4x-5$$

$$=2x^2+4x^2-5x+4x-5x-31-5$$

$$=6x^2-6x-36$$

For stationary points,

$$\frac{dy}{dx}=0$$

$$6x^2-6x-36=0$$ M1

$$6(x^2-x-6)=0$$

$$x^2-x-6=0$$

$$(x-3)(x+2)=0$$

$$x=3 \text{ or } x=-2$$ M1

When $x = 3$,

$$y=(3+1)\left[2(3)^2-5(3)-31\right]$$

$$=4(18-15-31)$$

$$=-112$$ M1

When $x = -2$,

$$y=(-2+1)\left[2(-2)^2-5(-2)-31\right]$$

$$=-(8+10-31)$$

$$=13$$ A1

$\therefore$ the stationary points are $(3,-112)$ and $(-2,13)$.

(b) Determine the nature of the stationary points. [3]

$$\frac{d^2y}{dx^2}=12x-6$$ M1

When $x = 3$,

$$\frac{d^2y}{dx^2}=12(3)-6$$

$$=30>0$$

$\therefore (3,-112)$ is a minimum point. A1

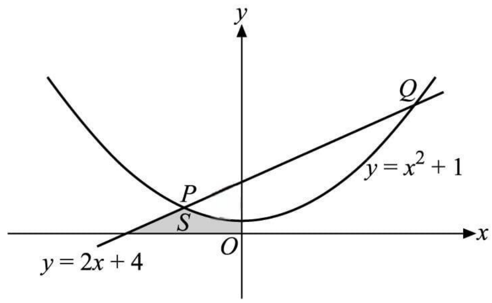
When $x = -2$,

$$\frac{d^2y}{dx^2}=12(-2)-6$$

$$=-30<0$$

$\therefore (-2,13)$ is a maximum point. A1

9 The diagram shows part of the curve $y = x^2 + 1$, the line $y = 2x + 4$.
The curve and the line intersect at the points P and Q.
Region S is bounded by the curve $y = x^2 + 1$, the line $y = 2x + 4$, the x-axis and the y-axis.



(a) Find the coordinates of P and Q. [3]

$$y = 2x + 4 \ldots\ldots\ldots(1)$$
$$y = x^2 + 1 \ldots\ldots\ldots(2)$$

Sub (1) into (2) : $2x + 4 = x^2 + 1$ M1

$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$
$$x = 3 \text{ or } x = -1$$ M1

Substituting $x = -1$ into (1),

$y = 2(-1) + 4$

$\quad = 2$

Substituting $x = 3$ into (1),

$y = 2(3) + 4$

$\quad = 10$

$\therefore$ the coordinates of P and Q are $(-1, 2)$ and $(3, 10)$ respectively. A1

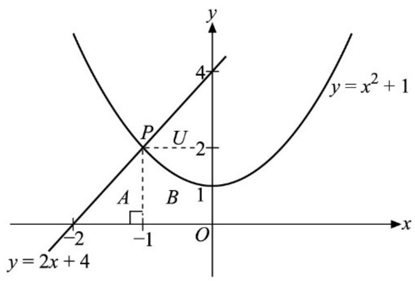
(b) Find the area of the shaded region S. [6]

For $y = 2x + 4$, when $y = 0$,

$$0 = 2x + 4$$

$$x = -2,$$

M1



Let $S = A + B$.

From the diagram,

$$\text{Area of } A = \frac{1}{2}(1)(2)$$

$$= 1 \text{ unit}^2$$

M1

$$\text{Area of } B = \int_{-1}^{0} \left(x^2 + 1\right) dx$$

M1

$$= \left[\frac{x^3}{3} + x\right]_{-1}^{0}$$

$$= -\left(-\frac{1}{3} - 1\right)$$

M1

$$= 1\frac{1}{3} \text{ units}^2$$

$$\therefore \text{area of } S = 1 + 1\frac{1}{3}$$

M1

$$= 2\frac{1}{3} \text{ units}^2$$

A1

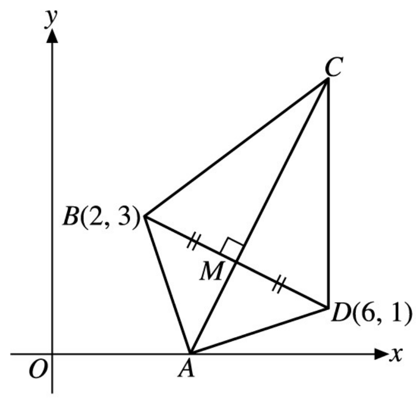
`**10** **(a)** Show that (cosec 2*x*) (cos 2*x* + 1) = cot *x*. [5]

$$\text{LHS} = (\operatorname{cosec} 2x)(\cos 2x + 1)$$
$$= \frac{\cos 2x + 1}{\sin 2x} \quad \text{M1}$$
$$= \frac{2\cos^2 x - 1 + 1}{2\sin x \cos x} \quad \text{M2}$$
$$= \frac{2\cos^2 x}{2\sin x \cos x} \quad \text{M1}$$
$$= \frac{\cos x}{\sin x}$$
$$= \cot x \quad \text{A1}$$
$$= \text{RHS (shown)}$$

(b) Hence or otherwise, solve the equation (cosec 2*x*) (cos 2*x* + 1) = $\sqrt{3}$ [4]
for $-180^{o} \le x \le 180^{o}$.

$$(\operatorname{cosec} 2x)(\cos 2x + 1) = \sqrt{3}$$
$$\therefore \cot x = \sqrt{3} \quad \text{M1}$$
$$\tan x = \frac{1}{\sqrt{3}} \quad \text{M1}$$
$$\alpha = 30° \quad \text{M1}$$
$$x = -150° \text{ or } 30° \quad \text{A1}$$

11 In the diagram below, *ABCD* is a quadrilateral where *B* is $(2,3)$, *D* is $(6,1)$ and *A* lies on the x-axis. *AC* is the perpendicular bisector of *BD*, *M* is the mid-point of *BD* and *CD* is parallel to y-axis.



Find

(a) the coordinates of *M*, [2]

Coordinates of $M = \left(\frac{2+6}{2}, \frac{3+1}{2}\right)$ | M1

$= (4,\ 2)$ | A1

(b) the equation of *AC*, [4]

Gradient of $BD = \frac{3-1}{2-6}$ | M1

$= -\frac{1}{2}$

$\therefore$ gradient of $AC = -1 \div \left(-\frac{1}{2}\right)$ | M1

$= 2$

Let equation of *AC* be $y = 2x + c$.

Since *AC* passes through $M(4,2)$,

$2 = 2(4) + c$

$c = -6$ | M1

$\therefore$ the equation of *AC* is $y = 2x - 6$. | A1

(c) the coordinates of A and C, [3]

Since A lies on the x-axis, the y-coordinate of A is 0.
Substituting $y = 0$ into the equation of AC,

$$0 = 2x - 6$$
$$x = 3$$

$\therefore$ the coordinates of A are $(3,\ 0)$. B1

Since CD is // to the y-axis, the x-coordinate of C is 6.
Substituting $x = 6$ into the equation of AC, M1

$$y = 2(6) - 6$$
$$= 6$$ A1

$\therefore$ the coordinates of C are $(6,\ 6)$.

(d) the area of $ABCD$. [2]

$$\text{Area of } ABCD = \frac{1}{2}\begin{vmatrix} 3 & 6 & 6 & 2 & 3 \\ 0 & 1 & 6 & 3 & 0 \end{vmatrix}$$ M1

$$= \frac{1}{2}\left[(3 + 36 + 18) - (6 + 12 + 9)\right]$$

$$= 15 \text{ units}^2$$ A1

END OF PAPER