RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 5

Additional Practice Questions for Chapter 1: Basics

1 Find the set of values of p for which the equation $x^2+2x+p=3px-1$ has no real roots.

[$\left(0,\dfrac{16}{9}\right)$]

Solution

$x^2+2x+p=3px-1$

$x^2+(2-3p)x+p+1=0$

For no real roots,

$(2-3p)^2-4(p+1)<0$

$9p^2-12p+4-4p-4<0$

$9p^2-16p<0$

$p(9p-16)<0$

$0<p<\dfrac{16}{9}$

$\therefore$ Set of values of p is $\left(0,\dfrac{16}{9}\right)$.

2 Find the range of values of t for which the equation $x+\dfrac{2}{x}=t$ has real roots, leaving your answer in surd form.

$\left[t\le -2\sqrt{2} \text{ or } t\ge 2\sqrt{2}\right]$

Solution

$x+\dfrac{2}{x}=t$

$x^2-tx+2=0$

For $x^2-tx+2=0$ to have real roots, $t^2-4(2)\ge 0$

$\left(t+2\sqrt{2}\right)\left(t-2\sqrt{2}\right)\ge 0$

$t\le -2\sqrt{2}$ or $t\ge 2\sqrt{2}$

3 Given that q is a constant, show that $(2+qx)^2+x(2x-q)$ is positive for all real values of x.

Solution

$$(2+qx)^2+x(2x-q)=\left(q^2x^2+4qx+4\right)+2x^2-qx=\left(q^2+2\right)x^2+\left(3q\right)x+4$$

$$\begin{aligned}\text{Discriminant} &= \left(3q\right)^2-4\left(q^2+2\right)\left(4\right)\\ &= -7q^2-32\\ &\le -32 \text{ (Since } q^2\ge 0, \text{ so } -7q^2\le 0)\\ &< 0\end{aligned}$$

In addition, coefficient of $x^2=q^2+2>0$,

Hence $(2+qx)^2+x(2x-q)$ is positive for all real values of x.

Alternatively, complete the square to get

$(2+qx)^2+x(2x-q)$

$=\left(q^2+2\right)x^2+\left(3q\right)x+4$

$=\left(q^2+2\right)\left(x+\dfrac{3q}{2\left(q^2+2\right)}\right)^2-\left(q^2+2\right)\left[\dfrac{3q}{2\left(q^2+2\right)}\right]^2+4$

$=\left(q^2+2\right)\left(x+\dfrac{3q}{2\left(q^2+2\right)}\right)^2+\dfrac{-9q^2+16\left(q^2+2\right)}{4\left(q^2+2\right)}$

$=\left(q^2+2\right)\left(x+\dfrac{3q}{2\left(q^2+2\right)}\right)^2+\dfrac{7q^2+32}{4\left(q^2+2\right)}$

$\ge\dfrac{7q^2+32}{4\left(q^2+2\right)}>0$ for all q since $\left(q^2+2\right)>0$ and $\left(x+\dfrac{3q}{2\left(q^2+2\right)}\right)^2\ge 0$

4 Express $\dfrac{1-2x+5x^2}{\left(1-2x\right)\left(1+x^2\right)}$ as partial fractions.

[$\dfrac{1}{1-2x}-\dfrac{2x}{1+x^2}$]

Solution

Let $\dfrac{1-2x+5x^2}{\left(1-2x\right)\left(1+x^2\right)}\equiv\dfrac{A}{1-2x}+\dfrac{Bx+C}{1+x^2}$

$1-2x+5x^2\equiv A\left(1+x^2\right)+\left(Bx+C\right)\left(1-2x\right)$

When $x=\dfrac{1}{2}$, $\dfrac{5}{4}A=\dfrac{5}{4}\Rightarrow A=1$.

When $x=0$, $1=\left(1\right)+\left(C\right)\left(1\right)\Rightarrow C=0$

By comparing coefficients of x^2, $5=1-2B\Rightarrow B=-2$

$\therefore\dfrac{1-2x+5x^2}{\left(1-2x\right)\left(1+x^2\right)}\equiv\dfrac{1}{1-2x}-\dfrac{2x}{1+x^2}$

5 **J2021/Cambridge O level Additional Mathematics/4037/11/Q8**

A curve has equation $y=\left(2-\sqrt{3}\right)x^2+x-1$. The x-coordinate of a point A on the curve is $\dfrac{\sqrt{3}+1}{2-\sqrt{3}}$.

(a) Show that the coordinates of A can be written in the form $\left(p+q\sqrt{3},r+s\sqrt{3}\right)$, where p, q, r and s are integers.

(b) Find the x-coordinate of the stationary point on the curve, giving your answers in the form $a+b\sqrt{3}$, where a and b are rational numbers.

$\left[\text{(a)}\left(5+3\sqrt{3},18+11\sqrt{3}\right)\ \text{(b)}\ -1-\frac{1}{2}\sqrt{3}\right]$

Solution

(a) $\dfrac{\sqrt{3}+1}{2-\sqrt{3}}=\dfrac{\sqrt{3}+1}{2-\sqrt{3}}\times\dfrac{2+\sqrt{3}}{2+\sqrt{3}}$

$$=\frac{2\sqrt{3}+3+2+\sqrt{3}}{4-3}$$

$$=5+3\sqrt{3}$$

Substitute $x=5+3\sqrt{3}$ into $y=\left(2-\sqrt{3}\right)x^2+x-1$:

$$y=\left(2-\sqrt{3}\right)\left(5+3\sqrt{3}\right)^2+5+3\sqrt{3}-1$$

$$=\left(2-\sqrt{3}\right)\left(25+27+30\sqrt{3}\right)+4+3\sqrt{3}$$

$$=\left(2-\sqrt{3}\right)\left(52+30\sqrt{3}\right)+4+3\sqrt{3}$$

$$=\left(104-90\right)+\sqrt{3}\left(60-52\right)+4+3\sqrt{3}$$

$$=18+11\sqrt{3}$$

$\therefore$ Coordinates of A are $\left(5+3\sqrt{3},18+11\sqrt{3}\right)$

(b) $y=\left(2-\sqrt{3}\right)x^2+x-1$

$$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=2\left(2-\sqrt{3}\right)x+1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=0$$

$$\Rightarrow x=-\frac{1}{2\left(2-\sqrt{3}\right)}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=-\frac{2+\sqrt{3}}{2\left(4-3\right)}=-\frac{2+\sqrt{3}}{2}=-1-\frac{1}{2}\sqrt{3}$$

The x-coordinate of the stationary point is $-1-\dfrac{1}{2}\sqrt{3}$

6 J2021/Cambridge O level Additional Mathematics/4037/21/Q8

In this question, a, b, c and d are positive constants.

(a) (i) It is given that $y=\log_a(x+3)+\log_a(2x-1)$. Explain why x must be greater than $\frac{1}{2}$.

(ii) Find the exact solution of the equation $\dfrac{\log_a 6}{\log_a(y+3)}=2$.

(b) Write the expression $\log_a 9+(\log_a b)(\log_{\sqrt{b}} 9a)$ in the form $c+d\log_a 9$, where c and d are integers.

$\left[\textbf{(ai)}\ -3+\sqrt{6}\quad \textbf{(b)}\ 2+3\log_a 9\right]$

Solution

(ai) $y=\log_a(x+3)+\log_a(2x-1)$

For above logarithm functions to be defined, $x+3>0$ and $2x-1>0$

$\Rightarrow x>-3$ and $x>\dfrac{1}{2}$

$\therefore x>\dfrac{1}{2}$

(aii) $\dfrac{\log_a 6}{\log_a(y+3)}=2$

$\Rightarrow \log_a 6=2\log_a(y+3)$

$\Rightarrow \log_a 6-\log_a(y+3)^2=0$

$\Rightarrow \log_a\dfrac{6}{(y+3)^2}=0$

$\Rightarrow \dfrac{6}{(y+3)^2}=a^0=1$

$\Rightarrow (y+3)^2=6$

$\therefore y=-3\pm\sqrt{6}$

Since $y+3>0$ for logarithm function to be defined, $y=-3+\sqrt{6}$

Note that $\dfrac{\log_a c}{\log_a b}\neq\log_a c-\log_a b$, where $a,b,c>0$

although $\log_a\dfrac{c}{b}=\log_a c-\log_a b$

(b)
$$\begin{aligned}\log_a 9+(\log_a b)(\log_{\sqrt{b}} 9a)&=\log_a 9+(\log_a b)\left(\frac{\log_a 9a}{\log_a\sqrt{b}}\right)\\&=\log_a 9+(\log_a b)\left(\frac{\log_a 9+\log_a a}{\frac{1}{2}\log_a b}\right)\\&=\log_a 9+2(\log_a 9+1)\\&=2+3\log_a 9\end{aligned}$$

7 Let $\mathrm{f}(t)=1-\frac{1}{2}\mathrm{e}^{-2t}$.

(a) By solving $\mathrm{f}(t)=0$, show that $t=-0.347$, correct to 3 significant figures.

(b) Sketch the graph of $y=\mathrm{f}(t)$ for $t\geq -1$, showing the equations of any asymptotes and the coordinates of any intersections with the axes.

Solution

(a)

$$1-\frac{1}{2}\mathrm{e}^{-2t}=0$$

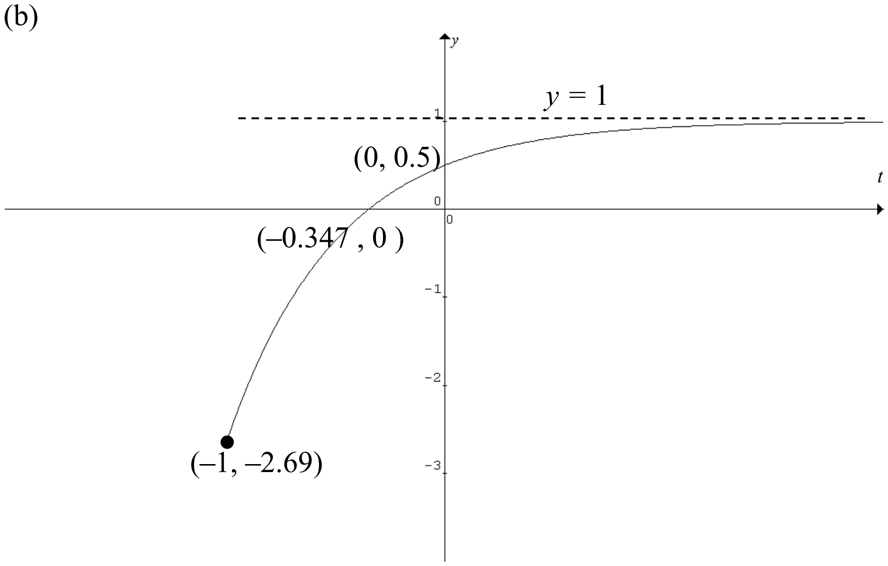
$$\mathrm{e}^{-2t}=2$$

$$-2t=\ln 2$$

$$t=-\frac{\ln 2}{2}$$

$$t\approx -0.34657=-0.347 \text{ (to 3 s.f) (shown)}$$

(b)



8 Solve the equation $4\cos 2x+\operatorname{cosec} x-4=0$ for $0\leq x\leq 2\pi$.

$$\left[\frac{\pi}{6} \text{ or } \frac{5\pi}{6}\right]$$

Solution

$$4\cos 2x+\operatorname{cosec} x-4=0$$

$$4\left[1-2\sin^2 x\right]+\frac{1}{\sin x}-4=0$$

$$-8\sin^3 x+1=0$$

$$\sin^3 x=\frac{1}{8}$$

$$\sin x=\frac{1}{2}\Rightarrow x=\frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

9 Find all the angles between 0 and 2π exclusive which satisfy the equation $2\sin^3 x = 3\cos x \sin x$.

$[\frac{\pi}{3}, \pi, \frac{5\pi}{3}]$

Solution
$2\sin^3 x = 3\cos x\sin x$ $2\sin^3 x - 3\cos x\sin x = 0$ $\sin x(2\sin^2 x - 3\cos x) = 0$ $\sin x = 0$ or $2(1-\cos^2 x) - 3\cos x = 0$ $x = \pi$ or $2\cos^2 x + 3\cos x - 2 = 0$ or $(2\cos x - 1)(\cos x + 2) = 0$ $\cos x = \frac{1}{2}$ or $\cos x = -2$ (reject since $\lvert\cos x\rvert \le 1$) $x = \frac{\pi}{3}, \frac{5\pi}{3}$ Thus $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

10 **J2021/Cambridge O level Additional Mathematics/4037/21/Q10**

Relative to an origin O, the position vectors of the points A, B, C and D are

$$\overrightarrow{OA} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 10 \\ 3 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } \overrightarrow{OD} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}.$$

(a) Find the unit vector in the direction of $\overrightarrow{AB}$.

(b) The point A is the mid-point of BC. Find the value of x and of y.

(c) The point E lies on OD such that $OE : OD = 1 : 1+\lambda$. Find the value of λ such that $\overrightarrow{BE}$ is parallel to the x-axis.

$\left[\textbf{(a)}\frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2 \end{pmatrix} \textbf{ (b) } x = 2, y = -13 \textbf{ (c)}\frac{4}{3}\right]$

Solution
(a) $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 8 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ Unit vector in the direction of $\overrightarrow{AB}$ is $\frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$
(b) Point A is the mid-point of BC $\Rightarrow \overrightarrow{BA} = \overrightarrow{AC}$ $\Rightarrow -4\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} x-6 \\ y+5 \end{pmatrix}$ $\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -13 \end{pmatrix}$ $\Rightarrow x = 2, y = -13$

(c) $OE : OD = 1 : 1+\lambda$

$$\Rightarrow \overrightarrow{OE} = \frac{1}{1+\lambda}\overrightarrow{OD} = \frac{1}{1+\lambda}\begin{pmatrix}12\\7\end{pmatrix} \quad \text{and} \quad \overrightarrow{BE} = \frac{1}{1+\lambda}\begin{pmatrix}12\\7\end{pmatrix} - \begin{pmatrix}10\\3\end{pmatrix}$$

For $\overrightarrow{BE}$ to be parallel to the x-axis, $\overrightarrow{BE}$ has to be parallel to $\begin{pmatrix}1\\0\end{pmatrix}$

$$\Rightarrow \frac{1}{1+\lambda}\begin{pmatrix}12\\7\end{pmatrix} - \begin{pmatrix}10\\3\end{pmatrix} = \alpha\begin{pmatrix}1\\0\end{pmatrix} \text{ for some real } \alpha$$

$$\Rightarrow \frac{7}{1+\lambda} = 3 \Rightarrow \lambda = \frac{7}{3} - 1 = \frac{4}{3}$$

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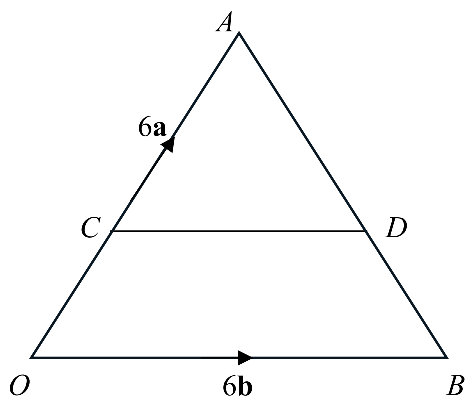


In the above diagram, $\overrightarrow{OA} = 6\mathbf{a}$ and $\overrightarrow{OB} = 6\mathbf{b}$. The points C and D lie on OA and BA respectively such that $OC : OA = BD : BA = 1 : 3$.

(a) Express as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

(i) $\overrightarrow{BD}$,

(ii) $\overrightarrow{CD}$.

(b) What type of quadrilateral is *OBDC*?

(c) State the ratio of the area of triangle *AOB* to area of triangle *ACD*.

[(a)(i) $2(\mathbf{a}-\mathbf{b})$ (ii) $4\mathbf{b}$ (c) 9:4]

Solution

(a) (i) $\overrightarrow{BA} = 6\mathbf{a} - 6\mathbf{b} = 6(\mathbf{a}-\mathbf{b})$. $\overrightarrow{BD} = \frac{1}{3}\overrightarrow{BA} = 2(\mathbf{a}-\mathbf{b})$

(ii) $\overrightarrow{OC} = \frac{1}{3}\overrightarrow{OA} = 2\mathbf{a}$.

$$\begin{aligned}\overrightarrow{CD} &= \overrightarrow{CO} + \overrightarrow{OB} + \overrightarrow{BD}\\ &= -2\mathbf{a} + 6\mathbf{b} + 2(\mathbf{a}-\mathbf{b})\\ &= 4\mathbf{b}\end{aligned}$$

(b) Since $\overrightarrow{CD}$ and $\overrightarrow{OB}$ are parallel, *OBDC* is a trapezium.

(c) Since $OC : OA = 1 : 3$, $OA : CA = 3 : 2$.

Triangles *ACD* and *AOB* are similar, so ratio of area of triangle *AOB* to area of triangle *ACD* is 9:4

12 Solve the following equations.

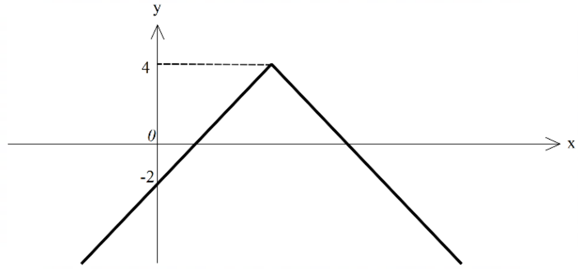
(a) $\left|x^2-2\right|=1$ **(b)** $|2x-3|=|x+5|$ **(c)** $2|x-1|=|4-4x|-6$

[**(a)** $x=\pm\sqrt{3}$ or ± 1 **(b)** $x=-\dfrac{2}{3}$ or 8 **(c)** $x=-2$ or 4]

Solution
(a) $\left|x^2-2\right|=1$ $x^2-2=\pm 1$ $x^2=3$ or $x^2=1$ $x=\pm\sqrt{3}$ or ± 1
(b) $|2x-3|=|x+5|$ $|2x-3|^2=|x+5|^2$ $(2x-3)^2-(x+5)^2=0$ $(3x+2)(x-8)=0$ $x=-\dfrac{2}{3}$ or 8
(c) $2|x-1|=|4-4x|-6$ $2|x-1|=4|x-1|-6$ $|x-1|=3$ $x-1=\pm 3$ $x=4$ or -2

13 **2020/Nan Hua High School/4047/02/Q5(i)-(iv)**

The diagram below shows the graph of $y = a - 2|x - b|$, where a and b are constants. It is given that the y-intercept of the graph is -2 and the maximum value of y is 4.



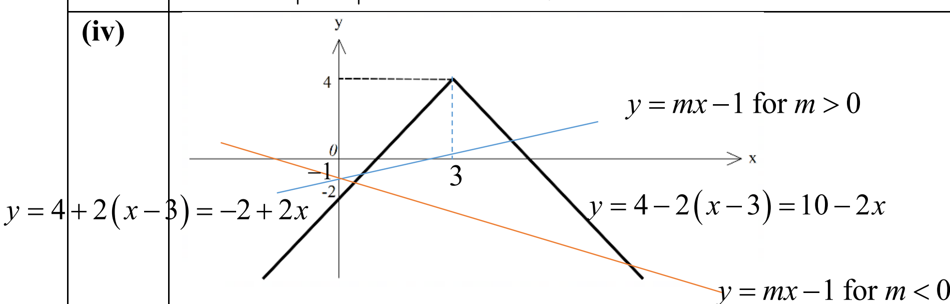
(i) State the value of a. [1]

(ii) Find the value of b. [2]

(iii) State the value of k for which the equation $a - 2|x - b| = k$ has only one solution. [1]

(iv) State the range of values of m for which the equation $a - 2|x - b| = mx - 1$ has exactly two distinct solutions. [2]

[**(i)** 4 **(ii)** 3 **(iii)** 4 **(iv)** $-2 < m < 2$]

Solution	
(i)	$a = 4$
(ii)	$y = 4 - 2\|x - b\|$ When x=0, y=-2. $-2 = 4 - 2\|-b\|$ Since b>0 from graph (when x=b, y=4), $-2 = 4 - 2b$. $\therefore b = 3$
(iii)	For $4 - 2\|x - 3\| = k$ to have only one solution, k=4.
(iv)	 For $4 - 2\|x - 3\| = mx - 1$ to have exactly 2 distinct solutions, If $m > 0$, $m < \frac{4-(-1)}{3-0} = \frac{5}{3}$ If $m < 0$, $m > -2$ Since the line $y = -1$ cuts the graph of $y = 4 - 2\|x - 3\|$ at 2 distinct points, m can be 0. $\therefore -2 < m < \frac{5}{3}$.