RAFFLES INSTITUTION
2025 YEAR 6 PRELIMINARY EXAMINATION
Higher 2

MATHEMATICS 9758/01

Paper 1

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **8** printed pages.

RAFFLES INSTITUTION

Mathematics Department

© RI 2025

1 A curve has equation $y = ax + b + \dfrac{c}{x^2 - 1}$, where a, b and c are real constants. It is given that the curve crosses the x-axis at $x = 2$. The normal to the curve at the point $(0, 3)$ meets the x-axis at $x = 7.5$. Find the values of a, b and c. [4]

2 The point P travels along the curve C with equation $y = x\sin^{-1}x$, $-1 < x < 1$. Let the gradient of the curve C at the point P be m.
If the x-coordinate of P is increasing at the rate of 9 units per second when $x = \dfrac{1}{2}$, find the exact value of the rate at which m is changing at this instant. [4]

3 Relative to the origin O, the points A and C have position vectors $\mathbf{a}$ and $2\mathbf{b}$ such that

$$\mathbf{a} = 5p\,\mathbf{i} - 2p\,\mathbf{j} + 4p\,\mathbf{k} \text{ and } \mathbf{b} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k},$$

where p is a positive constant. It is given that $|\mathbf{a}| = 2|\mathbf{b}|$.

(a) Find the exact value of p. [2]

(b) Evaluate $(\mathbf{a} + 2\mathbf{b})\cdot(\mathbf{a} - 2\mathbf{b})$. [1]

(c) Evaluate $\mathbf{a} \times \mathbf{b}$ and hence find the area of triangle OAC. [3]

(d) Use a geometrical reason to explain why $|\mathbf{a} \times \mathbf{b}| = \dfrac{1}{4}(|\mathbf{a} + 2\mathbf{b}|)(|\mathbf{a} - 2\mathbf{b}|)$. [2]

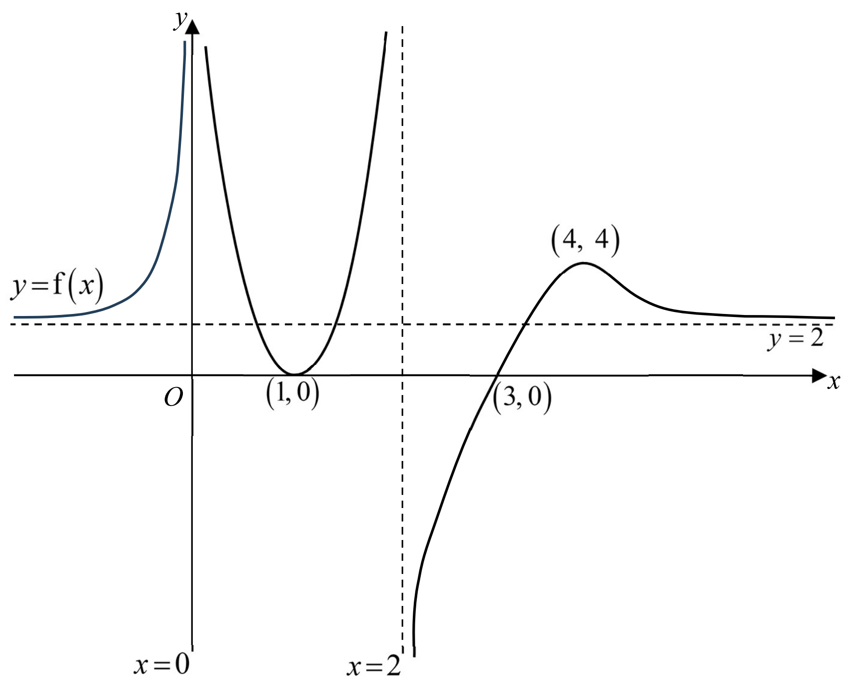
4 **(a)** Find $\displaystyle\int \frac{9x}{(2x-1)(x+1)^2}\,\mathrm{d}x$. [4]

(b) **(i)** Differentiate $\dfrac{1}{x^2 + 1}$ with respect to x. [1]

(ii) Differentiate $\ln\sqrt{x^2 + 1}$ with respect to x. [1]

(iii) Hence find $\displaystyle\int \frac{x\ln\sqrt{x^2+1}}{(x^2+1)^2}\,\mathrm{d}x$. [4]

5 **(a)** The diagram below shows a sketch of the graph of $y = \mathrm{f}(x)$. The curve passes through the points with coordinates $(1, 0)$ and $(3, 0)$, and has turning points at $(1, 0)$ and $(4, 4)$. The asymptotes are $x = 0$, $x = 2$ and $y = 2$.



Sketch on separate diagrams, the graphs of

(i) $y = \mathrm{f}'(x)$, [3]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

showing clearly the main features of the graphs.

(b) Describe a sequence of transformations which transform the graph of $y = \dfrac{x^2+1}{x}$ to the graph of $y = \dfrac{2x^2-5x+4}{x-2}$. [3]

6 **(a)** A geometric series has first term 2 and common ratio 0.8. Find algebraically, the least value of m for the sum of the first $4m$ terms of the series to be greater than 99% of the sum to infinity. [3]

(b) A finite arithmetic progression A has n terms, first term a and common difference d.

In another progression B, the kth term is obtained by adding the kth positive odd integer to the corresponding term of A. That is, the first term of B is obtained by adding 1 to the first term of A, the second term of B is obtained by adding 3 to the second term of A and so on.

It is given that the twelfth term of A is 25, the sum of all the terms of A is 676 and the sum of all the terms of B is twice the sum of all the terms of A.

(i) Find the values of n, a and d. [4]

(ii) Obtain the sum of the first ten terms of B. [2]

7 The function f is defined by $\mathrm{f}: x \mapsto 1+\dfrac{3}{\mathrm{e}^{x}-2}$ for $x \in \mathbb{R}$, $x > \ln 2$.

(a) Find $\mathrm{f}^{-1}(x)$ and state its domain. [3]

Another function g is defined by

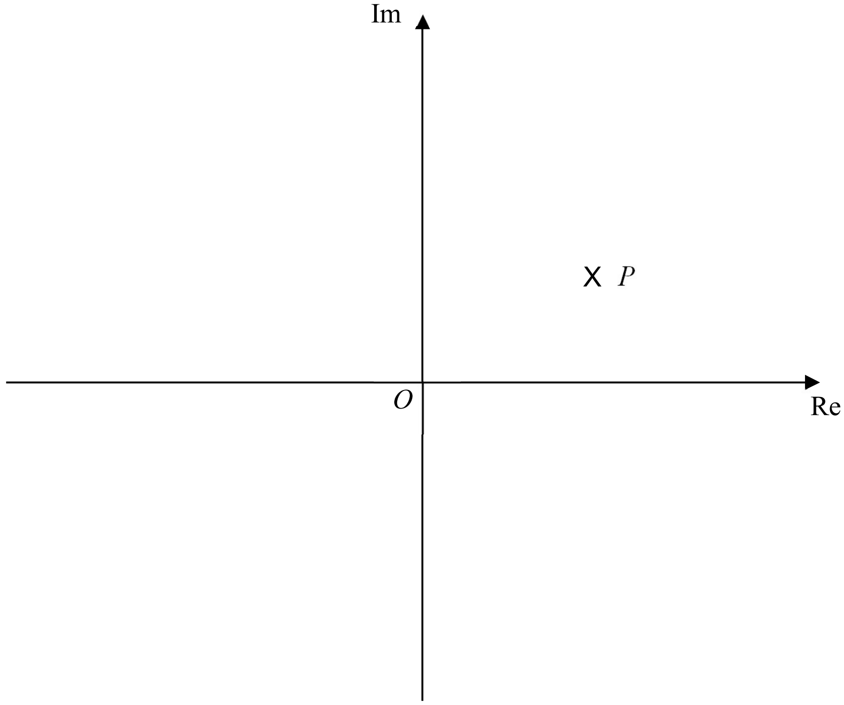
$$\mathrm{g}: x \mapsto \begin{cases} 2(x-1)^2+2 & \text{for } x \leq 0, \\ 2+|2-x| & \text{for } 0 < x \leq 3. \end{cases}$$

(b) Sketch the graph of $y=\mathrm{g}(x)$ and explain why the composite function $\mathrm{f}^{-1}\mathrm{g}$ exists. [4]

(c) Hence find the exact value of k such that $\mathrm{f}^{-1}\mathrm{g}(k)=\ln\dfrac{5}{2}$. [3]

8 **Do not use a calculator in answering this question.**

(a)



The point P on the Argand diagram represents the complex number w given by $w = u + \mathrm{i}v$, where u and v are real and positive.

(i) Explain algebraically why $\arg(kw) = \arg(w)$ for any real constant $k > 1$. [1]

Points Q and R represent the complex numbers kw and $\mathrm{i}kw$, where k is a real constant and $k > 1$.

(ii) On the same Argand diagram in the Printed Answer Booklet, plot the points Q and R. Show clearly the geometrical relationship between the points P, Q and R. [2]

(b) In another Argand diagram, the points A, B and C represent the complex numbers z, $\mathrm{f}(z)$ and $\mathrm{f}(\mathrm{f}(z))$ respectively, where $\mathrm{f}(z) = z^2 - 2z$.

(i) Show that $\mathrm{f}(\mathrm{f}(z)) - \mathrm{f}(z) = z(z-2)(z-3)(z+1)$. [2]

It is given that ABC is a right-angled triangle, described in an anticlockwise sense, with a right angle at B, and $BC = mBA$, where m is a positive real constant.

(ii) By considering $\dfrac{\mathrm{f}(\mathrm{f}(z)) - \mathrm{f}(z)}{\mathrm{f}(z) - z}$, or otherwise, show that

$$(z-2)(z+1) = m\mathrm{i}.$$ [2]

(iii) In the case where $z = x + 2\mathrm{i}$, where x is a positive real number, find x and m. Hence obtain the complex number represented by the point B. [5]

9 Decontamination is a water treatment method to remove a certain undesirable chemical from contaminated water. As the liquid agent is continuously added to the contaminated water in a large tank, the undesirable chemical in the water is removed gradually.

The rate of decrease of the concentration of the undesirable chemical is proportional to the product of its current concentration and the rate at which the liquid agent flows into the tank. At time t minutes after the start of the treatment process, the concentration of the undesirable chemical is C mg/L and the liquid agent flows into the tank at a rate of $\dfrac{1}{1+t^2}$ L/min. It is known that the initial concentration of the undesirable chemical in the tank is 50 mg/L and after 10 minutes, its concentration is measured to be 20 mg/L.

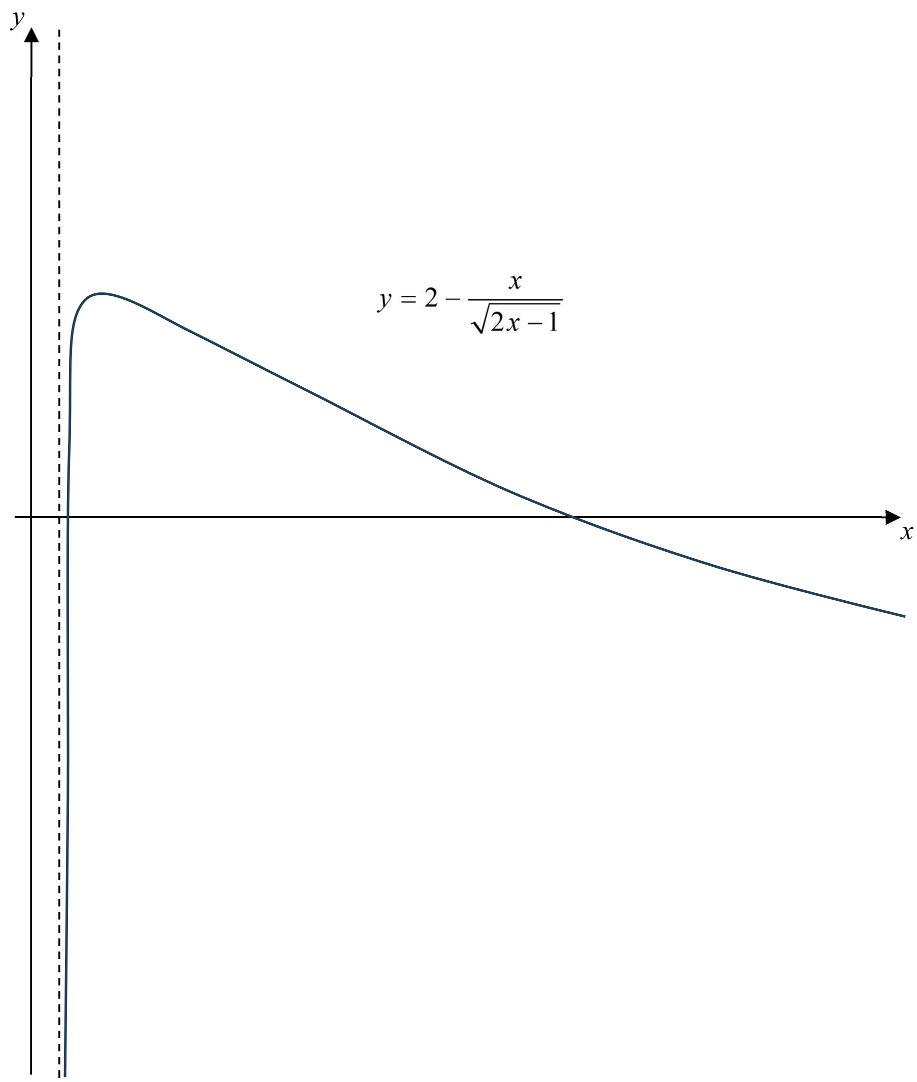
(a) By setting up and solving a differential equation relating C and t, show that $C = 50\mathrm{e}^{q\tan^{-1}t}$, giving the value of q correct to 5 decimal places. [7]

(b) Determine the value of C when $t = 50$. [1]

(c) Sketch the graph of C against t and state what happens to C in the tank for large values of t. [3]

10 **(a)** Using the substitution $u = \sqrt{2x-1}$, show that $\int \frac{x}{\sqrt{2x-1}}\,\mathrm{d}x = \int \frac{u^2+1}{2}\,\mathrm{d}u$. [2]

(b) **(i)** The diagram below shows the graph of $y = 2 - \frac{x}{\sqrt{2x-1}}$. Indicate on the same diagram in the Printed Answer Booklet, the equation of the asymptote of the curve and the coordinates of its turning point and points of intersection with the x-axis. [2]



(ii) The region R is bounded by the curve $y = 2 - \frac{x}{\sqrt{2x-1}}$ and the lines $x = 1$, $x = 3$ and $y = \frac{1}{2}$. Find the exact area of R. [5]

(c) The region S is bounded by the curves $y=2-\dfrac{x}{\sqrt{2x-1}}$, $x=2(y-1)^2+1$, the lines $x=1$, $x=4$ and the x-axis.

(i) Express $x=2(y-1)^2+1$ in the form $y=\mathrm{f}(x)$. [1]

(ii) On the same diagram as in part **(b)(i)**, sketch the graph of $x=2(y-1)^2+1$. [2]

(iii) Find the volume of the solid generated when S is rotated through 2π radians about the x-axis. Give your answer correct to 3 decimal places. [3]

11 A curve C has parametric equations

$$x=2\cos\theta+\cos 2\theta, \qquad y=2\sin\theta-\sin 2\theta,$$

where $0<\theta<2\pi$ and $\theta\neq\dfrac{2\pi}{3},\ \pi,\dfrac{4\pi}{3}$.

The point P with parameter p lies on C, and the lines M_p and N_p are the tangent and normal to C at P respectively.

(a) It is given that

$$\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \quad \text{and}$$
$$\cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}.$$

Using these two results, show that the gradient of M_p is $-\tan\left(\dfrac{p}{2}\right)$. [2]

(b) For $p=\dfrac{\pi}{3}$, M_p and N_p cut the x-axis at points Q and R respectively. Find the area of ΔPQR. [4]

(c) The point S with parameter s lies on C, and the line N_s is the normal to C at S. It is given that M_p and N_s are parallel.

Given $p\neq\dfrac{\pi}{3},\ \dfrac{5\pi}{3}$ and $p<s$, by using the result $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$, find an equation relating p and s. [2]