10 WAVE MOTION

H2 Physics 9478

Content **Page**

Learning Outcomes

Candidates should be able to:

(a) show an understanding that mechanical waves involve the oscillations of particles within a material medium, such as a string or a fluid, and electromagnetic waves involve the oscillations of electromagnetic fields in space and time.

(b) show an understanding of and use the terms displacement, amplitude, period, frequency, phase, phase difference, wavelength, and speed.

(c) deduce, from the definitions of speed, frequency and wavelength, the equation $v = f\lambda$.

(d) recall and use the equation $v = f\lambda$.

(e) analyse and interpret graphical representations of transverse and longitudinal waves with respect to variations in time and position (space).

(f) show an understanding that energy is transferred due to a progressive wave without matter being transferred.

(g) recall and use the term intensity as the power transferred (radiated) by a wave per unit area, and the relationship intensity $\propto$ (amplitude)^2 for a progressive wave.

(h) show an understanding of and apply the concept, that the intensity of a wave from a point source and travelling without loss of energy obeys an inverse square law, to solve problems.

(i) show an understanding that polarisation is a phenomenon associated with transverse waves.

(j) recall and use Malus' law (intensity $\propto \cos^2\theta$) to calculate the amplitude and intensity of a plane polarised electromagnetic wave after transmission through a polarising filter.

10.1 Progressive Waves

A wave can be described as a **disturbance** that travels through a medium from one location to another location. Waves can be considered as an **energy** transport phenomenon because energy is transferred from one point to another some distance away in the direction of wave propagation. This chapter will focus on **progressive waves**.

Progressive Wave

A progressive wave transports energy from one point to another in the direction of wave propagation.

❖ Wave Terminologies

Displacement x of a particle is the distance in a specific direction of a particle of a wave from its equilibrium position.

Amplitude x_0 or A is the magnitude of the maximum displacement of a particle in the wave from its equilibrium position.

Period T is the time taken for a particle of a wave to complete one oscillation.

Frequency f of a wave is the number of oscillations made per unit time.

$\boxed{f = \frac{1}{T}}$, unit: hertz (Hz)

Wavelength λ of a wave is the distance between two consecutive points which are in phase.

Speed v of a wave is the speed at which the wave shape (or wave profile) moves.

Phase ϕ of a wave is an angle that gives a measure of the fraction of a cycle that has been completed by an oscillating particle or by a wave.

One oscillation cycle is equivalent to a phase angle of 2π rad.

Phase difference $\Delta\phi$ between two particles in a wave or between two waves at a point is a measure of the fraction of a cycle which one is ahead of the other.

Wavefront is a line or surface joining points on a wave that are **in phase**. The wave travels in a direction perpendicular to the wavefront.

❖ Mechanical Waves and Electromagnetic Waves

Mechanical waves are waves that require a medium to propagate. An example is a rope connected to an oscillatory source, producing waves that travel along the rope, as shown in Fig. 10.1(a).

Electromagnetic (EM) waves do not require a medium to propagate, and hence are not mechanical waves. This is how EM waves, such as visible light, are able to travel from the Sun to reach the Earth through vacuum. Fig. 10.1(b) shows an EM wave travelling in the *z* direction, with perpendicular electric and magnetic fields in the *x*- and *y* directions.

Fig. 10.1(a)

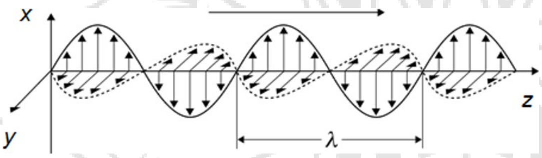


Fig. 10.1(b)

❖ Oscillations in waves

In a progressive wave, particles **oscillate** about their equilibrium positions. The particles in the medium **do not move** along with the wave. Hence, a progressive wave **transports energy** from one point to another, but **do not transport matter**.

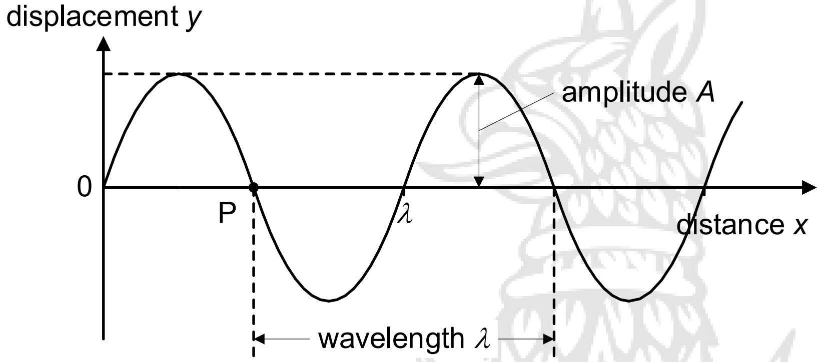
In the video (scan QR code to view), we can observe how the wave travels to the right, but the particles in the video do not move in the horizontal direction – they oscillate vertically about their equilibrium positions, and not in the direction of the wave propagation.

Illustration of how particles in a progressive wave oscillate

In an electromagnetic (EM) wave, there is no particle involved in oscillations. Instead, the electric and magnetic fields of the EM wave oscillate perpendicularly to each other.

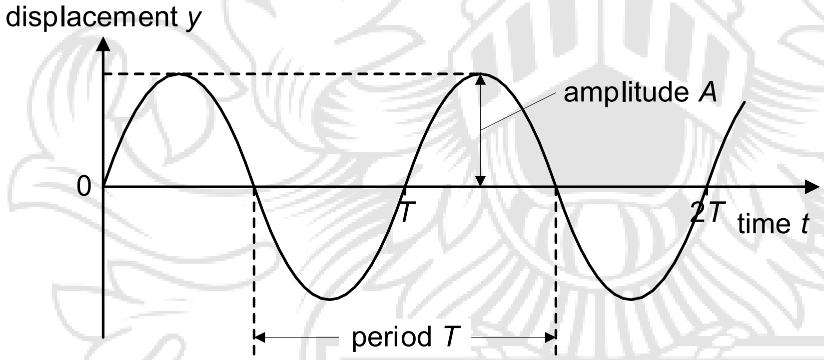
❖ **Graphical Representation of Waves**

Waves can be represented graphically in two ways:



This is a snapshot of a wave.

Displacement-distance graph shows the displacement of **ALL** particles in a wave at a particular instant of time.

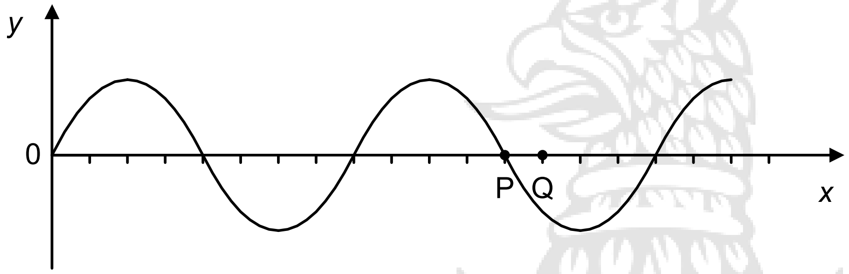


Displacement-time graph shows the displacement of **ONE** particle (P) in a wave over time.

Example 1

The diagram shows the variation with distance x of the displacement y of a transverse wave at a particular instant. The wave is travelling to the right. The frequency of the wave is 12.5 Hz.

Hint: Think about the distance between point P and point Q in terms of the wavelength of the wave.



At the instant shown, the displacement is zero at the point P.

Determine the shortest time to elapse before the displacement is zero at point Q.

[Solution]

❖ **Speed of a Progressive Wave**

The particles of a wave do not move with the propagation of the wave.

They only oscillate about their equilibrium positions with different phases within one wavelength.

The result is that a waveform is created that moves in the direction of energy transfer.

In a time of one period T, the waveform moves a distance of one wavelength λ.

The speed of the wave

$$v = \frac{\text{distance}}{\text{time}} = \frac{\lambda}{T}$$

Since $f = \frac{1}{T}$,

$$\boxed{v = f\lambda}$$

❖ Phase and Phase Difference

The **phase** of a particle in oscillation is the fraction of an oscillation cycle it is at relative to its starting displacement.

Refer to pg. 3 of Oscillations lecture note.

The **phase difference** between two particles is how much one particle is "ahead" or "behind" the other as a fraction of an oscillation cycle at any given instant. It is typically measured in radian or degree.

Consider the particles A, B, C & D in a wave (Fig. 10.2). All the particles are in simple harmonic motion about their equilibrium positions. The wave profile at the next instance is illustrated by the dotted line.

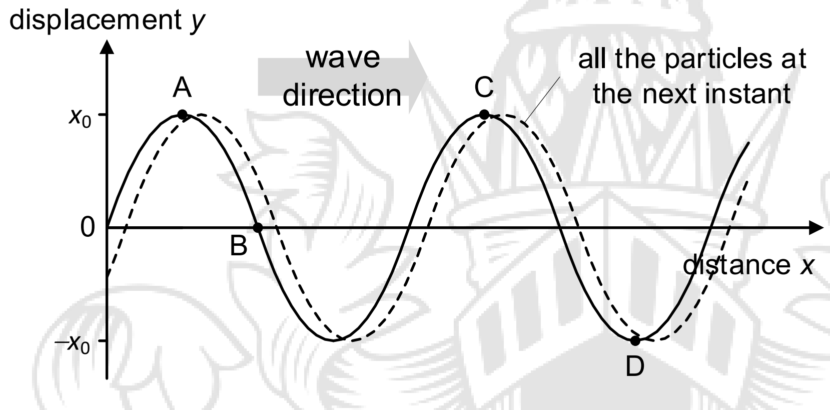


Fig. 10.2

	displacement		velocity		acceleration	
	mag.	dir.	mag.	dir.	mag.	dir.
A	x_0	up	0	-	$\omega^2 x_0$	down
B	0	-	ωx_0	up	0	-
C	x_0	up	0	-	$\omega^2 x_0$	down
D	x_0	down	0	-	$\omega^2 x_0$	up

Two particles in a wave oscillate **in phase** when they are at the same fraction of their oscillation cycle relative to their starting displacements. These particles have a phase difference of 0 rad or $0°$.

The phase of a particle is independent of its amplitude.

Particles that are at different fractions of an oscillation cycle are **out of phase** with one other. These particles have a non-zero **phase difference**.

Two particles in a wave oscillate **in anti-phase** when there is a difference of 1/2 an oscillation cycle in their phase. These particles have a phase difference of π rad or $180°$.

One oscillation cycle corresponds to a phase angle of 2π rad.

If the starting displacements of all particles are at their respective equilibrium positions, particles **A** and **C**, are both at 1/4 of their oscillation cycle. Hence, **A** and **C** are **in phase**.

On the other hand, particle **B** is at 1/2 of its oscillation cycle while particle **D** is at 3/4 of its oscillation cycle. Hence **A** (or **C**) and **D** have a phase difference of π rad while **A** (or **C**) and **B** have a phase difference of $\pi/2$ rad.

❖ Calculating Phase Difference

The phase difference between two particles in a wave can be determined in two ways.

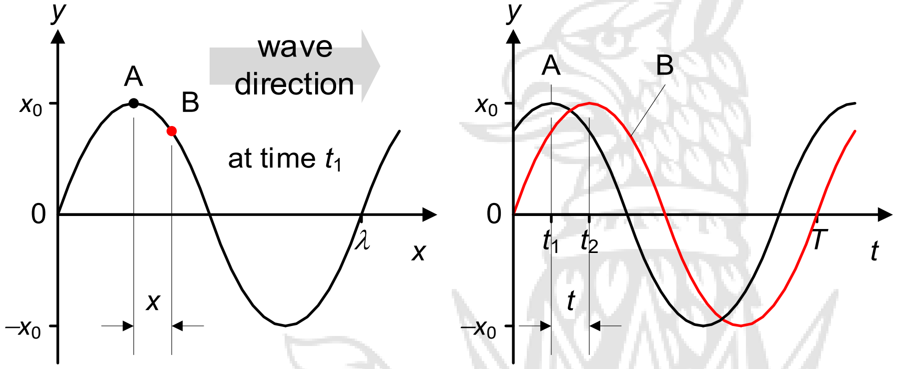


Fig. 10.3(a) **Fig. 10.3(b)**

Fig. 10.3(a) shows two particles **A** and **B** in a wave separated by a distance *x* at time t_1.

B is lagging behind A by a distance *x*.

The phase difference between A and B is

$$\Delta\phi = \frac{x}{\lambda} \times 2\pi$$

Fig. 10.3(b) shows the displacement-time graph of A and B. From this representation, the phase difference between A and B is

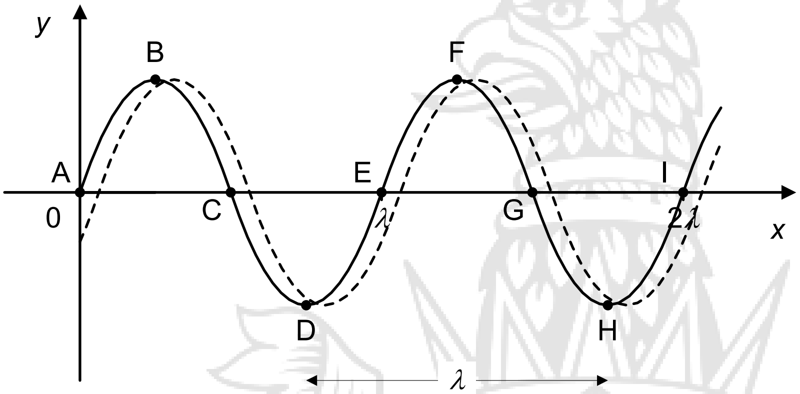
B is lagging behind A by a time *t*.

$$\Delta\phi = \frac{t}{T} \times 2\pi$$

Note that both methods of calculating the phase difference between two particles yield identical results.

Example 2

Consider a wave moving to the right at a certain instant as shown below where the diagram represents the variation with distance *x* of displacement *y* at time t_1.



9 particles on the wave are labelled A to I.

The dotted line shows the position of the wave at time t_2 (a short time interval later) and the motion of the 9 particles during this time interval are indicated by the arrows.

Complete the table below.

Particle		Particle
A	is in phase with	
B	is in phase with	
C	is in phase with	
D	is in phase with	
E	is in phase with	

Note

In a progressive wave, all particles within a wavelength are **out of phase** with one another.

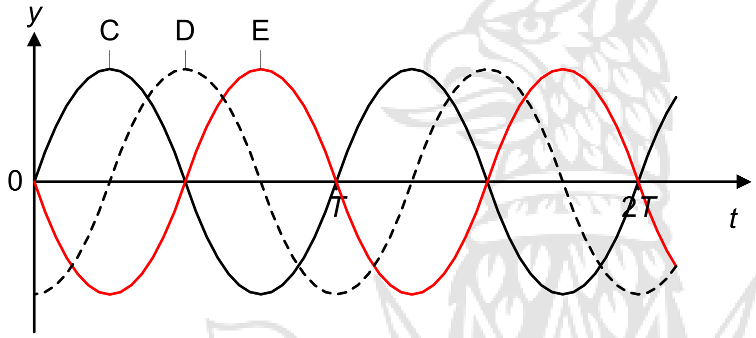
A has maximum downward velocity while **C** has maximum upward velocity.

B and **D** are momentarily at rest.

Example 3

Refer to the points **C**, **D** and **E** in Example 2.

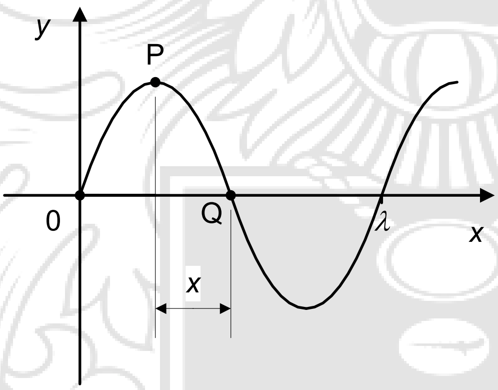
The diagram below shows the displacement-time graph of point **C**.



On the same axes, sketch a graph which represents the point **D** and **E**.

Example 4

For a wave that propagates to the right, determine the phase difference $\Delta\phi$ between points P and Q.



[Solution]

10.2 Intensity of a Wave

The intensity I of a wave is defined as the **rate of transfer of energy per unit area normal** to the **direction** of **energy transfer** of the wave.

$$I = \frac{P}{S}$$

where P is the power of the source and S is the surface area.

The energy of a wave comes from the oscillations of its electric and magnetic fields (electromagnetic radiation) or from the oscillation of the particles in the medium (mechanical waves).

For mechanical waves, the total energy of an oscillating particle is

$$E = \tfrac{1}{2} m\omega^2 A^2$$

if it is oscillating in simple harmonic motion.

Hence, the intensity I is also proportional to the square of its amplitude A and the square of its frequency f:

$$I \propto A^2 f^2$$

If frequency f is constant,

$$I = kA^2$$

where k is a proportionality constant.

Intensity of a Wave
The intensity I of a wave is defined as the **rate of transfer of energy per unit area normal** to the **direction** of **energy transfer** of the wave.

❖ Point Source of Wave & the Inverse Square Law

Fig. 10.4 shows a point source emitting waves that spread out radially in all directions. Energy will spread over an expanding spherical wavefront with an increasingly larger area as the wave travels further away from the source. Hence, the intensity of the wave decreases.

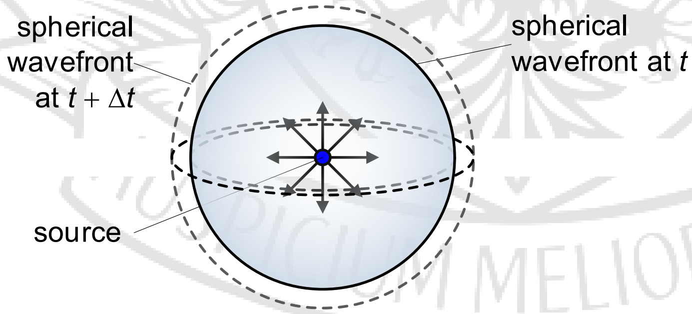


Fig. 10.4

The intensity at a point on a spherical wavefront is

$$I = \frac{P}{4\pi r^2}$$

where P is the power of the source, and r is the distance from the source.

If the power P of the source remains constant, then the intensity is inversely proportional to the square of the distance from the source r:

$$I = \frac{k}{r^2}$$

This is known as the **inverse square law**.

Example 5

A constant power source of 500 W emits light uniformly in all directions.

(a) Calculate the intensity of the light detected by a man located 3.0 m from the source.

(b) The power of the source is now halved. In order to detect the same intensity as before, determine the distance of the man from the source.

[Solution]

❖ Plane Waves & Cylindrical Waves

Plane waves are waves whose wavefronts are parallel planes normal to the direction of energy transfer. At large distance from a point source, spherical waves gradually become plane waves. The intensity of the plane waves remains constant with distance.

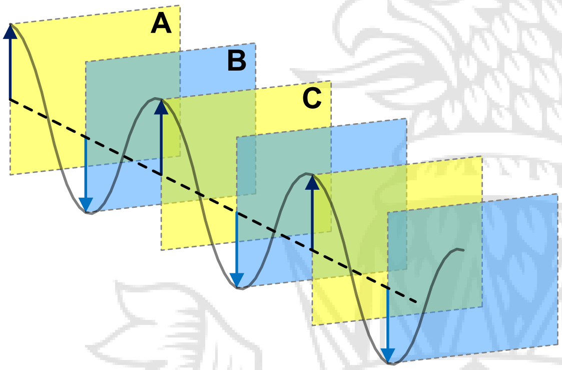


Fig. 10.5

Wavefronts **A** and **B** are **anti-phase**. Wavefronts **A** and **C** are **in phase**.

Cylindrical waves are waves whose wavefronts form a family of coaxial cylinders. Cylindrical waves can be formed when plane wavefronts undergo diffraction after passing through a narrow, rectangular slit. The intensity of the cylindrical waves is **inversely proportional to distance** from source.

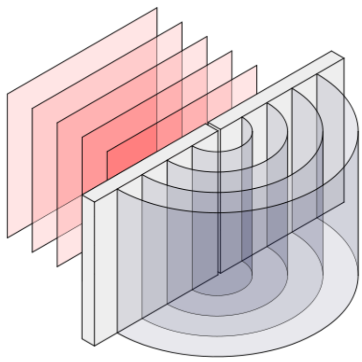


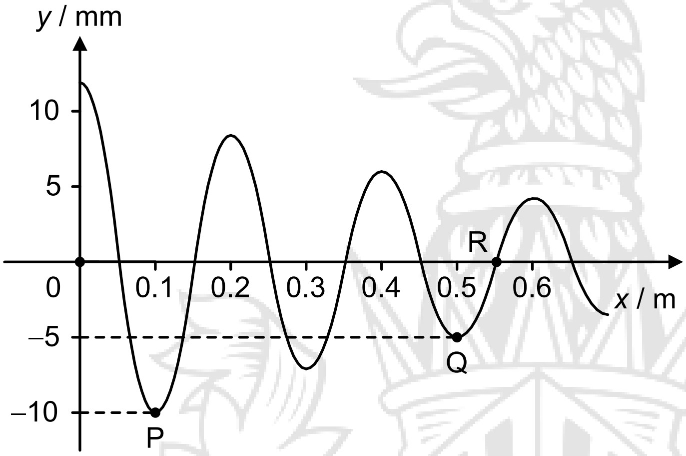
Fig. 10.6

A plane wave undergoing diffraction to become a **cylindrical wave**.

Example 6

The diagram below shows a portion of the displacement *y* against distance *x* graph of a wave.

Use the diagram to answer the following questions



(a) State the wavelength of the wave.

(b) The frequency of the wave is 10.0 Hz.

Calculate the speed of the wave.

(c) Calculate the phase difference between particles P and R.

(d) Calculate the ratio $\dfrac{\text{intensity at Q}}{\text{intensity at P}}$.

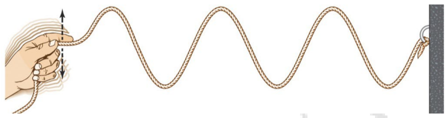
[Solution]

10.3 Transverse and Longitudinal Waves

❖ Comparisons Between Transverse and Longitudinal Waves

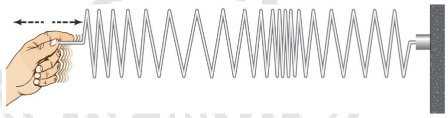
A **transverse wave** is one in which its particles oscillate in a direction **perpendicular** to the direction of energy transfer.

Example: electromagnetic (EM) waves, pulses in ropes



A **longitudinal wave** is one in which its particles oscillate in a direction **parallel** to the direction of energy transfer.

Example: sound, longitudinal pulses in springs



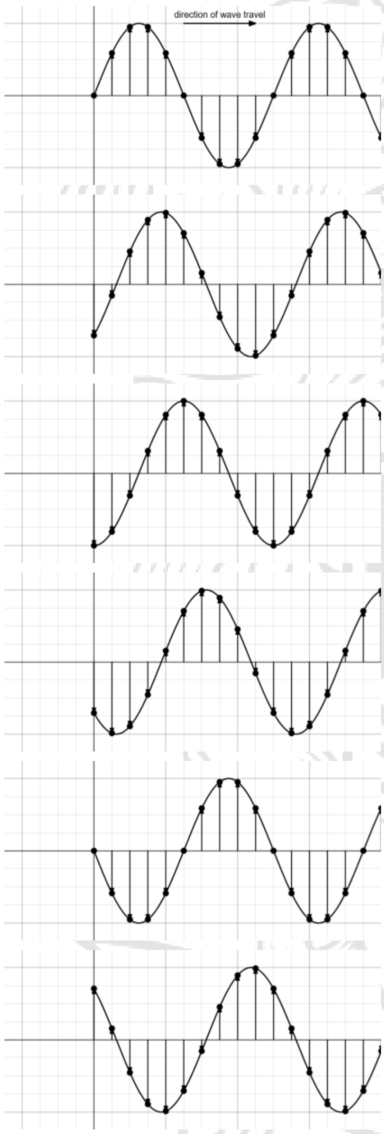


Fig. 10.7(a)

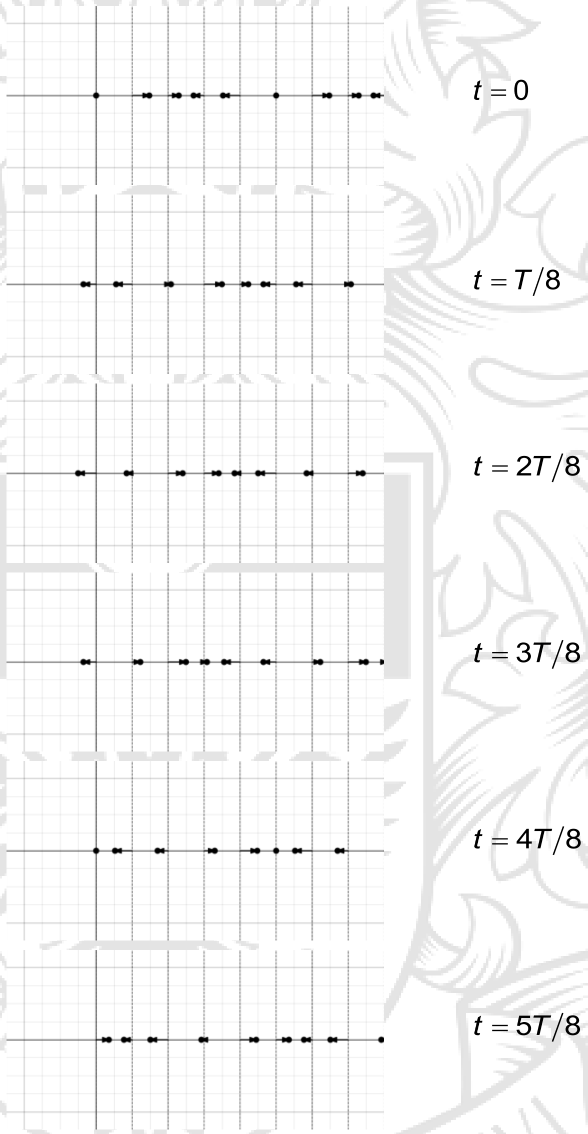


Fig. 10.7(b)

❖ Electromagnetic (EM) Waves

The most important and common type of transverse waves is the electromagnetic (EM) waves. They are generated by oscillating electric charges and are made up of oscillating electric and magnetic field vectors which are perpendicular to each other and the direction of energy transfer.

EM Waves

Practical Uses of EM Waves

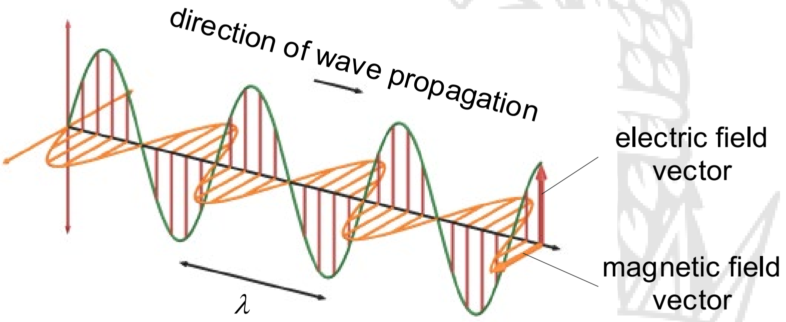


Fig. 10.8

❖ Properties of Electromagnetic Waves

1. EM waves do not require a medium to propagate; they can travel through vacuum.
2. EM waves travel at the speed of $c = 3.00 \times 10^{8}$ m s^{-1} in vacuum. (No object can travel at the speed of light.)
3. EM waves are transverse waves, and hence they can be polarised.

❖ The Electromagnetic Spectrum

Electromagnetic wave is classified into 7 types according to the frequency of the wave.

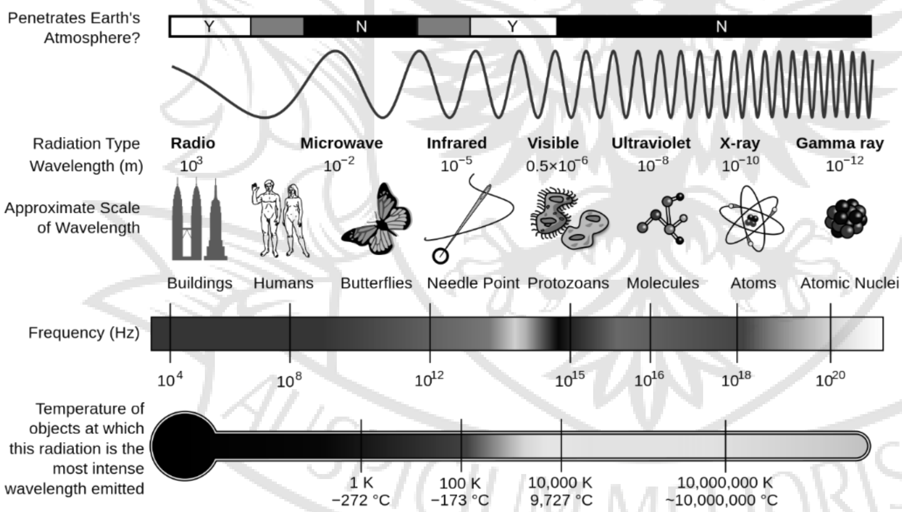


Fig. 10.9

❖ **Sound Waves**

The most important and common type of longitudinal waves is the sound waves.

The particles of the medium in which sound wave propagates, oscillate about their equilibrium positions in a direction **parallel** to the direction of energy transfer of the wave.

The positions of the air particles in a sound wave at an instant is shown in Fig. 10. 10. The equilibrium positions of the particles are represented by the dashed vertical lines, and their displacement with respect to their equilibrium positions are indicated by the arrows.

The **compression** are regions where air particles are closest together and these regions are at a pressure higher than atmospheric pressure. The **rarefactions** are regions where the air particles are spread furthest apart and these regions are at a pressure lower than atmospheric pressure. The regions at the midpoint between adjacent compression and rarefaction is at atmospheric pressure (pressure variation $\Delta p = 0$).

Since a sound wave consists of a repeating pattern of high-pressure and low-pressure regions moving through a medium, it is sometimes referred to as a pressure wave.

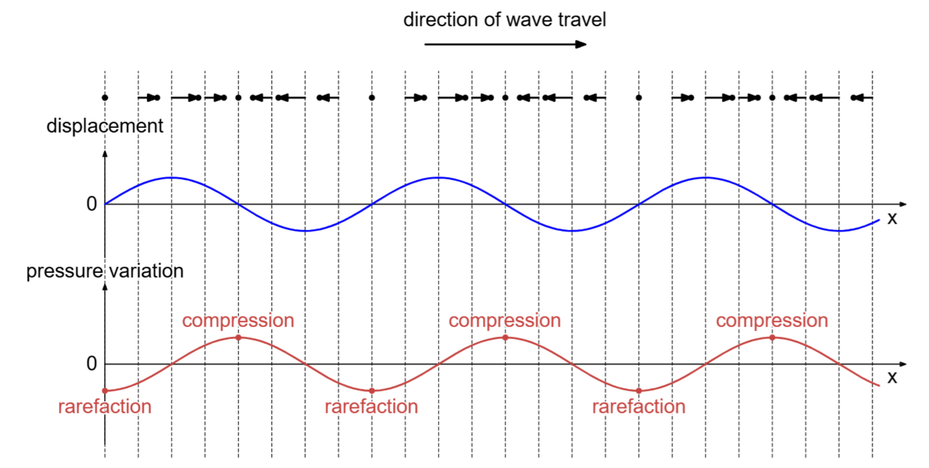


Fig. 10.10

At compression, the pressure variation $\Delta p =$ positive .

At rarefactions, the pressure variation $\Delta p =$ negative .

10.4 Polarisation

Polarisation is a phenomenon associated only with **transverse wave** whereby the oscillations of the wave particles are restricted to only one direction which is **perpendicular** to the direction of energy transfer.

Longitudinal waves cannot be polarised as their particles oscillate **parallel** to the direction of energy transfer.

❖ Polarisation of Light (EM Wave)

Since the oscillation of the electric field vector of light (EM wave) is perpendicular to the direction of energy transfer, light can be polarised. The **direction of polarisation** of light is defined to be the direction of oscillation of the **electric field vector**.

In general, EM waves are unpolarised, i.e., all planes of polarisation are present. Fig. 10.11 shows an unpolarised EM wave where the electric field vectors have components in all directions.

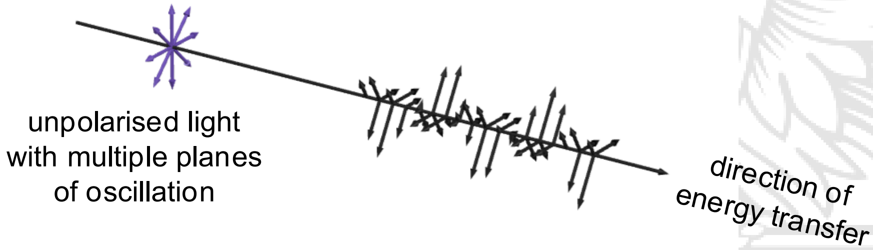


Fig. 10.11

❖ Intensity of Plane Polarised Light

When an unpolarised light (with multiple planes of oscillation) passes through a linear polariser (Fig. 10.12), a **linearly polarised light** (with one plane of oscillation) is produced. This light is also said to be **linearly polarised**.

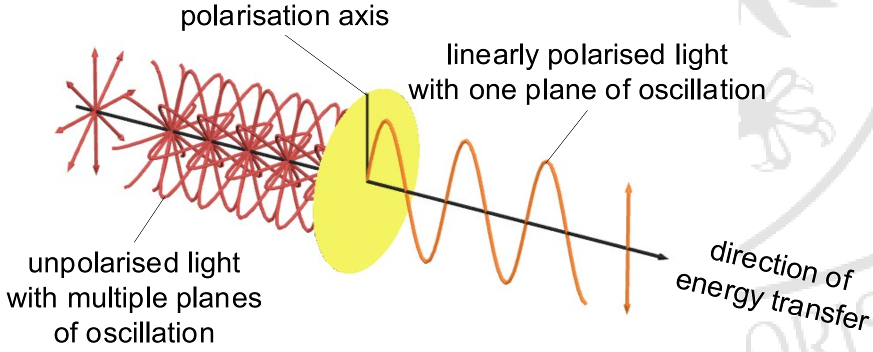


Fig. 10.12

It is not possible to compare amplitudes between unpolarised and polarised light.

The intensity of the transmitted light is exactly **half** that of the incident unpolarised light regardless of the direction of polarising axis. This is because the unpolarised light has a random mixture of all polarisation directions and the components of the electric field vectors that are perpendicular or parallel to the polarising axis must be equal.

Since only the component of the electric field vectors parallel to the polarising axis is transmitted, the intensity of the polarised light is halved.

❖ Malus' Law

When polarised light is incident on a second polariser (usually called an analyser) placed with its polarising axis at an angle θ with the polarising axis of the first polariser, only the electric field component parallel to the polarising axis of the analyser will be transmitted.

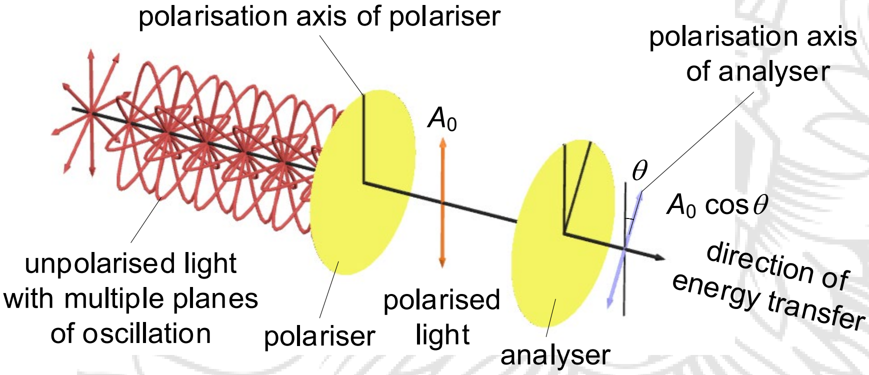


Fig. 10.13

The resultant amplitude A of the transmitted wave is therefore

$$A = A_0 \cos\theta$$

The amplitude of the transmitted wave is greatest when $\theta = 0$ and zero when the polariser and analyser are crossed so that $\theta = 90°$.

Intensity of an electromagnetic wave is proportional to the square of the amplitude of the wave. This implies that the intensity I of the transmitted wave can be expressed as

$$I = I_0 \cos^2\theta$$

where I_0 is the intensity of the light before passing through the analyser.

This equation is known as **Malus' Law**.

Example 7

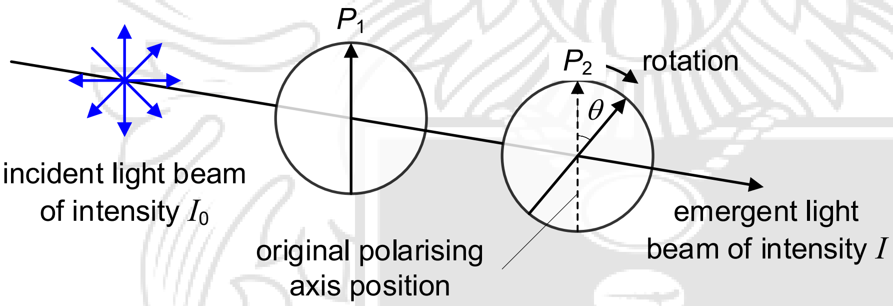
A beam of plane polarised light of intensity I_0 falls normally onto a thin sheet of polaroid. The transmitted beam has an intensity of $I_0/4$.

Determine the angle θ between the plane of incident polarisation and the polarising axis of the polaroid.

[Solution]

Example 8

A beam of unpolarised light with intensity I_0 passes in turn through polaroids P_1 and P_2. P_1 is fixed in position while P_2 is initially aligned with P_1. P_2 is then rotated through $360°$ as shown.

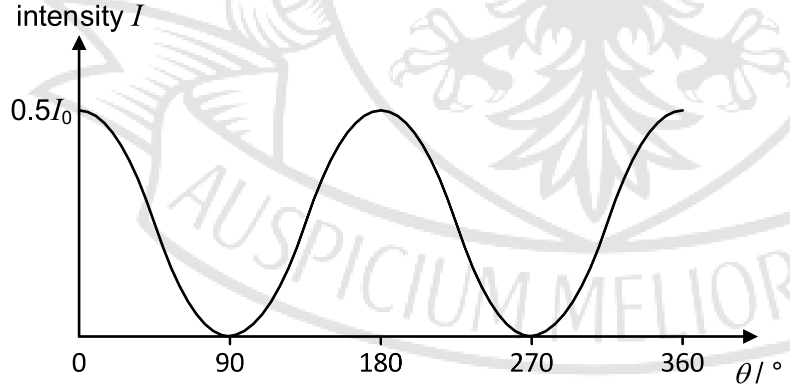


Sketch a graph on the axes provided to show how the intensity I of the emergent beam varies with the angle of rotation.

[Solution]

According to Malus' Law,

$$I = I_0 \cos^2 \theta$$



10.5 Appendix

❖ Measuring Frequency of a Sound Wave

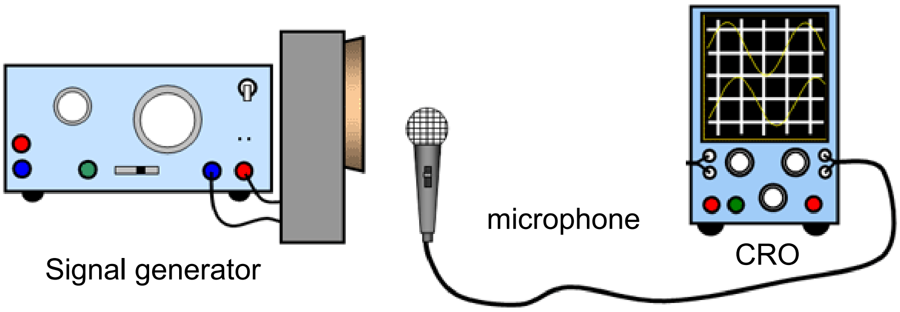
An oscilloscope is a type of electronic test instrument that graphically displays varying signal voltages, usually as a two-dimensional plot of one or more signals as a function of time. Other signals (such as **sound or vibration**) can be converted to voltages and displayed.

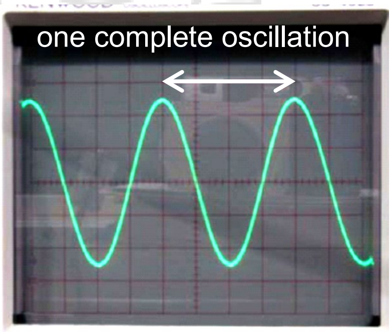
Oscilloscopes display the change of an electrical signal over time, with voltage and time as the *y* and *x* axes, respectively, on a calibrated scale. The waveform can then be analysed for properties such as amplitude, frequency, rise time, time interval, distortion, and others.

Modern digital instruments may calculate and display these properties directly. For the H2 syllabus, the calculation of these values requires measuring the waveform against the scales built into the screen of the instrument.

As discussed earlier, sound waves produce regions of compressions and rarefactions as they travel through air. This gives rise to pressure variations as the wave travels.

Hence by placing a microphone in front of a sound source (or a signal generator), the microphone should be able to detect a continuous series of compressions and rarefactions over time. If the microphone is connected to a cathode ray oscilloscope (CRO), the CRO will be able to display a variation of the pressure experienced by the microphone with respect to time.





When a signal is viewed on the CRO display, the period of the signal can be measured by counting the number of horizontal divisions that the signal "covers" on the display and multiplying by the scale of each division. This scale is the “time base”.

For the display show, the time base has been set to 1 ms div^{-1}.

Hence, $T = 4 \text{ divisions} \times 1 \text{ ms div}^{-1} = 4 \text{ ms}$

$$\therefore f = \frac{1}{T} = \frac{1}{0.004} = 250 \text{ Hz}$$