CHIJ ST. THERESA'S CONVENT

PRELIMINARY EXAMINATION 2022

SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/2

Paper 1

30 August 2022

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 A spherical bath bomb of radius r cm is dissolving in such a way that the total surface area, A cm^2, is decreasing at a rate of 12 cm^2/s. Assuming that the bath bomb retains its shape, calculate the rate of change of r when $r = 5$, leaving your answer in terms of π. [4]

2 The equation of a curve is $y = 2x^2 - kx - 5$, where k is a constant, and the equation of a line is $y - 3x - 10 = 0$.

(i) In the case where k = 4, find the coordinates of the points of intersection of the line with the curve. [3]

(ii) Show that, for all real values of k, the line intersects the curve at two distinct points. [2]

3 **(i)** Given that the constant term in the binomial expansion of $\left(x^3+\frac{k}{x}\right)^8$ is $\frac{7}{16}$, find the value of the positive constant k. [4]

(ii) Using the value of k found in part **(i)**, show that there is no constant term in the expansion of $(1-7x^4)\left(x^3+\frac{k}{x}\right)^8$. [4]

4 The equation of a curve is $y = 3x^2 + (2m+1)x + 3 + c$, where m and c are constants. The line $y = mx + c$ is a tangent to the curve at the point P.

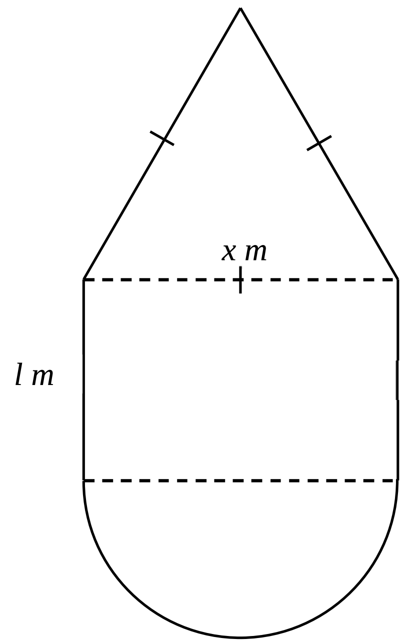
(i) Find the positive value of m. [4]

(ii) Using this value of m, and given that the curve passes through (2, 38), find the coordinates of P. [4]

The straight line L meets the curve at one point only.

(iii) Given that L is **not** a tangent to the curve, what can be deduced about L? [1]

5



A farmer uses 120 m of fencing to enclose a plot of the shape shown above. The shape consists of an equilateral triangle of side x m, a rectangle with sides x m and l m and a semicircle of diameter x m.

(i) Show that the area of the plot is $\left[60x+\left(\frac{\sqrt{3}}{4}-1-\frac{\pi}{8}\right)\right]x^2$ m^2. [3]

(ii) Given that x can vary, find the value of x for which the area of the plot is stationary. [4]

(iii) Explain why this value of x gives the farmer the largest possible area. [1]

6 It is given that $f(x) = -4\sin 3x + 2$.

(i) State the amplitude of $f(x)$. [1]

(ii) State the least and greatest values of $f(x)$. [2]

(iii) State the period of $f(x)$. [1]

(iv) Sketch the graph of $y = f(x)$ for $0° \leq x \leq 180°$. [3]

7 **(i)** Express $\cos 3\theta$ as an expression in terms of $\cos\theta$. [3]

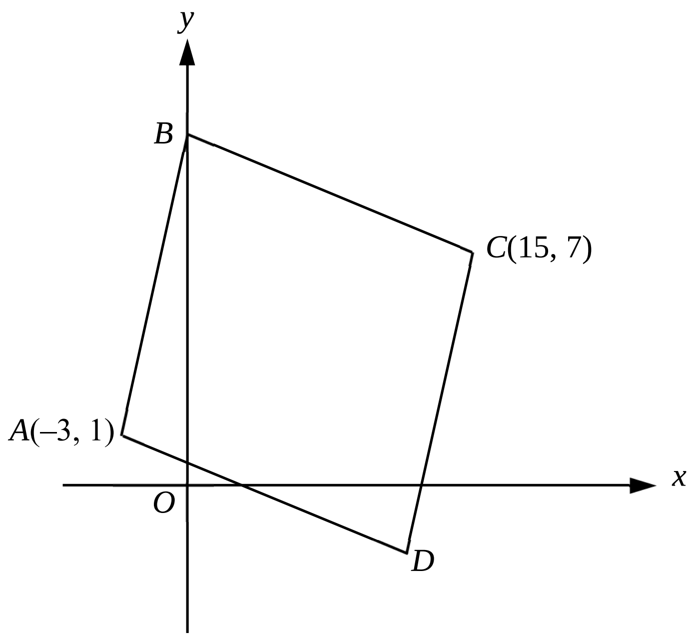
(ii) Use your answer in part **(i)** to find, for $0° \le \theta \le 360°$, the solutions of the equation $2\cos 3\theta = 3 - 4\cos^2\theta$. [4]

8 **(i)** Express $\dfrac{4x}{2x+5}$ in the form $a+\dfrac{b}{2x+5}$, where a and b are integers. [2]

(ii) Differentiate $x\ln(2x+5)$ with respect to x. [2]

(iii) Using the results in parts **(i)** and **(ii)**, determine $\int \ln(2x+5)\,\mathrm{d}x$. [3]

9 The diagram shows a rhombus $ABCD$ in which the coordinates of the points A and C are $(-3, 1)$ and $(15, 7)$ respectively.

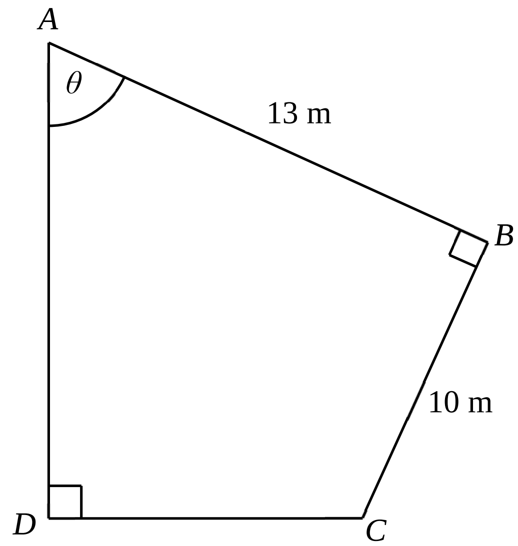


Given that the point B lies on the y-axis, find

(i) the coordinates of B and of D, [6]

(ii) the area of the rhombus. [2]

10 The diagram shows a field in the shape of a quadrilateral in which angles ABC and ADC are right angles. The lengths of AB and BC are 13 m and 10 m respectively. The acute angle BAD is θ radians.



(i) Show that the perimeter, P cm, is given by $P = 23 + 3\cos\theta + 23\sin\theta$. [3]

(ii) Express P in the form $c + R\sin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iii) Hence find the maximum perimeter of the field, giving your answer correct to two decimal places. [2]

11 **(a)** Solve the equation $\log_2(2x-3)-\log_2(x+1)=2+\log_2\frac{1}{5}$. [4]

(b) The graph of $y = \log_a x$ passes through the points with coordinates (16, 2), (1, b) and (c, −1.5).

(i) Determine the value of each of the constants a, b and c. [3]

(ii) Sketch the graph of $y = \log_a x$. [2]

12 A curve is such that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 24x - 14$ and the curve passes through the point $P(2, 5)$. The gradient of the curve at P is 23.

(i) Find the equation of the curve. [6]

(ii) Find the coordinates of the point where the normal to the curve at $x = 1$ cuts the x-axis. [4]

~~~ End of Paper ~~~
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