



ANDERSON SERANGOON JUNIOR COLLEGE

H2 MATHEMATICS

9758

JC1 Promo Paper (100 marks)

7 Oct 2025

3 hours

Additional Material(s): List of Formulae (MF 27)

CANDIDATE
NAME

--

CLASS

		/		
--	--	---	--	--

READ THESE INSTRUCTIONS FIRST

Write your name and class in the boxes above.

Please write clearly and use capital letters.

Write in dark blue or black pen. HB pencil may be used for graphs and diagrams only.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions and write your answers in this booklet.

Do not tear out any part of this booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

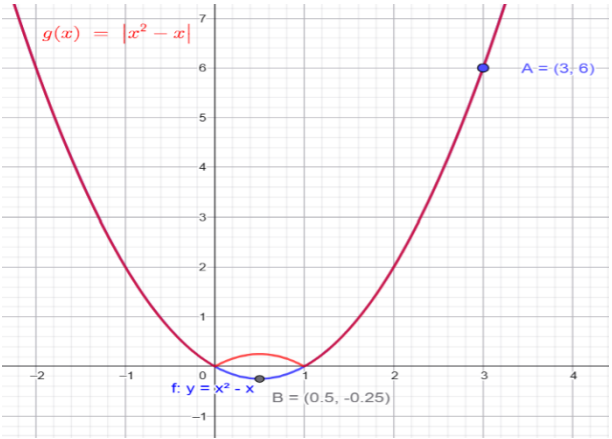
You are expected to use an approved graphing calculator.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

All work must be handed in at the end of the examination. If you have used any additional paper, please insert them inside this booklet.

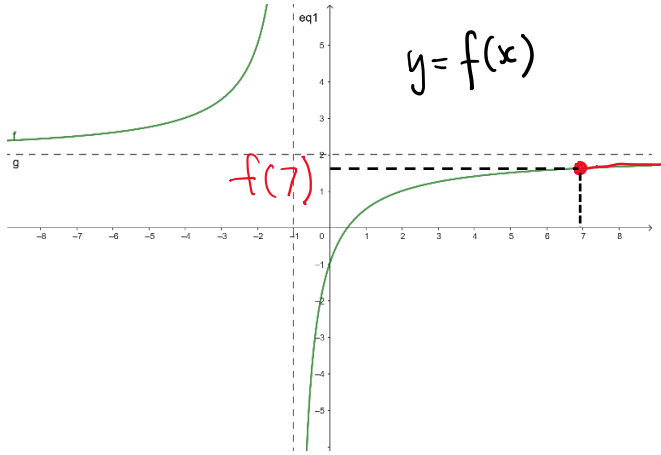
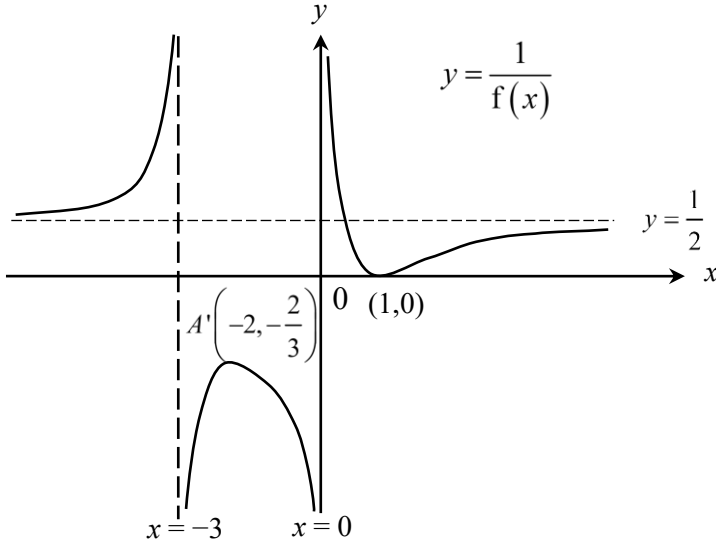
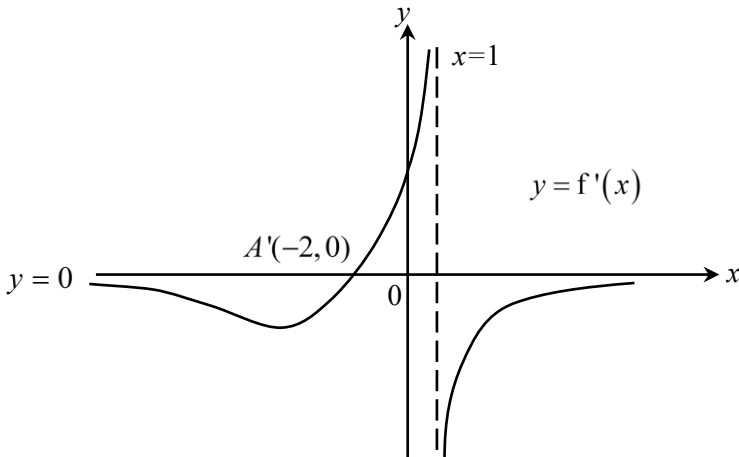
The number of marks is given in brackets [] at the end of each question or part question.

Question number	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total	

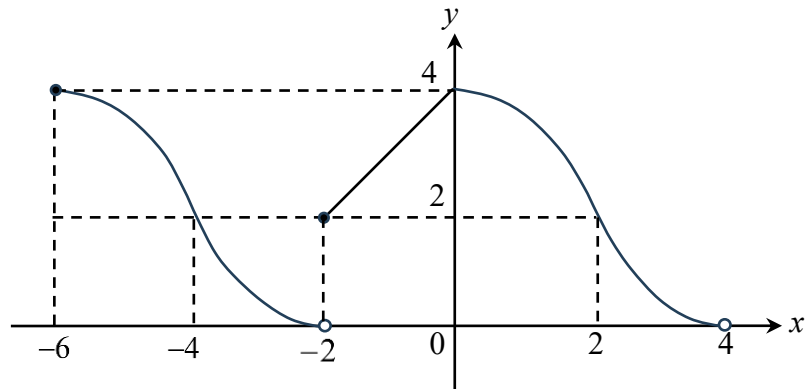
1	(a) $\int \left(\operatorname{cosec} 2x \cot 2x - \frac{1}{\sqrt{9-4x^2}} \right) dx$ $= \int \left(\operatorname{cosec} 2x \cot 2x - \frac{1}{\sqrt{3^2-(2x)^2}} \right) dx$ $= -\frac{1}{2} \operatorname{cosec} 2x - \frac{1}{2} \sin^{-1} \left(\frac{2x}{3} \right) + C$	
	(b) $\int_0^3 x^2 - x dx = \int_0^1 -(x^2 - x) dx + \int_1^3 (x^2 - x) dx$	
	$= \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 + \left[\frac{x^3}{3} - \frac{x^2}{2} \right]_1^3$ $= \left(\frac{1}{2} - \frac{1}{3} \right) + \left[\left(9 - \frac{9}{2} \right) - \left(\frac{1}{3} - \frac{1}{2} \right) \right]$ $= \frac{29}{6}$ 	
2	(a) Diff wrt x, $\Rightarrow 4x + 3 \left(x \frac{dy}{dx} + y \right) + 2y \frac{dy}{dx} = 0$	
	$\Rightarrow (3x + 2y) \frac{dy}{dx} = -(4x + 3y)$ $\Rightarrow \frac{dy}{dx} = -\frac{4x + 3y}{3x + 2y}$	
	Gradient of normal at (1,2) $= -\frac{1}{\left(-\frac{10}{7} \right)} = \frac{7}{10}$	
	Equation of normal at (1,2): $y - 2 = \frac{7}{10}(x - 1)$	
	$y = \frac{7}{10}x + \frac{13}{10}$	
	(b) When $x = 0$, $y = \frac{13}{10}$ and $y = 0$, $x = -\frac{13}{7}$	
	Area of triangle formed $= \frac{1}{2} \left(\frac{13}{7} \right) \left(\frac{13}{10} \right) = \frac{169}{140} = 1 \frac{29}{140}$	
3	$3 + \frac{10}{x-1} \leq \frac{1}{x+2}$	

	$\frac{3(x-1)(x+2)+10(x+2)-(x-1)}{(x-1)(x+2)} \leq 0$	
	$\frac{3x^2+3x-6+10x+20-x+1}{(x-1)(x+2)} \leq 0$	
	$\frac{x^2+4x+5}{(x-1)(x+2)} \leq 0$	
	Method 1	
	$\frac{(x+2)^2+1}{(x-1)(x+2)} \leq 0$ $\frac{1}{(x-1)(x+2)} \leq 0 \quad \left(\because (x+2)^2+1 > 0 \quad \forall x \in \mathbb{R} \right)$	
	Method 2	
	Consider $x^2+4x+5=0$ Discriminant $= 4^2-4(5) = -4$ AND Coeff of x^2 is positive Hence $x^2+4x+5 > 0 \quad \forall x \in \mathbb{R}$ $\therefore \frac{1}{(x-1)(x+2)} \leq 0$	
	$\begin{array}{c} + \quad \quad - \quad \quad + \\ \text{---} \oplus \text{---} \oplus \text{---} \\ -2 \quad \quad 1 \end{array}$	
	So $-2 < x < 1$.	
	Replace x in $3 + \frac{10}{x-1} \leq \frac{1}{x+2}$ by e^{-x}	
	So $-2 < e^{-x} < 1$ $e^{-x} < 1$ (since $e^{-x} > 0 \quad \forall x \in \mathbb{R}$)	
	$\Rightarrow x > 0$	
4	(ai) Vertical asymptote: $x=1 \Rightarrow 2x=2 \Rightarrow 2x-2=0$ $d=-2$	
	(ii) $y = \frac{ax^2+bx+c}{2x-2} = 2-2x + \frac{k}{2x-2}$	
	$\frac{ax^2+bx+c}{2x-2} = \frac{(2-2x)(2x-2)+k}{2x-2}$	
	$\frac{ax^2+bx+c}{2x-2} = \frac{-4x^2+8x-4+k}{2x-2}$	
	By comparison, $a = -4$ and $b = 8$.	
	When $x=0$, $y = -2.5$,	

	$-2.5 = \frac{-4(0)^2 + 8(0) + c}{2(0) - 2}$ $c = 5$	
	(b) $y = \frac{4x^2 - 4x + 2}{2x - 1}$	
	$y = 2x - 1 + \frac{1}{2x - 1}$	
	$y = 2x - 1 + \frac{1}{2x - 1} \xrightarrow{\text{Replace } x \text{ by } \frac{1}{2}x} y = x - 1 + \frac{1}{x - 1}$ $y = x - 1 + \frac{1}{x - 1} \xrightarrow{\text{Replace } x \text{ by } x+1} y = x + \frac{1}{x}$	
	The sequence of transformation is	
	(1) Scaling parallel to x axis by a factor of 2	
	(2) Translation of 1 unit in the negative x direction	
5	(a)(i) $y = f(x) = \frac{2x + a}{x + b}$	
	$yx + by = a + 2x$ $x(y - 2) = a - by$	
	$x = \frac{a - by}{y - 2} = f^{-1}(y)$ $f^{-1}(x) = \frac{a - bx}{x - 2}$	
	$y = \frac{2x + a}{x + b} = 2 + \frac{a - 2b}{x + b},$ $\therefore D_{f^{-1}} = R_f = \mathbb{R} \setminus \{2\}$	
	(ii) $f(x) = f^{-1}(x)$	
	$\frac{a - bx}{x - 2} = \frac{2x + a}{x + b}$	
	Comparing, $b = -2$	
	$\therefore \frac{a}{2} \neq -2 \Rightarrow a \neq -4$	
	(b)(i) $g(x) = 7 + \sqrt{x - 3}, x \geq 3$	
	$R_g = [7, \infty)$	
	$D_f = (-\infty, -1) \cup (-1, \infty)$	
	Since $R_g \subseteq D_f \Rightarrow fg(x)$ exists.	
	(ii) $fg(7) = f(7 + \sqrt{4}) = f(7 + 2)$	
	$= \frac{2(9) - 1}{9 + 1}$ $= 1.7$	

	(iii) 	
	$R_g = [7, \infty) \rightarrow \text{New } R_f = [f(7), 2) = R_{fg}$ $= \text{New } D_f$	
	$R_{fg} = \left[\frac{2(7)-1}{7+1}, 2 \right) = \left[\frac{13}{8}, 2 \right)$	
6	(a)(i) 	
	(ii) 	

(b)



$$\begin{aligned}
 g(2025) &= g(337 \times 6 + 3) \\
 &= g(3) \\
 &= 2 \cos\left(\frac{3\pi}{4}\right) + 2 = 2 - \sqrt{2}
 \end{aligned}$$

7

(a) $S_n = 2n^2 - n + \ln(3^n)$

$$S_{n-1} = 2(n-1)^2 - (n-1) + \ln(3^{n-1})$$

$$u_n = S_n - S_{n-1}, \quad n \geq 2$$

$$= [2n^2 - n + \ln(3^n)] - [2(n-1)^2 - (n-1) + \ln(3^{n-1})]$$

$$u_n = S_n - S_{n-1}$$

$$= [2n^2 - n + \ln(3^n)] - [2(n^2 - 2n + 1) - (n-1) + \ln(3^{n-1})]$$

$$= 4n - 3 + \ln\left(\frac{3^n}{3^{n-1}}\right)$$

$$= 4n - 3 + \ln 3$$

Since $u_1 = S_1 = 1 + \ln 3 = 4(1) - 3 + \ln 3$,

$$\therefore u_n = 4n - 3 + \ln 3 \text{ for } n \geq 1.$$

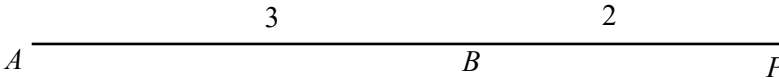
$$u_{n-1} = 4(n-1) - 3 + \ln 3$$

$$\therefore u_n - u_{n-1} = [4n - 3 + \ln 3] - [4(n-1) - 3 + \ln 3] = 4 \text{ (independent of } n)$$

Since $u_n - u_{n-1}$ is a constant, the sequence is an arithmetic progression for $n \geq 1$

(bi)

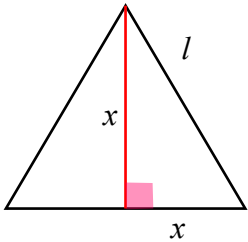
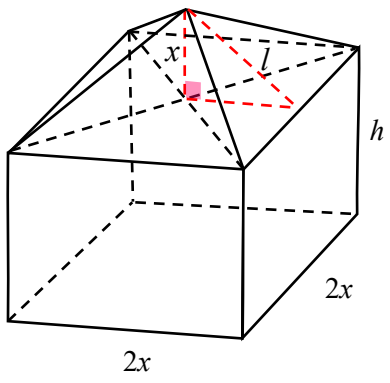
Month (n)	Amount at the start of month	Amount at the end of month
1	1000	$1000(1.001)$
2	$1000(1.001) + 200$	$1000(1.001)^2 + 200(1.001)$

	3	$1000(1.001)^2 + 200(1.001) + 200$	$1000(1.001)^3 + 200(1.001)^2 + 200(1.001)$		
	n		$1000(1.001)^n + 200(1.001)^{n-1} + \dots + 200(1.001)^2 + 200(1.001)$		
	Amount at the end of n month				
	$= 1000(1.001)^n + 200(1.001)^{n-1} + \dots + 200(1.001)^2 + 200(1.001)$				
	$= 1000(1.001)^n + \frac{200(1.001)(1.001^{n-1} - 1)}{1.001 - 1}$				
	$= 1000(1.001)^n + 200200(1.001^{n-1} - 1)$				
	$= 1000(1.001)^n + 200200(1.001^{n-1}) - 200200$				
	$= 1.001^{n-1}[1000(1.001) + 200200] - 200200$				
	$= 201201(1.001^{n-1}) - 200200$				
	(ii) $201201(1.001^{n-1}) - 200200 > 6500$				
	$1.001^{n-1} > \frac{206700}{201201}$ $n-1 > \frac{\ln\left(\frac{206700}{201201}\right)}{\ln(1.001)}$ $n > 27.977538$				
	On the April 2027 (the 28 th Month), Mr A's account first became greater than \$6500.				
8					
	Using ratio theorem,				
	(a) $\vec{OB} = \frac{2\vec{OA} + 3\vec{OP}}{5}$ $\vec{OP} = \frac{1}{3}\left(5\vec{OB} - 2\vec{OA}\right) = \frac{5}{3}\mathbf{b} - \frac{2}{3}\mathbf{a}$				
	(b) $\vec{AP} = \vec{OP} - \vec{OA} = \left(\frac{5}{3}\mathbf{b} - \frac{2}{3}\mathbf{a}\right) - \mathbf{a} = \frac{5}{3}\mathbf{b} - \frac{5}{3}\mathbf{a} = \frac{5}{3}(\mathbf{b} - \mathbf{a})$				
	$\vec{AC} = \vec{OC} - \vec{OA} = \left(\frac{7}{2}\mathbf{b} - \frac{5}{2}\mathbf{a}\right) - \mathbf{a} = \frac{7}{2}\mathbf{b} - \frac{7}{2}\mathbf{a} = \frac{7}{2}(\mathbf{b} - \mathbf{a})$				
	Since $\vec{AP}$ is parallel to $\vec{AC}$ and A is a common point, the points A , C and P are collinear.				
	(c) Exact area of triangle OBC				
	$= \frac{1}{2} \mathbf{c} \times \mathbf{b} $				
	$= \frac{1}{2}\left \left(\frac{7}{2}\mathbf{b} - \frac{5}{2}\mathbf{a}\right) \times \mathbf{b}\right $				

	$= \frac{1}{2} \left \frac{7}{2} \mathbf{b} \times \mathbf{b} - \frac{5}{2} \mathbf{a} \times \mathbf{b} \right $	
	$= \frac{5}{4} \mathbf{a} \times \mathbf{b} \quad (\because \mathbf{b} \times \mathbf{b} = \mathbf{0})$	
	$= \frac{5}{4} \mathbf{a} \mathbf{b} \sin\left(\frac{5}{6}\pi\right) \hat{\mathbf{n}} $	
	$= \frac{5}{4} (2\sqrt{3})(1)\left(\frac{1}{2}\right)(1)$	
	$= \frac{5}{4} \sqrt{3}$	
	(d) Length of projection of $\mathbf{c}$ on $\mathbf{a}$	
	$= \frac{ \mathbf{c} \cdot \mathbf{a} }{ \mathbf{a} }$	
	$= \frac{\left \left(\frac{7}{2} \mathbf{b} - \frac{5}{2} \mathbf{a} \right) \cdot \mathbf{a} \right }{ \mathbf{a} }$	
	$= \frac{\left \frac{7}{2} \mathbf{b} \cdot \mathbf{a} - \frac{5}{2} \mathbf{a} \cdot \mathbf{a} \right }{ \mathbf{a} }$	
	$= \frac{\left \frac{7}{2} \mathbf{b} \mathbf{a} \cos\left(\frac{5}{6}\pi\right) - \frac{5}{2} \mathbf{a} ^2 \right }{ \mathbf{a} }$	
	$= \left \frac{7}{2} \mathbf{b} \cos\left(\frac{5}{6}\pi\right) - \frac{5}{2} \mathbf{a} \right $	
	$= \left \frac{7}{2} (1) \left(-\frac{\sqrt{3}}{2} \right) - \frac{5}{2} (2\sqrt{3}) \right = \frac{27}{4} \sqrt{3}$	
9	(a) $1 - x = -\frac{1}{2}(2x - 4) - 1$	
	$\int \frac{1-x}{x^2-4x+1} dx$	
	$= -\frac{1}{2} \int \frac{2x-4}{x^2-4x+1} dx - \int \frac{1}{(x-2)^2-3} dx$	
	$= -\frac{1}{2} \ln x^2-4x+1 - \frac{1}{2(\sqrt{3})} \ln \left \frac{(x-2)-\sqrt{3}}{(x-2)+\sqrt{3}} \right + c$	
	$= -\frac{1}{2} \ln x^2-4x+1 - \frac{\sqrt{3}}{6} \ln \left \frac{(x-2)-\sqrt{3}}{(x-2)+\sqrt{3}} \right + c$	
	(bi) $\frac{d}{dx} \left(e^{\frac{1}{x}} \right) = -\frac{1}{x^2} e^{\frac{1}{x}}$	

	(ii) $\int_{\frac{1}{2}}^1 \frac{1}{x^3} e^{\frac{1}{x}} dx = - \int_{\frac{1}{2}}^1 \frac{1}{x} \left(-\frac{1}{x^2} e^{\frac{1}{x}} \right) dx$	
	$= - \left(\left[\frac{1}{x} e^{\frac{1}{x}} \right]_{\frac{1}{2}}^1 - \int_{\frac{1}{2}}^1 -\frac{1}{x^2} e^{\frac{1}{x}} dx \right)$	
	$= - \left(\left[e - 2e^2 \right] - \left[e^{\frac{1}{x}} \right]_{\frac{1}{2}}^1 \right)$	
	$= - (e - 2e^2 - (e - e^2)) = e^2$	
	(c) Given $x = 3 \sec \theta \Rightarrow \frac{dx}{d\theta} = 3 \sec \theta \tan \theta$	
	$x = 3, \sec \theta = 1 \Rightarrow \cos \theta = 1 \Rightarrow \theta = 0$ $x = 6, \sec \theta = 2 \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$	
	$\int_0^{\frac{\pi}{3}} \sqrt{9 \sec^2 \theta - 9} \left(\frac{1}{3 \sec \theta} \right) (3 \sec \theta \tan \theta) d\theta$	
	$= 3 \int_0^{\frac{\pi}{3}} \tan^2 \theta d\theta$	
	$= 3 \int_0^{\frac{\pi}{3}} \sec^2 \theta - 1 d\theta$	
	$= 3 [\tan \theta - \theta]_0^{\frac{\pi}{3}}$	
	$= 3 \left(\sqrt{3} - \frac{\pi}{3} \right)$	
	$= 3\sqrt{3} - \pi \quad (a = 3, b = 3, c = -1)$	
10	(a) Let θ be the acute angle between the l_1 and p_1 .	
	$\sin \theta = \frac{\left \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix} \right }{\sqrt{1^2 + 1^2 + 1^2} \sqrt{(-2)^2 + 3^2 + (-2)^2}}$	
	$= \left \frac{-1}{\sqrt{51}} \right $	
	$= \frac{1}{\sqrt{51}}$	
	$\theta = 8.0^\circ$ (1 d.p.)	
	(b) Let the foot of perpendicular from the point A to p_1 be N .	

	$l_{AN} : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}$	
	Since N lies on the line l_{AN} ,	
	$\overrightarrow{ON} = \begin{pmatrix} 1+\lambda \\ \lambda \\ \lambda \end{pmatrix}$, for some $\lambda \in \mathbb{R}$.	
	Since N lies on the plane p_1 ,	
	$\begin{pmatrix} 1+\lambda \\ \lambda \\ \lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 3$	
	$1 + \lambda + \lambda + \lambda = 3$	
	$\lambda = \frac{2}{3}$	
	Therefore coordinates of N are $\left(\frac{5}{3}, \frac{2}{3}, \frac{2}{3}\right)$.	
	(c) Since B lies on l_1 , $\overrightarrow{OB} = \lambda \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix}$ for some $\lambda \in \mathbb{R}$.	
	$D(3,0,0)$ is a point on p_1	
	$\frac{\left \begin{pmatrix} 3+2\lambda \\ -3\lambda \\ 2\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right }{\sqrt{1^2 + 1^2 + 1^2}} = \sqrt{27}$	
	$ 3 + \lambda = 9$	
	$3 + \lambda = \pm 9$ $\lambda = 6$ or -12	
	Possible coordinates of B are $(-12, 18, -12)$ or $(24, -36, 24)$	
	(d) Subst $(1,1,1)$ into P_2 : L.H.S = $3(1) + 4(1) + \alpha = 7 + \alpha = \text{R.H.S}$	
	$p_1 : \mathbf{r} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 3, \quad p_2 : \mathbf{r} \cdot \begin{pmatrix} 3 \\ 4 \\ \alpha \end{pmatrix} = \alpha + 7$	
	$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 4 \\ \alpha \end{pmatrix} = \begin{pmatrix} \alpha - 4 \\ 3 - \alpha \\ 1 \end{pmatrix}$	

	$l_2: \mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} \alpha - 4 \\ 3 - \alpha \\ 1 \end{pmatrix}, \mu \in \mathbb{R}$	
11	(a) $l = \sqrt{2}x$  	
	Total surface area = area of triangular surfaces + area of square base + area of rectangular sides	
	$72 = 4 \times \frac{1}{2} (2x) (\sqrt{2}x) + 4x^2 + 4 \times (2xh)$ $72 = 4\sqrt{2}x^2 + 4x^2 + 8xh$ $h = \frac{72 - 4\sqrt{2}x^2 - 4x^2}{8x}$ $h = \frac{18 - (1 + \sqrt{2})x^2}{2x}$	
	V = volume of cuboid + volume of pyramid roof $= (2x)^2 h + \frac{1}{3} (2x)^2 x$ $= 4x^2 \left[\frac{18 - (1 + \sqrt{2})x^2}{2x} \right] + \frac{4}{3} x^3$ $= 36x - 2(1 + \sqrt{2})x^3 + \frac{4}{3} x^3$ $= 36x - \left(\frac{2}{3} + 2\sqrt{2} \right) x^3 \text{ (shown)}$	
	(b) $V = 36x - \left(\frac{2}{3} + 2\sqrt{2} \right) x^3$ $\frac{dV}{dx} = 36 - 3 \left(\frac{2}{3} + 2\sqrt{2} \right) x^2 = 0$ $x^2 = \frac{36}{3 \left(\frac{2}{3} + 2\sqrt{2} \right)} = \frac{36}{3 \times \frac{2}{3} (1 + 3\sqrt{2})} = \frac{18}{1 + 3\sqrt{2}}$ $x = \sqrt{\frac{18}{1 + 3\sqrt{2}}} \text{ m or } x = \sqrt{\frac{18}{17}} (3\sqrt{2} - 1) \text{ m}$	

	$\frac{d^2V}{dx^2} = -6\left(\frac{2}{3} + 2\sqrt{2}\right)x^2 < 0 \quad (\because x^2 > 0)$ Thus V is maximised when $x = \sqrt{\frac{18}{1+3\sqrt{2}}}$ (i.e. 1.85 m)	
	$V = 36\left(\sqrt{\frac{18}{1+3\sqrt{2}}}\right) - \left(\frac{2}{3} + 2\sqrt{2}\right)\left(\sqrt{\frac{18}{1+3\sqrt{2}}}\right)^3$ $= 44.47054 = 44.5 \text{ m}^3 \text{ (3 s.f)}$ Since $44.5 \text{ m}^3 > 36 \text{ m}^3$, the above volume meets the needs of the garden.	
	(c) $V_{\text{(cuboid part)}} = (2x)^2(h) = (2 \times 1.8)^2 \left[\frac{18 - (1 + \sqrt{2})(1.8)^2}{2(1.8)} \right] = 36.64061 \text{ m}^3$ $\frac{dV}{dt} = 1 \text{ m}^3 \text{min}^{-1} \Rightarrow \frac{(36.64061 - 0) \text{ m}^3}{\text{time taken}} = 1 \text{ m}^3 \text{min}^{-1}$ $\Rightarrow \text{time taken} = 36.64061 = 36.6 \text{ min}$	
	When $x = 1.8$, $h = \frac{18 - (1 + \sqrt{2})(1.8)^2}{2(1.8)} = 2.82721 = 2.83 \text{ m}$ $\frac{dV}{dt} = 1 \text{ m}^3 \text{min}^{-1} \Rightarrow V_{\text{(cuboid part)}} = 4x^2h \Rightarrow \frac{dV}{dt} = 4x^2 \frac{dh}{dt}$ $\Rightarrow 1 \text{ m}^3 \text{min}^{-1} = \left[4(1.8)^2 \text{ m}^2 \right] \frac{(2.82721 - 0) \text{ m}}{\text{time taken}}$ $\text{Thus time taken} = \frac{2.82721 \text{ m} \left[4(1.8)^2 \text{ m}^2 \right]}{1 \text{ m}^3 \text{min}^{-1}} = 36.64064 = 36.6 \text{ min}$	