| Name : | Index No.: | Class: |
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# PRESBYTERIAN HIGH SCHOOL

## ADDITIONAL MATHEMATICS
**Paper 2** — **4051/02**

7 August 2025 — Thursday — 1 hour 45 minutes



## 2025 SECONDARY FOUR NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

**DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.**

**INSTRUCTIONS TO CANDIDATES**

Write your name, index number and class in the spaces provided above.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 70.

| *For Examiner's Use* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Qn | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | *Marks Deducted* |
| Marks | | | | | | | | | | | |

| Category | Accuracy | Units | Symbols | Others |
|---|---|---|---|---|
| Question No. | | | | |

| TOTAL MARKS |
|---|
| 70 |

Setter: Mrs Yvonne Ng
Vetter: Miss Li Feiqing

This question paper consists of **17** printed pages and **1** blank page.

*Mathematical Formulae*

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

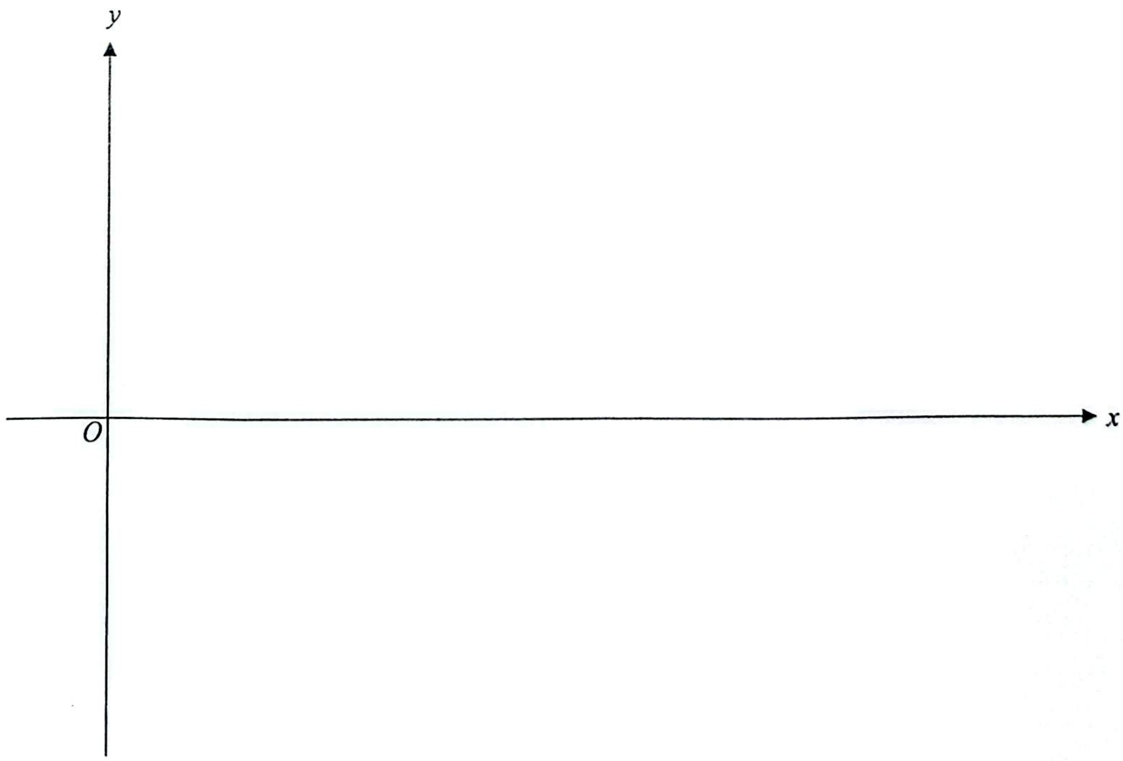
**Answer all questions.**

**1** The graph of $y = a\cos bx + c$ where $a > 0$, has an amplitude of 4 and period of $\pi$.

**(a)** State the value of $a$ and $b$. [2]

**(b)** Given that the maximum value of $y$ is 7, find the value of $c$. [1]

**(c)** Sketch the graph of $y = a\cos bx + c$ for $0 \le x \le 2\pi$. [2]



**2** It is given that $y = \frac{2x+3}{1+4x} - x$ for $x > -\frac{1}{4}$.

**(a)** Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

**(b)** Hence explain why $y$ can never be an increasing function. [2]

3 The polynomial $f(x) = x^3 + ax^2 + bx + 70$ for all real $x$. Given that $x - 2$ is a factor of $f(x)$ and that when $f(x)$ is divided by $x - 3$ the remainder is $-32$.

**(a)** Find the value of each of the constants $a$ and $b$. [4]

**(b)** Hence, solve $f(x) = 0$. [3]

4 A scientist measures the height of a mini rocket at different times after it launches. The height, $h$, above the ground is measured in metres and the time, $t$ is measured in seconds. The mini rocket was launched 45 m above the ground. The rocket returned to the ground 9 seconds after it had been launched.

The model of the motion is given as $h = -12t^2 + 103t + 45$.

**(a)** Explain why the model is suitable for the measurements found. [2]

**(b)** Find the range of values of $t$ for which the height of the rocket above the ground is greater than 136 m. [3]

**(c)** Explain whether the rocket can reach 300 m above the ground. [2]

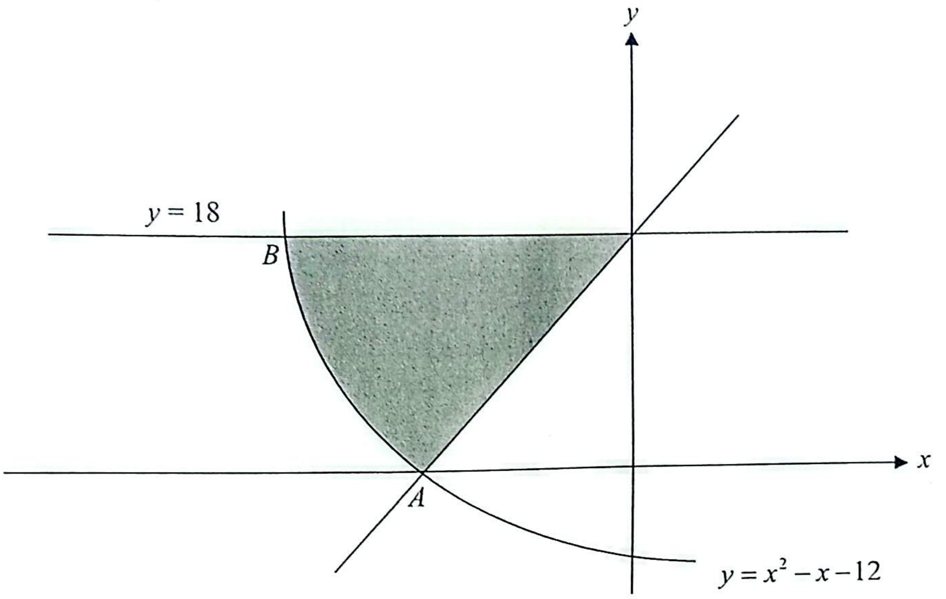
5 (a) Express $6\sin t + 8\cos t$ in the form $R\sin(t+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

(b) An electric voltage from a battery, $V$ volts varies with time and is given by $V = 6\sin t + 8\cos t$, where $t$ is the time in milliseconds.

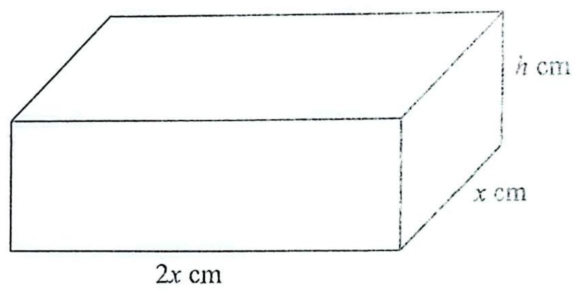
(i) An electrical drill requires a voltage exceeding 10 volts to work. Determine if the electrical drill can be used with the battery. [2]

(ii) Find the value of $t$ when the electric voltage is 4 volts at the first instant. [3]

6 The curve $y = x^2 - x - 12$ cuts the $x$-axis at $A$ and the line $y = 18$ at $B$. Calculate the area of the shaded region. [5]



7 The diagram shows an open box with a rectangular base $x$ cm by $2x$ cm and four vertical sides of height $h$ cm. The external surface area of the box is 36 $\text{cm}^2$.




(a) Show that the volume, $V$ cubic centimetres, of the box is

$$V = 12x - \frac{2}{3}x^3.$$

[3]

**(b)** Given that $x$ and $h$ can vary, find the value of $x$ for which $V$ has a stationary value and explain why this value of $x$ gives the maximum value of volume. [5]

8 (a) Prove that $\dfrac{\cot A \cos A}{\cot A + \cos A} = \dfrac{1 - \sin A}{\cos A}$. [4]

**(b)** Solve the equation $2\sin x\cos x + 4\cos^2 x = 2$ for $0° \leqslant x \leqslant 180°$. [5]

9 The points $A$ and $B$ have coordinates $(0, a)$ and $(4, 0)$ respectively, where $a > 0$. The perpendicular bisector of $AB$ crosses the line $x = -1$ at $C$.

**(a)** Show that the $y$-coordinate of $C$ is $\dfrac{a^2 - 24}{2a}$. [5]

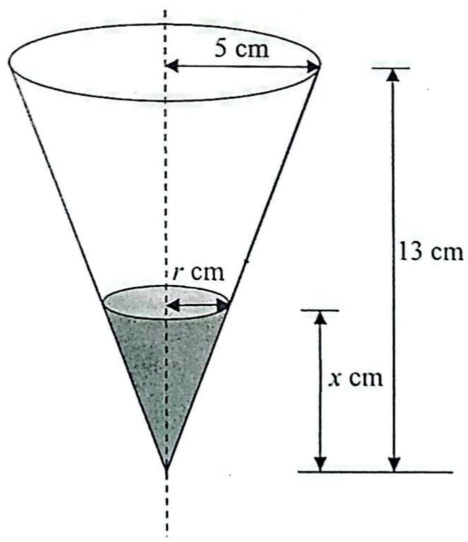
**(b)** Given that the area of triangle $ABC$ is $7.5a$ square units, find the value of $a$. [3]

**10** [The volume of a cone with base radius $r$ cm and vertical height $h$ cm is given by $\frac{1}{3}\pi r^2 h$.]

To make drip coffee, coffee is poured into an inverted funnel of radius 5 cm and height 13 cm. The coffee flows in at a constant rate of $\alpha$ cm$^3$/s and dripped out into a cup at a constant rate of $\beta$ cm$^3$/s where $\alpha > \beta$. Initially, the funnel is empty.

**(a)** Express $\frac{\mathrm{d}V}{\mathrm{d}t}$ in terms of $\alpha$ and $\beta$. [1]

At time $t$ s, the coffee surface has radius $r$ cm and is at a vertical height of $x$ cm above the vertex

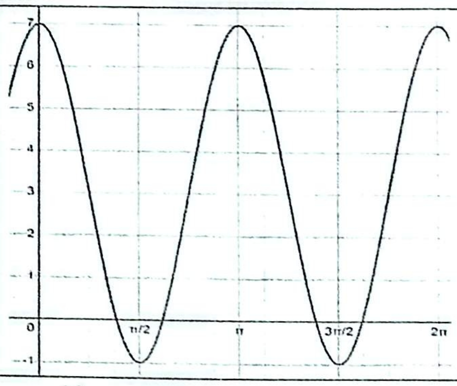



**(b)** Show that the volume of coffee in the cone at any time, $t$, is given by $V = \frac{25}{507}\pi x^3$, where $x$ is the depth of the water. [2]

**(c)** Given that $\alpha = 6$ and $\beta = 2$, find the rate of change of the vertical height of the coffee surface when $x = 5.2$. [5]

**END OF PAPER**

**Answer Key**

| | | | |
|---|---|---|---|
| **1(a)** | $a = 4$, $b = 2$ | **5a** | $10\sin(t+0.927)$ |
| **1(b)** | $c = 3$ | **5b(i)** | Since max V=10 volts, the drill will not be able to work |
| **1(c)** |  | **5b(ii)** | 1.80 milliseconds |
| | | **6** | $\frac{139}{3}$ unit$^2$ |
| | | **7b** | Volume is a maximum at $x=\sqrt{6}$ |
| **2(a)** | $\frac{-10}{(1+4x)^2}-1$ | **8b** | 58.3°, 148.3° |
| **2(b)** | Since $(1+4x)^2>0$ for $x>-\frac{1}{4}$,<br>$\frac{-10}{(1+4x)^2}<0$<br>$\frac{-10}{(1+4x)^2}-1<-1$<br>Since $\frac{dy}{dx}<-1$, it means that $\frac{dy}{dx}$ is always negative,<br>$\therefore$ $y$ can never be an increasing function. | **9b** | $a=2$ |
| | | **10a** | $\frac{dV}{dt}=\alpha-\beta$ |
| | | **10c** | 0.318 cm/s |
| **3(b)** | $x=2$, $x=-5$, $x=7$ | | |
| **4(a)** | When $t=0$, $h=45$<br>When $h=0$, $t=9$<br>Since the 2 conditions given are fulfilled, the model is suitable. | | |
| **4(b)** | $1<t<\frac{91}{12}$ | | |
| **4(c)** | $-12t^2+103t+45=300$<br>$-12t^2+103t-255=0$<br>$b^2-4ac$<br>$=(103)^2-4(-12)(-255)$<br>$=-1631<0$<br>Since discriminant < 0,<br>the object cannot reach 300 m above the ground. | | |