H

millennia institute

CANDIDATE NAME

CLASS

ADMIN NUMBER

2025 Preliminary Examination

Pre-University 3

MATHEMATICS **9758/01**

Paper 1 **August/September 2025**

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** the questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

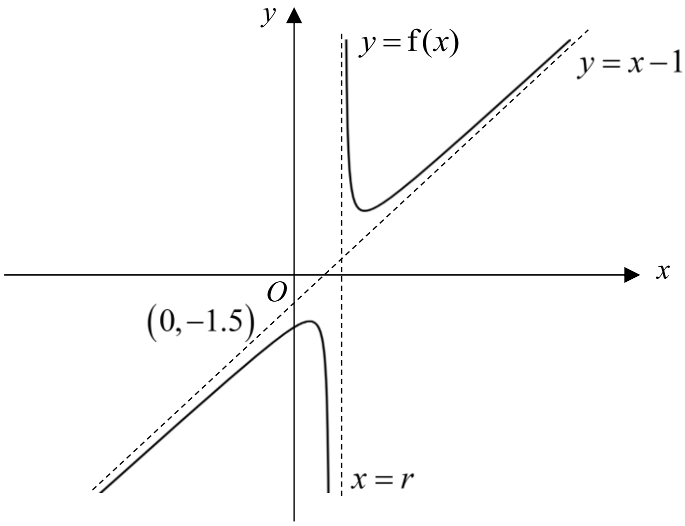
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

1 In this question you may use expansions from the List of Formulae (MF27).

It is given that $\mathrm{f}(\theta)=\dfrac{1}{3-\sin\theta}$, for $\theta>0$. Show that when θ is sufficiently small, $\mathrm{f}(\theta)\approx p+q\theta+r\theta^2$, where p, q and r are exact constants to be determined. [4]

2 The diagram below shows the curve with equation $y=\mathrm{f}(x)$ where $\mathrm{f}(x)=\dfrac{x^2+px+q}{x-2}$ for some constants p and q. The curve passes through the point with coordinates $(0,-1.5)$ and has asymptotes $y=x-1$ and $x=r$ for some constant r.



(i) Write down the value of r and show that $p=-3$ and $q=3$. [3]

(ii) Solve the inequality $\mathrm{f}(x)>\dfrac{7}{2}$. [1]

3 **(a)** Find $\displaystyle\int \frac{x}{\sqrt{x^2-4}}\,\mathrm{d}x$. [2]

(b) Find $\displaystyle\int x^2\ln(2x)\,\mathrm{d}x$. [3]

(c) Find $\displaystyle\int \frac{\sin 2x}{1+\cos^2 x}\,\mathrm{d}x$. [2]

4 Three different blends of coffee beans – Blend A, Blend B and Blend C – are sold at both M Café and I Café. The usual selling prices of the blends are the same at both cafés. The total cost of buying one package each of Blend A, Blend B and Blend C is \$176. During a holiday sale, the two cafés offered the following discounts:

Café	**Discounts given for each package**			**Total price after the discount**
	Blend A	**Blend B**	**Blend C**	
M Café	15%	10%	5%	\$161.80
I Café	7%	5%	2%	\$169.56

(i) Find the usual selling price of each package of Blend A, Blend B and Blend C coffee beans respectively. [3]

Loyal customers of I Café receive an additional discount of x% on the usual selling price of the packages for Blend A and Blend B only. (This loyalty discount is calculated as x% of the usual price of each package and is subtracted from the sale prices above.)

I Café wants the total cost for its loyal customers to be less than the total cost at M Café when buying one of each package of the three blends.

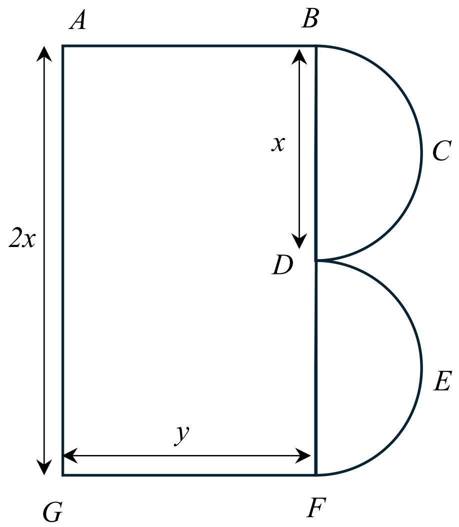
(ii) Form an inequality and find the smallest integer value of x. [3]

5 **(i)** Using the substitution $u = x^2 + 3$, show that

$$\int \frac{x^3}{\sqrt{x^2+3}}\,\mathrm{d}x = \frac{1}{3}\sqrt{\left(x^2+3\right)^3} - 3\sqrt{\left(x^2+3\right)} + C. \qquad [4]$$

(ii) Hence find the exact value of $\int_{-1}^{3} \left| \frac{3x^3}{\sqrt{x^2+3}} \right| \mathrm{d}x$, leaving your answer in the form $a + b\sqrt{3}$ where a and b are integers to be determined. [3]

6 A company produces customised badges. The diagram below shows the design of one such badge which consists of a rectangle and two identical semicircles.



It is given that $AG = 2x$ cm , $FG = y$ cm , $BD = DF = x$ cm and the outer perimeter $ABCDEFGA$ of the badge is 25 cm.

(i) Show that the area of the badge, S cm^2 is given by the formula

$$S = 25x - 2x^2 - \frac{3}{4}\pi x^2 .$$ [3]

(ii) Use calculus to find the exact value of x which gives the maximum value of S. [4]

7 Epidemiologists are modelling the spread of an infectious disease in a town. The following assumptions are made:

- At Week 0, there are 500 new infections.
- In each subsequent week, the number of new infections is 60% the number of new infections in the previous week, due to ongoing community transmission.

Let u_n denote the number of new infections in Week *n*, for $n \geq 0,\ n \in \mathbb{Z}$.

(i) Write down the value u_1 and verify that $u_2 = 180$. [2]

(ii) If S_n denotes the total cumulative number of new infections after *n* weeks, show that $S_n = 1250\left(1-0.6^{n+1}\right)$. [2]

The local healthcare facility has a capacity of 1240 beds. You may assume that each infected person occupies one bed, and that none are discharged during the period being considered.

(iii) Find the smallest value of *n* such that the total number of infections up to and including Week *n* first exceeds 1240. [1]

The healthcare facility later increases its capacity to 1280 beds.

(iv) Based on the given model, comment on whether this new capacity is sufficient to accommodate all infected persons over a prolonged period of time. [2]

8 It is given that $y = \mathrm{e}^{2\sin^{-1}x}$.

(i) Show that $\sqrt{1-x^2}\,\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2y$. [2]

(ii) By further differentiation of the result in part **(i)**, find the Maclaurin series for *y* up to and including the term in x^2. [4]

(iii) Using your result from part (**ii**), find an approximate value for $\int_0^{0.1} x\mathrm{e}^{2\sin^{-1}x}\,\mathrm{d}x$, giving your answer to 4 significant figures. [2]

9 The coordinates of point A and B are (1, 2, 3) and (17, 10, 19) respectively. The line l_1 passes through both points A and B and is parallel to the vector $\begin{pmatrix} k \\ 1 \\ 2 \end{pmatrix}$, where k is a constant.

(i) Show that $k = 2$. [2]

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ where μ is a parameter.

(ii) Determine whether the lines l_1 and l_2 intersect. If they do, find the position vector of their point of intersection; otherwise, explain why they do not intersect. [4]

The plane p has equation $\mathbf{r} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 16$.

(iii) Let α be the acute angle between l_1 and p. Find the exact value of $\sin\alpha$ and $\cos\alpha$. [3]

(iv) Find the coordinates of point F, the foot of the perpendicular of the point B to the plane p. [3]

10 The curves C_1 and C_2 are given by the equations $y = \dfrac{1}{1+4x^2}$ and $y = \dfrac{x^2}{5}$ respectively.

(i) Find the coordinates of the intersection points for C_1 and C_2. [2]

The region R is bounded by C_1 and C_2.

(ii) Find the exact area of R. [5]

(iii) Find the exact volume generated when R is rotated through π radians about the y-axis. [5]

11 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel vectors. The point C lies on OA such that $OC : CA = 1:2$. The point D lies on OB such that $OB : DB = 3:1$.

(i) Find the position vectors $\overrightarrow{OC}$ and $\overrightarrow{OD}$, giving your answers in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(ii) Show that the point E where the lines AD and BC meet has position vector $\frac{1}{7}\mathbf{a}+\frac{4}{7}\mathbf{b}$. [5]

(iii) Show that the area of triangle OAE can be written as $k|\mathbf{a}\times\mathbf{b}|$, where k is a constant to be found. [3]

It is further given that $\mathbf{b}$ is a unit vector.

(iv) Give the geometrical meaning of $|\mathbf{a}\cdot\mathbf{b}|$. [1]

(v) If $|\mathbf{a}\cdot\mathbf{b}| = \frac{1}{2}|\mathbf{a}|$, find the perpendicular distance of point A to OB, leaving your answer in terms of $|\mathbf{a}|$. [3]

12 A tank initially contains 100 litres of pure water. A brine solution, with a salt concentration of 0.2 kg per litre, is pumped into the tank continuously at a constant rate of 5 litres per minute. At the same time, a well-stirred mixture of the tank's contents is continuously drained from the tank at the same rate of 5 litres per minute. At time t minutes, the mass of salt in the tank is denoted by Q kilograms.

(i) Explain briefly why the volume of the mixture in the tank remains constant. [1]

(ii) By considering the salt concentration of the brine solution, show that salt enters the tank at a constant rate of 1 kg per minute. [1]

(iii) Hence, by also considering the mass of salt leaving the tank, show that the rate of change of the mass of salt in the tank can be expressed as

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = 1 - \frac{Q}{20}.$$ [2]

(iv) Solve the differential equation found in part **(iii)** to find Q in terms of t. [5]

(v) Determine the time taken for the salt concentration in the tank to reach 0.1 kg per litre. [2]

(vi) What happens to Q for large values of t? [1]

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