

Class	Register Number	Name
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南洋女子中學校
NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2020
Secondary 3

INTEGRATED MATHEMATICS 1

2 hours

TUESDAY

13 October 2020

0845 – 1045

READ THESE INSTRUCTIONS FIRST

1. Write your name, register number and class on all the work you hand in.
2. Answer **all** questions.
3. Write your answers and working on the separate writing paper provided, unless otherwise stated.
4. Write in dark blue or black ink on both sides of the paper.
5. You may use an HB pencil for any diagrams or graphs.
6. Do not use staples, paper clips, glue or correction tape/fluid.
7. Omission of essential working will result in loss of marks.
8. The use of an electronic calculator is expected, where appropriate.
9. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .
10. At the end of the examination, fasten all your work securely together.
11. The number of marks is given in brackets [] at the end of each question or part question.
12. The total number of marks for this paper is 80.

Setter: CH	This document consists of 10 printed pages, including this cover page. NANYANG GIRLS' HIGH SCHOOL	[Turn over
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Mathematical Formulae***MENSURATION***

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

TRIGONOMETRY

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

- 1 (a) It is given that $N = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$, evaluate N^2 . [1]
- (b) The table below shows the volume of petrol sold at two different petrol stations during a time interval of two hours on a particular day.

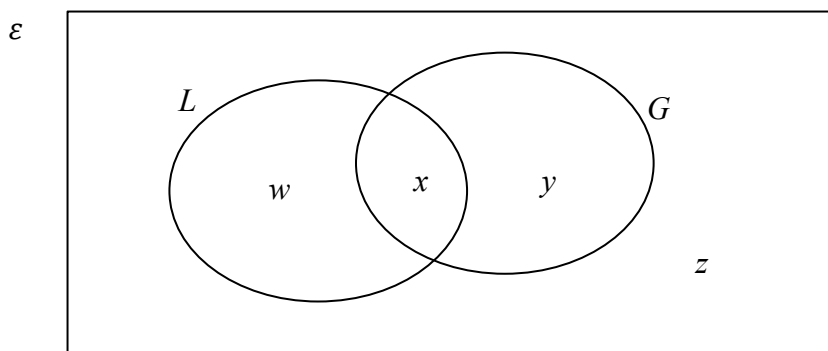
	Volume of Petrol Sold (litres)		
	Regular	Power	Supreme
Beatty Road Station	325	289	210
Cassin Road Station	236	277	312

The prices of the different grades of petrol are as follows:

Regular at \$1.48/litre, Power at \$1.63/litre and Supreme at \$1.87/litre.

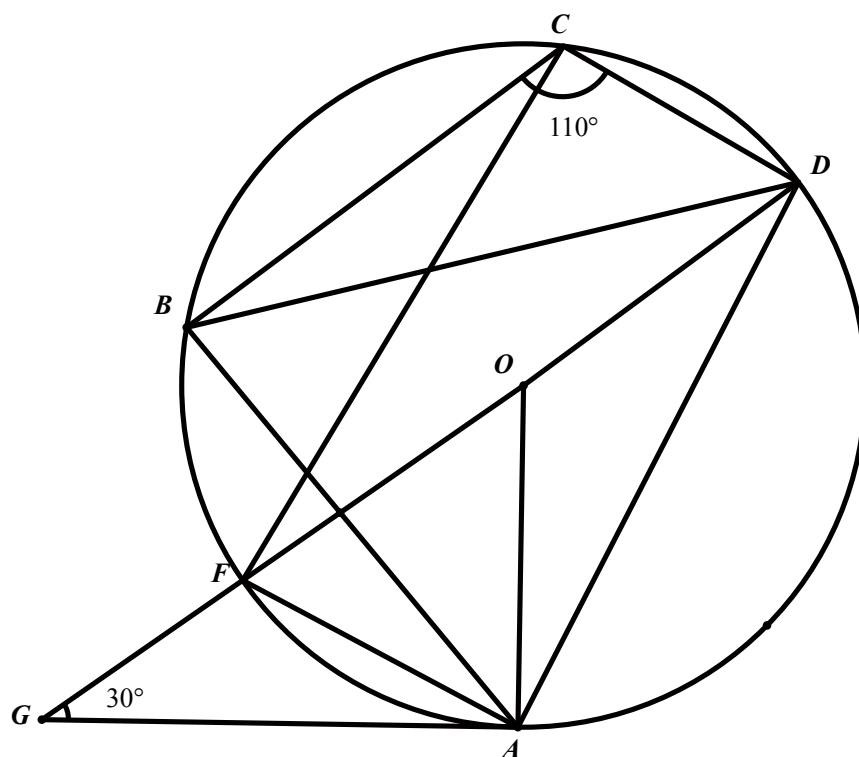
- (i) Write down two matrices whose product gives the total amount of money received by each petrol station during the 2-hour interval. Evaluate this matrix product. [2]
- (ii) The Beatty Road Station operates 20 hours a day whereas the Cassin Road Station operates 24 hours a day. The sales volume per two hours remains constant for the entire day for both petrol stations. Find, by matrix multiplication, the total amount of money collected by both stations for the day. [2]

- 2 The Venn diagram shows the set of pupils in the Secondary Three classes of a school. L is the set of pupils who study Literature. G is the set of pupils who study Geography.



Given that $n(\epsilon) = 100$, $n(G) = 48$ and $n(L) = 59$,

- (a) express w in terms of x , [1]
- (b) Given that every student in these classes studies either Literature or Geography, find the value of x and y , [2]
- (c) find the greatest possible value of z . [1]



In the diagram, O is the centre of the circle, DFG and AB are straight lines and GA is a tangent to the circle at A . Angle $AGD = 30^\circ$ and angle $BCD = 110^\circ$.

(a) Find, giving reasons for each answer,

(i) angle GOA , [2]

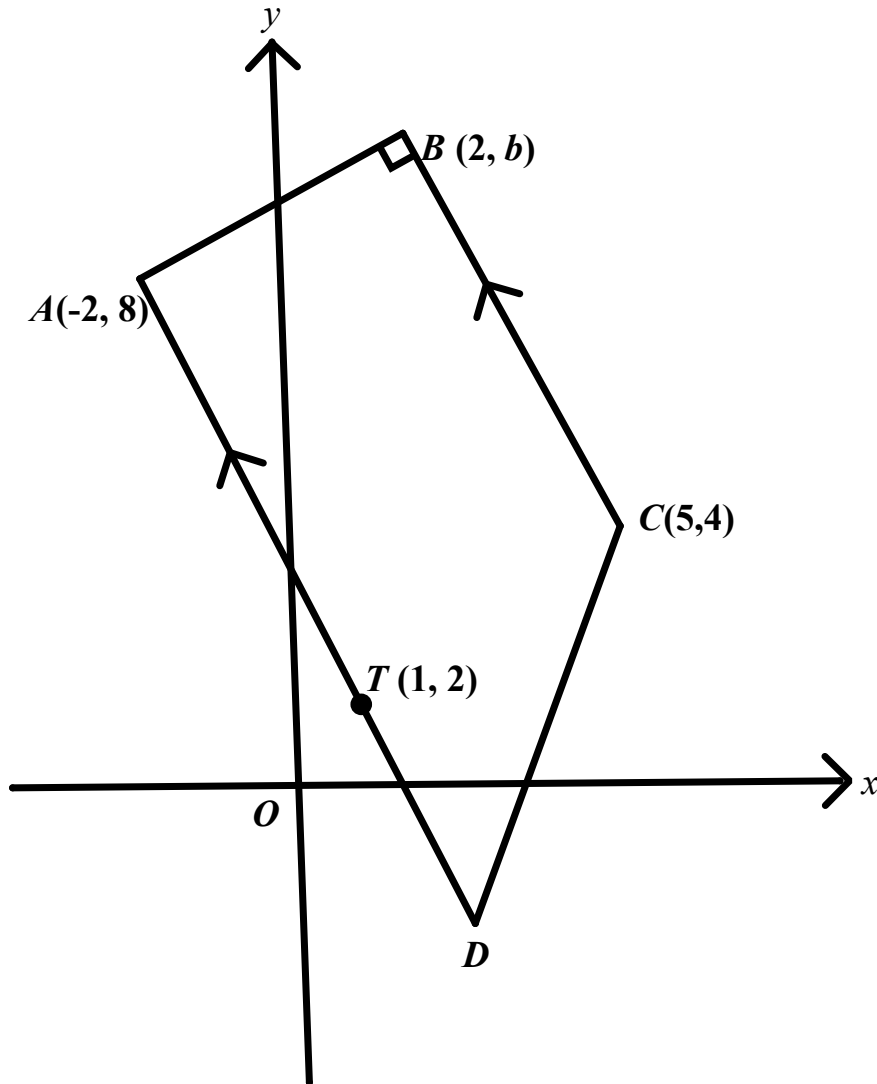
(ii) angle BDF , [2]

(iii) angle BAD , [1]

(iv) angle BAG . [2]

(b) Explain, with suitable working, if $FG = FA$. [2]

4 Solutions to this question by accurate drawing will not be accepted.

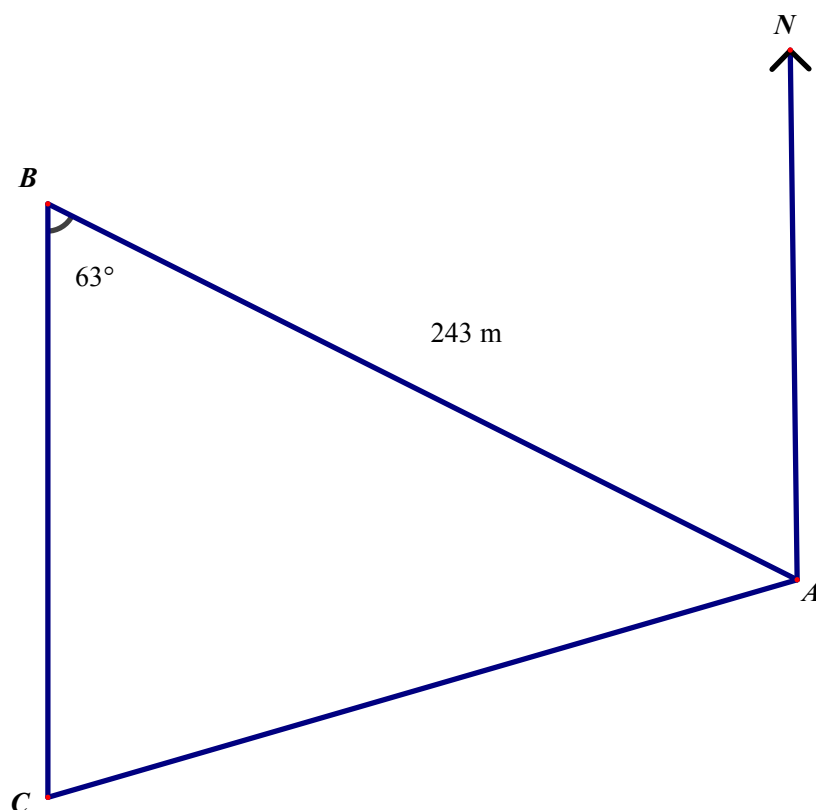


The diagram (not drawn to scale) shows a trapezium $ABCD$ in which BC is parallel to AD and AB is perpendicular to AD and BC . The coordinates of A , B and C are $(-2, 8)$, $(2, b)$ and $(5, 4)$ respectively. The point $T(1, 2)$ is on AD .

- (a) Calculate the value of b . [3]
- (b) Find the equation of the perpendicular bisector of AB . [3]
- (c) Given further that $AT : TD$ is $3 : 2$, find the coordinates of D . [2]
- (d) Find the area of trapezium $ABCD$. [2]
- (e) A point, directly above D , lies on the perpendicular bisector of AB .
Find the coordinates of this point. [2]

[Turn over]

5



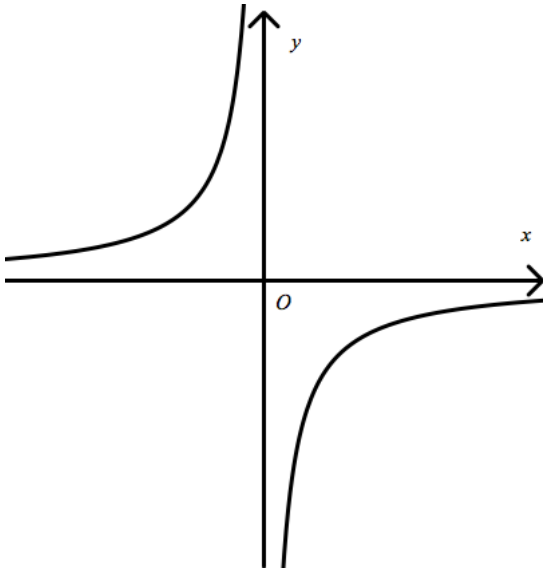
ABC is a triangular piece of land where an exercise station will be built. B is due north of C , angle $ABC = 63^\circ$ and $AB = 243$ m. The bearing of C from A is 255° .

- (a) Calculate the length of AC . [2]
- (b) Calculate the bearing of B from A . [1]
- (c) A drone is hovering directly above B while Mr Lim is walking from point A to point C . What distance must Mr Lim walk in order to get to the point where the angle of elevation of the drone from Mr Lim is the greatest? [2]

6 The equation of a circle is $x^2 + y^2 + 8x - 20y + 91 = 0$.

- (a) Find the coordinates of the centre of the circle and the radius of the circle. [3]
- (b) Given that the equation of the diameter to the circle at a point, T , is $3y + 4x = k$, show that $k = 14$. [1]
- (c) Show that the two possible pairs of coordinates of T are $(-7, 14)$ and $(-1, 6)$. [4]
- (d) The diameter to the circle in part (b) meets the y -axis at another point, Q .
 - (i) Calculate the coordinates of Q . [1]
 - (ii) Given that T lies between Q and the centre of the circle, find the length of TQ . [3]

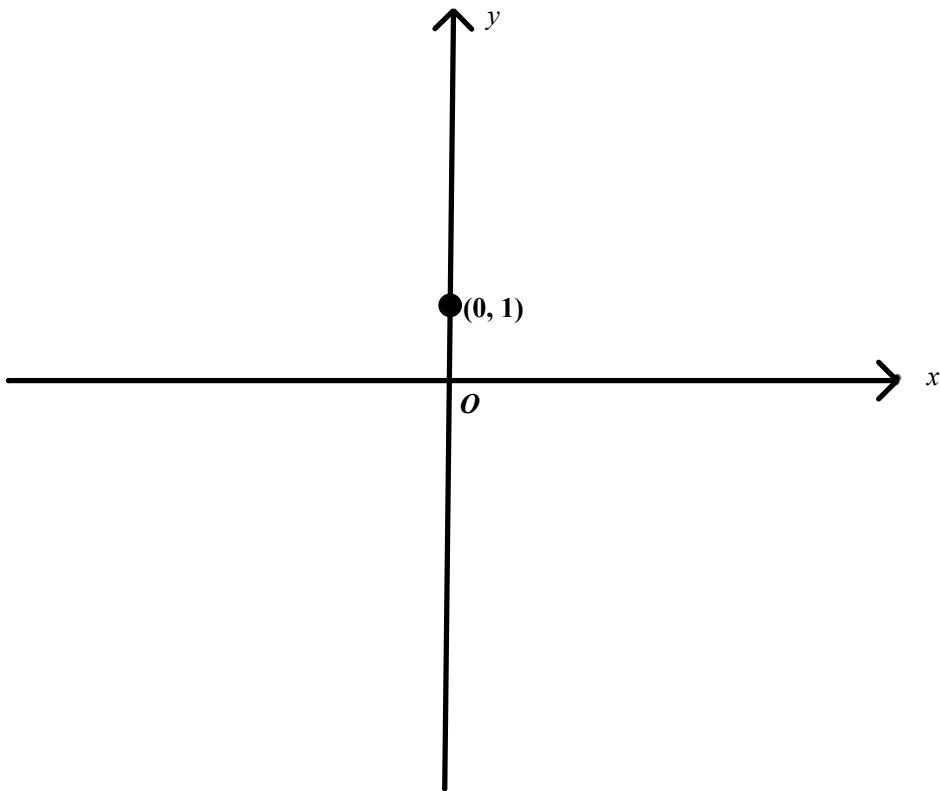
- 7(a) Given that the graph $y = \frac{k}{x^n}$ is shown below, write down possible values of k and n . [2]



- 7(b) Answer this part question on the insert provided.

On the axes provided, the point (0, 1) is shown.

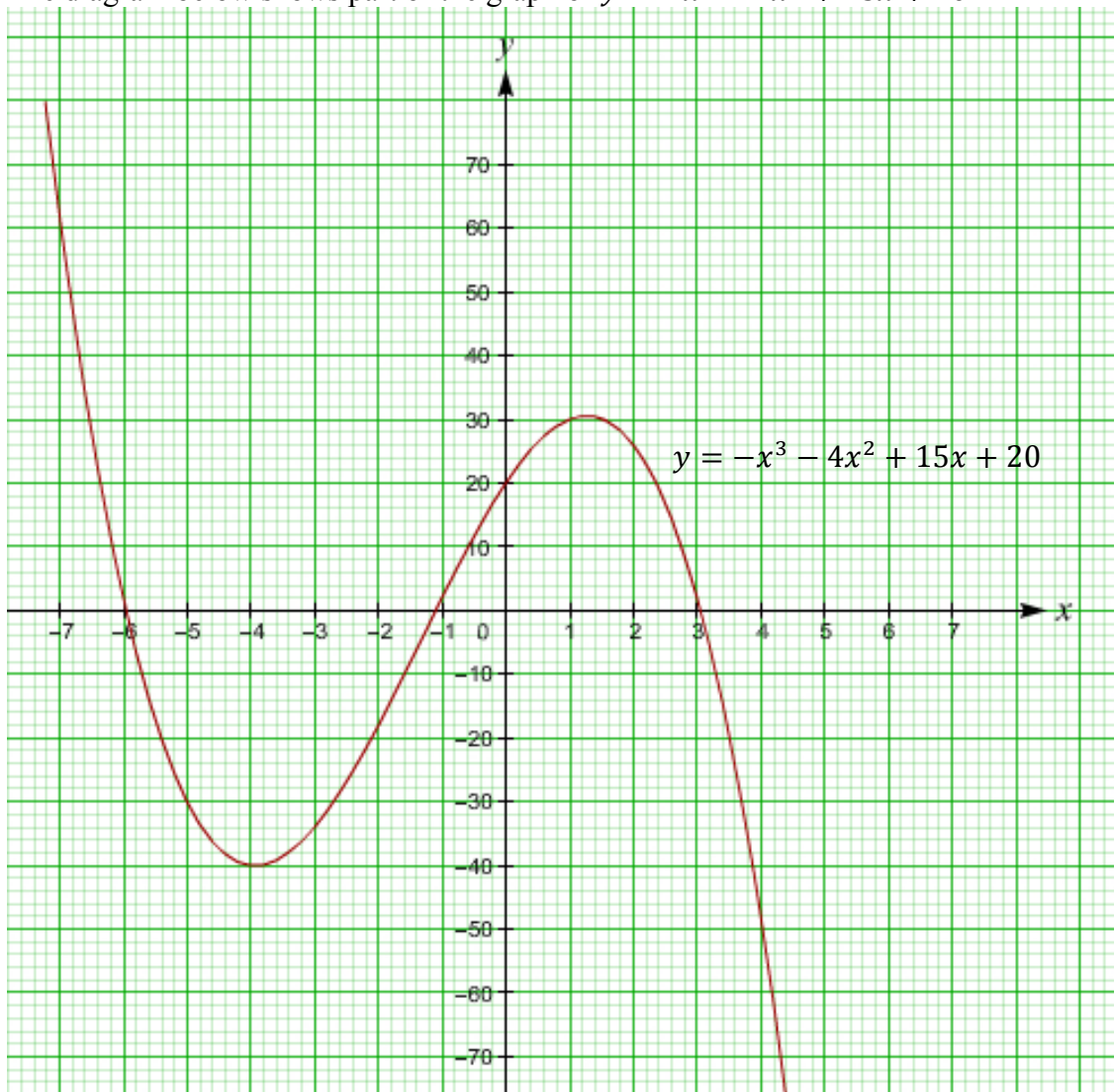
Sketch the graph of $y = -\frac{1}{3}x^3 + 2$, indicating only the y-intercept. [2]



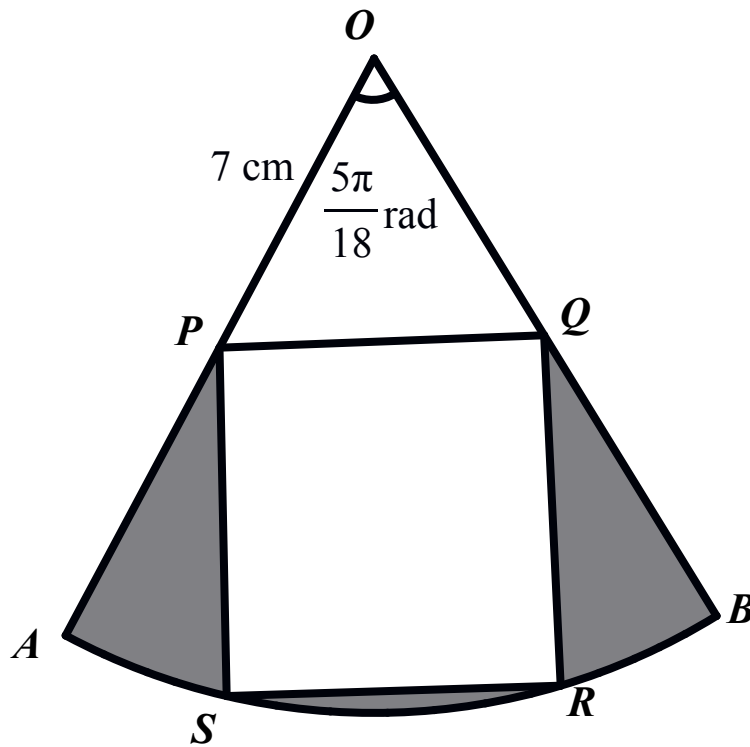
[Turn over]

8 Answer the whole of this question on the insert provided.

The diagram below shows part of the graph of $y = -x^3 - 4x^2 + 15x + 20$.



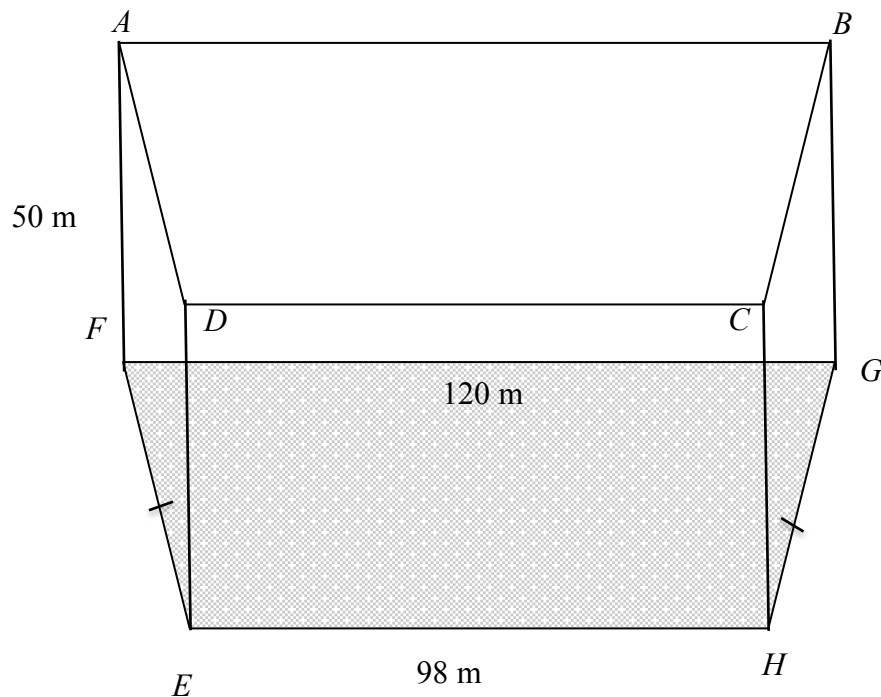
- (a) Using the graph,
- (i) find the ranges of values of x for which y is increasing, [1]
 - (ii) state the coordinates of one point on the graph where the gradient is 0. [1]
- (b) The line $y = kx - 10$ is a tangent to the graph at the point R . By drawing a suitable straight line on the same axes, use the graph to
- (i) find the x -coordinate of R , [2]
 - (ii) find the value of k . [2]
- (c) By drawing suitable straight lines, use your graph to solve
- (i) the inequality $-x^3 - 4x^2 + 15x < -30$, [3]
 - (ii) the equation $-50 = -2x^3 - 8x^2 + 36x$. [3]



A company is thinking of a new product design where a square $PQRS$ is inscribed in a sector of a circle OAB , with centre O and radius 13 cm . Angle $AOB = \frac{5\pi}{18}\text{ rad}$ and $OP = 7\text{ cm}$.

- (a) Find length PQ . [2]
- (b) (i) Show that angle $SOR \approx 0.5$ radian. [2]
- (ii) Calculate arc length RS . [2]
- (c) Calculate the area of the shaded region. [4]

10



A drama hall $ABCDEFGH$ is in the shape of a prism with a trapezium as a base. There is no elevated stage and the actors will move around on the stage $EHGF$ as shaded in the diagram. The height of the drama hall is 50 m. $EH = 98$ m, $FG = 120$ m and $FE = GH$. A theatrical company rented the hall for one of its productions.

- (a) In a scene at a recent drama production, an actress stood at a point along EH and looked at an imaginary friend at A . The angle of elevation of A from where the actress stood was 23° . After that, she ran to point F , which was directly below A . Calculate the distance she ran. [2]
- (b) The shortest distance from H to FG is 34 m, calculate the value of $\tan \angle GHE$. [2]
- (c) One of the props for this production required a string of lights hung taut from G to D . Calculate the length of the string of lights. [3]

END OF PAPER

Sec 3 IM1

EOY 2020 Answers only

1 (a) $\begin{pmatrix} 1 & -18 \\ 6 & 13 \end{pmatrix}$

1 (b) (i) $\begin{pmatrix} 1344.77 \\ 1384.23 \end{pmatrix}$

1 (b) (ii) (30058.46)

Amount of money = \$30 058.46

2 (a) $w = 59 - x$

2 (b) $x = 7$

$y = 41$

2 (c) 41

3 (a) (i) 60°

3 (a) (ii) 20°

3 (a) (iii) 70°

3 (a) (iv) 50°

3 (b) $\angle FAG = 30^\circ$ (tangent perpendicular to radius)

Since angle $FGA = \text{angle } FAG$, therefore triangle AFG is an isosceles triangle, and $GF = FA$.

4 (a) $b = 10$

4 (b) $y - 9 = -2x$

4 (c) D (3, -2)

4 (d) 40 sq units or units²

4 (e) (3, 3)

5 (a) 224 m

5 (b) 297°

5 (c) $181m$

6 (a) centre of circle $(-4, 10)$
radius of the circle = 5 units

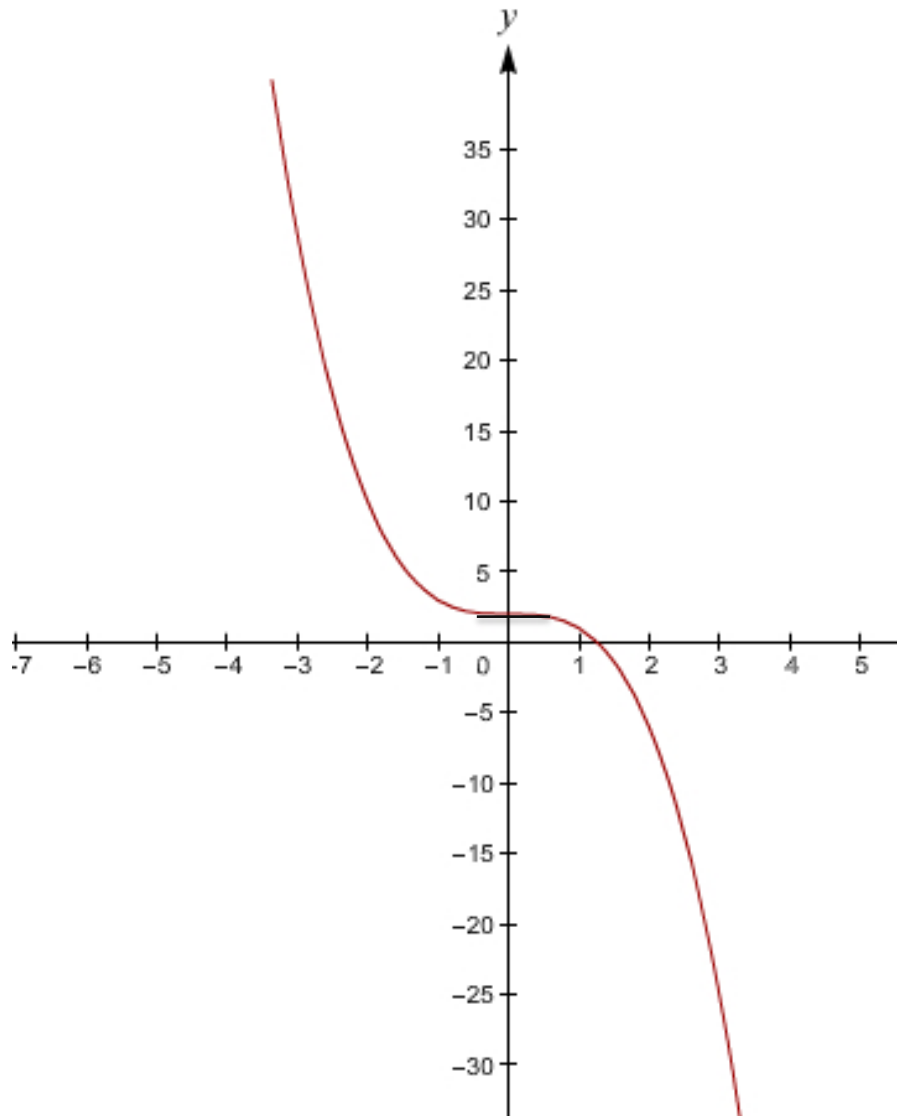
6 (c) $x = -1$ or -7
Sub. values of x into (2)
 $y = 6$ or 14
 $T(-1, 6)$ or $T(-7, 14)$ (shown)

6 (di) $Q(0, \frac{4}{3})$

6 (dii) $1\frac{2}{3}$ units

7a $k = \text{any negative value}$
 $n = \text{any odd integer}$

7b



8(ai) $-4 < x < 1.3$

8(aii) $(1.3, 30)$ or $(-4, -40)$

8(bi) -3.4

8(bii) 8.33

8(ci) $-5.7 < x < -1.6$ and $x > 3.3$

8(cii) $x = 3.4, -1.2$ and -6.3

9(a) 5.92 cm

9(bii) 5.97 cm

9(c) 20.0 cm^2

10 (a) $118m$

10 (b) $-\frac{34}{11}$

10 (c) 125 m

END OF ANSWER SCHEME