Name:____________________________________ **Class:**_____________

JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2025

MATHEMATICS **9758/01**
Higher 2 **3 Sept 2025**

Paper 1 **3 hours**

Additional materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given by [ ] at the end of each question or part question.

1 The curve C has equation $y = \dfrac{1}{4x - x^2}$.

(i) Find $\dfrac{dy}{dx}$. Hence, find the x-coordinate, $x = x_1$, of the turning point on C and determine its nature. [3]

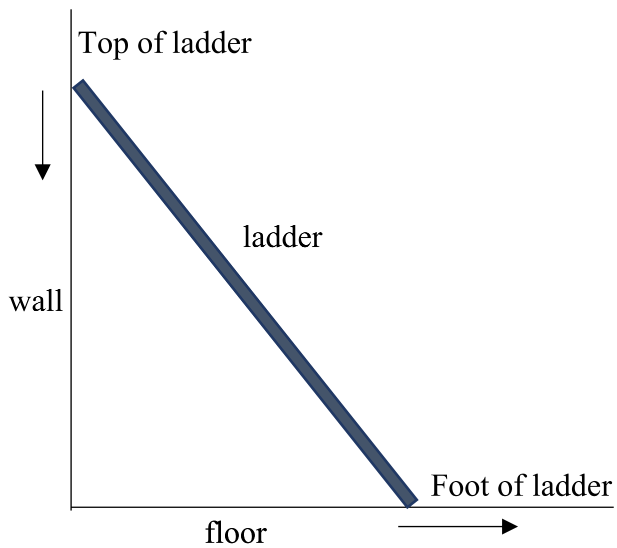
(ii) Using calculus, find the exact area of the region between C, the x-axis and the lines with equations $x = 1$ and $x = x_1$. [3]

2 **(i)** Find, in terms of a, the roots of the equation $\dfrac{1}{(x-a)^2} = |x - a|$. [3]

(ii) On the same axes, sketch the curves with equations $y = \dfrac{1}{(x-a)^2}$ and $y = |x - a|$, where $a > 1$.

Hence solve the inequality $\dfrac{1}{(x-a)^2} > |x - a|$. [3]

3



A ladder of length 3.12 m is sliding down a vertical wall such that the foot of the ladder is moving along the floor at a constant rate of 0.2 m/s (see diagram). Find the rate at which the top of the ladder is sliding down the wall when it is 1.2 m above the floor. [4]

4 Functions f and g are defined by

$$\mathrm{f}: x \mapsto [\ln(x-1)]^2 + 2, \quad x \geq a,$$
$$\mathrm{g}: x \mapsto 4 + 3x - x^2, \quad x \leq \frac{3}{2}.$$

(i) It is given that the function f^{-1} exists. State the smallest value of a. [1]

(ii) Find an expression for $\mathrm{g}^{-1}(x)$, stating its domain. [3]

Using the value of a found in part **(i)**,

(iii) determine whether the composite function $\mathrm{g}^{-1}\mathrm{f}^{-1}$ exists. [1]

5 **(a)** The curves C_1 and C_2 have equations $\frac{x^2}{9} - \frac{y^2}{4} = 1$ and $y^2 + x^2 = k^2$ respectively, where k is a positive constant.

(i) Sketch C_1 and C_2 on the same diagram, stating the coordinates of any points of intersection with the axes and the equations of any asymptotes. [3]

(ii) State the range of values of k for C_1 and C_2 to intersect. [1]

(iii) State the equations of the common lines of symmetry for both C_1 and C_2. [1]

(b) The function f, with domain the set of all real values, is given by

$$\mathrm{f}(x) = \begin{cases} -2x + 6 & \text{for } 0 < x \leq 3, \\ 3x - 9 & \text{for } 3 < x \leq 5, \end{cases}$$

and that $\mathrm{f}(x) = \mathrm{f}(x+5)$.

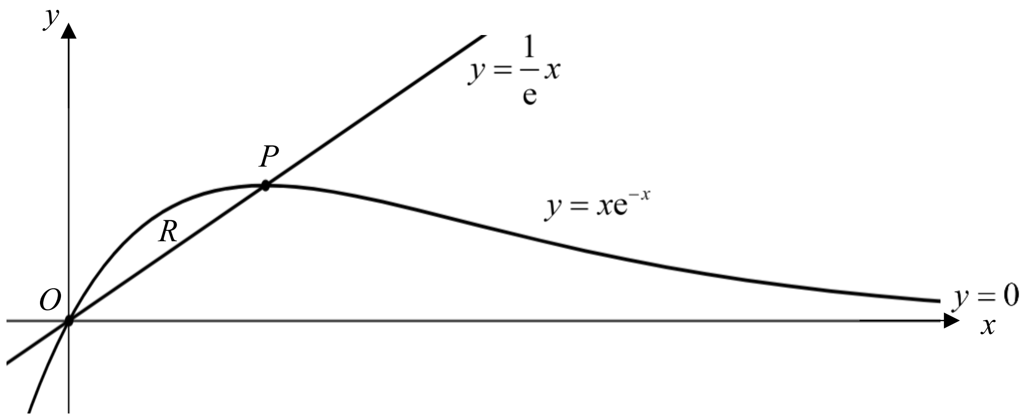
(i) Find $\mathrm{f}(46)$. [1]

(ii) Sketch the graph of $y = \mathrm{f}(x)$ for $-5 \leq x \leq 5$. [2]

(iii) Hence, state the roots of $\mathrm{f}(-x) = 0$ for $-5 \leq x \leq 5$. [1]

6 **(i)** Find $\int x^2 e^{-2x}\,dx$. [4]

(ii) The curve with equation $y = xe^{-x}$ and the line with equation $y = \frac{1}{e}x$ meet at the origin O and the point P with x-coordinate 1. The region R is bounded by the curve and the line (see diagram).



Find the exact volume of the solid formed when R is rotated through $360°$ about the x-axis. [4]

7 **(a)** **(i)** Find $\int \frac{x}{\sqrt{25-x^2}}\,dx$. [2]

(ii) Hence, given that $\int_{\alpha}^{4} \left|\frac{x}{\sqrt{25-x^2}}\right| dx = 3$, where $\alpha < 0$, find α algebraically. [3]

(b) Using the substitution $x = 4\tan\theta$, evaluate $\int_0^4 \sqrt{\frac{x^2}{16+x^2}}\,dx$ exactly. [5]

8 **(a)** The first three terms of a sequence are given by $u_1 = 9$, $u_2 = 27$ and $u_3 = 55$. Given that u_n is a quadratic polynomial in n, find u_n in terms of n. [3]

(b) It is given $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$ and $u_r = 2r^3 + 5$.

(i) Find $\sum_{r=1}^{n} u_r$. [2]

(ii) Hence, or otherwise, find $\sum_{r=2}^{n} \left(2(r+2)^3 + 5\right)$. [3]

(iii) Explain why the series $\sum_{r=1}^{\infty} u_r$ does not converge. [1]

9 It is given that $\mathrm{f}(x)=\dfrac{1}{4+9x^2}$.

(i) Find $\int \mathrm{f}(x)\,\mathrm{d}x$. [2]

(ii) Find the binomial expansion for f(x), up to and including the term in x^4. Give the coefficients as exact fractions in their simplest form. [2]

(iii) Hence, find the Maclaurin series for $\tan^{-1}\dfrac{3x}{2}$. Give the coefficients as exact fractions in their simplest form. [3]

(iv) Use your series from part **(iii)** to estimate $\int_0^{0.5} \tan^{-1}\dfrac{3x}{2}\,\mathrm{d}x$, correct to 3 decimal places. [1]

(v) Use your calculator to find $\int_0^{0.5} \tan^{-1}\dfrac{3x}{2}\,\mathrm{d}x$, correct to 3 decimal places. [1]

(vi) Comparing your answers to parts **(iv)** and **(v)**, comment on the accuracy of your estimate in **(iv)** and how it can be improved. [2]

10 **(a)** Show that $y=\ln\left(\dfrac{\mathrm{e}^2}{3x}\right)$ can be written in the form $y=a+b\ln(cx)$, where a, b and c are integers to be found. Hence, state a sequence of transformations which transform the graph of $y=\ln x$ onto the graph of $y=\ln\left(\dfrac{\mathrm{e}^2}{3x}\right)$. [4]

(b) The curve $y=\mathrm{f}(x)$ passes through the point P with coordinates (a,b), where $b\neq 0$. The tangent to the curve at P has gradient 5. When $y=\mathrm{f}(x)$ is transformed onto the curve $y=\mathrm{g}(x)$, P corresponds to the point R on $y=\mathrm{g}(x)$. For each of the following curves, state the coordinates of R and find the gradient of the curve at R.

(i) $\mathrm{g}(x)=2\mathrm{f}(x-1)$ [3]

(ii) $\mathrm{g}(x)=\dfrac{1}{\mathrm{f}(x)}$ [2]

11 A company posted a video on a social media platform to advertise a new product. The video is uploaded at the start of 1st April and the number of daily views is recorded at the end of each day. Let u_n, where $n \geq 1$, denotes the number of daily views recorded each day. It is given that

$$u_{n+1} = (1+k)u_n,$$

where k is a positive constant.

(i) Explain why the sequence $\{u_n\}$ is a geometric progression. [1]

(ii) Given that the number of daily views recorded at the end of 1st April is 311 and the total number of daily views recorded from 1st April to 3rd April is 4043. Find the value of k. [3]

(iii) Explain why there is no limit to the total number of daily views in the long run. [1]

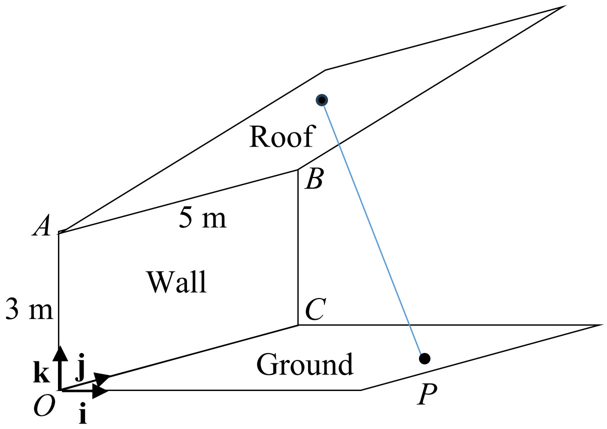
The company also looks at the number of comments being posted on the social media platform. The number of daily comments is recorded at the end of each day. It is given that v_r, where $r \geq 1$, denotes the number of daily comments recorded and it is defined by the following relation:

$$v_r = \begin{cases} u_r & \text{for } 1 \leq r \leq 4, \\ v_{r-1} + 80 & \text{for } r \geq 5. \end{cases}$$

(iv) Show that the total number of comments up to the rth day, where $r \geq 5$, is $40r^2 + 8117r - t$, where t is a constant to be determined. [3]

(v) Find the least number of days required for the total number of daily comments to exceed 100 000. [3]

12 A designer wants to construct an inclined roof that is attached to a vertical rectangular wall, positioned above a horizontal flat ground, as shown in the diagram below. Assume that the roof and wall are of negligible thickness.



Points (x, y, z) are defined relative to a corner of the wall at $O(0,0,0)$, where units are metres. Unit vectors $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$ are in the directions shown in the diagram. The other corners of the wall are A, B and C, such that AB is 5 m and A and B are 3 m vertically above O and C respectively. The roof is attached to the wall along AB and is modelled by a plane parallel to $4\mathbf{i}+5\mathbf{j}+\mathbf{k}$.

(i) Show that the roof is defined by the plane $x-4z=-12$. [3]

(ii) Find the obtuse angle between the wall and the roof, correct to the nearest degree. [3]

To support the roof, a wooden strut, of negligible thickness, is constructed. It has one end at point P with coordinates $(4,3,0)$ on the horizontal ground and the other end at the roof, such that the length of the wooden strut is a minimum.

(iii) Find the exact length of the wooden strut. [2]

(iv) A ray of light is emitted in the direction $7\mathbf{i}-4\mathbf{j}+4\mathbf{k}$ from a light source at C. Show that this light ray meets the wooden strut. Hence, find the coordinates of the point where the light ray meets the wooden strut. [4]

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