Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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MA304.5.4 – More Trigonometric Functions

At the end of this unit, you should be able to

- **understand and use the functions secant, cosecant and cotangent**
- **sketch the graphs of the functions secant, cosecant and cotangent**
- **solve simple trigonometric equations involving secant, cosecant and cotangent**
- **derive and use the identity** $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

By taking the reciprocals of the three trigonometric ratios, we obtain three more trigonometric functions.

$\sec\theta = \dfrac{1}{\cos\theta}$ $\quad$ $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ $\quad$ $\cot\theta = \dfrac{1}{\tan\theta}$

E1. Determine the set of values of θ such that

(a) $\sec\theta$ is not defined.

$\theta = 90^\circ$ or 270°, $0^\circ \leq \theta \leq 360^\circ$

$= 90^\circ + 180^\circ n$

$\theta \in \{(90 + 180n)^\circ \mid n \in \mathbb{Z}\}$

(b) $\operatorname{cosec}\theta$ is not defined

$\theta = 0^\circ$ or 180°, $0^\circ \leq \theta \leq 360^\circ$

$= 180^\circ n$

$\theta \in \{(180n)^\circ \mid n \in \mathbb{Z}\}$

(c) $\cot\theta$ is not defined

$\theta = 0^\circ$ or 180°, $0^\circ \leq \theta \leq 360^\circ$

$= (180n)^\circ$

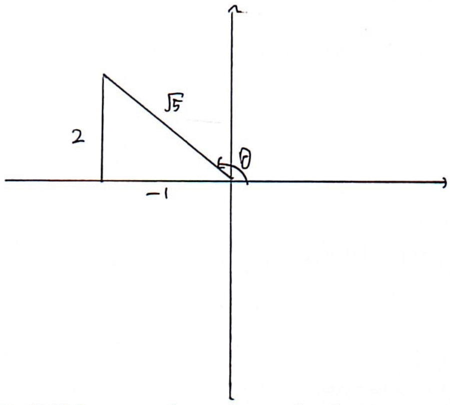
$\theta \in \{(180n)^\circ \mid n \in \mathbb{Z}\}$

P1. Recall and write down the relationship between $\tan\theta$ and $\tan(90^\circ-\theta)$. How will $\cot\theta$ relate with $\tan(90^\circ-\theta)$?

$$\tan\theta = \frac{1}{\tan(90^\circ-\theta)}$$

$$\cot\theta = \tan(90^\circ-\theta)$$

E2. Given that $\tan\theta=-2$ and $\sin\theta$ is positive, determine the exact value of $\sec\theta$, $\operatorname{cosec}\theta$ and $\cot\theta$.



Since $\tan\theta$ is negative and $\sin\theta$ is positive, θ must be in the second quadrant.

$\sqrt{2^2+1^2} = \sqrt{5}$ units.

$\sec\theta = -\sqrt{5}$

$\operatorname{cosec}\theta = \frac{\sqrt{5}}{2}$

$\cot\theta = -\frac{1}{2}$

E3. Without using the calculator, evaluate

(a) $\sec 150^\circ \cot 30^\circ$

$= -\frac{\sqrt{3}}{3}\cdot\sqrt{3}$

$= -\frac{3}{3}$

$= -1$

(b) $\operatorname{cosec} 60^\circ \sec(-300^\circ)$

$= \frac{2\sqrt{3}}{3}\times 2$

$= \frac{4\sqrt{3}}{3}$

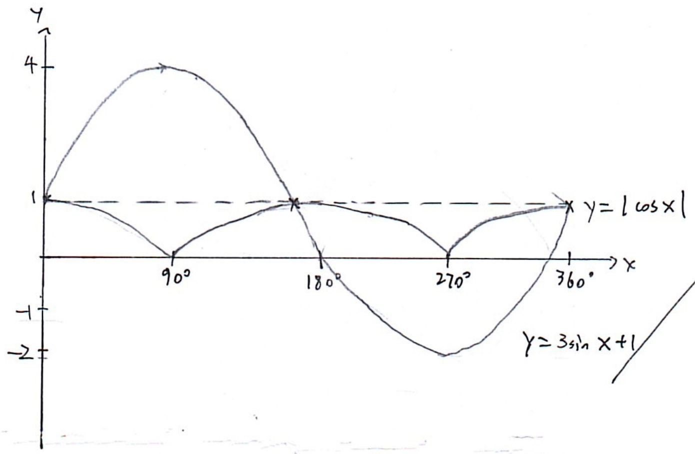
P2. Evaluate $\sin\frac{\pi}{3}\cos\frac{2\pi}{3}+2\tan\frac{\pi}{3}-\operatorname{cosec}\frac{3\pi}{2}$ and leave your answer in exact form.

$= \frac{\sqrt{3}}{2}\left(-\frac{1}{2}\right)+2\sqrt{3}+1$

$= -\frac{\sqrt{3}}{4}+2\sqrt{3}+1$

$= \frac{7\sqrt{3}}{4}+1$

E4. Sketch the graphs of $y=|\cos x|$ and $y=3\sin x+1$ on the same diagram for the domain $0^\circ \le x \le 360^\circ$.



State the number of solutions in this domain for the following equations:

(a) $|\cos x|+2=3(\sin x+1)$

3 solutions.

(b) $|\sec x(3\sin x+1)|=1$

5 solutions.

P3. Solve the following equations for $-\pi \le x \le \pi$,

(a) $\sec x-1=\operatorname{cosec} 70^\circ$

$\sec x - 1 = \frac{1}{\sin 70^\circ}$

$\frac{1}{\cos x} - 1 = \frac{1}{\sin 70^\circ}$

$\cos x = \frac{\sin 70^\circ}{\sin 70^\circ + 1}$

Let α be the basic angle of x.

$\alpha = \cos^{-1}\left(\frac{\sin 70^\circ}{\sin 70^\circ + 1}\right)$

$= 61.0^\circ$ (1d.p.)

$x = 61.0^\circ$ or -61.0°

$= 1.07$ or -1.07

(b) $|5\cot x+2|=3$

$\pm(5\cot x + 2) = 3$

Case 1:

$5\cot x + 2 = 3$

$5\cot x = 1$

$\cot x = \frac{1}{5}$

$\tan x = 5$

Case 2:

$5\cot x + 2 = -3$

$5\cot x = -5$

$\cot x = -1$

$\tan x = -1$

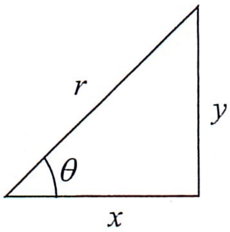
Let the basic angle of x be α.

$\alpha = \tan(-5)$ or $\tan^{-1}(1)$

$= 1.17$ (3sf.) or $\frac{\pi}{4}$

$x = 1.37$ (3sf.) or -1.77 (3sf.) or $-\frac{\pi}{4}$ or $\frac{3\pi}{4}$

E5. Using the right-angled triangle, express $\sin\theta$, $\cos\theta$ and $\tan\theta$ in terms of x, y and r.



$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

(a) Hence show that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\cos\theta \neq 0$.

$\frac{y}{x} = \frac{y}{r} \cdot \frac{r}{x}$

$= \frac{y}{x}$ (shown)

(b) Express $\cot\theta$ in terms of $\sin\theta$ and $\cos\theta$.

$\cot\theta = \frac{1}{\tan\theta}$

$= \frac{\cos\theta}{\sin\theta}$.

P4. Solve for $0 \le x \le 2\pi$ when $\cos x \sin x = 2\sin\frac{\pi}{3}\cos^2 x$.

$\cos x \sin x - 2\sin\frac{\pi}{3}\cos^2 x = 0$

$\cos x(\sin x - 2\sin\frac{\pi}{3}\cos x) = 0$

Case 1: $\cos x = 0$

$x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

Case 2: $\sin x - 2\sin\frac{\pi}{3}\cos x = 0$

$\sin x = 2\sin\frac{\pi}{3}\cos x$

$\tan x = \sqrt{3}$

$x = \tan^{-1}(\sqrt{3})$

$= \frac{\pi}{3}$ or $\frac{4\pi}{3}$