

NAME: _____ ()

CLASS: _____

**FAIRFIELD METHODIST SCHOOL (SECONDARY)****PRELIMINARY EXAMINATION 2025
SECONDARY 4 EXPRESS****ADDITIONAL MATHEMATICS****4049/01****Paper 1****Date: 27 August 2025****Duration: 2 hours 15 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

Table of Penalties		Question Number		90
Presentation	☐ 1 ☐ 2			
Rounding off	☐ 1		Parent's/Guardian's Signature	

Setter: Mdm Haliza

This question paper consists of 22 printed pages including the cover page.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

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Answer **all** the questions.

1 The equation of a curve is $y = 3x^2 - 12x + 10$.

(a) By expressing $y = 3x^2 - 12x + 10$ in the form $a(x+b)^2 + c$, where a , b and c are constants, find the coordinates of the stationary point on the curve. [2]

(b) If the curve is translated 3 units to the left and 2 units down, write the new equation in the form $y = a(x+d)^2 + e$, where a , d and e are constants. [1]

(c) The line $y = 3x + c$ is a tangent to the curve. Find the value of c . [3]

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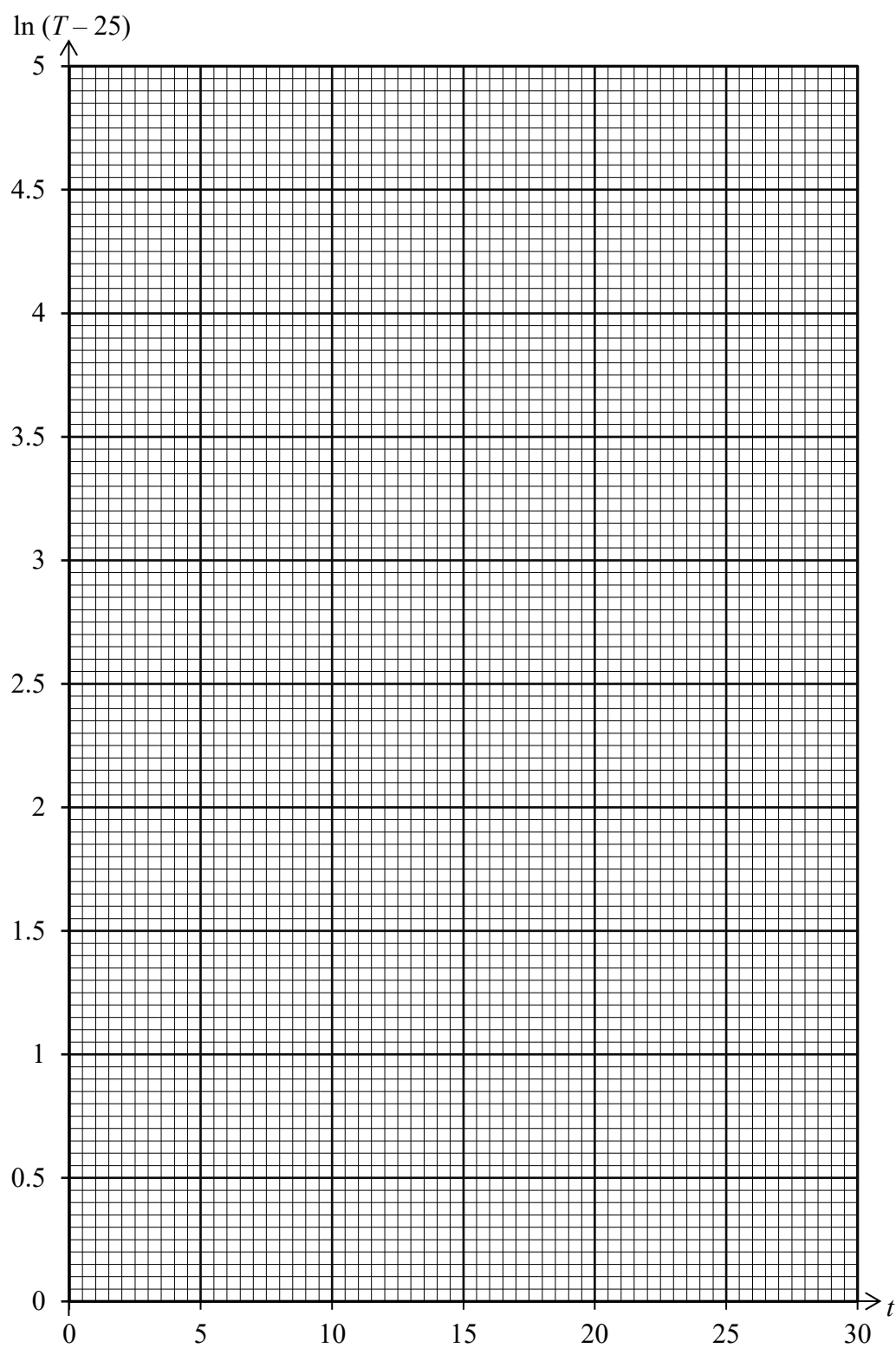
- 2 A group of students investigated the cooling rate of a cup of hot chocolate in a cafe. The temperature of the cafe was 25°C . The temperature of the hot chocolate, $T^{\circ}\text{C}$, after time t minutes is modelled by the formula $T = Ae^{-kt} + 25$ where A and k are constants. Measurements of T and t are shown in the table below.

t (min)	5	10	15	20	25
T ($^{\circ}\text{C}$)	72	58	47	39	35

- (a) On the next page, plot $\ln(T - 25)$ against t and draw a straight line graph to illustrate the information. [3]

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2 **(b)** Use your graph to estimate the value of A and the value of k . [2]

(c) If the ideal drinking temperature is 45°C , use your graph to estimate when to serve the hot chocolate. [1]

(d) The hot chocolate was stirred constantly.
Predict whether the value of k would increase or decrease. Justify your answer. [1]

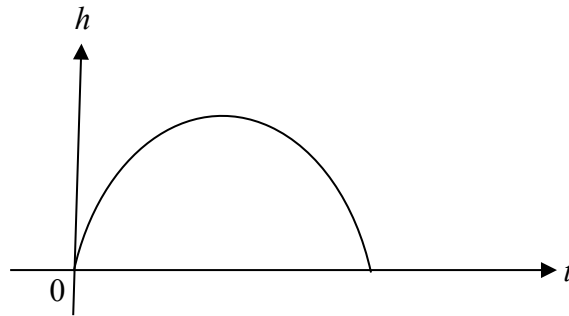
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3 (a) Divide $2x^3 + 5x^2 - 3x + 4$ by $x^3 + 2x^2 - x$. [1]

(b) Express $\frac{2x^3 + 5x^2 - 3x + 4}{x^3 + 2x^2 - x}$ in partial fractions. [5]

4



A swing follows a parabolic path, where its height, h metres, at time t seconds is given by the equation $h = -t^2 + 6t$. The swing starts at rest at $t = 0$, reaches maximum height, and falls back down. At two moments during the swing's motion, the heights are recorded at times k and $3k$ seconds, where $0 < k < 2$.

- (a) Find the range of values of k for which the height at time $3k$ is greater than the height at time k . [3]

- (b) Hence, by referring to the graph or otherwise, explain why the speed of the swing at time k seconds is always greater than the speed at time $3k$ seconds. [2]

- 5** **(a)** Find the first 4 terms in the expansion of $\left(x^2 - \frac{2}{x}\right)^8$ in descending powers of x , simplifying each term. [2]

- (b)** Given that the coefficient of x^{10} in the expansion of $(kx^3 + 5)\left(x^2 - \frac{2}{x}\right)^8$ is -336 , find the value of k . [2]

- (c)** Explain why there is no constant term in the expansion of $\left(x^2 - \frac{2}{x}\right)^8$. [2]

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6 **(a)** Find the amplitude and period of

(i) $\frac{3}{2} \sin x$, [1]

(ii) $1 - 3 \cos 2x$. [2]

(b) Sketch, on the same diagram, the curves $y = \frac{3}{2} \sin x$ and $y = 1 - 3 \cos 2x$ for $0 \leq x \leq 2\pi$. [3]

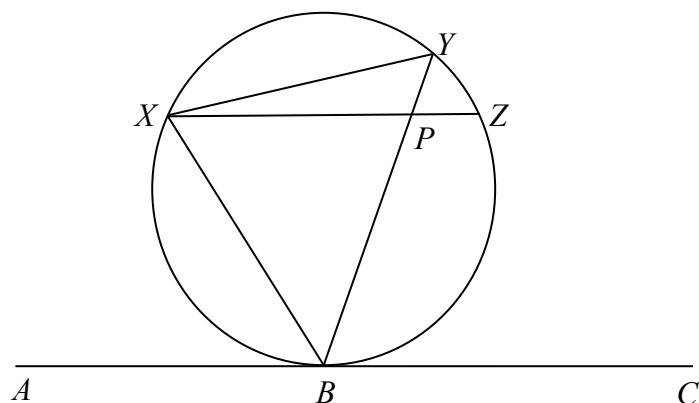


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- 7 A curve is such that $\frac{d^2y}{dx^2} = 3\cos x - 2e^{-x}$. The curve passes through $S(0, 4)$ with a gradient of 2 at S . Find the equation of the curve. [7]

- 8 In the diagram, ABC is a tangent to the circle at point B . The points X , Y and Z lie on the circle. The chords XZ and BY intersect at P and $\angle BXY = \angle BPX$.



Prove that

- (a) triangle BXY is similar to triangle BPX , [2]

- (b) $BX^2 = BP \times BY$, [2]

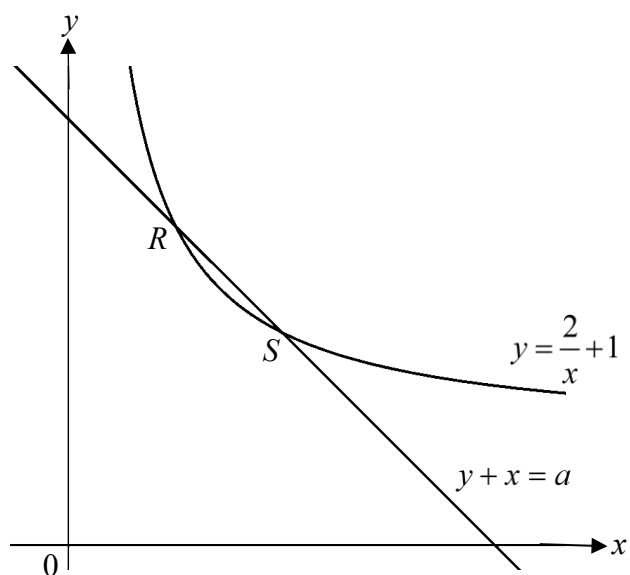
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8 (c) XZ is parallel to AC .

[2]

- 9 The diagram shows a part of a curve $y = \frac{2}{x} + 1$ and the line $y + x = a$ where a is a constant. The line intersects the curve at the points R and S .



- (a) In the case where $a = 4$, find the coordinates of R and S .

[4]

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- 9 (b) Find the set of values of a for which the line does not intersect the curve.
Give your answer in the exact form. [4]

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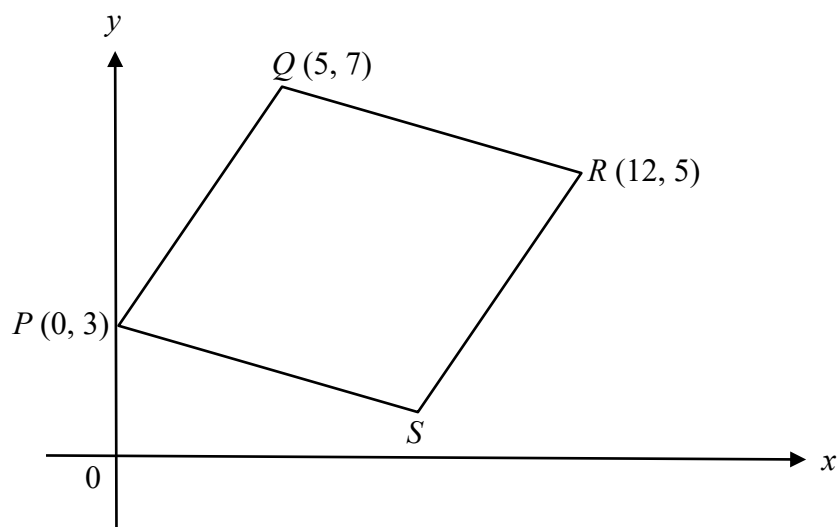
10 The equation of a curve is $y = -x^3 + px^2 + qx + 3$, where p and q are constants.

(a) Show that $p^2 + 3q < 0$ for which y is decreasing. [3]

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- 10 (b) In the case where $p = -1$ and $q = 7$, find the x -coordinate of each of the three points at which the curve intersects the x -axis. [6]

11 Solution to this question by accurate drawing will not be accepted.



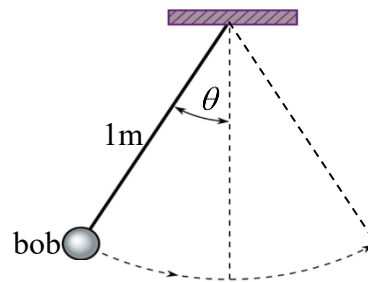
Given that $PQRS$ is a parallelogram, where P is $(0, 3)$, Q is $(5, 7)$ and R is $(12, 5)$, find
(a) the equation of the perpendicular bisector of PR , [3]

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11 **(b)** the coordinates of S , and hence find the area of $PQRS$. [4]

12



A pendulum of length 1m swings back and forth such that its angular displacement θ radians from the vertical is modelled by the function $\theta(t) = a \cos bt$ where t is the time in seconds. The pendulum completes one full swing (left to right and back) in 4 seconds and the maximum angle reached is 0.5 radians.

- (a) Explain why $a = 0.5$ and show that $b = \frac{1}{2}\pi$. [3]

- (b) Obtain an expression for the velocity of the pendulum at time t and hence deduce its maximum speed in m/s. [2]

- (c) Find the magnitude of the acceleration of the pendulum when $\theta = 0.5$ radians after 1 cycle. [2]

13 The curve $y = \frac{3x+5}{x+2}$ is defined for $x > -2$.

(a) Show that the curve has no stationary points. [4]

(b) The point P lies on the curve and the gradient of the curve at P is $\frac{1}{4}$. Find the equation of the normal to the curve at P and determine where it intersects the x -axis and y -axis. [4]

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Continuation of working space for question **13(b)**.

- (c) Calculate the area of the triangle formed by the normal line and the two axes.
[2]

~ End of Paper ~

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Additional Mathematics Paper 1

Answer Key

1(a)	$(2, -2)$	1(b)	$y = 3(x+1)^2 - 4$
1(c)	$c = -8.75$	2(b)	accept $4.2 \leq \ln A \leq 4.4 \Rightarrow 66.686 \leq A \leq 81.45$ $k = 0.0775$ (dependent on graph)
2(c)	accept $15.5 \leq t \leq 16.5$		
2(d)	<u>Stirring increases heat transfer/ causes faster cooling</u> which results in a <u>larger k (more negative slope)</u> . <u>Or k would increase as stirring increase the rate of cooling/causes the gradient to be steeper.</u>		
3(a)	$2 + \frac{x^2 - x + 4}{x^3 + 2x^2 - x}$	3(b)	$2 - \frac{4}{x} + \frac{5x+7}{x^2 + 2x - 1}$
4(a)	$0 < k < 1.5$		
4(b)	For $0 < k < 1.5 \Rightarrow 0 < 3k < 4.5$. The height at $t = k$ is always lower than the height at $t = 3k$. Hence the gradient of the curve at $t = k$ is greater than at $t = 3k$. Hence the speed of the swing at time k seconds is always greater than the speed at time $3k$ seconds. OR $\frac{dh}{dt} = -2t + 6$ For $0 < k < 1.5 \Rightarrow 0 < 3k < 4.5$. At $t = k$, $\frac{dh}{dt} = 6 - 2k$ while at $t = 3k$, $\frac{dh}{dt} = 6 - 6k$. Hence, the speed of the swing at time k seconds is always greater than the speed at time $3k$ seconds.		
5(a)	$x^{16} - 16x^{13} + 112x^{10} - 448x^7$	5(b)	$k = 2$
5(c)	For term independent of x, $r = \frac{16}{3}$ Since r is not a positive integer, there is no term independent of x.		
6(a)(i)	Amplitude = $\frac{3}{2}$ Period = 2π or 360°	6(a)(i)	Amplitude = 3 Period = π or 180°
6 (b)			

7	$y = -3\cos x - 2e^{-x} + 9$	8(a)	To prove $\triangle BXY$ is similar to $\triangle BPX$: $\angle B$ is a common angle $\angle BXY = \angle BPX$ (given) Therefore, $\triangle BXY$ is similar to $\triangle BPX$ (AA)
8(b)	To prove $BX^2 = PB \times BY$: Since $\triangle BXY$ is similar to $\triangle BPX$ from (a), $\frac{BX}{BP} = \frac{BY}{BX}$ $BX^2 = BP \times BY$ (Shown)	8(c)	$\angle CBY = \angle BXY$ (Alt. segment theorem) Since $\angle BXY = \angle BPX$ (given), $\therefore \angle CBY = \angle BPX$ Since $\angle CBY = \angle BPX$, by the converse of alternate angles, XZ is parallel to AC .
9(a)	$R(1, 3)$ and $S(2, 2)$	9(b)	$1 - 2\sqrt{2} < a < 1 + 2\sqrt{2}$
10(a)	$\frac{dy}{dx} = -3\left(x - \frac{1}{3}p\right)^2 + \frac{1}{3}p^2 + q$ Since $-3\left(x - \frac{1}{3}p\right)^2 \leq 0$, then $\frac{1}{3}p^2 + q < 0$ $p^2 + 3q < 0$ (shown)	Or	$\frac{dy}{dx} = -3x^2 + 2px + q$ For y to be decreasing, $\frac{dy}{dx} < 0$. Thus $b^2 - 4ac < 0$. $(2p)^2 - 4(-3)(q) < 0$ $4p^2 + 12q < 0$ $p^2 + 3q < 0$ (Shown)
10(b)	$-3, 1 + \sqrt{2}$ and $1 - \sqrt{2}$	11(a)	$y = -6x + 40$
11(b)	S is $(7, 1)$. Area is 38 units^2	12(a)	<u>Amplitude</u> , a , refers to the <u>maximum angle/displacement</u> of the pendulum swing from the vertical. So $a = 0.5$ (shown) Period = $\frac{2\pi}{b} = 4$ $b = \frac{2\pi}{4}$ $= \frac{1}{2}\pi$ (shown)
12(b)	$v = -\frac{\pi}{4} \sin \frac{\pi}{2}t$ Maximum speed = $\frac{\pi}{4} \text{ m/s}$ or 0.785 m/s (to 3 s.f.)	12(c)	$a = -\frac{\pi^2}{8}$ Magnitude of acceleration is $\frac{\pi^2}{8} \text{ m/s}^2$ or 1.23 m/s^2 (to 3 s.f.)
13(a)	$\frac{dy}{dx} = \frac{1}{(x+2)^2}$ Since $\frac{dy}{dx} \neq 0$ for all real values of x , where $x \neq -2$, the curve does not have a stationary point.	13(b)	Equation of normal at P is $y = -4x + 2.5$ The normal intersects the x -axis at $x = \frac{5}{8}$ or 0.625 and the y -axis at $y = 2.5$.
13(c)	$\frac{25}{32}$ or 0.78125 units^2		