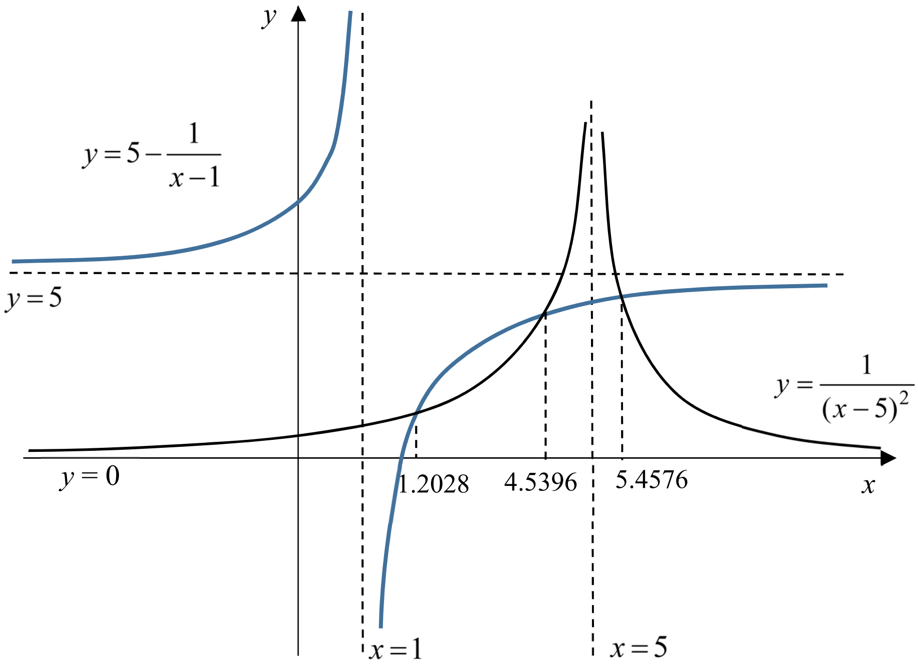
Q1	Suggested Answers
(a)	
(b)	Using GC, the graphs in **(a)** intersect at $x=1.2028,\ 4.5396,\ 5.4576$ For $5-\dfrac{1}{x-1}<\dfrac{1}{(x-5)^2}$ $1<x<1.20$ or $4.54<x<5$ or $5<x<5.46$

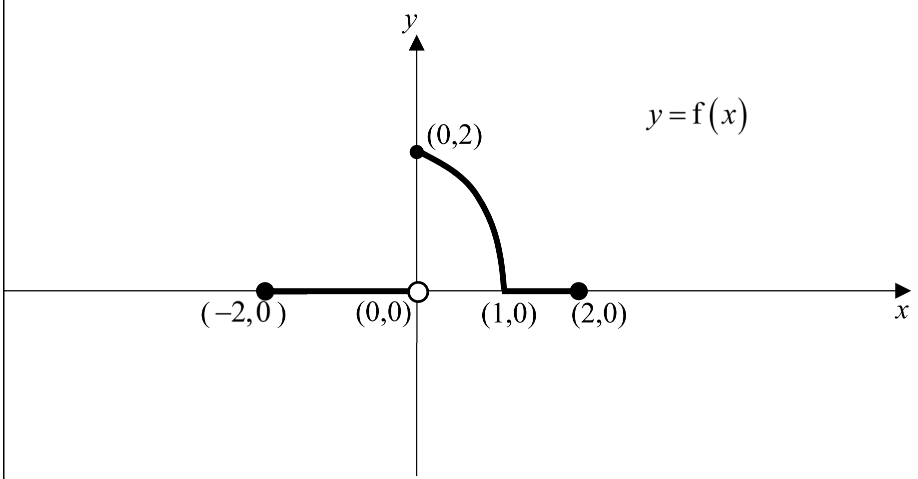
Q2	Suggested Answers
(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(x+\sqrt{1+x^2}\right)\right)$ $=\frac{1}{x+\sqrt{1+x^2}}\times\left(1+\frac{1}{2}\times\frac{2x}{\sqrt{1+x^2}}\right)$ $=\frac{1}{x+\sqrt{1+x^2}}\times\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\right)$ $=\frac{1}{\sqrt{1+x^2}}$
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\tan^{-1}\left(\frac{2+x}{1-2x}\right)=\frac{1}{1+(\frac{2+x}{1-2x})^2}\cdot\frac{(1-2x)-(2+x)(-2)}{(1-2x)^2}$ $=\frac{1-2x+4+2x}{(1-2x)^2+(2+x)^2}$ $=\frac{5}{5x^2+5}=\frac{1}{x^2+1}$

Q3	Suggested Answers
(a)	O, A, C and B forms a parallelogram. Or O, A, C and B are vertices of a parallelogram.
(b)	$\overrightarrow{OC} = \mathbf{a}+\mathbf{b}$ $\overrightarrow{OM} = \frac{1}{3}\left(2\overrightarrow{OA}+\overrightarrow{OC}\right) = \frac{1}{3}(2\mathbf{a}+\mathbf{a}+\mathbf{b}) = \frac{1}{3}(3\mathbf{a}+\mathbf{b}) = \mathbf{a}+\frac{\mathbf{b}}{3}$ Area of triangle OAM $= \frac{1}{2}\left\lvert\overrightarrow{OA}\times\overrightarrow{OM}\right\rvert = \frac{1}{2}\left\lvert\mathbf{a}\times\left(\mathbf{a}+\frac{\mathbf{b}}{3}\right)\right\rvert$ $= \frac{1}{2}\left\lvert\mathbf{a}\times\mathbf{a}+\mathbf{a}\times\frac{\mathbf{b}}{3}\right\rvert = \frac{1}{2}\left\lvert\mathbf{0}+\mathbf{a}\times\frac{\mathbf{b}}{3}\right\rvert$ $= \frac{1}{6}\lvert\mathbf{a}\times\mathbf{b}\rvert$
(b)	**Alternative method:** Area of triangle OAC $= \frac{1}{2}\left\lvert\overrightarrow{OA}\times\overrightarrow{OC}\right\rvert = \frac{1}{2}\lvert\mathbf{a}\times(\mathbf{a}+\mathbf{b})\rvert$ $= \frac{1}{2}\lvert\mathbf{a}\times\mathbf{a}+\mathbf{a}\times\mathbf{b}\rvert = \frac{1}{2}\lvert\mathbf{0}+\mathbf{a}\times\mathbf{b}\rvert$ $= \frac{1}{2}\lvert\mathbf{a}\times\mathbf{b}\rvert$ Area of triangle OAM $= \frac{1}{3}\times\text{area of triangle } OAC$ $= \frac{1}{6}\lvert\mathbf{a}\times\mathbf{b}\rvert$
(c)	The shortest distance from A to OB = $\lvert\mathbf{a}\times\hat{\mathbf{b}}\rvert = \left\lvert\mathbf{a}\times\frac{\mathbf{b}}{\lvert\mathbf{b}\rvert}\right\rvert = \sqrt{3}$ $\lvert\mathbf{a}\times\mathbf{b}\rvert = 5\sqrt{3}$ Area of triangle OAM $= \frac{1}{6}\lvert\mathbf{a}\times\mathbf{b}\rvert = \frac{1}{6}\times5\sqrt{3} = \frac{5\sqrt{3}}{6}$
(c)	**Alternative method 1:** Area of triangle OAM $= \frac{1}{6}\lvert\mathbf{a}\times\mathbf{b}\rvert = \frac{1}{6}\times$ area of parallelogram $OACB$ $= \frac{1}{6}\times$ base $OB\times$ height of parallelogram $= \frac{1}{6}\times5\times\sqrt{3} = \frac{5\sqrt{3}}{6}$ **Alternative method 2:**

	Let θ be the angle between **a** and **b** Area of triangle OAM $=\frac{1}{6}\lvert\mathbf{a}\times\mathbf{b}\rvert$ OR $=\frac{1}{6}\times(2\times\text{area of }\triangle AOB)$ $=\frac{1}{6}\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta$ $=\frac{1}{6}\times(2\times\frac{1}{2}\times5\times\sqrt{3})$ $=\frac{1}{6}\times\lvert\mathbf{a}\rvert\times5\times\frac{\sqrt{3}}{\lvert\mathbf{a}\rvert}=\frac{5\sqrt{3}}{6}$ $=\frac{5\sqrt{3}}{6}$
(d)	Since $ON:NM=3:1$, $\overrightarrow{ON}=\frac{3}{4}\overrightarrow{OM}=\frac{3}{4}\left(\mathbf{a}+\frac{\mathbf{b}}{3}\right)=\frac{3}{4}\mathbf{a}+\frac{1}{4}\mathbf{b}$ $\overrightarrow{AN}=\overrightarrow{ON}-\overrightarrow{OA}$ $=\frac{3}{4}\mathbf{a}+\frac{1}{4}\mathbf{b}-\mathbf{a}=\frac{1}{4}(\mathbf{b}-\mathbf{a})=\frac{1}{4}\overrightarrow{AB}$ Since $\overrightarrow{AN}$ is parallel to $\overrightarrow{AB}$ and A is a common point, A, N and B are collinear. $AN:NB=1:3$

<table>
<tr><th>Q4</th><th>Suggested Answers</th></tr>
<tr><td>(a)</td><td>(1) Translate the curve by 1 unit in the negative x-direction
(2) Reflect the curve in the y-axis
(3) Scale the curve by factor 2 parallel to the y-axis

(1) Reflect the curve in the y-axis
(2) Translate the curve by 1 unit in the positive x-direction
(3) Scale the curve by factor 2 parallel to the y-axis

(1) Translate the curve by 4 units in the negative x-direction
(2) Reflect the curve in the y-axis
(3) Scale the curve by factor 1/4 parallel to the x-axis</td></tr>
<tr><td>(b)(i)</td><td>
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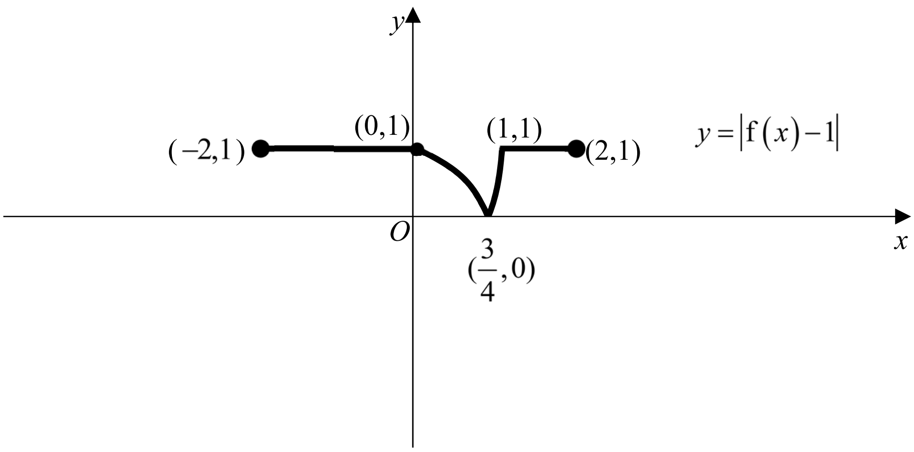
<tr><td>(b)(ii)</td><td>
</td></tr>
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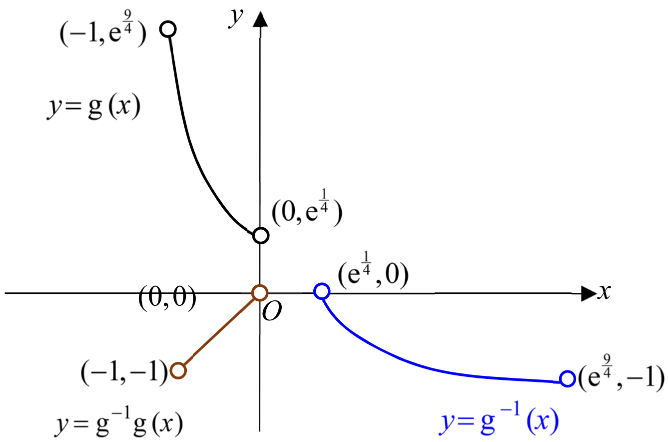
Q5	Suggested Answers
(a)	Greatest value of $a = ½$ <u>Explanation (for students)</u> Since $\mathrm{g}(x)=\mathrm{e}^{\left(x-\frac{1}{2}\right)^2}$ is an even function that is symmetrical about $x=\dfrac{1}{2}$, for g to be a 1-1 function, the largest $\mathrm{D}_g=(-1,\frac{1}{2})$. Alternatively, use GC to find the minimum point of $y=\mathrm{g}(x)$.
(b)	Given $a=0$. So,  Note that $\mathrm{D}_{\mathrm{g}^{-1}\mathrm{g}}=\mathrm{D}_{\mathrm{g}}=(-1,0)$
(c)(i)	$\mathrm{hg}(x)=\mathrm{h}\left(\mathrm{e}^{\left(x-\frac{1}{2}\right)^2}\right)=1+\ln\left(\mathrm{e}^{\left(x-\frac{1}{2}\right)^2}\right)=1+\left(x-\dfrac{1}{2}\right)^2$ $\mathrm{D}_{\mathrm{hg}}=(-1,0)$
(c)(ii)	"Hence" method: Let $x=(\mathrm{hg})^{-1}\left(\frac{3}{2}\right)$ $(\mathrm{hg})(x)=\frac{3}{2},\quad -1<x<0$ $1+\left(x-\dfrac{1}{2}\right)^2=\dfrac{3}{2}$ $\left(x-\dfrac{1}{2}\right)^2=\dfrac{1}{2}$ $x=\dfrac{1}{2}\pm\dfrac{1}{\sqrt{2}}$ (reject $x=\dfrac{1}{2}+\dfrac{1}{\sqrt{2}}$ since $-1<x<0$) $\therefore x=(\mathrm{hg})^{-1}\left(\frac{3}{2}\right)=\dfrac{1}{2}-\dfrac{1}{\sqrt{2}}=\dfrac{1-\sqrt{2}}{2}$
(c)(ii)	"Otherwise" method:

	Let $y = \mathrm{hg}(x) = 1 + \left(x - \frac{1}{2}\right)^2, -1 < x < 0$ $\left(x - \frac{1}{2}\right)^2 = y - 1$ $x = \frac{1}{2} \pm \sqrt{y-1}$ (Reject $x = \frac{1}{2} + \sqrt{y-1}$ since $-1 < x < 0$) $\Rightarrow x = (\mathrm{hg})^{-1}(y) = \frac{1}{2} - \sqrt{y-1}$ $\Rightarrow (\mathrm{hg})^{-1}(x) = \frac{1}{2} - \sqrt{x-1}$ $\therefore (\mathrm{hg})^{-1}\left(\frac{3}{2}\right) = \frac{1}{2} - \sqrt{\frac{3}{2} - 1} = \frac{1}{2} - \frac{1}{\sqrt{2}} = \frac{1-\sqrt{2}}{2}$

<table>
<tr><th>6</th><th>Suggested Answers</th></tr>
<tr><td>(a)</td><td>

$$\begin{aligned}
\mathrm{f}(r-1)-2\mathrm{f}(r)+\mathrm{f}(r+1) &= \ln(r-1)-2\ln(r)+\ln(r+1) \\
&= \ln\left(\frac{(r-1)(r+1)}{r^2}\right) \\
&= \ln\left(\frac{r^2-1}{r^2}\right) = \ln\left(1-\frac{1}{r^2}\right)
\end{aligned}$$

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<tr><td>(b)</td><td>

$$\sum_{r=2}^{N}\ln\left(1-\frac{1}{r^2}\right) = \sum_{r=2}^{N}\left[\mathrm{f}(r-1)-2\mathrm{f}(r)+\mathrm{f}(r+1)\right]$$

$$\begin{aligned}
&= \mathrm{f}(1)-2\mathrm{f}(2)+\mathrm{f}(3) \\
&\quad + \mathrm{f}(2)-2\mathrm{f}(3)+\mathrm{f}(4) \\
&\quad + \mathrm{f}(3)-2\mathrm{f}(4)+\mathrm{f}(5) \\
&\quad + \mathrm{f}(4)-2\mathrm{f}(5)+\mathrm{f}(6) \\
&\quad +\ldots \\
&\quad + \mathrm{f}(N-3)-2\mathrm{f}(N-2)+\mathrm{f}(N-1) \\
&\quad + \mathrm{f}(N-2)-2\mathrm{f}(N-1)+\mathrm{f}(N) \\
&\quad + \mathrm{f}(N-1)-2\mathrm{f}(N)+\mathrm{f}(N+1) \\
&= \mathrm{f}(1)-2\mathrm{f}(2)+\mathrm{f}(2)+\mathrm{f}(N)-2\mathrm{f}(N)+\mathrm{f}(N+1) \\
&= \ln 1-\ln 2-\ln N+\ln(N+1) \\
&= \ln\left(\frac{N+1}{2N}\right)
\end{aligned}$$

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<tr><td>(c)</td><td>

$$\frac{N+1}{2N} = \frac{1}{2}+\frac{1}{2N}$$

As $N\to\infty$, $\frac{1}{2N}\to 0$ thus $\frac{1}{2}+\frac{1}{2N}\to\frac{1}{2}$.

Hence $\sum_{r=2}^{N}\ln\left(1-\frac{1}{r^2}\right)\to\ln\left(\frac{1}{2}\right)$ which is a constant/finite value.

Hence the series converges.

$$\sum_{r=2}^{\infty}\ln\left(1-\frac{1}{r^2}\right) = \ln\left(\frac{1}{2}\right)$$

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<tr><td>(d)</td><td>

Method 1:

$$\ln\left(1-\frac{1}{(2k+1)^2}\right)+\ln\left(1-\frac{1}{(2k+2)^2}\right)+\ln\left(1-\frac{1}{(2k+3)^2}\right)+\ldots$$

$$\begin{aligned}
&= \sum_{r=2k+1}^{\infty}\ln\left(1-\frac{1}{r^2}\right) \\
&= \sum_{r=2}^{\infty}\ln\left(1-\frac{1}{r^2}\right)-\sum_{r=2}^{2k}\ln\left(1-\frac{1}{r^2}\right) \\
&= \ln\left(\frac{1}{2}\right)-\ln\left(\frac{2k+1}{2(2k)}\right) \\
&= \ln\left(\frac{4k}{2(2k+1)}\right) = \ln\left(\frac{2k}{2k+1}\right)
\end{aligned}$$

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	Method 2 (Change of index): $\ln\left(1-\frac{1}{(2k+1)^2}\right)+\ln\left(1-\frac{1}{(2k+2)^2}\right)+\ln\left(1-\frac{1}{(2k+3)^2}\right)+\ldots$ $=\sum_{r=1}^{\infty}\ln\left(1-\frac{1}{(2k+r)^2}\right)$ Replace r with $r-2k$ $=\sum_{r-2k=1}^{\infty}\ln\left(1-\frac{1}{(2k+r-2k)^2}\right)$ $=\sum_{r=2k+1}^{\infty}\ln\left(1-\frac{1}{r^2}\right)$ $=\sum_{r=2}^{\infty}\ln\left(1-\frac{1}{r^2}\right)-\sum_{r=2}^{2k}\ln\left(1-\frac{1}{r^2}\right)$ $=\ln\left(\frac{1}{2}\right)-\ln\left(\frac{2k+1}{2(2k)}\right)$ $=\ln\left(\frac{4k}{2(2k+1)}\right)$ $=\ln\left(\frac{2k}{2k+1}\right)$

Q7	Suggested Answers
(a)	$\dfrac{dx}{dt}=\dfrac{1}{2}e^{t}+2e^{-t}$ $\dfrac{dy}{dt}=e^{t}-2e^{-t}$ $\dfrac{dy}{dx}=\dfrac{e^{t}-2e^{-t}}{\frac{1}{2}e^{t}+2e^{-t}}$ At $t=\ln 4$, $\dfrac{dy}{dx}=\dfrac{e^{\ln 4}-2e^{-\ln 4}}{\frac{1}{2}e^{\ln 4}+2e^{-\ln 4}}=\dfrac{7}{5}$ Gradient of normal at $t=\ln 4$ is $-\dfrac{5}{7}$
(b)	At $t=\ln 4$, $x=\dfrac{3}{2}$, $y=\dfrac{9}{2}$. Equation of normal is $y-\dfrac{9}{2}=-\dfrac{5}{7}\left(x-\dfrac{3}{2}\right)$ $14y-63=-10x+15$ $5x+7y=39$ (shown)
(c)	Sub $x=\dfrac{1}{2}e^{t}-2e^{-t}$ and $y=e^{t}+2e^{-t}$ into $5x+7y=39$ $5\left(\dfrac{1}{2}e^{t}-2e^{-t}\right)+7\left(e^{t}+2e^{-t}\right)=39$ $19e^{t}+8e^{-t}-78=0$ Let $a=e^{t}$ $19a+\dfrac{8}{a}-78=0$ $19a^{2}-78a+8=0$ $(a-4)(19a-2)=0$ $a=4$ or $a=\dfrac{2}{19}$ $t=\ln 4$ or $t=\ln\dfrac{2}{19}$ (reject)

Q8	Suggested Answers
(a)	**Method 1:** $f(x)=\tan(\alpha+\beta x)$ $f'(x)=\beta\sec^2(\alpha+\beta x)$ $\quad=\beta\left[1+\tan^2(\alpha+\beta x)\right]$ $\quad=\beta(1+y^2)$ (shown) **Method 2:** $y=\tan(\alpha+\beta x)$ $\tan^{-1}y=\alpha+\beta x$ $\dfrac{1}{1+y^2}\dfrac{dy}{dx}=\beta$ $f'(x)=\dfrac{dy}{dx}=\beta(1+y^2)$ (shown) $f''(x)=\beta(2y)\dfrac{dy}{dx}$ ------(1) $\quad=2\beta y\left[\beta(1+y^2)\right]$ $\quad=2\beta^2y(1+y^2)$-----(2) or $f''(x)=2\beta^2(y+y^3)$----(3) Differentiate (1) w.r.t. x, $f'''(x)=2\beta\left(\dfrac{dy}{dx}\right)^2+2\beta y\dfrac{d^2y}{dx^2}$ $=2\beta(\beta^2)(1+y^2)^2+2\beta y(2\beta^2y)(1+y^2)$ $=2\beta^3(1+y^2)(1+y^2+2y^2)$ $=2\beta^3(1+y^2)(1+3y^2)$ or differentiate (2) w.r.t. x, $f'''(x)=(2\beta^2y)\left(2y\dfrac{dy}{dx}\right)+2\beta^2\dfrac{dy}{dx}(1+y^2)$ $\quad=2\beta^2\dfrac{dy}{dx}(2y^2+1+y^2)$ $\quad=2\beta^2\left[\beta(1+y^2)\right](2y^2+1+y^2)$ $\quad=2\beta^3(3y^2+1)(1+y^2)$ or differentiate (3) w.r.t. x,

	$\mathrm{f}'''(x)=\left(2\beta^2\right)\left(\frac{\mathrm{d}y}{\mathrm{d}x}+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ $=2\beta^2\frac{\mathrm{d}y}{\mathrm{d}x}\left(1+3y^2\right)$ $=2\beta^2\left[\beta\left(1+y^2\right)\right]\left(1+3y^2\right)$ $=2\beta^3\left(1+3y^2\right)\left(1+y^2\right)$
(b)	For $\alpha=\frac{\pi}{4}$, When x = 0, $\mathrm{f}(0)=\tan(\alpha)=\tan\left(\frac{\pi}{4}\right)=1$ $\mathrm{f}'(0)=\beta\left(1+(1)^2\right)=2\beta$, $\mathrm{f}''(0)=2\beta^2(1)\left(1+(1)^2\right)=4\beta^2$, $\mathrm{f}'''(0)=2\beta^3\left(1+3(1)^2\right)\left(1+(1)^2\right)=16\beta^3$ Hence the Maclaurin series for f(x) is $\mathrm{f}(x)=1+2\beta x+\frac{4\beta^2}{2!}x^2+\frac{16\beta^3}{3!}x^3+\cdots$, i.e. $\mathrm{f}(x)=1+2\beta x+2\beta^2x^2+\frac{8\beta^3}{3}x^3+\cdots$
(c)	Let $\beta=3$ Equation of tangent to curve of $y=\tan\left(\frac{\pi}{4}+3x\right)$ is $y=1+6x$
(d)	When x = 0, cot $3x$ is undefined. Hence Maclaurin expansion for $y=\cot 3x$ cannot be found.

Q9	Suggested Answers
(a)	$\overrightarrow{AC}=\begin{pmatrix}2\\ \alpha\\ \sqrt{2}\end{pmatrix}-\begin{pmatrix}3\\ -1\\ 0\end{pmatrix}=\begin{pmatrix}-1\\ \alpha+1\\ \sqrt{2}\end{pmatrix}$ $\left\lvert\overrightarrow{AC}\cdot\begin{pmatrix}1\\ 0\\ 0\end{pmatrix}\right\rvert=\left\lvert\overrightarrow{AC}\right\rvert\cdot 1\cdot\cos 60^{\circ}$ $\left\lvert\begin{pmatrix}-1\\ \alpha+1\\ \sqrt{2}\end{pmatrix}\cdot\begin{pmatrix}1\\ 0\\ 0\end{pmatrix}\right\rvert=\sqrt{1+(\alpha+1)^2+2}\cdot\frac{1}{2}$ $\lvert -1\rvert=\sqrt{\alpha^2+2\alpha+4}\cdot\frac{1}{2}$ $\alpha^2+2\alpha=0$ $\alpha(\alpha+2)=0$ Since α is a non-zero constant, $\alpha=-2$
(b)	$\overrightarrow{AB}=\begin{pmatrix}5\\ 3\\ 4\end{pmatrix}-\begin{pmatrix}3\\ -1\\ 0\end{pmatrix}=\begin{pmatrix}2\\ 4\\ 4\end{pmatrix}//\begin{pmatrix}1\\ 2\\ 2\end{pmatrix}$ Line l: $\mathbf{r}=\begin{pmatrix}3\\ -1\\ 0\end{pmatrix}+\lambda\begin{pmatrix}1\\ 2\\ 2\end{pmatrix}$, $\lambda\in\mathbb{R}$ Since Q lies on l, $\overrightarrow{OQ}=\begin{pmatrix}3\\ -1\\ 0\end{pmatrix}+\lambda\begin{pmatrix}1\\ 2\\ 2\end{pmatrix}$ for some $\lambda\in\mathbb{R}$ $\overrightarrow{AQ}=\lambda\begin{pmatrix}1\\ 2\\ 2\end{pmatrix}$ $\left\lvert\overrightarrow{AQ}\right\rvert=3 \quad\Rightarrow\quad \left\lvert\lambda\begin{pmatrix}1\\ 2\\ 2\end{pmatrix}\right\rvert=3$ $\lvert\lambda\rvert=1$ $\lambda=\pm 1$ $\lambda=1$: $\quad \overrightarrow{OQ}=\begin{pmatrix}3\\ -1\\ 0\end{pmatrix}+\begin{pmatrix}1\\ 2\\ 2\end{pmatrix}=\begin{pmatrix}4\\ 1\\ 2\end{pmatrix}$ $\lambda=-1$: $\quad \overrightarrow{OQ}=\begin{pmatrix}3\\ -1\\ 0\end{pmatrix}-\begin{pmatrix}1\\ 2\\ 2\end{pmatrix}=\begin{pmatrix}2\\ -3\\ -2\end{pmatrix}$

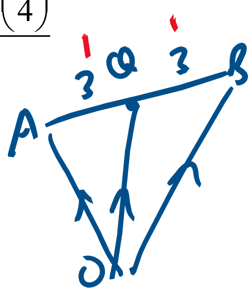
Alternative Method:

$$\overrightarrow{AB}=\begin{pmatrix}5\\3\\4\end{pmatrix}-\begin{pmatrix}3\\-1\\0\end{pmatrix}=\begin{pmatrix}2\\4\\4\end{pmatrix}$$

$$\left|\overrightarrow{AB}\right|=\left|\begin{pmatrix}2\\4\\4\end{pmatrix}\right|=6$$

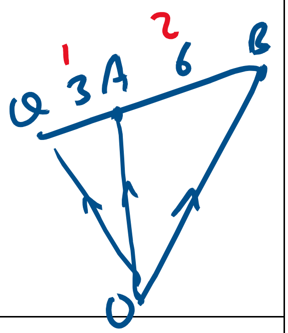
By ratio theorem,

$$\overrightarrow{OQ}=\frac{\overrightarrow{OA}+\overrightarrow{OB}}{2}=\frac{\begin{pmatrix}3\\-1\\0\end{pmatrix}+\begin{pmatrix}5\\3\\4\end{pmatrix}}{2}=\begin{pmatrix}4\\1\\2\end{pmatrix}$$



$$\overrightarrow{OA}=\frac{2\overrightarrow{OQ}+\overrightarrow{OB}}{3}$$

$$\overrightarrow{OQ}=\frac{3\overrightarrow{OA}-\overrightarrow{OB}}{2}=\frac{3\begin{pmatrix}3\\-1\\0\end{pmatrix}-\begin{pmatrix}5\\3\\4\end{pmatrix}}{2}=\begin{pmatrix}2\\-3\\-2\end{pmatrix}$$



(c) **<u>Standard Method</u>:**

Line BF: $\mathbf{r}=\begin{pmatrix}5\\3\\4\end{pmatrix}+\mu\begin{pmatrix}1\\0\\2\end{pmatrix},\ \mu\in\mathbb{R}$ Plane π : $\mathbf{r}\bullet\begin{pmatrix}1\\0\\2\end{pmatrix}=3$

$$\overrightarrow{OF}=\begin{pmatrix}5\\3\\4\end{pmatrix}+\mu\begin{pmatrix}1\\0\\2\end{pmatrix}\text{ for some }\mu\in\mathbb{R}$$

$$\left(\begin{pmatrix}5\\3\\4\end{pmatrix}+\mu\begin{pmatrix}1\\0\\2\end{pmatrix}\right)\bullet\begin{pmatrix}1\\0\\2\end{pmatrix}=3$$

$$(5+8)+\mu(1+4)=3$$

$$\mu=-2$$

$$\overrightarrow{OF}=\begin{pmatrix}5\\3\\4\end{pmatrix}+(-2)\begin{pmatrix}1\\0\\2\end{pmatrix}=\begin{pmatrix}3\\3\\0\end{pmatrix}$$

<u>Alternative method</u>: using projection

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$$\overrightarrow{FB} = \left(\overrightarrow{AB}\cdot\hat{\mathbf{n}}\right)\hat{\mathbf{n}}$$

$$= \left(\begin{pmatrix}2\\4\\4\end{pmatrix}\cdot\frac{1}{\sqrt{5}}\begin{pmatrix}1\\0\\2\end{pmatrix}\right)\frac{1}{\sqrt{5}}\begin{pmatrix}1\\0\\2\end{pmatrix}$$

$$= \frac{10}{5}\begin{pmatrix}1\\0\\2\end{pmatrix}$$

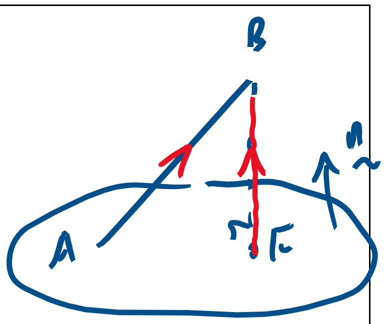
$$= \begin{pmatrix}2\\0\\4\end{pmatrix}$$

$$\overrightarrow{OB} = \overrightarrow{OF} + \overrightarrow{FB}$$

Hence $\overrightarrow{OF} = \overrightarrow{OB} - \overrightarrow{FB}$

$$= \begin{pmatrix}5\\3\\4\end{pmatrix} - \begin{pmatrix}2\\0\\4\end{pmatrix}$$

$$= \begin{pmatrix}3\\3\\0\end{pmatrix}$$



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<tr>
<td>(d)</td>
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Let B' be the reflection of B in π .

$$\overrightarrow{OB}' = \overrightarrow{OB} + 2\overrightarrow{BF}$$

$$= \overrightarrow{OB} - 2\overrightarrow{FB}$$

$$= \begin{pmatrix}5\\3\\4\end{pmatrix} - 2\begin{pmatrix}2\\0\\4\end{pmatrix}$$

$$= \begin{pmatrix}1\\3\\-4\end{pmatrix}$$

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<tr>
<td>(d)</td>
<td>

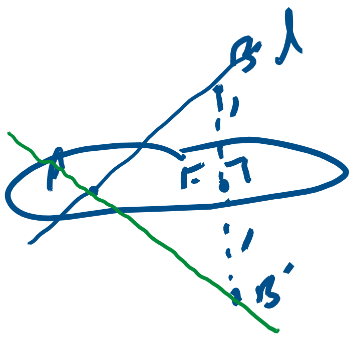
Let B' be the reflection of B in π .

$$\overrightarrow{OF} = \frac{1}{2}\left(\overrightarrow{OB} + \overrightarrow{OB}'\right)$$

$$\overrightarrow{OB}' = 2\overrightarrow{OF} - \overrightarrow{OB}$$

$$= 2\begin{pmatrix}3\\3\\0\end{pmatrix} - \begin{pmatrix}5\\3\\4\end{pmatrix}$$

$$= \begin{pmatrix}1\\3\\-4\end{pmatrix}$$



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	$\overrightarrow{AB'} = \overrightarrow{OB'} - \overrightarrow{OA}$ $= \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$ $= \begin{pmatrix} -2 \\ 4 \\ -4 \end{pmatrix} // \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$ Reflection of l in $\pi : \mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$, $t \in \mathbb{R}$

10	Suggested Answers
(a)	$6+(n-1)(8)=140$ $n=17.75$ Since n is not an integer, it is not possible to have a disc with that base area.
(b)	$h\left(\frac{22}{2}(2(6)+(22-1)(8))\right)=4950$ $h=2.5$
(c)	$3(0.95)^{n-1}\le 2$ $n\ge 8.90$ Hence he needs a minimum of 9 attempts
(d)	see below

(d)

Day (n)	Prize Money at start of day
1	200
2	$200(0.8)+150$
3	$200(0.8)^2+150(0.8)+150$

Prize money on start of Day n

$$=200(0.8)^{n-1}+150(0.8)^{n-2}+150(0.8)^{n-3}+\ldots+150$$

$$=200(0.8)^{n-1}+\underbrace{150(0.8)^{0}+\ldots+150(0.8)^{n-3}+150(0.8)^{n-2}}_{\text{GP with } n-1 \text{ terms}}$$

$$=200(0.8)^{n-1}+\left(\frac{150(1-0.8^{n-1})}{1-0.8}\right)$$

$$=200(0.8)^{n-1}+750-750(0.8)^{n-1}$$

$$=750-550(0.8)^{n-1}$$

(e)

Prize money won by first winner on Day $k=$ $750-550(0.8)^{k-1}$

Prize money won by second winner on Day 24

$$=750-550(0.8)^{(24-k)-1}$$

$$=750-550(0.8)^{(23-k)}$$

$$\left[750-550(0.8)^{k-1}\right]-\left[750-550(0.8)^{23-k}\right]\ge 100$$

$$550(0.8)^{23-k}-550(0.8)^{k-1}-100\ge 0$$

Using GC,

k	$550(0.8)^{23-k}-550(0.8)^{k-1}-100$
16	-4.0080
17	28.698

Hence the set of possible values of k is $\{k\in\mathbb{Z}^+ : 17\le k\le 23\}$

Q11	Suggested Answers
(a)(i)	By Pythagoras Theorem, $\left(\frac{1}{2}a\right)^2+\left(\frac{1}{2}a+h\right)^2=20^2$ $\left(\frac{1}{2}a+h\right)^2=20^2-\left(\frac{1}{2}a\right)^2$ $h=\sqrt{400-0.25a^2}-0.5a$ -------------- (*) Volume $V=a^2\left(\sqrt{400-0.25a^2}-0.5a\right)$ $=a^2\sqrt{400-0.25a^2}-0.5a^3$ (shown) <u>Alternatively</u> $a^2+(a+2h)^2=40^2$ $4h^2+4ah+2a^2-1600=0$ $h=\dfrac{-4a\pm\sqrt{16a^2-4(4)(2a^2-1600)}}{2(4)}$ $=-\frac{a}{2}\pm\frac{1}{8}\sqrt{16\times1600-16a^2}$ $=-\frac{a}{2}\pm\frac{1}{8}\sqrt{64\left(400-\frac{1}{4}a^2\right)}$ $=-\frac{a}{2}+\sqrt{400-0.25a^2}$ (reject $-\frac{a}{2}-\sqrt{400-0.25a^2}$ as $h>0$) Thus Volume $V=a^2\left(\sqrt{400-0.25a^2}-0.5a\right)$
(a)(ii)	$\frac{\mathrm{d}V}{\mathrm{d}a}=2a\sqrt{400-0.25a^2}+a^2\times\dfrac{-0.5a}{2\sqrt{400-0.25a^2}}-1.5a^2$ At stationary points, $\frac{\mathrm{d}V}{\mathrm{d}a}=0$ $2a\sqrt{400-0.25a^2}-\dfrac{0.5a^3}{2\sqrt{400-0.25a^2}}-1.5a^2=0$ Using GC, $a=19.585\approx19.6$ Using (*), $h=7.646\approx7.6$
(b)	$V=\frac{1}{3}\pi r^2x$ Using similar triangles, $\frac{r}{x}=\frac{10}{16}$

	$r = \frac{5}{8}x$ $V = \frac{1}{3}\pi\left(\frac{5}{8}x\right)^2 x = \frac{25}{192}\pi x^3$ Differentiate w.r.t t $\frac{dV}{dt} = \frac{25}{64}\pi x^2 \frac{dx}{dt}$ When $x = 12$ cm, $5 = \frac{25}{64}\pi(12)^2 \frac{dx}{dt}$ $\frac{dx}{dt} = \frac{4}{45\pi}$ cm/s Or $\frac{dx}{dt} = 0.0283$ cm/s