

Class	Register Number	Name
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南洋女子中學校

Nanyang Girls' High School

End-of-Year Examination 2023 Secondary 3

INTEGRATED MATHEMATICS 1

Thursday 5 October

Additional Material: Insert

2 hours
0845 – 1045

READ THESE INSTRUCTIONS FIRST

1. Write your name, register number and class on all the work you hand in.
2. Write in dark blue or black ink.
3. You may use an HB pencil for any diagrams or graphs.
4. Do not use staples, paper clips, glue or correction tape/ fluid.
5. Write your answers and working on the separate writing paper provided, unless otherwise stated.
6. Answer **all** questions.
7. Omission of essential working will result in loss of marks.
8. The use of an approved scientific calculator is expected, where appropriate.
9. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degree to one decimal place. For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .
10. At the end of the examination, fasten all your work securely together.
11. The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **9** printed pages and **1** blank page.

Mathematical Formulae

Mensuration

$$\text{Area of triangle } ABC = \frac{1}{2}ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

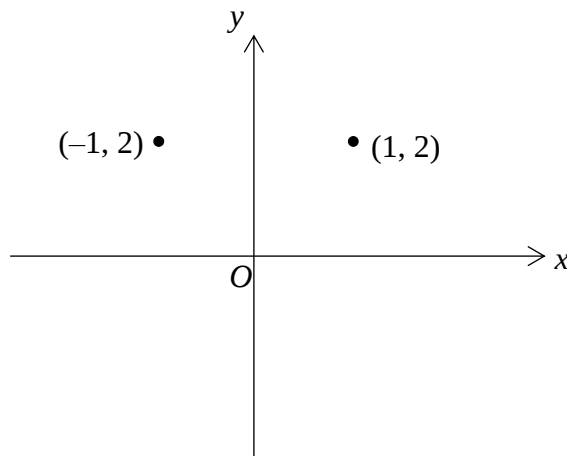
$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

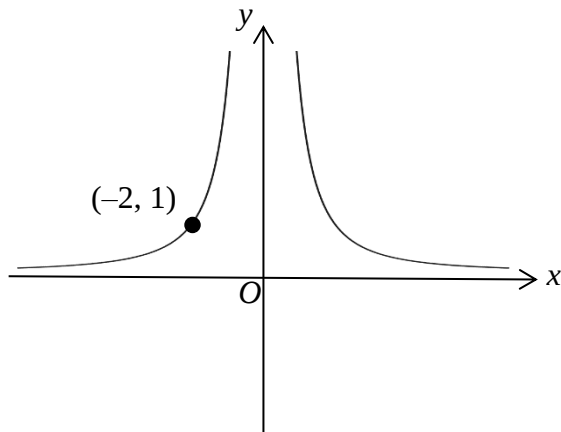
$$a^2 = b^2 + c^2 - 2bc \cos A$$

- 1 Answer the whole of this question on the insert provided.



- (a) On the diagram above, sketch the graph of $y^2 = 4x$. [2]

- (b) The diagram below shows the graph of $y = kx^n$.
Write down a possible value of n and the corresponding value of k . [2]



2 The diagram shows a circle $ABCDE$, centre O .

TAP and TBQ are tangents to the circle.

Angle $AOB = 130^\circ$ and angle $BEC = 30^\circ$.

(a) Giving reasons for each answer, find

(i) angle BDC , [1]

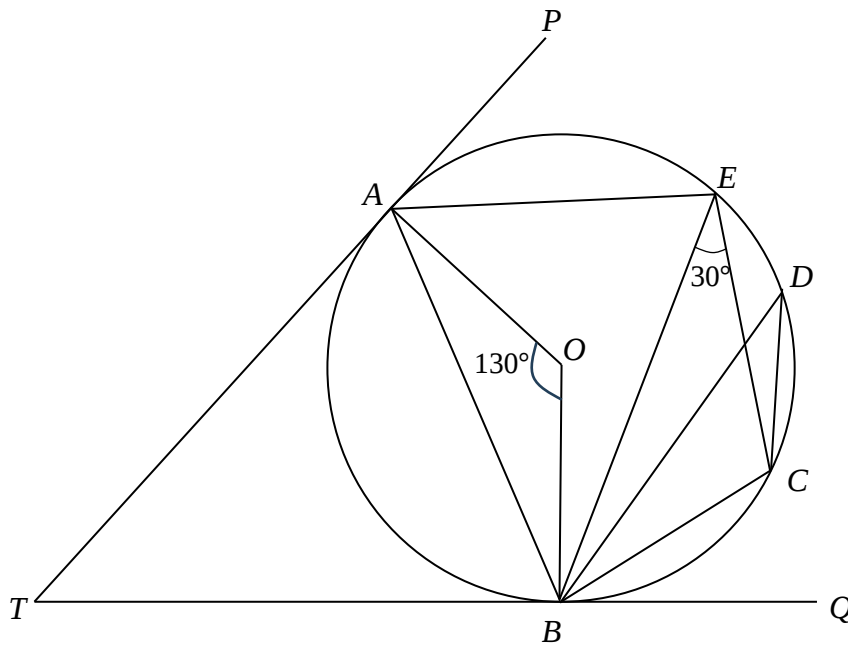
(ii) angle AEB , [1]

(iii) angle ABC , [1]

(iv) angle TAB , [1]

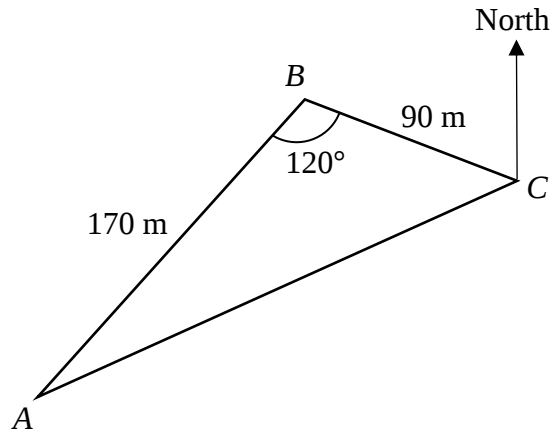
(v) angle ATB . [1]

(b) Explain why a semicircle, with OT as the diameter, passes through A . [2]



- 3 (a) $EFGH$ is a quadrilateral with coordinates $E(2, 1)$, $F(9, -5)$, $G(7, 4)$ and $H(3, 5)$.
- (i) Find the area of $EFGH$. [2]
- (ii) Given that P divides EG internally such that $EP : PG$ is $2 : 3$, find the coordinates of P . [2]
- (b) A is the point $(1, 1)$, B is the point $(1, \sqrt{3})$ and O is the origin.
- (i) Explain why the line AB is parallel to the y -axis. [1]
- (ii) Given that $OATB$ is a parallelogram, find the coordinates of T . [2]
- (iii) Find the angle AOB . [2]
- 4 The variables x and y are related in such a way that when a graph of $y - x$ against x^2y is drawn, the resulting straight line passes through the points $(2, 7)$ and $(5, 1)$.
- (a) Express y in terms of x . [4]
- (b) Find the values of x when $y = 3$. [3]
- (c) If the variables x and y are plotted on another graph of $y + 2x^2y$ against x , find the gradient of the new straight line. [1]
- 5 The equation of a circle, C_1 , is $x^2 + y^2 - 6x - 10y + 15 = 0$.
- (a) Find the radius and the coordinates of the centre of C_1 . [3]
- (b) Determine whether the x -axis is a tangent to C_1 . [2]
- The equation of the tangent to a second circle, C_2 , at point A is $3y = -4x + 9$.
 B is $(5, 13)$ and AB is a diameter of C_2 .
- (c) Show that the coordinates of A are $(-3, 7)$. [4]
- (d) Find the equation of C_2 . [3]

- 6 In the diagram, A , B and C represent three points on level ground.
 $AB = 170$ m and $BC = 90$ m.
 Angle $ABC = 120^\circ$. The bearing of C from B is 110° .



Calculate

- (a) AC , [3]
- (b) the area of triangle ABC , [2]
- (c) the bearing of A from C . [4]

A tower, TB , of height 30 m, stands vertically at B .

- (d) A man is at a point on AC . Calculate the greatest angle of depression of the man from the top of the tower. [3]

- 7 (a) Given that $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} x \\ y \end{pmatrix}$, and that $\mathbf{A}^2\mathbf{B} = \begin{pmatrix} 11 \\ 23 \end{pmatrix}$, find the value of x and of y . [4]

- (b) A coffee company sets up two coffee stalls, X and Y , with both serving three types of coffee. The table shows the number of cups of each type of coffee served during a day and the selling price of each cup of coffee.

	Americano	Cappuccino	Latte
Stall X	32	20	65
Stall Y	48	35	50
Selling price (per cup)	\$4.50	\$5.00	\$6.00

- (i) Given that $\mathbf{A} = \begin{pmatrix} 32 & 20 & 65 \\ 48 & 35 & 50 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4.5 \\ 5 \\ 6 \end{pmatrix}$, evaluate the product $\mathbf{AB}$. [1]
- (ii) Explain what the elements of $\mathbf{AB}$ represent. [2]

A cup of Americano is made from 18 grams of coffee beans.

A cup of Cappuccino is made from 16 grams of coffee beans.

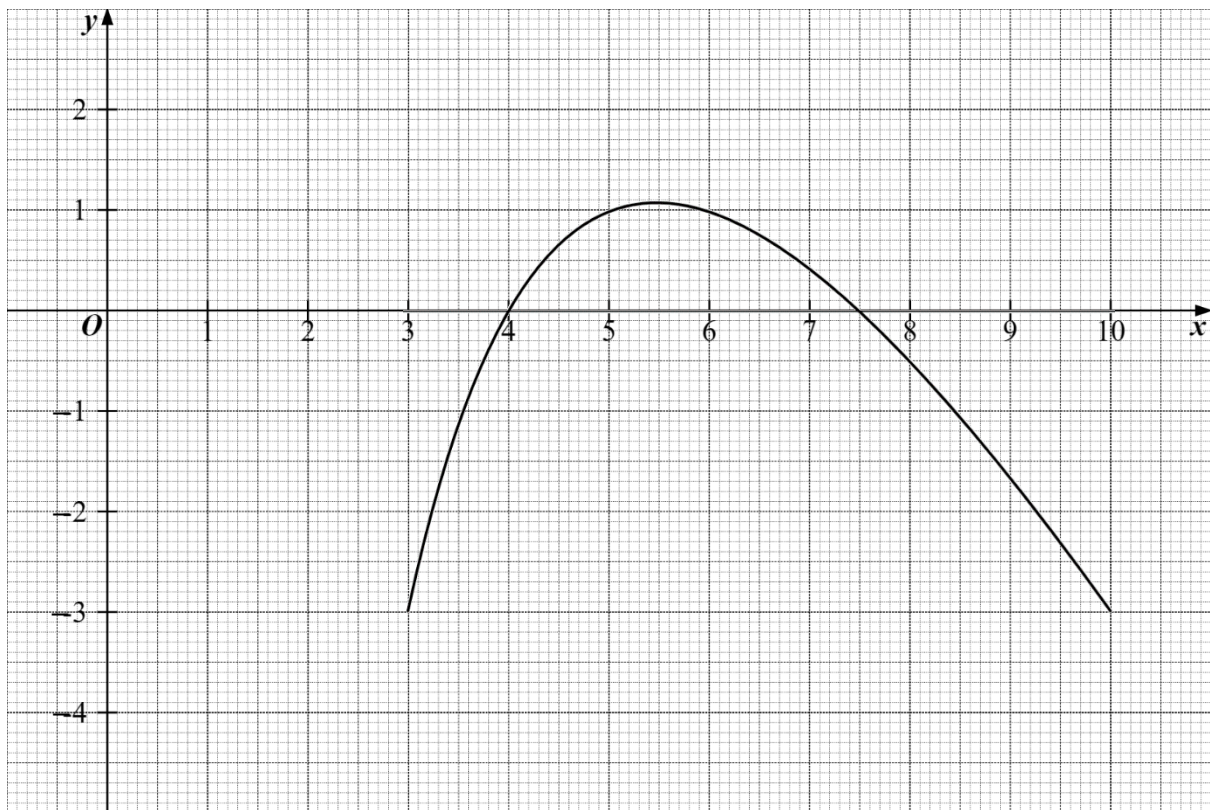
A cup of Latte is made from 20 grams of coffee beans.

- (iii) Stall X operates 5 days a week and Stall Y operates 7 days a week.
Write down two matrices $\mathbf{P}$ and $\mathbf{Q}$ such that $\mathbf{PAQ}$ will give the **total** mass of coffee beans used by the coffee company in a week. [2]
- (iv) Evaluate this product $\mathbf{PAQ}$. [2]

8 Answer the whole of this question on the insert provided.

The diagram below shows the graph of $y = \frac{23x - 60 - 2x^2}{x}$ for $3 \leq x \leq 10$.

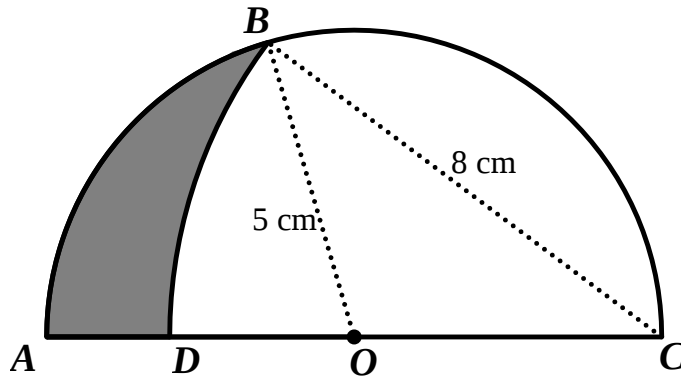
- (a) The equation $\frac{23x - 60 - 2x^2}{x} = 1.1$ has only one solution. Explain how this can be seen from your graph. [2]
- (b) The gradient of the tangent to the curve at point P is 0.4. Use your graph to find the x -coordinate of point P . [2]
- (c) The equation $23x - 60 - 2x^2 = -x^2 + 2x$ can be solved by drawing a suitable straight line on the same axes.
- (i) Find the equation of this straight line. [1]
- (ii) By drawing this straight line, solve the equation $23x - 60 - 2x^2 = -x^2 + 2x$. [2]



9 In the diagram, ABC is a semicircle with centre O and radius 5 cm.

D lies on AC such that CBD is a sector with centre C and radius 8 cm.

- (a) Show that the angle BOA is 1.287 radians, correct to 4 significant figures. [3]
 (b) Find the perimeter of the shaded region. [3]
 (c) Find the area of the shaded region. [4]



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2023 Sec 3 IM1 EOY Answers

1a	
1b	$n = -2$ (or any negative even integer) $k = 4$ (or $2^{-(\text{their } n)}$)
2ai	$\angle BDC = 30^\circ$ ($\angle$ s in same segment)
2aii	$\angle AEB = 65^\circ$ ($\angle$ at centre = $2 \times \angle$ at $\odot$ ce)
2aiii	$\angle ABC = 85^\circ$ ($\angle$ s in opp. segments)
2aiv	$\angle TAB = \angle AEB = 65^\circ$ (Alt. Segment Theorem)
2av	$\angle ATB = 50^\circ$ (tangents from ext. pt.)
2b	$\angle OAT = 90^\circ$ (tangent $\perp$ radius) Since OT is the diameter and $\angle OAT = 90^\circ$, it satisfies the property “right angle in a semicircle”. Therefore, a semicircle, with OT as diameter, passes through A .
3ai	Area = 34 units ²
3aii	$P(4, \frac{11}{5})$
3bi	Points A and B have the same x -coordinate, 1, therefore AB is a vertical line, $x = 1$. Since the gradient of the line AB and the y -axis are both <u>undefined</u> , therefore, the line AB is parallel to the y -axis.
3bii	$T(2, \sqrt{3} + 1)$
3biii	$\angle AOB = 15^\circ$
4a	$y = \frac{x+11}{1+2x^2}$
4b	$x = 1.24$ or -1.07 (3 s.f.)
4c	Gradient = 1

5a	Centre (3, 5) radius = $\sqrt{19} = 4.36$ units (3 s.f.)
5b	The lowest point of the circle is (3, 0.64), which is above the x -axis. Therefore, the x -axis is not a tangent to C_1 .
5c	Equation of tangent at A is $y = -\frac{4}{3}x + 3$ ----- (1) Equation of diameter AB is $y = \frac{3}{4}x + \frac{37}{4}$ ----- (2) A (-3, 7)
5d	Equation of C_2 is $(x-1)^2 + (y-10)^2 = 25$
6a	$AC = 229$ m (3 s.f.)
6b	Area of $\triangle ABC = 6630$ m ² (3 s.f.)
6c	Bearing of A from C = 249.9° (to 1 dp)
6d	Greatest angle of depression = 27.4° (1 d.p.)
7a	$x = 3, y = -1$
7bi	$\begin{pmatrix} 634 \\ 691 \end{pmatrix}$
7bii	The elements of AB represent the total sales for a day of Stall X and Y respectively, with \$634 for Stall X and \$691 for Stall Y.
7biii	$\mathbf{P} = \begin{pmatrix} 5 & 7 \end{pmatrix}$ $\mathbf{Q} = \begin{pmatrix} 18 \\ 16 \\ 20 \end{pmatrix}$
7biv	$\mathbf{PAQ} = \begin{pmatrix} 5 & 7 \end{pmatrix} \begin{pmatrix} 32 & 20 & 65 \\ 48 & 35 & 50 \end{pmatrix} \begin{pmatrix} 18 \\ 16 \\ 20 \end{pmatrix}$ $= (27948)$
8a	Draw the line $y = 1.1$. The line $y = 1.1$ meets the curve $y = \frac{23x - 60 - 2x^2}{x}$ at only one point, therefore, the equation $\frac{23x - 60 - 2x^2}{x} = 1.1$ has only one solution

8b	Draw a line with gradient 0.4 that is tangent to the curve. Then, x -coordinate of point P is 5.
8ci	Equation is $y = -x + 2$
8cii	Draw the line $y = -x + 2$. $x = 3.4$ (Other acceptable answers: 3.3, 3.35, 3.45 or 3.5)
9a	$\angle BOA = 1.28699$ (6 s.f.) $= 1.287$ rad (4 s.f.) shown
9b	Perimeter of shaded region = 13.6 cm (3 s.f.)
9c	Area of shaded region = 7.50 cm^2 (3.s.f.)