2026

SECONDARY 4

G3 Express Exam Paper

G3 A-Math

(Full Paper)

1	Anglican High	Prelim
2	Canberra Sec	Prelim
3	Chung Cheng High	Prelim
4	Compassvale Sec	Prelim
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ANGLICAN HIGH SCHOOL
SECONDARY 4
PRELIMINARY EXAMINATIONS

2026S4

CANDIDATE NAME

CLASS | | **INDEX NUMBER** | |

ADDITIONAL MATHEMATICS
4049/01
Paper 1

21 August 2026
2 hours 15 minutes

Candidates answer on Question Paper.
No Additional Matèrials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and full name on the front cover of your paper.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiners' Use:

Question	1	2	3	4	5	6	7	8	9	10
Marks										
Question	11	12	13	14	15					

Clarity/Logic	Accuracy/Precision	Units		
				90

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1 **(a)** Express $7-6x-3x^2$ in the form of $a(x+b)^2+c$ where a, b, and c are constants. [2]

(b) Hence, find the range of values of the constant h for which the equation $7-6x-3x^2=h$ does not have two distinct roots. [2]

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2 A solid cylinder has a radius of $\left(3+\sqrt{2}\right)$ cm. The volume of the cylinder is $\pi\left(32+13\sqrt{2}\right)$ cm^3. Without using a calculator, express the total surface area of the cylinder in the form of $2\pi\left(a+b\sqrt{2}\right)$, where a and b are constants. [7]

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3 Solve the equation $3\sec^2\theta + 11\tan\theta = 7$ where $0 \leq \theta \leq 2\pi$. [4]

6·28

4 Solve the equation $216^x = \dfrac{5}{\sqrt{6^x}}$. [4]

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5 Find the range of values of the constant k for which the line $y = kx + 1$ does not intersect the curve $y = x^2 - 3kx + 1 - 2k$. [5]

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6 The line $x-2y+7=0$ cuts the curve $3x^2+2y^2=35$ at the points A and B.

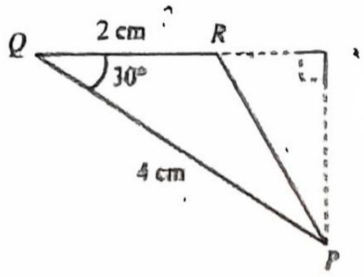
(a) Find the coordinates of A and of B. [5]

(b) Find the area of triangle OAB, where O is the origin. [2]

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7 The diagram shows a triangle PQR in which $PQ = 4$ cm, $QR = 2$ cm and $\angle PQR = 30°$.

26



The point S lies on the line QR produced such that $\angle PSR = 90°$.

(a) Express PR^2 in the form $a + b\sqrt{3}$, where a and b are constants. [2]

(b) Show that $\angle PRS = \tan^{-1}\left(\frac{\sqrt{3}+1}{2}\right)$. [4]

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8 (a) The term independent of x in the expansion of $\left(2x^2+\frac{p}{x}\right)^{12}$, where p is a positive constant, is 7920. Find the value of p. [4]

(b) Using your value of p, find the coefficient of the term in x^3 in the expansion of $\left(1+x^4\right)\left(2x^2+\frac{p}{x}\right)^{12}$. [4]

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9 Given that $f(x) = 7\cos^2 x - 3\sin^2 x$, where $0 \le x \le \pi$

(a) express $f(x)$ in the form of $a\cos 2x + b$, where a and b are constants. [3]

(b) state the greatest and least values of $f(x)$, [2]

(c) state the period and amplitude of $f(x)$. [2]

(d) Sketch the graph of $y = f(x)$ for $0 \le x \le \pi$. [2]

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10 **(a)** Prove that $\frac{\cos\theta}{1-\sin\theta}-\frac{1}{\cos\theta}=\tan\theta$. [4]

(b) Hence solve the equation $\frac{\cos\theta}{1-\sin\theta}-\frac{1}{\cos\theta}=2\sin\theta$ for $180° \le \theta \le 360°$. [4]

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11 A triangular prism has its base an equilateral triangle of side x cm and a height of y cm. The total surface area of the prism is 30 cm^2.

(a) Show that $y = \dfrac{60 - \sqrt{3}x^2}{6x}$. [2]

(b) Hence show that the volume, V cm^3, is given by the equation $V = \dfrac{5\sqrt{3}x}{2} - \dfrac{x^3}{8}$. [1]

(c) Given that x can vary, find the stationary value of V and determine its nature. [4]

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12 Given that $y = \ln\left(\cos^5 x\right)$, where $\frac{-\pi}{2} < x < \frac{\pi}{2}$, without finding any coordinates of the stationary points, explain clearly why the curve only has maximum points. [3]

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13 The gradient function of a curve is given by $\frac{dy}{dx} = 3\sin\left(3x - \frac{\pi}{2}\right) + \frac{\pi}{(x-\pi)^2}$. This curve passes through the point $(2\pi, 4)$. Find the equation of the curve. [5]

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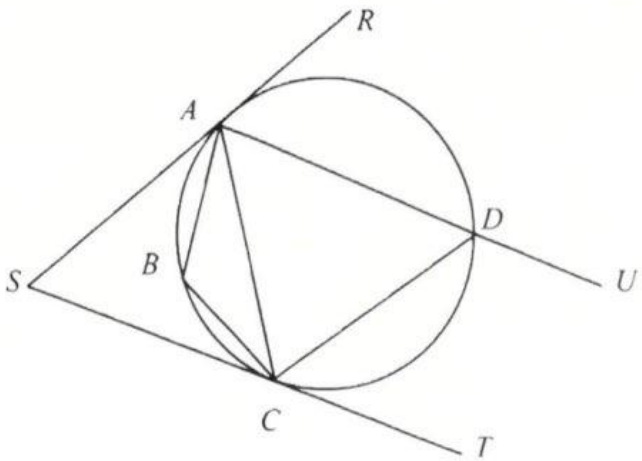
14 The equation of a curve is $y=(x-2)\sqrt{4x+1}$ where $x>-\frac{1}{4}$.

(a) Show that $\frac{dy}{dx}=\frac{3(2x-1)}{\sqrt{4x+1}}$ [3]

(b) Hence find the range of values of x for which the function $y=(x-2)\sqrt{4x+1}$ is decreasing. [3]

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15 The diagram shows a quadrilateral, $ABCD$, inscribed in a circle. The lines SAR and TCS are tangents to the circle at A and C respectively. The lines ADU and SCT are parallel.



(a) Prove that angle ACD = angle ASC [2]

(b) Prove that angle ABC = angle UDC [2]

[3]

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(c) Given that A, S and C lie on a circle, and B is the centre of this circle, prove that the triangle ASC is equilateral. [3]

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SECONDARY 4
PRELIMINARY EXAMINATIONS

2026S4

CANDIDATE NAME	
CLASS	
INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/02**

Paper 2

25th August 2026
2 hours 15 minutes

Candidates answer on Question Paper.
No Additional Materials are required.

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Marks										
Question	11	12	13	14						

Clarity/Logic	Accuracy/Precision	Units		
				90

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1 Express $\dfrac{3x^3-11x^2+8x-4}{x^3-4x^2+4x}$ in partial fractions. [6]

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2 Solve the equation $2^x - 2^{2-x} = 3$. [4]

3 Solve the equation $\dfrac{2\log_5(x-1)}{\log_5 3} - 1 = \log_3(x+5)$. [5]

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4 The cubic polynomial $f(x)$ is such that the roots of $f(x)=0$ are -2, 1 and k. It is given that $f(x)$ has a remainder of -24 when divided by x, and a remainder of -18 when divided by $(x+1)$.

(a) Find the value of k and an expression for $f(x)$. [5]

(b) Hence, or otherwise, solve $f(1-2y)=0$. [2]

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5 The point $P(-4,6)$ and $Q(-7,3)$ lie on the circumference of a circle, C, with centre (a,b) and radius r units. The centre of the circle lies on the line $y=4x+34$.

(a) Find the equation of the circle. [7]

(b) The point $H(k,7)$ lies inside the circle. Find the set of possible values of k. [2]

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6 Find the exact gradient of the curve $y = \dfrac{e^{5x+7}}{(6x-11)^3}$ at the point $\left(2, e^{17}\right)$. [3]

---

7 (a) Given that $y = 3x\,e^{1-2x}$, find $\dfrac{dy}{dx}$. [2]

(b) Hence find $\int xe^{1-2x}\,dx$. [3]

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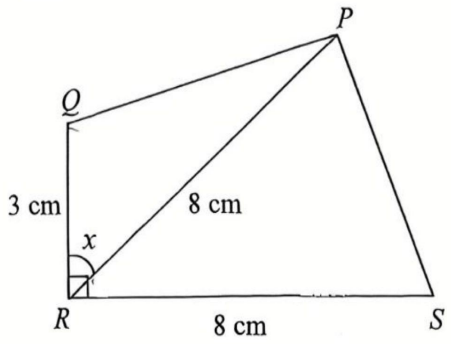
8 Water is leaking from a container at a constant rate of 0.4 cm^3/s. When the depth of the water in the container is x cm, the volume, V cm^3, of water in the container is given by

$$V = \frac{1-2x}{x-3} + 5 \quad (x \neq 3)$$

Find the rate of change of the depth of water when $x = 4$ cm. [5]

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9 The figure $PQRS$ is formed by triangle PQR and triangle PRS. Angle $QRS = 90°$, angle $PRQ = x$, $RQ = 3$ cm, $RP = RS = 8$ cm. The area of the figure is $PQRS$ is A cm^2.



(a) Show that $A = 32\cos x + 12\sin x$. [2]

(b) Express A in the form of $R\cos(x - \alpha)$. [3]

(c) Hence find the maximum value of A and the corresponding value of x. [2]

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10 A ball is rolled down a skateboard ramp. Its height h m, above the ground at t seconds after being released is given by the formula $h = 0.1t^2 - t + 3$.

(a) State the height of the ramp from which the ball is first released. [1]

(b) Express h in the form of $a(t+b)^2 + c$ where a, b and c are constants to be determined. [3]

(c) State the minimum height attained by the ball and the time at which this occurs. [2]

(d) Find the times at which the ball is 2.5 m above the ground. [3]

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11 Given that $\sin A = \frac{5}{13}$, where $0 \leq A \leq \frac{\pi}{2}$ and $\cot B = -\frac{24}{7}$, where $\frac{\pi}{2} \leq B \leq \pi$, without solving for A and B, find the exact value for

(a) $\cos B$, [1]

(b) $\sec A$, [2]

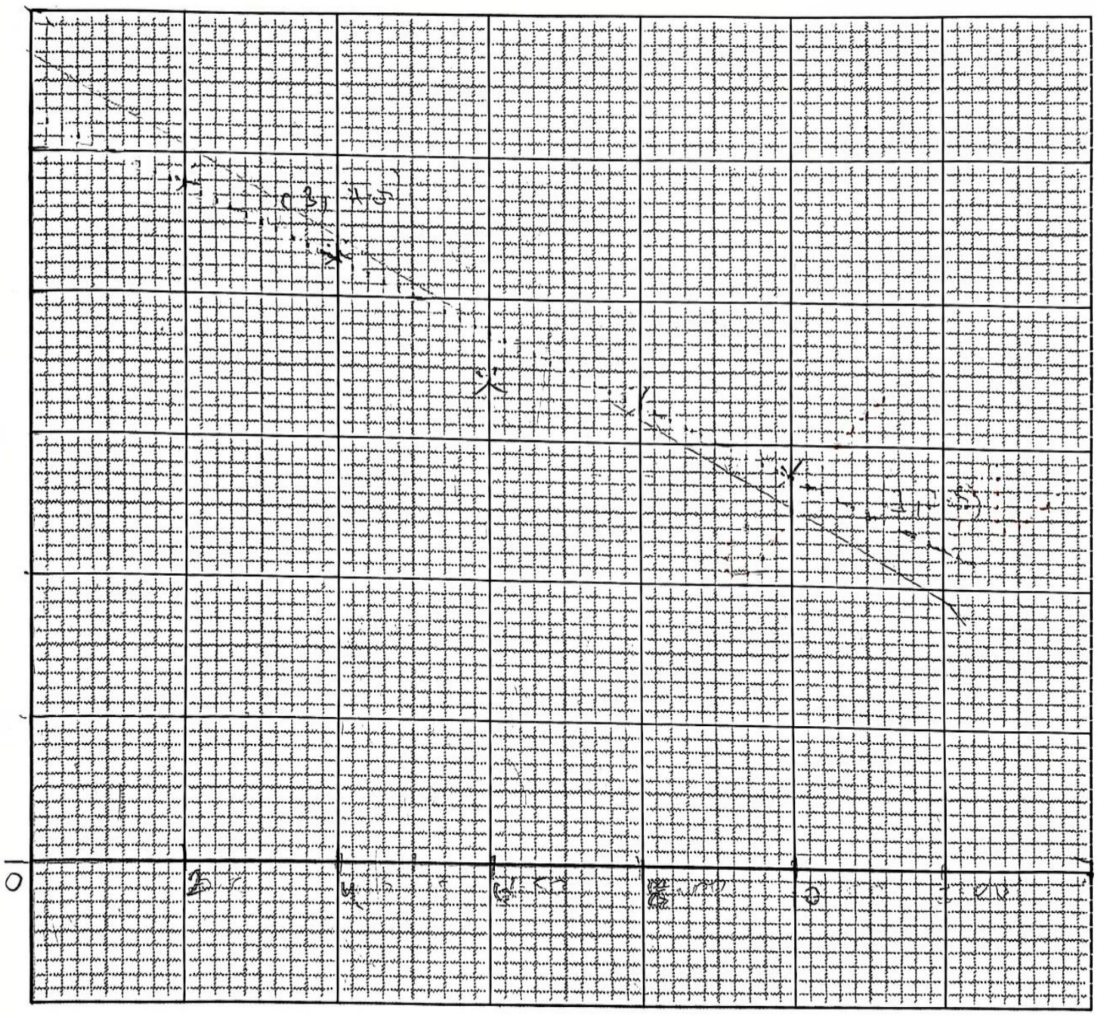
(c) $\sin(A+B)$ [2]

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12 (a) An oceanographer is studying how light intensity decreases at different depths in the ocean. For a specific location, the light intensity, I (in lumens), at a depth of d meters is modelled by the formula $I = Pe^{-kd}$, where P and k are constants. The table below shows corresponding values of d and I. It is believed that an error was made in recording one of the values of I.

d (metres)	2	4	6	8	10
I (lumens)	122	74	30	27	17

Explain how a straight line graph can be drawn to represent the formula and draw it for the given data. [4]



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Using your straight line graph,

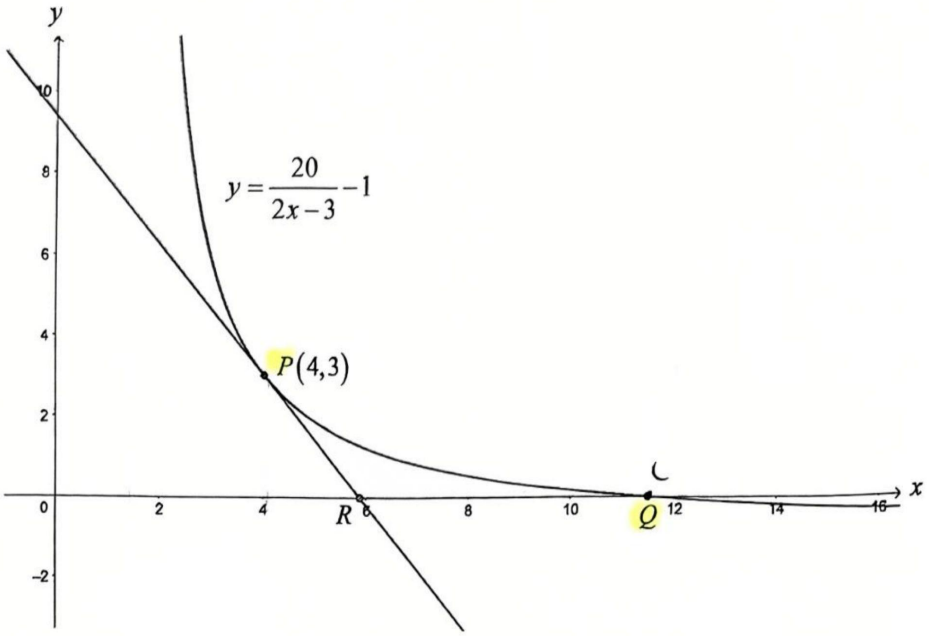
(b) identify the incorrect reading and suggest a corrected value for I, [2]

(c) estimate the value of k, and [2]

(d) deduce the value of P. [2]

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13



The above diagram shows part of the curve $y=\frac{20}{2x-3}-1$ passing through the point $P(4,3)$. The curve meets the x-axis at the point $Q(11.5,0)$. The tangent to the curve at P meets the x-axis at the point R.

(a) Show that the x-coordinate of R is 5.875. [4]

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Find the shaded area PQR.

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14 A particle, travelling along a straight path, passes a point P and, 5 seconds later, passes a point Q. The velocity, v m/s, t seconds after passing P, is given by

$$v = 3t^2 - 13t + 12$$

(a) Find the time when the particle is instantaneously at rest. [2]

(b) Find the distance travelled from P to Q. [5]

--- END OF PAPER ---

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SECONDARY 4
PRELIMINARY EXAMINATIONS

2026S4

CANDIDATE NAME

CLASS **INDEX NUMBER**

ADDITIONAL MATHEMATICS
4049/01
Paper 1

21 August 2026
2 hours 15 minutes

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Markers' Report

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

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1 **(a)** Express $7-6x-3x^2$ in the form of $a(x+b)^2+c$ where a, b, and c are constants. [2]

	$7-6x-3x^2 = -3\left(x^2+2x\right)+7$ $= -3\left[(x+1)^2-1^2\right]+7$ $= -3(x+1)^2+3+7$ $= -3(x+1)^2+10$	
	Marker's comments: Well done. Most students were able to complete the square.	

(b) Hence, find the range of values of the constant h for which the equation $7-6x-3x^2=h$ does not have two distinct roots. [2]

	Max point for $y=7-6x-3x^2$ is $(-1,10)$. Consider the intersection of the curve and the line $y=h$. Does not have two distinct roots means that there is 1 intersection point or no intersection point. Hence, $h \geq 10$
	Marker's comments: Badly done. Many were not able to use the maximum point to explain. Due to "hence", discriminant method, the method that most students used, was not accepted.

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2 A solid cylinder has a radius of $\left(3+\sqrt{2}\right)$ cm. The volume of the cylinder is $\pi\left(32+13\sqrt{2}\right)$ cm^3. Without using a calculator, express the total surface area of the cylinder in the form of $2\pi\left(a+b\sqrt{2}\right)$, where a and b are constants. [7]

$$\pi\left(32+13\sqrt{2}\right)=\pi\left(3+\sqrt{2}\right)^2 h$$

$$=\pi\left(9+2+3(2)\sqrt{2}\right)h$$

$$=\pi\left(11+6\sqrt{2}\right)h$$

$$h=\frac{32+13\sqrt{2}}{11+6\sqrt{2}}\times\frac{11-6\sqrt{2}}{11-6\sqrt{2}}$$

$$=\frac{32(11)+32\left(-6\sqrt{2}\right)+13\sqrt{2}(11)+13\sqrt{2}\left(-6\sqrt{2}\right)}{11^2-\left(6\sqrt{2}\right)^2}$$

$$=\frac{352-192\sqrt{2}+143\sqrt{2}-156}{121-36(2)}$$

$$=\frac{196-49\sqrt{2}}{49}$$

$$=\left(4-\sqrt{2}\right)\text{ cm}$$

Total surface area

$$=2\pi\left(11+6\sqrt{2}\right)+2\pi\left(3+\sqrt{2}\right)\left(4-\sqrt{2}\right)$$

$$=2\pi\left[11+6\sqrt{2}+12-3\sqrt{2}+4\sqrt{2}-2\right]$$

$$=2\pi\left(21+7\sqrt{2}\right)\text{cm}^2$$

Marker's comments:
Well done although there were a handful who did not know the formula for volume of cylinder. Common mistakes were formula for circumference and considering 1 circle instead of 2.

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3 Solve the equation $3\sec^2\theta + 11\tan\theta = 7$ where $0 \le \theta \le 2\pi$. [4]

	$3\sec^2\theta + 11\tan\theta = 7$ $3\left(1+\tan^2\theta\right) + 11\tan\theta - 7 = 0$ $3\tan^2\theta + 11\tan\theta - 4 = 0$	
	$(3\tan\theta - 1)(\tan\theta + 4) = 0$ $\tan\theta = \frac{1}{3}$ *or* $\tan\theta = -4$	
	For $\tan\theta = \frac{1}{3}$ $\theta = 0.32175$, $\pi + 0.32175$ $= 0.322$, 3.46 For $\tan\theta = -4$ $\theta = \pi - 1.32582$, $2\pi - 1.32582$ $= 1.82$, 4.96	
	Marker's comments: Not well done. Common mistakes: 1. changed $\sec^2\theta$ to $1/\cos^2\theta$ and could not get the correct quadratic equation to solve. 2. found the value of $\tan\theta$ and did not solve for θ. 3. wrong quadrants for $\tan\theta = -4$ 4. consider $\tan^{-1}(-4)$ instead of $\tan^{-1}(4)$.	

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4 Solve the equation $216^x = \dfrac{5}{\sqrt{6^x}}$. [4]

	$216^x = \dfrac{5}{\sqrt{6^x}}$ $\left(6^3\right)^x = \dfrac{5}{\left[6^x\right]^{\frac{1}{2}}}$ $6^{3x} = \dfrac{5}{6^{\frac{x}{2}}}$ $6^{3x}\,6^{\frac{x}{2}} = 5$ $6^{\frac{7x}{2}} = 5$ $\ln 6^{\frac{7x}{2}} = \ln 5$ $\dfrac{7x}{2}\ln 6 = \ln 5$ $\dfrac{7x}{2} = \dfrac{\ln 5}{\ln 6}$ $x = \dfrac{2}{7}\times\dfrac{\ln 5}{\ln 6}$ $= 0.256641 = 0.257$	Alternatively, $216^x = \dfrac{5}{\sqrt{6^x}}$ $\ln 216^x = \ln\dfrac{5}{\sqrt{6^x}}$ $x\ln 216 = \ln 5 - \ln\sqrt{6^x}$ $x\ln 216 = \ln 5 - \ln 6^{\frac{x}{2}}$ $x\ln 216 = \ln 5 - \dfrac{x}{2}\ln 6$ $x\ln 216 + \dfrac{x}{2}\ln 6 = \ln 5$ $x\left(\ln 216 + \dfrac{1}{2}\ln 6\right) = \ln 5$ $x = \ln 5 \div \left(\ln 216 + \dfrac{1}{2}\ln 6\right)$ $= 0.256641 = 0.257$
	Well done though some students were taking too many steps to try to solve the equation. Common errors: 1. changing 216^x to 6^3 2. when simplifying terms with base 6, considered the wrong law of indices and got $6^{2.5x}$ 3. write log instead of lg 4. able to get answer from a log that is not lg or ln	

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5 Find the range of values of the constant k for which the line $y = kx + 1$ does not intersect the curve $y = x^2 - 3kx + 1 - 2k$. [5]

	$x^2 - 3kx + 1 - 2k = kx + 1$ $x^2 - 4kx - 2k = 0$ Discriminant < 0 $(-4k)^2 - 4(1)(-2k) < 0$ $16k^2 + 8k < 0$ $8k(2k+1) < 0$  From graph, $-\frac{1}{2} < k < 0$
	Marker's comments: Well done. However there is quite a number who still could not solve quadratic inequalities. Common mistake: k<0 and 2k+1<0.

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6 The line $x-2y+7=0$ cuts the curve $3x^2+2y^2=35$ at the points A and B.

(a) Find the coordinates of A and of B. [5]

$x-2y+7=0$

$x=2y-7$ --- (1)

$3x^2+2y^2=35$ --- (2)

Substitute (1) into (2) yields

$$3(2y-7)^2+2y^2=35$$

$$3(4y^2-28y+49)+2y^2-35=0$$

$$14y^2-84y+112=0$$

$$y^2-6y+8=0$$

$$(y-4)(y-2)=0$$

$y=4$ or 2

Sub $y=2$ into (1)

$x=2(2)-7=-3$

Sub $y=4$ into (1)

$x=2(4)-7=1$

Hence coordinates of A and B are $(-3, 2)$ and $(1, 4)$.

Markers' Comments :

- Many students displayed a lack of good strategy in solving this part of the question, e,g, chose to use "y" as the subject from linear equation rather than "x", obtaining $y=\frac{x}{2}+\frac{7}{2}$

As a result, fractions were introduced and algebraic manipulations become more challenging when this was substituted into the second equation :

$3x^2+2\left(\frac{x}{2}+\frac{7}{2}\right)^2=35$.

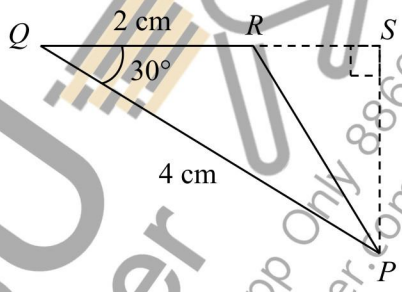
- Careless mistakes like $3(2y-7)+2y^2=35$, $x=2y+7$ occurred frequently in those answers that failed to meet the mark.
- Considerations were given if students showed evidences of doing substitutions, able to obtain quadratic equation of the form $ax^2+bx+c=0$, and showed correct method of solving, either by factorization or general solution. However, no consideration will be given if by error in algebraic manipulations, the equation was reduced to $x^2=3$, for instance.
If method for solving was not shown, then no mark will be awarded for method and answers. No mark if the students mixed up the x and y-coordinates.

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(b) Find the area of triangle OAB, where O is the origin. [2]

	Area of triangle OAB $= \frac{1}{2}\begin{vmatrix} 0 & 1 & -3 & 0 \\ 0 & 4 & 2 & 0 \end{vmatrix}$ $= \frac{1}{2}\left[(0)(4)+(1)(2)+(-3)(0)-(0)(1)-(4)(-3)-(2)(0)\right]$ $= \frac{1}{2}(2+12)$ $= 7 \text{ units}^2$
	Marker's Comments : - Many students did not realise that the shoe-lace method requires them to align the coordinates of points in the anticlockwise direction. No mark will be awarded if they do it in the clockwise direction, even if the answer or calculations is correct. - For those who got the wrong answers from part (a), 1 mark will still be awarded if they align the coordinates in the correct order. - Other common mistakes include not multiplying by $\frac{1}{2}$ or showing only three columns in the formula.

7 The diagram shows a triangle PQR in which $PQ = 4$ cm, $QR = 2$ cm and $\angle PQR = 30°$.



The point S lies on the line QR produced such that $\angle PSR = 90°$.

(a) Express PR^2 in the form $a + b\sqrt{3}$, where a and b are constants. [2]

(i)	$PR^2 = 2^2 + 4^2 - 2(2)(4)\cos 30°$ $PR^2 = 20 - 16\left(\frac{\sqrt{3}}{2}\right)$ $PR^2 = 20 - 8\sqrt{3} \ cm$
	Marker's Comments : - Most students did not use the method above for this part. Instead they found the values for RS and PS, then applied Pythagoras' Theorem to find PR^2. - For those who used the above method in the MS, the most common mistake would be using the wrong formula.

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(b) Show that $\angle PRS = \tan^{-1}\left(\frac{\sqrt{3}+1}{2}\right)$. [4]

(ii)	$\sin 30° = \frac{PS}{4}$ $PS = 2$ $\cos 30° = \frac{RS+2}{4}$ $RS = 2\sqrt{3}-2$ $\tan \angle PRS = \frac{2}{2\sqrt{3}-2}$ $= \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ $= \frac{\sqrt{3}+1}{2}$ $\angle PRS = \tan^{-1}\left(\frac{\sqrt{3}+1}{2}\right)$
	Marker's Comments : - For those who already found the lengths of *PS* and *RS* in part(a), marks would still be awarded accordingly (marking scheme is adjusted). - Considerations will be given as long as students have the idea that $\tan\angle PRS = \frac{PS}{RS}$. - Although the rationalisation process is expected to be presented clearly, some students simplified their working, e.g. $\tan\angle PRS = \frac{2}{2\sqrt{3}-2}$ $= \frac{\sqrt{3}+1}{3-1}$ $= \frac{\sqrt{3}+1}{2}$ Method mark will still be awarded as long as there is some evidence in the working that the student is rationalising the surd, according to the marker's discretion. - Some students found expression for *RS* by using the part (a) answer : $RS^2 = PR^2 - PS^2$ $= 20 - 8\sqrt{3} - 2^2$ $= 16 - 8\sqrt{3}$ Then $RS = \sqrt{16 - 8\sqrt{3}}$ However, though method mark was given to this, it will prove difficult for students to rationalise this when they simplify $\tan\angle PRS = \frac{PS}{RS}$. - Some students stopped their presentation after they showed $tan\angle PRS = \frac{\sqrt{3}+1}{2}$, and failed to show the exact results required. This will be penalised under Clarity.

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8 **(a)** The term independent of x in the expansion of $\left(2x^2+\frac{p}{x}\right)^{12}$, where p is a positive constant, is 7920. Find the value of p. [4]

(a)

$$T_{r+1}=\binom{12}{r}\left(2x^2\right)^{12-r}\left(\frac{p}{x}\right)^r$$

$$=\binom{12}{r}2^{12-r}\,p^r x^{24-3r}$$

For term independent of x,

$24-3r=0$

$r=8$

$$\binom{12}{8}2^{12-8}\,p^8=7920$$

$$p^8=\frac{7920}{7920}=1$$

Therefore $p=1$ (p is positive).

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Marker's Comments :
- Most students have the idea that they need to use the general term, but many seemed to have difficulties simplifying the expression with good precision, primarily because of their inherent weakness in simplifying indices. Mistakes like

$$T_{r+1}=\binom{12}{r}(2x^2)^{12-r}\left(\frac{p}{x}\right)^r$$

$$=\binom{12}{r}2p^r x^{24-r}p^r x^{-r}$$

are still common. Some other common problems like $T_{r+1}=\binom{n}{r}(2x^2)^{n-r}\left(\frac{p}{x}\right)^r$, $(2x^2)^{n-r}\left(\frac{p}{x}\right)^r$ $\left(2x^2+\frac{p}{x}\right)^{12}=\binom{12}{r}\left(2x^2\right)^{12-r}\left(\frac{p}{x}\right)^r$. Some others just provided the binomial expansion without showing any clear plan or strategy to find the value of p.
Marks are given for showing evidence to know the following : raw expression of general term, equating power of x to zero, equating the term independent of x to 7920.

(b) Using your value of p, find the coefficient of the term in x^3 in the expansion of $\left(1+x^4\right)\left(2x^2+\frac{p}{x}\right)^{12}$. [4]

(b)

$T_{r+1}=\binom{12}{r}2^{12-r}x^{24-3r}$ when $p=1$

Consider $\left(1+x^4\right)\left(2x^2+\frac{1}{x}\right)^{12}$:

To have a term in x^3,

Term of $1\times$ term in $\left(2x^2+\frac{1}{x}\right)^{12} = x^{24-3r}$

$24-3r=3$ implies $3r=21$ or $r=7$.

Term with $x^4\times$ term in $\left(2x^2+\frac{1}{x}\right)^{12} = x^{28-3r}$

$28-3r=3$ implies $3r=25$ or $r=\frac{25}{3}$

Hence the coefficient of x^3 occurs only when $r=7$, i.e.

$\binom{12}{7}2^5=25344$.

Marker's Comments :
- Most students were able to show evidence that they need to equate the power of x to 3, and obtained the first r value of 7, but many seemed to just overlooked or ignored the need to equate the power of x to -1. One mark will be deducted for such cases.
- Quite a number of students, after equating $24-3r=-1$, just left it there without any conclusion, or state that there is no solution; while some others obtained the wrong value of r, e.g. $r=\frac{23}{3}$. These will be penalized under Clarity.
- For students who managed to obtain the answer by doing expansion, most of them left out terms by using dots to represent the non-relevant terms in their expansion, as there were just too many of them. One mark will be deducted due to ambiguities and incompleteness in their presentation.

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9 Given that $f(x) = 7\cos^2 x - 3\sin^2 x$, where $0 \le x \le \pi$

(a) express $f(x)$ in the form of $a\cos 2x + b$, where a and b are constants. [3]

	$f(x) = 7\cos^2 x - 3\sin^2 x$ $= 7\cos^2 x - 3(1 - \cos^2 x)$ $= 10\cos^2 x - 3$ $= 5(2\cos^2 - 1) + 2$ $= 5\cos 2x + 2$
	Marker's comments: Well done with student's coming up with many varieties of working. Students who lost marks due to careless expansion or overthinking and complicating working.

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(b) state the greatest and least values of $f(x)$, [2]

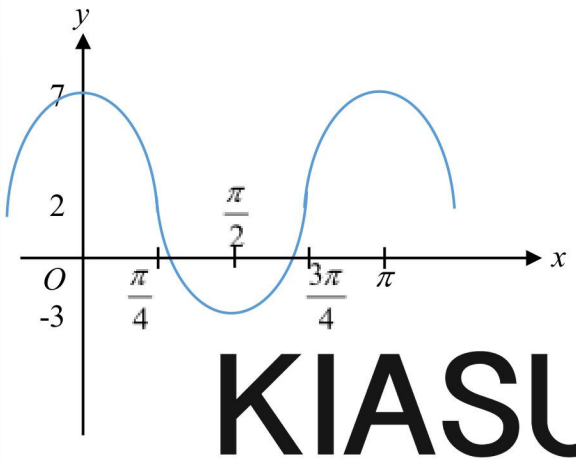
	$-1 \le \cos 2x \le 1$ $-5 \le 5\cos 2x \le 5$ $-3 \le 5\cos 2x + 2 \le 7$ Greatest value = 7 Least value = –3
	Marker's comments: Quite a handful of students did not label their values.

(c) state the period and amplitude of $f(x)$. [2]

	Period = π Amplitude = 5
	Marker's comments: 180° not accepted . Read the question.

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(d) Sketch the graph of $y = f(x)$ for $0 \le x \le \pi$. [2]



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Markers comments:
Students did not label corresponding horizontal values or axis titles.

Students casually marked $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ on horizontal axis and not on curve axis.

Students whose trigo curve looks like a quadratic curve are penalised.
If one cycle of cosine curve does not span between 0 and π, no marks awarded.

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10 **(a)** Prove that $\frac{\cos\theta}{1-\sin\theta}-\frac{1}{\cos\theta}=\tan\theta$. [4]

	$LHS=\frac{\cos\theta}{1-\sin\theta}-\frac{1}{\cos\theta}$ $=\frac{\cos^2\theta-(1-\sin\theta)}{(1-\sin\theta)\cos\theta}$ $=\frac{\cos^2\theta-1+\sin\theta}{(1-\sin\theta)\cos\theta}$ $=\frac{-\sin^2\theta+\sin\theta}{(1-\sin\theta)\cos\theta}$ $=\frac{\sin\theta(1-\sin\theta)}{(1-\sin\theta)\cos\theta}$ $=\frac{\sin\theta}{\cos\theta}$ $=\tan\theta$ $=RHS$	
	Marker's comments: Essential working must be seen for proving questions. Many students skipped crucial steps. Students showcased many alternatives but most are due to overthinking the question.	

(b) Hence solve the equation $\frac{\cos\theta}{1-\sin\theta}-\frac{1}{\cos\theta}=2\sin\theta$ for $180°\le\theta\le360°$. [4]

	$\frac{\cos\theta}{1-\sin\theta}-\frac{1}{\cos\theta}=2\sin\theta$ $\tan\theta=2\sin\theta$ $\frac{\sin\theta}{\cos\theta}=2\sin\theta$ $\sin\theta=2\sin\theta\cos\theta$ $\sin\theta-2\sin\theta\cos\theta=0$ $\sin\theta(1-2\cos\theta)=0$ $\sin\theta=0$ or $\cos\theta=\frac{1}{2}$ $\theta=180°,360°$ basic angle $=60°$ $\theta=360°-60°$ $=300°$ Therefore, $\theta=180°,\ 300°,\ 360°$	Marker's comments: Badly done because most students divided both sides by sin θ. Many reached: sin θ - sin 2θ = 0 and didn't know how to proceed. Students need to be reminded to state which quadrant the angles exist in before working out. Quite a number of students worked out 240°.

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11 A triangular prism has its base an equilateral triangle of side x cm and a height of y cm. The total surface area of the prism is 30 cm^2.

(a) Show that $y = \dfrac{60 - \sqrt{3}x^2}{6x}$. [2]

(a)	Surface area of the prism $3xy + 2\left(\frac{1}{2}x^2 \sin\frac{\pi}{3}\right) = 30$ $3xy + \frac{\sqrt{3}}{2}x^2 = 30$ $3xy = 30 - \frac{\sqrt{3}}{2}x^2$ $y = \left(30 - \frac{\sqrt{3}}{2}x^2\right)\left(\frac{1}{3x}\right)$ $= \frac{60 - \sqrt{3}x^2}{6x}$	
	Marker's Comments Several students did not know what a prism was. They scored zero marks for this part. The incorrect interpretation was that of a triangular pyramid or a tetrahedron.	

(b) Hence show that the volume, V cm^3, is given by the equation $V = \dfrac{5\sqrt{3}x}{2} - \dfrac{x^3}{8}$. [1]

(b)	Volume of the prism $V = y\left(\frac{1}{2}x^2 \sin\left(\frac{\pi}{3}\right)\right)$ $= \left(\frac{60 - \sqrt{3}x^2}{6x}\right)\left(\frac{\sqrt{3}}{4}x^2\right)$ $= \frac{5\sqrt{3}}{2}x - \frac{x^3}{8}$	
	Marker's Comments The students who did not know what a prism was could not do this part.	

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(c) Given that x can vary, find the stationary value of V and determine its nature. [4]

(c)	$\frac{dV}{dx} = \frac{5\sqrt{3}}{2} - \frac{3}{8}x^2$	
	$\frac{dV}{dx} = 0$ $\frac{5\sqrt{3}}{2} - \frac{3}{8}x^2 = 0$ $x^2 = \frac{20}{\sqrt{3}}$	
	$x = -3.39808 \ (\text{NA})$ or $x = 3.398088$ Stationary value of V $V = \frac{5\sqrt{3}(3.298088)}{2} - \frac{(3.298088)^3}{8}$ $= 9.796819 = 9.80$	
	$\frac{d^2V}{dx^2} = -\frac{3}{4}x$	
	When $x = 3.398088$ $\frac{d^2V}{dx^2} = -\frac{3}{4}(3.398088)$ $= -2.55 < 0$ V is a maximum value.	
	Marker's Comments One error was writing $V = \frac{5\sqrt{3x}}{2} - \frac{x^3}{8}$. The first term has a $\sqrt{x}$. Another error was that several students did not find the value of V. Many students did show the working to determine the nature of the stationary value.	

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12 Given that $y = \ln\left(\cos^5 x\right)$, where $\frac{-\pi}{2} < x < \frac{\pi}{2}$, without finding any coordinates of the stationary points, explain clearly why the curve only has maximum points. [3]

	$y = \ln\left(\cos^5 x\right)$ $= 5\ln\left(\cos x\right)$ $\frac{dy}{dx} = (5)\frac{1}{\cos x}(-\sin x)$ $= -5\tan x$ $\frac{d^2y}{dx^2} = -5\sec^2 x$ $\sec^2 x = \left(\frac{1}{\cos x}\right)^2 > 0$ so $-5\sec^2 x < 0$ Since $\frac{d^2y}{dx^2} < 0$ for $\frac{-\pi}{2} < x < \frac{\pi}{2}$, by the second derivative test, the curve, $y = \ln\left(\cos^5 x\right)$, has only maximum points.	
	Marker's Comments Many students could not find the $\frac{dy}{dx}$. Many students did not carry on finding $\frac{d^2y}{dx^2}$. Students who reached $\frac{d^2y}{dx^2}$ but got it incorrect will get M1. Often students dd not simplify their answers which made the working complicated. E.g. not simplifying $\frac{\cos^4\theta}{\cos^5\theta}$ to $\frac{1}{\cos\theta}$	

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13 The gradient function of a curve is given by $\frac{dy}{dx} = 3\sin\left(3x - \frac{\pi}{2}\right) + \frac{\pi}{(x-\pi)^2}$. This curve passes through the point $(2\pi, 4)$. Find the equation of the curve. [5]

	$\frac{dy}{dx} = 3\sin\left(3x - \frac{\pi}{2}\right) + \frac{\pi}{(x-\pi)^2}$ $y = \int 3\sin\left(3x - \frac{\pi}{2}\right) + \frac{\pi}{(x-\pi)^2}\, dx$ $= -\cos\left(3x - \frac{\pi}{2}\right) - \frac{\pi}{x-\pi} + c$ Using $(2\pi, 4)$ $4 = -\cos\left(3(2\pi) - \frac{\pi}{2}\right) - \frac{\pi}{2\pi - \pi} + c$ $c = 5$ Equation of the curve is $y = -\cos\left(3x - \frac{\pi}{2}\right) - \frac{\pi}{x-\pi} + 5$	
	Marker's Comments Several students found the equation of the tangent to the curve, instead of the equation of the curve. These students scored zero marks. Several students integrated one or the other term wrong. Some students dd not evaluate $\cos\left(3(2\pi) - \frac{\pi}{2}\right)$ when finding the constant. They will not get the final A1. Often, students did not write "$y =$" or "dx".	

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14 The equation of a curve is $y=(x-2)\sqrt{4x+1}$ where $x>-\frac{1}{4}$.

(a) Show that $\frac{dy}{dx}=\frac{3(2x-1)}{\sqrt{4x+1}}$ [3]

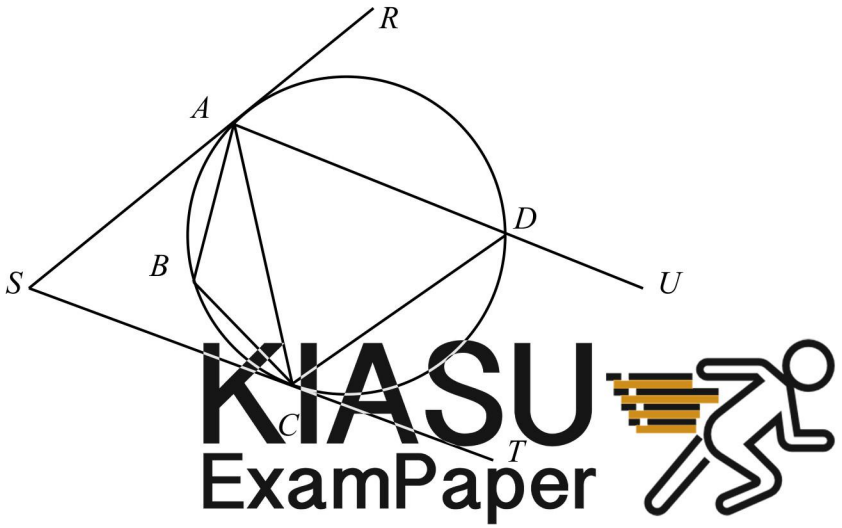
(a)	$y=(x-2)(4x+1)^{\frac{1}{2}}$ $\frac{dy}{dx}=(4x+1)^{\frac{1}{2}}+(x-2)\frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)$ $=\sqrt{4x+1}+\frac{2(x-2)}{\sqrt{4x+1}}$ $=\frac{4x+1+2x-4}{\sqrt{4x+1}}$ $=\frac{6x-3}{\sqrt{4x+1}}$ $=\frac{3(2x-1)}{\sqrt{4x+1}}$	
	Marker's Comments Generally well done.	

(b) Hence find the range of values of x for which the function $y=(x-2)\sqrt{4x+1}$ is decreasing. [3]

(b)	For the function $y=(x-2)\sqrt{4x+1}$ to be decreasing, $\frac{dy}{dx}=\frac{3(2x-1)}{\sqrt{4x+1}}<0$ Firstly, $4x+1>0$ so $x>-\frac{1}{4}$ (Given) Then, $2x-1<0$ so $x<\frac{1}{2}$ Hence the range required is $-\frac{1}{4}<x<\frac{1}{2}$.	
	Marker's Comments Several students did not add in the $x>-\frac{1}{4}$.	

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15 The diagram shows a quadrilateral, $ABCD$, inscribed in a circle. The lines SAR and TCS are tangents to the circle at A and C respectively. The lines ADU and SCT are parallel.



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(a) Prove that angle ACD = angle ASC [2]

(a)	$\angle RAD = \angle ACD$ (Alternate Segment Theorem) $\angle RAD = \angle ASC$ (Corresponding angles, ADU and TCS are parallel) Hence, angle ACD = angle ASC.	
	Marker's Comments Many students lose marks for not writing the correct reasons. Often the organisation of the proof was not sequential or not the most efficient. Some students assumed that SAB and CD were parallel.	

(b) Prove that angle ABC = angle UDC [2]

(b)	$\angle ABC + \angle CDA = 180°$ (Angles in opposite segments) $\angle UDC + \angle CDA = 180°$ (Sum of angles on a straight line) Hence angle ABC = angle UDC.	
	Marker's Comments Generally done better than (a).	

(c) Given that A, S and C lie on a circle, and B is the centre of this circle, prove that the triangle ASC is equilateral. [3]

(c)	Let $\angle ASC = x$ $SA = SC$ (Tangents from the same external point) Triangle ASC is isosceles. $\angle SCA = \frac{180° - x}{2}$ (Base angles of isosceles triangle ASC) $\angle ABC = 2x$ (angle at centre = 2 × angle at the circumference) From (ii) $\angle ABC + \angle CDA = 180°$ (Angles in opposite segments)	

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	$\angle CDA = 180° - \angle ABC$ $= 180° - 2x$ $\angle SCA = \angle CDA$ (Alternate Segment Theorem) $\frac{180° - x}{2} = 180° - 2x$ $180° - x = 360° - 4x$ $3x = 180°$ $x = 60°$ $\angle SCA = \frac{180° - 60°}{2}$ $= 60°$ $= \angle CAS$ Since all angles in triangle ASC are equal, triangle ASC is an equilateral triangle.	
	Marker's Comments Many students attempted this part. Often wrong assumptions were made, e.g. $CA = AS = SC$, or SAR was parallel to CD .	

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SECONDARY 4
PRELIMINARY EXAMINATIONS

2026S4

CANDIDATE NAME

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ADDITIONAL MATHEMATICS **4049/02**
Paper 2

25th August 2026
2 hours 15 minutes

Markers' Report

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Candidates answer on Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and full name on the front cover of your paper.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiners' Use:

Question	1	2	3	4	5	6	7	8	9	10
Marks										
Question	11	12	13	14						

Clarity/Logic	Accuracy/Precision	Units		90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

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1 Express $\dfrac{3x^3-11x^2+8x-4}{x^3-4x^2+4x}$ in partial fractions. [6]

	$\dfrac{3x^3-11x^2+8x-4}{x^3-4x^2+4x}$ is an improper fraction	**Marker's comments:** No marks are given for coincidental answers if long division was not done and partial fractions are wrong. About half the cohort did not do long division.
	$x^3-4x^2+4x \overline{)\,3x^3-11x^2+8x-4}$ 3 $-(3x^3-12x^2+12x)$ x^2-4x-4	
	$\dfrac{x^2-4x-4}{x^3-4x^2+4x} = \dfrac{x^2-4x-4}{x(x^2-4x+4)}$ $= \dfrac{x^2-4x-4}{x(x-2)^2}$	
	$\dfrac{x^2-4x-4}{x(x-2)^2} = \dfrac{A}{x}+\dfrac{B}{x-2}+\dfrac{C}{(x-2)^2}$ $x^2-4x-4 = A(x-2)^2+Bx(x-2)+Cx$ Substitute $x=0$ $(0)^2-4(0)-4 = A(0-2)^2+B(0)(0-2)+C(0)$ $A=-1$ Substitute $x=2$ $(2)^2-4(2)-4 = A(2-2)^2+B(2)(2-2)+C(2)$ $C=-4$ Comparing terms with x^2 $x^2 = -x^2+Bx^2 \qquad B=2$	
	$\dfrac{3x^3-11x^2+8x-4}{x^3-4x^2+4x} = 3-\dfrac{1}{x}+\dfrac{2}{(x-2)}\dfrac{4}{(x-2)^2}$	

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2 Solve the equation $2^x - 2^{2-x} = 3$. [4]

$2^x - 2^{2-x} = 3$

$2^x - \frac{2^2}{2^x} = 3$

$2^x - \frac{4}{2^x} = 3$

Let $u = 2^x$

$u - \frac{4}{u} = 3$

$u^2 - 4 = 3u$

$u^2 - 3u - 4 = 0$

$(u-4)(u+1) = 0$

$u = 4 \quad or \quad u = -1$

$2^x = 4$ or $2^x = -1$

$x = 2$ $\left(\text{No solution, } 2^x > 0\right)$

Marker's comments:

Relatively well done.
Several students take log on both sides immediately.

A lot of overthinking when solving:

$2^x = 4$

Students take log on both sides to solve ir.

Very few students were to give the correct reason as to why $2^x = -1$ is rejected.

3 Solve the equation $\frac{2\log_5(x-1)}{\log_5 3} - 1 = \log_3(x+5)$. [5]

$\frac{2\log_5(x-1)}{\log_5 3} - 1 = \log_3(x+5)$

$2\log_3(x-1) - \log_3 3 = \log_3(x+5)$

$\log_3(x-1)^2 - \log_3 3 = \log_3(x+5)$

$\log_3 \frac{(x-1)^2}{3} = \log_3(x+5)$

$\frac{(x-1)^2}{3} = (x+5)$

$x^2 - 2x + 1 = 3x + 15$

$x^2 - 5x - 14 = 0$

$(x+2)(x-7) = 0$

$x = -2 \ (rejected, x > 1)$ or $x = 7$

Marker's comments:

Relatively well done credited to lenient marking.

Very few students were able to do the reverse change of base.

No marks awarded for removing log if student show evidence of wrong use of logarithmic laws:

$$\frac{2\log_5(x-1)}{\log_5 3} = 2\log_5(x-1) - \log_5 3$$

$$2\log_3(x-1) - \log_3 3 = \log_3\left(\frac{x-1}{3}\right)^2$$

Students expected to show working for solving quadratic equation.

Very few students were able to explain why x = -2 is rejected.

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Alternative Method :

$\dfrac{\log_5(x-1)^2}{\log_5 3} - \log_3(x+5) = 1$ $\dfrac{\log_5(x-1)^2}{\log_5 3} - \dfrac{\log_5(x+5)}{\log_5 3} = 1$ $\log_5(x-1)^2 - \log_5(x+5) = \log_5 3$ $\log_5 \dfrac{(x-1)^2}{x+5} = \log_5 3$ $\dfrac{(x-1)^2}{x+5} = 3$ $\dfrac{(x-1)^2}{3} = (x+5)$ $x^2 - 2x + 1 = 3x + 15$ $x^2 - 5x - 14 = 0$ $(x+2)(x-7) = 0$	
$x = -2$ (NA) or $x = 7$	

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4 The cubic polynomial $f(x)$ is such that the roots of $f(x) = 0$ are –2, 1 and k. It is given that $f(x)$ has a remainder of –24 when divided by x, and a remainder of –18 when divided by $(x + 1)$.

(a) Find the value of k and an expression for $f(x)$. [5]

(b) Hence, or otherwise, solve $f(1-2y) = 0$. [2]

(a)	$f(x) = a(x+2)(x-1)(x-k)$ $f(0) = -24$ $2ak = -24$ $ak = -12$ $f(-1) = -18$ $-2a(-1-k) = -18$ $a + ak = -9$ $a - 12 = -9$ $a = 3$ $k = -4$ $f(x) = 3(x+2)(x-1)(x+4)$	Marker's comments: Badly done. Many did not recognise that the coefficient of x^3 does not need to be 1 and did not include a as the coefficient. Other mistakes: - Factor written as $(x+k)$ - even after $a = 3$ is found, it is not included in the calculation in finding k.
(b)	$f(1-2y) = 0$ $1-2y = -2,\ 1$ or -4 $y = \dfrac{3}{2},\ 0,\ \dfrac{5}{2}$	After ½ the cohort were able to recognise that the solution is linked to the original roots.

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5 The point $P(-4,6)$ and $Q(-7,3)$ lie on the circumference of a circle, C, with centre (a,b) and radius r units. The centre of the circle lies on the line $y=4x+34$.

(a) Find the equation of the circle. [7]

Gradient of $PQ = \dfrac{3-6}{-7-(-4)}$ $=1$ Midpoint of $PQ = \left(\dfrac{(-4)+(-7)}{2}, \dfrac{(3+6)}{2}\right)$ $= \left(-\dfrac{11}{2}, \dfrac{9}{2}\right)$ Gradient of the perpendicular bisector of PQ $= -\dfrac{1}{1}$ $=-1$ Equation of the perpendicular bisector of PQ $y-\dfrac{9}{2}=-1\left(x-\left(-\dfrac{11}{2}\right)\right)$ $y=-x-1$	Marker's comments: Those who could do this question did well while quite a handful did not know how to proceed. Common mistakes: - wrong gradient calculated for PQ - gradient of normal taken from gradient of line $y=4x+34$ instead of PQ - use equation of PQ instead of perpendicular bisector to find centre
The perpendicular bisector intersects $y=4x+34$ at the centre. $-x-1=4x+34$ $5x=-35$ $x=-7$ $y=-(-7)-1$ $=6$ Centre of the circle is $(-7,6)$ Radius $= \sqrt{(-7-(-4))^2+(6-6)^2}$ $=3$	
Equations of the circle is $(x-(-7))^2+(y-6)^2=3^2$ $(x+7)^2+(y-6)^2=9$ or $x^2+y^2+14x-12y+76=0$	

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(b) The point $H(k,7)$ lies inside the circle. Find the set of possible values of k. [2]

(b)	$H(k,7)$ is located along the line $y=7$. Consider the intersection of the circle and the line $y=7$. $(x+7)^2+(7-6)^2=9$ $(x+7)^2=8$ $x=-7+\sqrt{8}=-4.17157$ or $x=-7-\sqrt{8}=-9.82843$ The set of possible values of k: $-9.83<k<-4.17$ **Alternative method :** $(k+7)^2+(7-6)^2<9$ $k^2+14k+41<0$ Set $k^2+14k+41=0$ $k=-9.83$ or -4.17 Therefore $-9.83<k<-4.17$	Many were able to link to the radius. A handful did not read the question correctly – "lies inside" and use "≤"

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6 Find the exact gradient of the curve $y=\dfrac{e^{5x+7}}{(6x-11)^3}$ at the point $\left(2, e^{17}\right)$. [3]

	$y=\dfrac{e^{5x+7}}{(6x-11)^3}$ $\dfrac{dy}{dx}=\dfrac{(6x-11)^3\left(5e^{5x+7}\right)-e^{5x+7}3(6x-11)^2(6)}{(6x-11)^6}$ $=\dfrac{e^{5x+7}(6x-11)^2(5(6x-11)-18)}{(6x-11)^6}$ $=\dfrac{e^{5x+7}(30x-73)}{(6x-11)^4}$ When $x=2$ Gradient $=\dfrac{e^{5(2)+7}(30(2)-73)}{(6(2)-11)^4}$ $=-13e^{17}$	Marker's comments: Generally, well done. Many were able to apply quotient correctly though a handful mixed up the order of the numerator. Other mistakes: In using product rule, students did not differentiate the negative power correctly In the simplification when evaluating gradient, mistakes included calculation errors and not leaving answer as exact,

7 **(a)** Given that $y = 3x\,e^{1-2x}$, find $\dfrac{dy}{dx}$. [2]

(a)	$\dfrac{dy}{dx} = 3x\left(-2e^{1-2x}\right) + 3e^{1-2x}$ $= 3e^{1-2x} - 6xe^{1-2x}$	Well done

(b) Hence find $\int xe^{1-2x}\,dx$. [3]

(b)	$\int\left(3e^{1-2x} - 6xe^{1-2x}\right)dx = 3xe^{1-2x} + c_1$ $\int 3e^{1-2x}dx - 6\int xe^{1-2x}dx = 3xe^{1-2x} + c_1$ $6\int xe^{1-2x}dx = \int 3e^{1-2x}dx - 3xe^{1-2x} - c_1$ $6\int xe^{1-2x}dx = \dfrac{3e^{1-2x}}{-2} - 3xe^{1-2x} + c_2$ $\int xe^{1-2x}dx = -\dfrac{1}{4}e^{1-2x} - \dfrac{1}{2}xe^{1-2x} + c$	Many students were not able to apply integration as a reverse of differentiation here. Many could not form the correct equation. Many were also not careful with the constants.

8 Water is leaking from a container at a constant rate of 0.4 cm^3/s. When the depth of the water in the container is x cm, the volume, V cm^3, of water in the container is given by

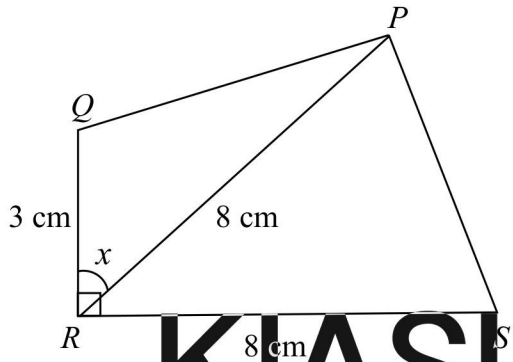
$$V = \frac{1-2x}{x-3} + 5 \quad (x \neq 3)$$

Find the rate of change of the depth of water when $x = 4$ cm. [5]

$\dfrac{dV}{dx} = \dfrac{(x-3)(-2)-(1-2x)}{(x-3)^2}$ $= \dfrac{-2x+6-1+2x}{(x-3)^2}$ $= \dfrac{5}{(x-3)^2}$ When $x = 4$, $\dfrac{dV}{dx} = \dfrac{5}{(4-3)^2}$ $= 5$ $\dfrac{dV}{dt} = \dfrac{dV}{dx} \times \dfrac{dx}{dt}$ $\dfrac{dx}{dt} = -0.4 \div 5$ $= -0.08$ cm/s	Marker's comments: Finding $\dfrac{dV}{dx}$ and later substituting $x = 4$ was generally well done. Only a small number of students made errors with the Quotient rule or the Product rule. Some students combined $\dfrac{1-2x}{x-3} + 5$ into a single term before differentiation. This was not necessary as differentiating the "5" would be zero. A common mistake was setting $\dfrac{dV}{dt} = 0.04$ instead of $\dfrac{dV}{dt} = -0.04$. A small but significant number of students did not give the correct connected rate of change equation. Some wrote $\dfrac{dt}{dx}$. These errors would result in loss of marks. Strangely, a few students used variables like $\dfrac{dV}{dh}$ or $\dfrac{dV}{dr}$.

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9 The figure $PQRS$ is formed by triangle PQR and triangle PRS. Angle $QRS = 90°$, angle $PRQ = x$, $RQ = 3$ cm, $RP = RS = 8$ cm. The area of the figure is $PQRS$ is A cm^2.



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(a) Show that $A = 32\cos x + 12\sin x$. [2]

(a)	$A = \frac{1}{2}(3)(8)\sin x + \frac{1}{2}(8)^2 \sin(90° - x)$ $= 12\sin x + 32\cos x$ $= 32\cos x + 12\sin x$ (shown)	Marker's comments: Many students can do this part using the formula for area of triangle. Some students drew the height of the triangle and added the various angles in the diagram. This was a good practice as it made it easier to follow their working.

(b) Express A in the form of $R\cos(x - \alpha)$. [3]

(b)	$R = \sqrt{32^2 + 12^2}$ $= 4\sqrt{73}$ or 34.176 $\tan\alpha = \frac{12}{32}$ $\alpha = 20.556°$ $\approx 20.6°$ $A = 4\sqrt{73}\cos(x - 20.6°)$ or $34.2\cos(x - 20.6°)$	Students were penalised one mark if they did not simplify the surd for the value of R. Often students did not write "$A =$".

(c) Hence find the maximum value of A and the corresponding value of x. [2]

(c)	$A = 4\sqrt{73}$ or 34.2 occurs when $\cos(x - 20.566°) = 1$ $x - 20.566° = 0°$ $x \approx 20.6$	Many students know how to use (b) to do (c). Some students used differentiation to find the maximum value. Other students found one value but not the other.

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10 A ball is rolled down a skateboard ramp. Its height h m, above the ground at t seconds after being released is given by the formula $h = 0.1t^2 - t + 3$.

(a) State the height of the ramp from which the ball is first released. [1]

(a)	The height of the ball is 3 m.	Marker's comments: Many students can do this part. Often the unit was left out.

(b) Express h in the form of $a(t+b)^2 + c$ where a, b and c are constants to be determined. [3]

(b)	$h = 0.1t^2 - t + 3$ $= 0.1\left(t^2 - 10t\right) + 3$ $= 0.1\left[\left(t-5\right)^2 - 5^2\right] + 3$ $= 0.1\left(t-5\right)^2 + 0.5$	Marker's comments: Many students can change $0.1t^2 - t + 3$ to $0.1\left(t^2 - 10t\right) + 3$ Algebraic errors arose when forming $= 0.1\left[\left(t-5\right)^2 - 5^2\right] + 3$.

(c) State the minimum height attained by the ball and the time at which this occurs. [2]

(c)	Min height = 0.5 m Time occurred = 5 sec	Marker's comments: Many students can use (b) to do (c). A small number of students set the minimum height to zero. Other students used differentiation to find the minimum height. Students who got (b) wrong but correctly used it in (c) were not penalized, unless the time or height were negative values. Often, the units were left out.

(d) Find the times at which the ball is 2.5 m above the ground. [3]

(d)	$0.1\left(t-5\right)^2 + 0.5 = 2.5$ $\left(t-5\right)^2 = 20$ $t - 5 = \pm\sqrt{20}$ $t = \pm 2\sqrt{5} + 5$ $= 9.472136$ or 0.5278640 $= 9.47$ or 0.528 The times are 9.47 seconds and 0.528 seconds.	Marker's comments: Many students can use (b) to do (d). Students who got (b) wrong but correctly used it in (d) were not penalized. A significant number of students use the given equation $h = 0.1t^2 - t + 3$ to do (d). This was acceptable. Students who set up an inequality were not given any credit. This was considered an incorrect interpretation.

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11 Given that $\sin A = \frac{5}{13}$, where $0 \le A \le \frac{\pi}{2}$ and $\cot B = -\frac{24}{7}$, where $\frac{\pi}{2} \le B \le \pi$, without solving for A and B, find the exact value for

(a) $\cos B$, [1]

(a)	$\cos B = -\frac{24}{25}$
	Marker's comments : - Not as well done as expected. Many could not seem to get the correct relationship with $\cot B = -\frac{24}{7}$. - Common mistakes also included not getting the correct quadrant. - Marker will not accept a positive and a negative answer.

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(b) $\sec A$, [2]

(b)	$\sec A = \frac{1}{\cos A}$ $= \frac{1}{\frac{12}{13}}$ $= \frac{13}{12}$
	Marker's Comments : - Not as well done as expected too but better than part (a). - Most students know the meaning of sec A. - Need to start with sec A = …., not 1/cos A = …. Penalise under Clarity.

,

(c) $\sin(A+B)$ [2]

(c)	$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $= \left(\frac{5}{13}\right)\left(-\frac{24}{25}\right) + \left(\frac{12}{13}\right)\left(\frac{7}{25}\right)$ $= -\frac{36}{325}$
	Marker's Comments : - Not well done. Some very basic misconceptions like sin A + sin B can still be seen for the weaker students. - Most students would be awarded a method mark for first step but minimally need to show substitution and not just quoting the formula.

12 **(a)** An oceanographer is studying how light intensity decreases at different depths in the ocean. For a specific location, the light intensity, I (in lumens), at a depth of d meters is modelled by the formula $I = Pe^{-kd}$, where P and k are constants. The table below shows corresponding values of d and I. It is believed that an error was made in recording one of the values of I.

d (metres)	2	4	6	8	10
I (lumens)	122	74	30	27	17

Explain how a straight line graph can be drawn to represent the formula and draw it for the given data. [4]

(a)

$I = Pe^{-kd}$

$\ln I = \ln(Pe^{-kd})$

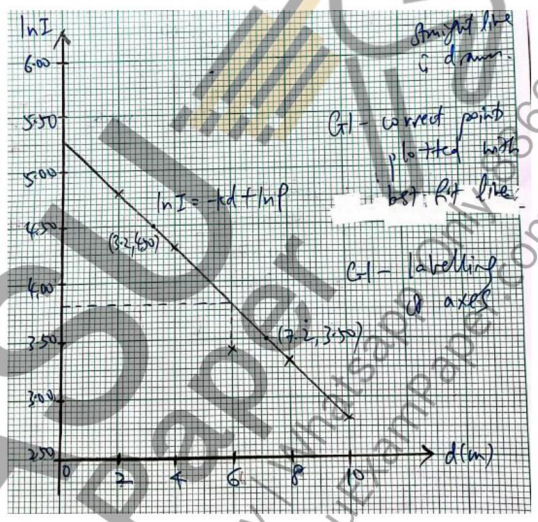
$\ln I = \ln P + \ln e^{-kd}$

$\ln I = -kd + \ln P$

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Draw $\ln I$ against d.

d (metres)	2	4	6	8	10
I (lumens)	122	74	30	27	17
$\ln I$	4.80	4.30	3.40	3.30	2.83



Marker's Comments :
- Not well done. While many students knew how to obtain the linear form, there is also a sizeable number who failed to do so due to non-familiarity with the logarithmic laws.
- Most students failed to address the part about explaining how a straight line graph can be drawn.
- Quite a number of students drew I against d. Some even mixed up the two axes.
- Curiously there was a significant number of students who plotted I against d, and still tried to draw a best-fit line, when it is obvious that the points plotted did not connect up to get a straight line.
- Many did not extend the line long enough to get the ln I-intercept.
- A huge number of students failed to draw a triangle or indicate the points they had used to calculate the gradient. This will be Key Learning Point.

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Using your straight line graph,

(b) identify the incorrect reading and suggest a corrected value for I, [2]

(b)	Incorrect reading : $d = 6, I = 30$ Correct value for I when $d = 6$: From graph, In $I = 3.80$ (range from 3.75 to 3.85) $I = e^{3.80}$ $= 44.701$(5 s.f.) $= 44.7$ lumens (3 s.f.) or 45 lumens (accept 43 to 47 lumens) (accept 42.5 to 47.0 lumens)
	Marker's comments : - Those who misinterpreted the question usually thought that the question is asking for the wrong value of d, but the question stated to "suggest a corrected value of I", which should give enough hint at what the question is asking for. Marks will only be given either for the value of I or for both the values of d and I. - Some ambiguities were also observed such as 30I lumens, or giving the value for the corrected value of I without showing how, e.g. I = 45 lumens, without showing $I = e^{3.80}$. Penalised under Clarity. - Answer for the corrected value of I could be either given to 3 significant figures or whole number.

(c) estimate the value of k, and [2]

(c)	$\text{Gradient} = \dfrac{2.83 - 4.80}{10 - 2}$ $= -0.24625$ $-k = -0.24625$ (5 s.f.) $k = 0.246$ (3 s.f.) (accept 0.196 to 0.296)
	Marker's Comments : - Many who got this part wrong thought that k represented the gradient, when its actually $-k$. These will only be awarded 1 method mark. - For those who failed to show the coordinates of the points used for calculating the gradient, either through labelling on the graph, or drawing a right-angled triangle joining the points, will be penalized under Clarity. - Students should be reminded that the gradient is an estimated gradient, and hence should be given to 3 significant figures, and not as an exact value like a fraction, which is a rational number. The row of values for In I should rightly reflected this also, though some gave the values of In I to more than 3 significant figures, which were not feasible for graph plotting. - Values of gradient calculated by substituting coordinates into the linear form of the equation will not be awarded with any mark since the question required a graphical method to do so.

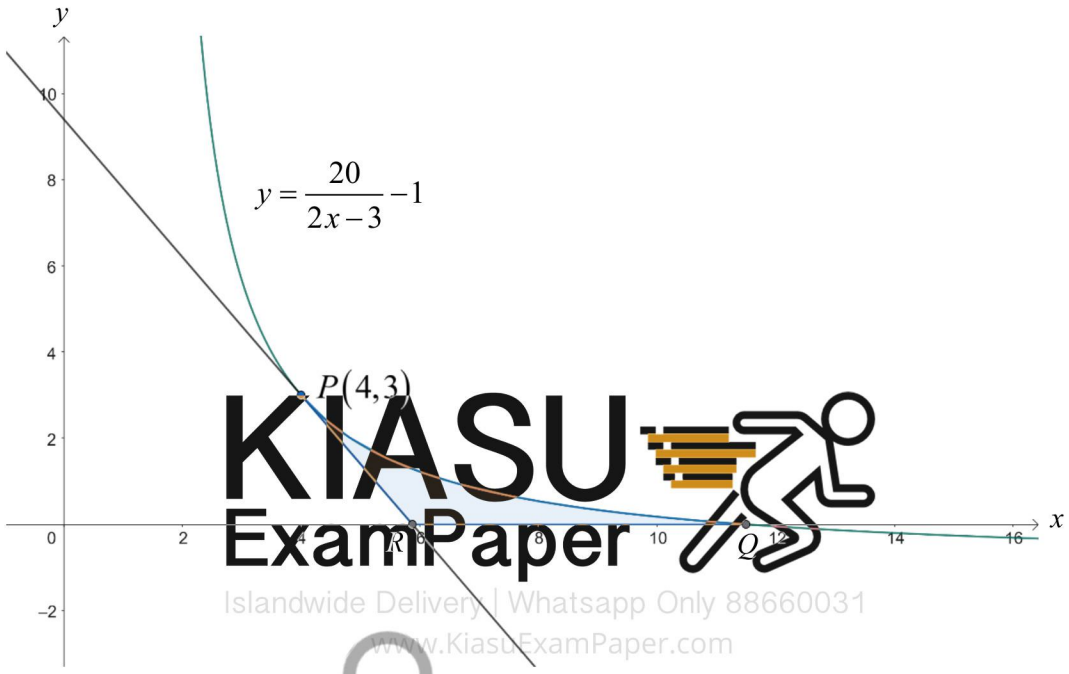
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(d) deduce the value of P. [2]

(d)	ln P = 5.25 (range from 5.20 to 5.30) $P = e^{5.25}$ $= 190.57$ (5 s.f.) $= 191$ (3 s.f.) (accept 181 to 200) .
	Marker's Comments : - A number of students who failed to show the ln P- intercept on the graph attempted to use substitution of points to obtain the value of P, which will not be accepted, since the question required a graphical method to do so. - Some students obtained answers out of range (181 to 200), but were given consideration due to a different scale used for the graph as compared to the marking scheme. On a case-by-case basis.

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13



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The above diagram shows part of the curve $y = \frac{20}{2x-3} - 1$ passing through the point $P(4,3)$.

The curve meets the x-axis at the point $Q(11.5,0)$. The tangent to the curve at P meets the x-axis at the point R.

(a) Show that the x-coordinate of R is 5.875. [4]

(a)

$$y = 20(2x-3)^{-1} - 1$$

$$\frac{dy}{dx} = -20(2x-3)^{-2}(2)$$

$$= -\frac{40}{(2x-3)^2}$$

At $P(4,3)$, gradient $= -\frac{40}{[2(4)-3]^2}$

$$= -\frac{40}{25}$$

$$= -\frac{8}{5}$$

Equation of tangent at $P(4,3)$:

$$y-3=-\frac{8}{5}(x-4)$$

$$y=-\frac{8}{5}x+\frac{47}{5}$$

At R, $y=0$

$$\frac{8}{5}x=\frac{47}{5}$$

$$x=\frac{47}{8}$$

$$=5.875 \text{ (shown).}$$

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Marker's comments :

- Most common problem started at the very first part on differentiating the given function. A number of the students actually tried to combine the two terms to obtain $y=\frac{23-2x}{2x-3}$, and then used quotient rule, to varying degree of success. They did not realise the original form of the function is actually easier to differentiate.
- Many students did not realise that they should substitute the x-coordinates of P ($x = 4$) into the gradient function to obtain the gradient at P. They still used the gradient function when finding the equation of the tangent at P.
- For the last part to show the x-coordinate of R is 5.875, marker will not accept answer which just states that $y = 0$. Some minimal working must be shown that led to the result like that shown above.

(b) Find the shaded area PQR. [4]

(b)

$$\text{Shaded area} = \int_4^{11.5}\left(\frac{20}{2x-3}-1\right)\mathrm{d}x - \frac{1}{2}(5.875-4)(3)$$

$$= \left[20\times\frac{1}{2}\ln(2x-3)-x\right]_4^{11.5} - 2.8125$$

$$= [10\ln(2\times11.5-3)-11.5]-[10\ln(2\times4-3)-4]-2.8125$$

$$= 10\ln 20 - 11.5 - 10\ln 5 + 4 - 2.8125$$

$$= 3.5504$$

$$= 3.55 \text{ units}^2 \quad (3 \text{ s.f.})$$

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Marker's Comments :
- Not so well done.

Common mistakes :

(1) $\int_4^{5.875}\left(\frac{20}{2x-3}-1\right)\mathrm{d}x - \frac{1}{2}(5.875-4)(4)$, or similar mistakes which either have the wrong upper and/or lower limit of the integral, or wrong calculation for the area of the triangle.

(2) $\int_4^{11.5}\left(\frac{20}{2x-3}-1\right)\mathrm{d}x = 20\int_4^{11.5}\left(\frac{1}{2x-3}-1\right)\mathrm{d}x$

(3) $\int_4^{11.5}\left(\frac{20}{2x-3}-1\right)\mathrm{d}x = [20\ln(2x-3)-x]_4^{11.5}$

(4) $\int_4^{11.5}\left(\frac{20}{2x-3}-1\right)\mathrm{d}x = [10\ln 2x-3]_4^{11.5}$ (no bracket for $2x-3$ and forgotten to integrate x)

(no mark for integration if the 'x' term is not integrated, but penalise under Clarity for forgetting the bracket for $2x-3$)

(5) Students are reminded to represent the integral as $\int_4^{11.5}\left(\frac{20}{2x-3}-1\right)\mathrm{d}x$,

instead of $\int_4^{11.5}\frac{20}{2x-3}-1\,\mathrm{d}x$. They are not penalised this round.

(6) Failure to show explicit substitution of the upper and lower limits into the integrated function, e.g. 10ln20 – 10 ln 5. Students will lose one method mark for this.

(7) Failure to evaluate non-simplified answer such as 10ln4 –10.3125 will be penalise under Accuracy.

14 A particle, travelling along a straight path, passes a point P and, 5 seconds later, passes a point Q. The velocity, v m/s, t seconds after passing P, is given by

$$v = 3t^2 - 13t + 12$$

(a) Find the time when the particle is instantaneously at rest. [2]

(a) When the particle is instantaneously at rest,

$3t^2 - 13t + 12 = 0$

$(3t - 4)(t - 3) = 0$

$t = \frac{4}{3}$ or $t = 3$

Marker's Comments :

Well done. Most students knew that they need to set $v = 0$, with only a few exceptions who still went to integrate v to obtain an expression for displacement s, before they set $s = 0$.

Common mistake : many left out the units for time, or rejected $t = \frac{4}{3}$ for no reason. The latter will be penalised under Accuracy.

(b) Find the distance travelled from P to Q. [5]

(b) The displacement,

$s = \int 3t^2 - 13t + 12 \, dt$

$= t^3 - \frac{13}{2}t^2 + 12t + c$

$t = 0, s = 0$ so $c = 0$

Hence $s = t^3 - \frac{13}{2}t^2 + 12t$

At $t = \frac{4}{3}$:

$s = \left(\frac{4}{3}\right)^3 - \frac{13}{2}\left(\frac{4}{3}\right)^2 + 12\left(\frac{4}{3}\right)$

$= 6\frac{22}{27}$ m

At $t = 3$,

$s = 3^3 - \frac{13}{2}\left(3^2\right) + 12(3)$

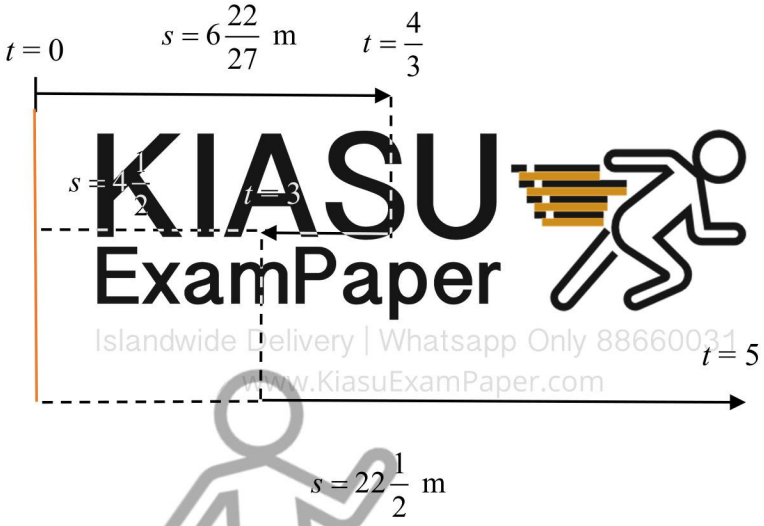
$= 4\frac{1}{2}$ m

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At $t = 5$,

$$s = 5^3 - \frac{13}{2}\left(5^2\right) + 12(5)$$

$$= 22\frac{1}{2} \text{ m}$$



Distance between P and Q

$$= 6\frac{22}{27} + \left(6\frac{22}{27} - 4\frac{1}{2}\right) + \left(22\frac{1}{2} - 4\frac{1}{2}\right)$$

$$= 6\frac{22}{27} + 2\frac{17}{54} + 18$$

$$= 27\frac{7}{54} \text{ m} \quad \text{or} \quad 27.1 \text{ m}$$

Marker's Comments :

- Most students knew that they need to integrate v to obtain s, but a big number just treated the constant c as zero, without any explanation. Marker will only award method mark for integration if the value of c is proven to be zero, for a complete presentation of the expression for s.

- Students need to be reminded that s represents displacement, and not distance, as this has great implications for their answers in the subsequent working.

- Interesting to note that a lot of students actually managed to find the displacements when $t = 0$, 3 and 5 seconds but not when $t = \frac{4}{3}$ s. Many simply added these displacements to find the total distance travelled.

- A sizeable number of students simply just evaluated the integral $s = \int_0^5 3t^2 - 13t + 12 \, \mathrm{d}t$ to find the total distance, neglecting the effect of the particle changing directions between $t = 0$ and $t = 5$ s.

- Most students did not sketch a diagram, which would have helped them to visualize better.

- Students are expected to write the units for each displacement found.

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Alternative method :

(b)

Distance travelled from $t = 0$ to $\frac{4}{3}$ s

$$= \int_0^{\frac{4}{3}} (3t^2 - 13t + 12)dt$$

$$= \left[t^3 - \frac{13}{2}t^2 + 12t \right]_0^{\frac{4}{3}}$$

$$= \left(\frac{4}{3}\right)^3 - \frac{13}{2}\left(\frac{4}{3}\right)^2 + 12\left(\frac{4}{3}\right) - 0$$

$$= \frac{184}{27}$$

$$= 6\frac{22}{27} \text{ m}$$

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Distance travelled from $t = \frac{4}{3}$ to 3s

$$= -\int_{\frac{4}{3}}^{3} (3t^2 - 13t + 12)dt$$

$$= -\left[t^3 - \frac{13}{2}t^2 + 12t \right]_{\frac{4}{3}}^{3}$$

$$= -\left[27 - \frac{117}{2} + 36 - 6\frac{22}{27} \right]$$

$$= \frac{125}{54}$$

$$= 2\frac{17}{54} \text{ m}$$

Distance travelled from $t = 3$ to 5s

$$= \int_3^5 (3t^2 - 13t + 12)dt$$

$$= \left[t^3 - \frac{13}{2}t^2 + 12t \right]_3^5$$

$$= [125 - 162.5 + 60 - 4.5]$$

$$= 18 \text{ m}$$

Total distance

$$= 6\frac{22}{27} + 2\frac{17}{54} + 18$$

or 27.1 m

$$= 27\frac{7}{54} \text{ m}$$

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Marker's Comments :

Most of the students who used this method enjoyed lesser success than they would if they have used the first method.

Common mistake :

Total distance travelled

$$= \int_0^{\frac{4}{3}} (3t^2 - 13t + 12)\ \mathrm{dt} + \int_{\frac{4}{3}}^{3} (3t^2 - 13t + 12)\ \mathrm{dt} + \int_3^5 (3t^2 - 13t + 12)\ \mathrm{dt}\,.$$

CANBERRA SECONDARY SCHOOL

2026 Preliminary Examination

Secondary Four Express

ADDITIONAL MATHEMATICS
4049/01

17 August 2026
2 hours 15 minutes
0815h – 1030h

Name: _____ ( ) **Class:** _____

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number on all work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

FOR MARKER'S USE		
	Marks Awarded	Max Marks
Total		90

HOM: Striving for Accuracy

"The quality of an individual is reflected in the standards they set for themselves." - Ray Kroc

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This question paper consists of 24 printed pages including the cover page.

Setter: Mrs Irving Long

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions

1 A particle moves along the curve $y = \ln(4x-1)$ in such a way that the y-coordinate of the particle is increasing at a rate of 0.05 units per second. Find the rate of change of the x-coordinate of the particle at the instant when $x = 6$. [3]

2 **(a)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x^2+2x+3}\right)$ [2]

(b) Hence find $\int \frac{5x+5}{\sqrt{x^2+2x+3}}\,\mathrm{d}x$. [2]

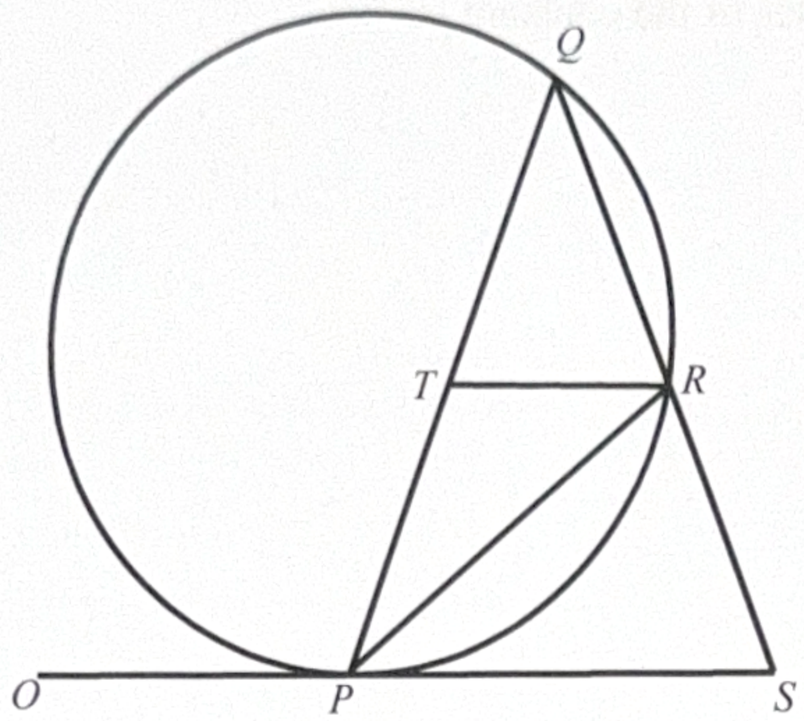
3 A, B and C are the angles of a triangle.

(i) Show that $\tan C = -\tan(A+B)$. [2]

(ii) Given that angle $A = 30°$ and angle $B = 45°$, find $\tan C$ in the form of $a+\sqrt{b}$, where a and b are integers.
You are to show your full working without the use of a calculator. [3]

4 The line $y-2x=12$ intersects the curve $x^2-xy+y^2=63$ at two points, A and B.
Find the coordinates of the midpoint of AB. [5]

5



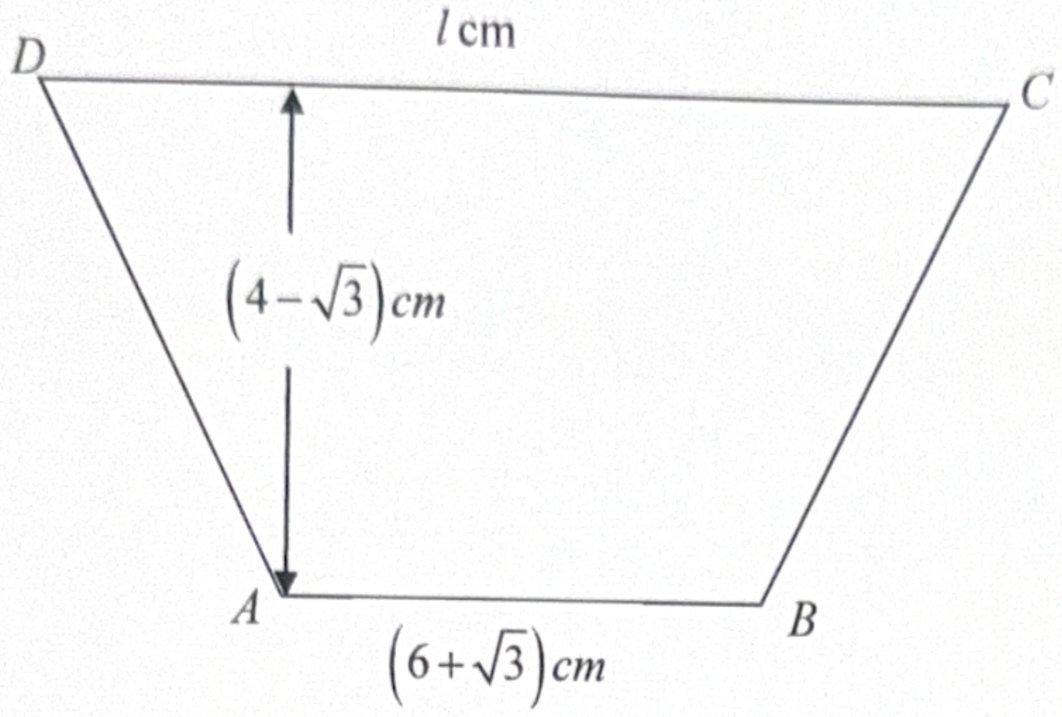
P, Q and R lie on the circumference of a circle. T and R are the midpoints of PQ and QS respectively. The line OS is a tangent to the circle at P.

By using similar triangles, prove that $PS^2 = QS \times RS$.. [3]

6 **(a)** Given that the coefficient of x^2 in the expansion of $(2x-1)^2(1-px)^6$ is 16, find the two possible values of the constant p. [4]

(b) For each value of p, find the coefficient of x^3 in the expansion of $(1-px)^6$. [3]

7 Do not use a calculator for this question.



The diagram shows a trapezium $ABCD$, AB and CD are parallel sides.
The length of AB is $\left(6+\sqrt{3}\right)cm$.

The perpendicular height of the trapezium is $\left(4-\sqrt{3}\right)$ cm and the area of the trapezium is 52 cm^2. Find the exact length of l, which is the length of CD in the form of $\left(a+b\sqrt{3}\right)cm$, where a and b are integers. [7]

Continuation of working space for **question 7.**

8 **(a)(i)** The equation of a curve is $y = 2x^2 - 8x + 11$.

By expressing the curve in the form of $y = a(x+b)^2 + c$, where a, b and c are constants, find the coordinates of the stationary point on the curve. [2]

(ii) Hence or otherwise, find the range of values of the constant k such that the equation $2x^2 - 8x + 11 = k$ does not have 2 distinct roots. [2]

(b) The equation of a curve is $y = p(x-q)^2 + r$, where p, q and r are constants.

The curve intersects with the x-axis at (2, 0) and (4,0) and the maximum value of y is 2. Find the values of p, q and r. [4]

9 It is given that $f(x) = a\sin x + 1$ and $g(x) = 3\cos 2x$.

(a) $f(x)$ has a maximum value of 3 and a minimum value of -1.
Find the value of a and state the line of axis for $f(x)$. [2]

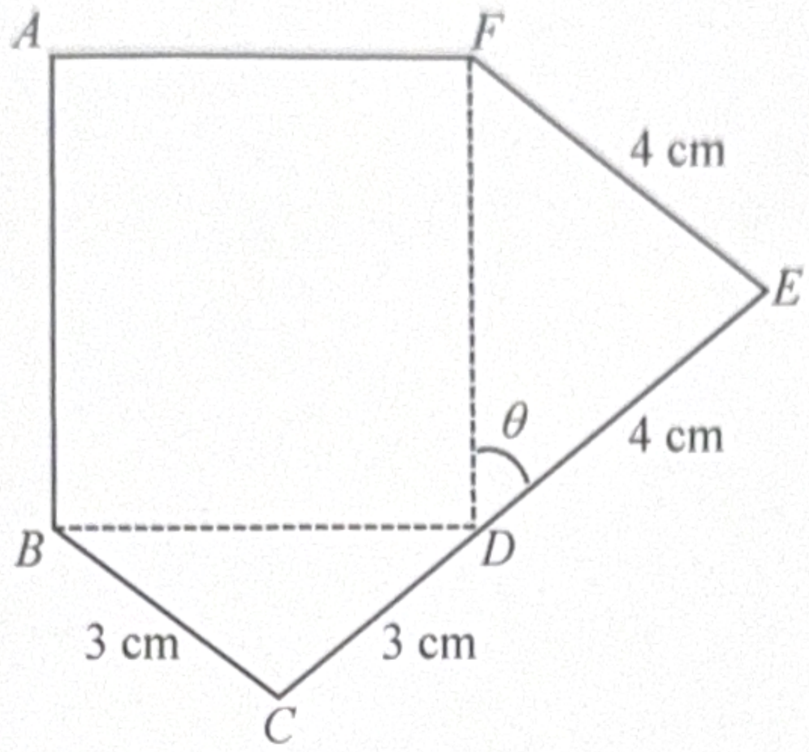
(b) Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0 \le x \le 2\pi$ radians. [3]

(c) Explain why the solutions to the equation $3\sin^2 x+\sin x-1=0$, for $0\le x\le \pi$ radians, can be found using the graphs in **(b)**. [3]

(d) Hence, state the number of solutions of the equation $3\sin^2 x+\sin x-1=0$ for $0\le x\le 2\pi$ radians. [1]

10 The diagram shows a pentagon $ABCEF$ formed by a rectangle $ABDF$ and two isosceles triangles BCD and DEF where CDE is a straight line.

The angle EDF is θ, $BC = CD = 3$ cm and $DE = EF = 4$ cm.



(a) Show that P cm, the perimeter of the pentagon, can be expressed in the form $14+6\sin\theta+8\cos\theta$. [2]

(b) Express P in the form of $k+R\sin(\theta+\alpha)$, where $R>0$ and α is an acute angle.[2]

(c) Write down the maximum possible value of P and find the value of θ at which this maximum value occurs. [2]

(d) Find the value(s) of θ for which $P = 23$ cm. [3]

11 **(a)** Show that $x-1$ is a factor of $f(x) = x^3+x^2-x-1$. Hence or otherwise, factorise $f(x)$. Show your working clearly. [3]

(b) Express $\dfrac{4}{x^3+x^2-x-1}$ as a sum of 3 partial fractions. [7]

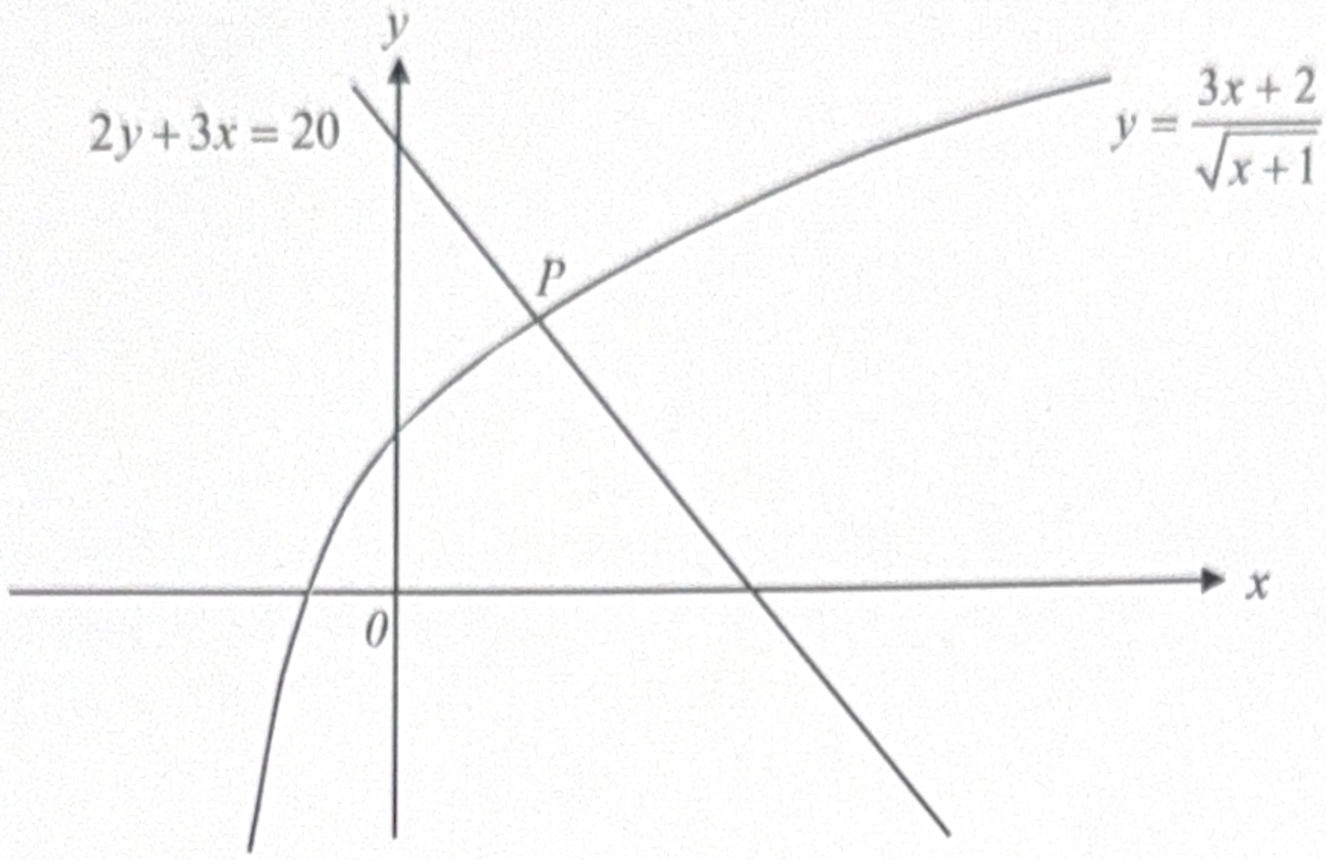
12 **(a)** Differentiate $x\cos x$ with respect to x. [2]

(b) Hence find $\int (x\sin x)\mathrm{d}x$. [2]

(c) Differentiate $x^2 \sin x$ with respect to x and hence, using the results in **(b)** and **(c)**, find $\int (x^2 \cos x) \mathrm{d}x$. [6]

13 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x\sqrt{x+1}) = \frac{3x+2}{2\sqrt{x+1}}$. [2]

(b)



The diagram shows part of the curve $y = \dfrac{3x+2}{\sqrt{x+1}}$. The curve intersects with the line $2y + 3x = 20$ at point P. The x-coordinate of the point P is 3.

Find the area of the shaded region bounded by the curve, the line and the x-axis. [8]

Continuation of working space for **13(b).**

~ End of Paper ~

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2026 Preliminary Examination

Secondary Four Express

ADDITIONAL MATHEMATICS
4049/02

19 Aug 2026
2 hours 15 minutes
1130h — 1345h

Name: _____ ( ) **Class:** _____

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Write your full name, class and index number on all work you hand in.
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This question paper consists of 21 printed pages including the cover page.

Setter: Mr Muhamad Lathif

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

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$$\Delta = \frac{1}{2} ab \sin C$$

Answer **all** the questions

1 **(a)** Find the range of values of h for which the graph $y = x^2 + (h+2)x + 1$ lies entirely above the x-axis. [3]

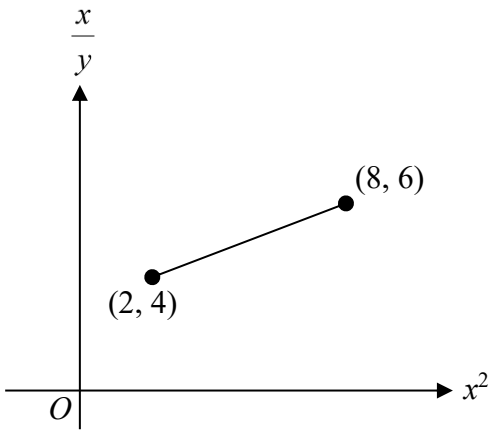
(b) Explain what happens when h is a positive number. [1]

[Turn Over

2 It is given that $y = \dfrac{Ax}{x^2 + B}$ where A and B are constants.

The diagram below shows the equation by plotting $\dfrac{x}{y}$ against x^2

Find the value of A and of B. [3]



3 An object is heated until it reaches a temperature of T degrees Celsius.

It is then allowed to cool.

Its temperature T degrees Celsius when it is cooling for time t minutes is given by the equation

$$T = 28 + 72e^{-0.1t}.$$

Find

(a) the value of t (in minutes and seconds) when $T = 50$, [2]

(b) the value of T when $t = 8$, [1]

(c) the rate at which T is decreasing when $t = 4$, [2]

(d) the highest temperature T_h and the approximate temperature T_A when t becomes very large. [2]

[Turn Over

4 Solve the following equations

(a) $\log_2 x - \dfrac{2}{\log_2 x} = 1$, [3]

(b) $\log_3(x+1) - \log_{\sqrt{3}}(x-1) = 1$. [5]

5 **(a)** Given that $\int_0^3 f(x)\,dx = 6$ and $\int_3^{10} f(x)\,dx = 10$,

Evaluate

(i) $\int_0^{10} f(x)\,dx$, [1]

(ii) the value of m for which $\int_0^3 [f(x) + mx^2]\,dx = 0$. [3]

(b) The expression $\frac{4x}{2x+1}$ can be written in the form $a+\frac{b}{2x+1}$, where a and b are integers

Find

(i) the values of a and b, [2]

(ii) $\int \frac{4x}{2x+1}dx$. [2]

[Turn Over

6 **(a)** Given that $y = \dfrac{2x}{\sqrt{4x+1}}$, find the value of h and of k for which $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{hx+k}{\sqrt{(4x+1)^3}}$. [4]

(b) The equation of a curve is $y = -\frac{2}{3}x^3 - x^2 + 12x + 4$.

Find the set of values of x for which y is decreasing. [4]

[Turn Over

7 A circle, C_1, has a diameter AB where A is the point $(12, -4)$ and B is the point $(24,12)$.

(a) Find the coordinates of the centre and the radius of C_1. [2]

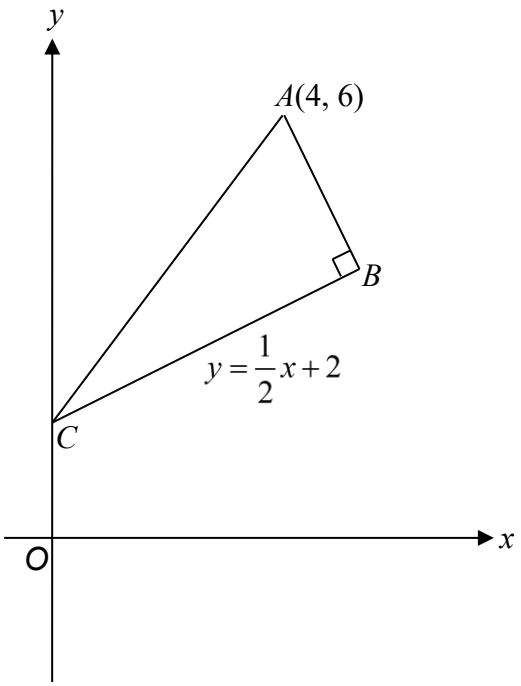
(b) Find the equation of C_1 in the form $x^2 + y^2 + Ax + By + C = 0$ [1]

(c) Find the equation of the tangent at A. [4]

A second circle, C_2, has equation $(x-6)^2+(y+1)^2=9$.

(d) Explain if the circles C_1 and C_2 intersect at two points , one point or does not intersect each other. [3]

8 Solution to this question by accurate drawing is not accepted.



The diagram (not to scale) shows a right-angled triangle ABC.

A is the point (4, 6) and $\angle ABC = 90°$

The equation of the line BC is $y = \frac{1}{2}x + 2$.

Find

(a) the equation of the line AB, [2]

(b) the coordinates of B. [2]

Q lies on AB produced such that $AB : BQ = 2 : 3$

(c) Find the coordinates of Q. [2]

(d) Calculate the area of triangle ACQ. [4]

[Turn Over

9 **(a)** Prove that cosec $2x$ + cot $2x$ = cot x. [4]

(b) Hence, deduce the value of cot 15° in surd form. [2]

(c) Solve the equation $\left(\cos ec2x + \cot 2x\right)^2 = 3$ for $0° \le x \le 360°$. [4]

10 **(a)** Given that $3x^2 - 2x + 4 = A(x-1)^2 + B(x-1) + C$ for all values of x, find the values of A, B and C. [3]

[Turn Over

(b) A polynomial $f(x)$ can be written in the form $f(x) = Q(x)(x-1)(x-2) + Ax + B$ where $Q(x)$ is a polynomial and A and B are constants.

When $f(x)$ is divided by $(x-1)$, the remainder is 3 and when $f(x)$ is divided by $(x-2)$, the remainder is 4.

Find the remainder when f(x) is divided by $(x-1)(x-2)$. [4]

(c) If $x + y = 5$ and $xy = 6$,

Find the value of

(i) $x^2 + y^2$, [2]

(ii) $x^3 + y^3$. [2]

11 A particle moves in a straight line and passes a fixed point O with a velocity of -8 m/s .

Its acceleration, a m/s^2 , t seconds after passing O is given by $a = 6 - 2t$.

Find

(a) an expression for the velocity of the particle after t seconds, [2]

(b) the maximum velocity of the particle, [3]

(c) the value(s) of t when the particle is instantaneously at rest, [2]

(d) the distance travelled by the particle in the first 8 seconds of its motion. [4]

- End of paper -

Qns	Suggested Solutions	Remarks
1	$y = \ln(4x-1)$ $\frac{dy}{dx} = \frac{4}{4x-1}$ When $x = 6$, $\frac{dy}{dx} = \frac{4}{23}$ $\frac{dy}{dt} = 0.05$ $\frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$ $= \frac{23}{4} \times 0.05$ $= \frac{23}{80}$ *units per sec* *Or* 0.2875 units per sec	M1 M1 M1 A1
2a	$y = \left(x^2+2x+3\right)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}\left(x^2+2x+3\right)^{-\frac{1}{2}}(2x+2)$ $= \frac{x+1}{\sqrt{x^2+2x+3}}$	M1 A1
2b	$\int \frac{x+1}{\sqrt{x^2+2x+3}} dx = \sqrt{x^2+2x+3} + c$ $\int \frac{5x+5}{\sqrt{x^2+2x+3}} dx = 5\sqrt{x^2+2x+3} + c$	M1 A1
3i	$C = 180° - (A+B)$ $\tan C = \tan[180° - (A+B)]$ $= \frac{\tan 180 - \tan(A+B)}{1 + \tan 180 \tan(A+B)}$ $= -\tan(A+B)$	M1 A1
3ii	$C = 180° - (30+45)$ $\tan C = -\tan(30+45)$ $= -\frac{\tan 30 + \tan 45}{1 - \tan 30 \tan 45}$	M1

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	$= -\dfrac{\frac{\sqrt{3}}{3}+1}{1-\frac{\sqrt{3}}{3}}$ $= -\dfrac{\sqrt{3}+3}{3-\sqrt{3}}$ $= \dfrac{\sqrt{3}+3}{\sqrt{3}-3}\times\dfrac{\sqrt{3}+3}{\sqrt{3}+3}$ $= \dfrac{6\sqrt{3}+3+9}{3-9}$ $= \underline{-2-\sqrt{3}}$	M1 A1
4	$x^2-xy+y^2=63$ $y=2x+12$ $x^2-x(2x+12)+(2x+12)^2=63$ $3x^2+36x+81=0$ $x^2+12x+27=0$ $(x+3)(x+9)=0$ $x=-3$ or $x=-9$ $y=6$ $y=-6$ A (–3,6) and B(–9,–6) Midpoint $= \left(\dfrac{-9-3}{2},\dfrac{-6+6}{2}\right)$ $= \underline{(-6,0)}$	M1 M1 M1 M1 A1
5	Angle QRT = Angle QSP (corr angles, RT//SP) Angle RQT = Angle RPS (alt segment theorem) Triangle QSP is similar to triangle PSR (AA) $\dfrac{PS}{RS}=\dfrac{QS}{PS}$ $PS^2=RS\times QS$	M1 M1 A1

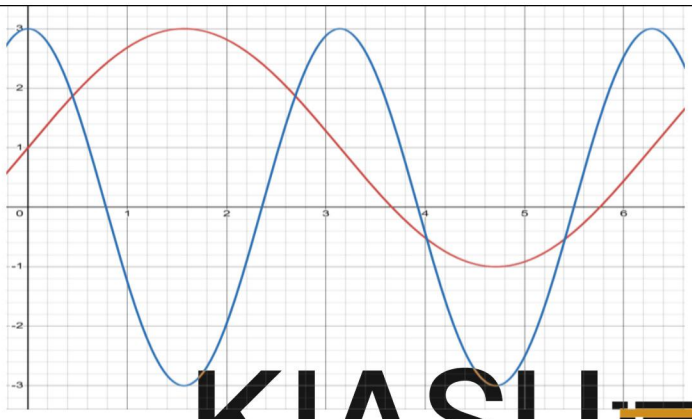
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6(a)	$(1-px)^6 = 1-6px+15p^2x^2+\ldots$ $(4x^2-4x+1)(1-px)^6$ $=(4x^2-4x+1)(1-6px+15p^2x^2+\ldots)$ $16 = 4 + 15p^2 + 24p$ $5p^2 + 8p - 4 = 0$ $(5p-2)(p+2)=0$ $p=\frac{2}{5}$ or $p=-2$	M1 M1 M1 A1
6(b)	When $p=\frac{2}{5}$, coefficient of $x^3 = -\binom{6}{3}\left(\frac{2}{5}\right)^3 = -\frac{32}{25}$ When $p=-2$, coefficient of $x^3 = \binom{6}{3}(2)^3 = 160$	M1 M1 A1
7	$52 = \frac{1}{2}(l+6+\sqrt{3})(4-\sqrt{3})$ $104 = (l+6+\sqrt{3})(4-\sqrt{3})$ $\frac{104}{4-\sqrt{3}} = (l+6+\sqrt{3})$ $\frac{104}{4-\sqrt{3}} \times \frac{4+\sqrt{3}}{4+\sqrt{3}} = (l+6+\sqrt{3})$ $\frac{416+104\sqrt{3}}{16-3} = (l+6+\sqrt{3})$ $32+8\sqrt{3} = (l+6+\sqrt{3})$ $32+8\sqrt{3}-6+\sqrt{3} = l$ $l = (26+7\sqrt{3})$ cm	M1 M1 M1 M1 M1 M1 A1
8(a)(i)	$y = 2(x^2-4x)+11$ $y = 2(x-2)^2+11-8$ $y = 2(x-2)^2+3$ Coordinates of stationary point = (2, 3)	M1 A1

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8(a)(ii)	$y = 2x^2 - 8x + 11 - k = 0$ Equation does not have 2 real and distinct roots, $b^2 - 4ac \le 0$ $64 - 4(2)(11-k) \le 0$ $64 - 88 + 8k \le 0$ $8k \le 24$ $k \le 3$	M1 A1
OR	Since minimum point of graph is (2, 3), There will be less than 2 distinct point if $k \le 3$	(M1) (A1)
8(b)	$y = p(x-2)(x-4)$ $y = px^2 - 6px + 8p$ $r = 2$ (maximum value of $y = 2$) $y = p(x-q)^2 + 2$ $px^2 - 2pqx + pq^2 + 2 = px^2 - 6px + 8p$ $-2pq = -6p$ $q = 3$ $pq^2 + 2 = 8p$ $9p + 2 = 8p$ $p = -2$	 M1 M1 M1 A1
OR	x intercepts are 2 and 4 centre = 3 hence $q = 3$ At (2, 0), $y = p(x-q)^2 + 2$ $0 = p(2-3)^2 + 2$ $p = -2$	M1 M1 M1 A1
9(a)	$a = 2$ Line of axis is $y = 1$	B1 B1

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9(b)		Shape 1m Max and Min 1m
9(c)	$2\sin x+1=3\cos 2x$ $2\sin x+1=3\left(1-2\sin^2 x\right)$ $2\sin x+1=3-6\sin^2 x$ $6\sin^2 x+2\sin x-2=0$ $3\sin^2 x+\sin x-1=0$ Hence solutions can be found using the graphs	M1 M1 M1
OR	$3\sin^2 x+\sin x-1=0$ $6\sin^2 x+2\sin x-2=0$ $2\sin x+1=3-6\sin^2 x$ $2\sin x+1=3\left(1-2\sin^2 x\right)$ $2\sin x+1=3\cos 2x$ Hence solutions can be found using the graphs	M1 M1 M1
9(d)	There are 4 solutions	B1
10(a)	$P=8\cos\theta+6\sin\theta+14$	B1 : 8cosθ + 6 sinθ B1 : 14
10(b)	$P=14+10\sin(\theta+53.1°)$	B1 : 10 B1 $\sin(\theta+53.1°)$
10(c)	Max $P=24$ cm $\sin(\theta+53.1°)=1$ $(\theta+53.1°)=\sin^{-1}1$ $\theta=36.9°$	M1 M1 A1
10(d)	$23=14+10\sin(\theta+53.1°)$ $9=10\sin(\theta+53.1°)$ $\frac{9}{10}=\sin(\theta+53.1°)$ $\sin^{-1}\frac{9}{10}=(\theta+53.1°)$	M1

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	$(\theta + 53.1°) = 64.2°$ or $115.8°$ $\theta = 64.2° - 53.1°$ or $115.8° - 53.1°$ $\theta = 11.1°$ or $62.7°$	M1 A1
11(a)	$f(x) = x^3 + x^2 - x - 1$ $f(1) = 1^3 + 1^2 - 1 - 1 = 0$ Hence $x - 1$ is a factor. $x^3 + x^2 - x - 1 = (x-1)(x^2 + ax + 1)$ Comparing coefficient of x^2, $1 = a - 1$ $a = 2$ $x^3 + x^2 - x - 1 = (x-1)(x^2 + 2x + 1)$ $x^3 + x^2 - x - 1 = (x-1)(x+1)^2$	M1 M1 M1 A1
11(b)	$\frac{4}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ $4 = A(x+1)^2 + B(x+1)(x-1) + C(x-1)$ When $x = 1$, $4 = 4A$ $A = 1$ When $x = -1$, $4 = -2C$ $C = -2$ When $x = 0$, $B = -1$ $\frac{4}{(x-1)(x+1)^2} = \frac{1}{x-1} - \frac{1}{x+1} - \frac{2}{(x+1)^2}$	M1 M1 M1 M1 A2 All correct A1 2 correct
12(a)	$y = x\cos x$ $\frac{dy}{dx} = x(-\sin x) + \cos x$ $= -x\sin x + \cos x$	M1 A1

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12(b)	$\int -x\sin x+\cos x\ dx = x\cos x + c$ $\int x\sin x\ \mathrm{d}x - \int \cos x\ dx = -x\cos x + c$ $\int x\sin x\ dx = -x\cos x + \int \cos x\ dx + c$ $\int x\sin x\ dx = -x\cos x + \sin x + c$	M1 A1
12(c)	$y = x^2\sin x$ $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2(\cos x) + 2x\sin x$ $\int x^2(\cos x) + 2x\sin x\ \mathrm{d}x = x^2\sin x$ $\int x^2\cos x\ \mathrm{d}x = x^2\sin x - \int 2x\sin x\ \mathrm{d}x$ $\int x^2\cos x\ \mathrm{d}x = x^2\sin x - 2(\sin x - x\cos x)$ $\int x^2\cos x\ \mathrm{d}x = x^2\sin x - 2\sin x + 2x\cos x + c$	M2 M1 M1 M1 A1
13(a)	$y = x(x+1)^{\frac{1}{2}}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = x\times\frac{1}{2}(x+1)^{-\frac{1}{2}} + \sqrt{x+1}$ $= \frac{x}{2\sqrt{x+1}} + \sqrt{x+1}$ $= \frac{x+2x+2}{2\sqrt{x+1}}$ $= \frac{3x+2}{2\sqrt{x+1}}$	M1 A1
13(b)	At P, when $x = 3$, $y = \frac{11}{2}$ $y = -\frac{3}{2}x + 10$ When $y = 0$, $x = 6\frac{2}{3}$ When $y = 0$, $\frac{3x+2}{\sqrt{x+1}} = 0$ $x = -\frac{2}{3}$ $\int_{-\frac{2}{3}}^{3}\frac{3x+2}{\sqrt{x+1}}dx = \left[2x\sqrt{x+1}\right]_{-\frac{2}{3}}^{3}$	M1 M1 M1 M1 M1 M1

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	$= 2\left(6 + \frac{2\sqrt{3}}{9}\right)$ Shaded area $= 2\left(6 + \frac{2\sqrt{3}}{9}\right) + \frac{1}{2} \times 3\frac{2}{3} \times \frac{11}{2}$ $=$ <u>22.9</u> square units	M1 A1

CANBERRA SECONDARY SCHOOL

O

2026 Preliminary Examination

Secondary Four Express

ADDITIONAL MATHEMATICS
4049/02

19 Aug 2026
2 hours 15 minutes
1130h — 1345h

Name: _____ **Class:** _____

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READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number on all work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

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	Marks Awarded	Max Marks
Total		90

This question paper consists of 17 printed pages including the cover page.

Setter: Mr Muhamad Lathif

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0, a \neq 0,$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$\Delta = \frac{1}{2}ab\sin C$$

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Answer **all** the questions

1 **(a)** Find the range of values of h for which the graph $y = x^2 + (h+2)x + 1$ lies entirely above the x-axis. [3]

$y = x^2 + (h+2)x + 1$

$b^2 - 4ac < 0$ M1

$(h+2)^2 - 4(1)(1) < 0$ M1

$h^2 + 4h < 0$

$h(h+4) < 0$

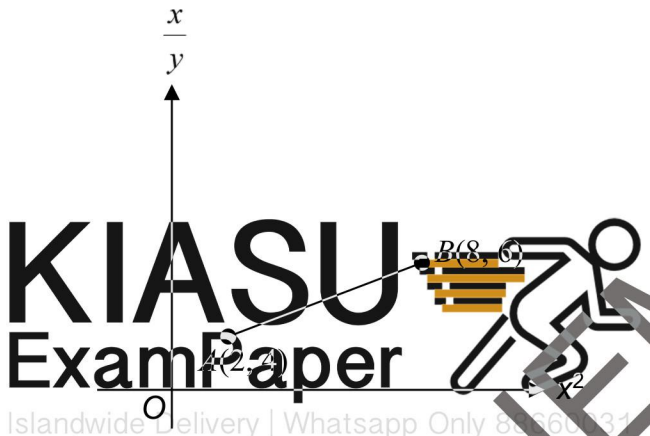
$-4 < h < 0$ A1

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(b) Explain what happens when h is a positive number. [1]

The curve will cut the x-axis at 2 points (there are 2 roots or solutions) B1

2 The diagram shows part of a straight line graph obtained by plotting $\frac{x}{y}$ against x^2 and the coordinates of the two of the points on the line. [3]



Express y in terms of x^2.

$$\text{Gradient} = \frac{6-4}{8-2} = \frac{2}{6} = \frac{1}{3} \qquad \text{M1}$$

$y = mx + c$

At point (2, 4),

$$4 = \frac{1}{3}(2) + c \qquad \text{M1}$$

$$\frac{10}{3} = c$$

$$\frac{x}{y} = \frac{1}{3}x^2 + \frac{10}{3}$$

$$\frac{3x}{y} = x^2 + 10 \qquad \text{A1}$$

$$y = \frac{3x}{x^2 + 10}$$

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3 An object is heated until it reaches a temperature of T degrees Celsius.

It is then allowed to cool.

Its temperature T degrees Celsius when it is cooling for time t minutes is given by the equation

$$T = 28 + 72e^{-0.1t}.$$

Find

(a) the value of t (in minutes and seconds) when $T = 50$, [2]

$50 = 28 + 72e^{-0.1t}$

$22 = 72e^{-0.1t}$

$e^{-0.1t} = \frac{22}{72}$

$-0.1t = \ln\left(\frac{22}{72}\right)$ M1

$t = 11.85$ minutes = 11 minutes 51 seconds A1

(b) the value of T when $t = 8$, [1]

$T = 60.4\ ^{o}C$ B1

(c) the rate at which T is decreasing when $t = 4$, [2]

$T = 28 + 72e^{-0.1t}$

$\frac{dT}{dt} = -7.2e^{-0.1t}$ M1

When $t = 4$, $\frac{dT}{dt} = -7.2e^{-0.1(4)} = -4.8\ ^{o}C/min$

There fore rate at which T is decreasing is 4.8 ^{o}C/min A1

(d) the highest temperature T_h and the approximate temperature T_A when t becomes very large. [2]

Highest temperature, T_h, occurs when $t = 0$, therefore $T = 28 + 72 = 100\ ^{o}C$ B1

T_A , the temperature after a very long time will be the ambient temperature $T = 28\ ^{o}C$ B1

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MARKING SCHEME

4 Solve the following equations

(a) $\log_2 x - \dfrac{2}{\log_2 x} = 1$. [3]

Let $y = \log_2 x$

$y^2 - y - 2 = 0$ M1

$(y-2)(y+1) = 0$

$y = 2$ or $y = -1$ M1

$\log_2 x = 2$ $\log_2 x = -1$

$x = 4$ $x = \dfrac{1}{2}$ A1

(b) $\log_3(x+1) - \log_{\sqrt{3}}(x-1) = 1$. [5]

$\log_3(x+1) - \log_{\sqrt{3}}(x-1) = 1$

$\log_3(x+1) - \dfrac{\log_3(x-1)}{\log_3 3^{\frac{1}{2}}} = 1$ M1

$\log_3(x+1) - \dfrac{\log_3(x-1)}{\frac{1}{2}\log_3 3} = 1$

$\log_3(x+1) - 2\log_3(x-1) = 1$ M1

$\log_3(x+1) - \log_3(x-1)^2 = 1$

$\log_3 \dfrac{x+1}{(x-1)^2} = 1$ M1

$\dfrac{x+1}{(x-1)^2} = 3^1$

$3(x-1)^2 - (x+1) = 0$

$3(x^2 - 2x + 1) - x - 1 = 0$

$3x^2 - 7x + 2 = 0$ M1

$(3x-1)(x-2) = 0$

$x = \dfrac{1}{3}$ (reject) or 2 A1

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5 **(a)** Given that $\int_0^3 f(x)\,dx = 6$ and $\int_3^{10} f(x)\,dx = 10$,

Evaluate

(i) $\int_0^{10} f(x)\,dx$, [1]

$6 + 10 = 16$ B1

(ii) the value of m for which $\int_0^3 [f(x) + mx^2]\,dx = 0$. [3]

$\int_0^3 [f(x) + mx^2]\,dx = 0$

$\int_0^3 f(x)dx + \int_0^3 mx^2 dx = 0$ M1

$6 + \int_0^3 mx^2 dx = 0$

$\int_0^3 mx^2 dx = -6$

$\left[\frac{1}{3}mx^3\right]_0^3 = -6$ M1

$9m = -6$

$m = -\frac{2}{3}$ A1

(b) The expression $\frac{4x}{2x+1}$ can be written in the form $a + \frac{b}{2x+1}$, where a and b are integers.

Find

(i) the values of a and b, [2]

By long division or comparing coefficients, $a = 2$ and $b = -2$ B2

(ii) $\int \frac{4x}{2x+1}\,dx$. [2]

$\int \frac{4x}{2x+1}dx = \int 2 - \frac{2}{2x+1}dx$ M1

$= 2x - \ln(2x+1) + c$ A1

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6 (a) Given that $y = \dfrac{2x}{\sqrt{4x+1}}$, find the value of h and of k for which $\dfrac{dy}{dx} = \dfrac{hx+k}{\sqrt{(4x+1)^3}}$. [4]

Let $u = 2x$ and $v = \sqrt{4x+1}$.

$\dfrac{du}{dx} = 2$ $\qquad$ $\dfrac{dv}{dx} = \dfrac{1}{2}(4x+1)^{-\frac{1}{2}}(4) = \dfrac{2}{\sqrt{4x+1}}$

$$\frac{dy}{dx} = \frac{2\sqrt{4x+1} - \dfrac{4x}{\sqrt{4x+1}}}{4x+1}$$ M1

$$= \frac{\dfrac{2(4x+1)}{\sqrt{4x+1}} - \dfrac{4x}{\sqrt{4x+1}}}{4x+1}$$ M1

$$= \frac{\dfrac{8x+2-4x}{\sqrt{4x+1}}}{4x+1}$$ M1

$$= \frac{4x+2}{\sqrt{(4x+1)^3}}$$

Hence, the values of h and k are 4 and 2 respectively. A1

(b) The equation of a curve is $y = -\dfrac{2}{3}x^3 - x^2 + 12x + 4$.

Find the set of values of x for which y is decreasing. [4]

$$y = -\frac{2}{3}x^3 + x^2 - 12x + 4$$

$$\frac{dy}{dx} = -2x^2 + 2x - 12$$ M1

Since y is decreasing, $\dfrac{dy}{dx} < 0$, M1

$$-2x^2 - 2x + 12 < 0$$

$$x^2 + x - 6 > 0$$ M1

$$(x+3)(x-2) > 0$$

$x < -3, x > 2$ A1

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MARKING SCHEME

7 A circle, C_1, has a diameter AB where A is the point $(12,-4)$ and B is the point $(24,12)$.

(a) Find the coordinates of the centre and the radius of C_1. [2]

$\text{centre} = \left(\frac{12+24}{2}, \frac{-4+12}{2}\right) = (18,\ 4)$ B1

$\text{radius} = \sqrt{(18-12)^2+(4+4)^2} = 10$ B1

(b) Find the equation of C_1 in the form $x^2+y^2+Ax+By+C=0$. [1]

$(x-18)^2+(y-4)^2 = 100$

$x^2+y^2-36x-8y+240=0$ B1

(c) Find the equation of the tangent at A.
[4]

gradient $AB = \frac{12+4}{24-12} = \frac{16}{12} = \frac{4}{3}$ M1

Gradient of tangent to $AB = -\frac{3}{4}$ M1

$-4 = -\frac{3}{4}(12)+c$

$-4 = -9+c$

$c = 5$ M1

$\therefore y = -\frac{3}{4}x+5$ or $4y = -3x+20$ or $4y+3x = 20$ A1

A second circle, C_2, has equation $(x-6)^2+(y+1)^2 = 9$.

(d) Explain if the circles C_1 and C_2 intersects at 2 points , 1 point or does not intersect each other. [3]

$(x-6)^2+(y+1)^2 = 9.$

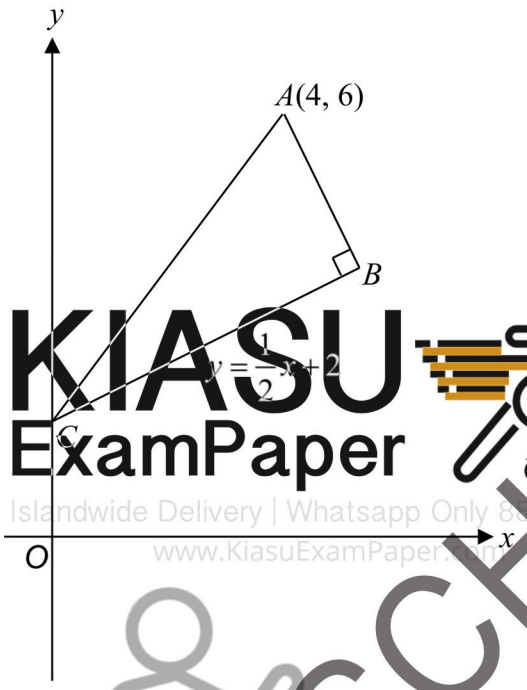
Centre $C_2(6, -1)$, Radius = 3 M1

length of points $C_1(18, 4)$ to $C_2(6, -1)$ $= \sqrt{(18-6)^2+(4+1)^2} = \sqrt{169} = 13$ M1

Since the distance between the centers = total of the radii =13, therefore they intersect at 1 point. A1

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8 Solution to this question by accurate drawing is not accepted.



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The diagram (not to scale) shows a right-angled triangle ABC.

A is the point (4, 6) and $\angle ABC = 90°$.

The equation of the line BC is $y = \frac{1}{2}x + 2$.

Find

(a) the equation of the line AB, [2]

$M_{AB} = -2$ M1

$6 = -2(4) + c$

$c = 14$

$y = -2x + 14$ A1

(b) the coordinates of B. [2]

$\frac{1}{2}x + 2 = -2x + 14$ M1

$\frac{5}{2}x = 12$

$x = 4.8$

$y = -2(4.8) + 14 = 4.4$

$B(4.8\ ,\ 4.4)$ A1

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Q lies on AB produced such that AB : BQ = 2 : 3

(c) Find the coordinates of Q. [2]

$A(4 , 6) \rightarrow B(4.8 , 4.4) \rightarrow C(x , y)$

x coordinates,

4.8 – 4 = 0.8
2U → 0.8
3U → 1.2

$x = 4.8 + 1.2 = 6$ B1

y coordinates,

4.4 – 6 = -1.6
2U → -1.6
3U → -2.4

$y = 4.4 – 2.4 = 2$ B1

$Q(6 , 2)$

(d) Calculate the area of triangle ACQ.
[4]

$C(0 , 2)$ M1

Area of triangle $= \frac{1}{2}\begin{bmatrix} 4 & 0 & 6 & 4 \\ 6 & 2 & 2 & 6 \end{bmatrix}$ M1

Area of triangle $= \frac{1}{2}\left[(8+36)-(12+8)\right]$ M1

Area of triangle = 12 units^2 A1

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MARKING SCHEME

9 **(a)** Prove that $\operatorname{cosec} 2x + \cot 2x = \cot x$. [4]

$$\text{LHS} = \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x} \quad \text{M1}$$

$$= \frac{1+\cos 2x}{\sin 2x} \quad \text{M1}$$

$$= \frac{1+2\cos^2 x - 1}{2\sin x \cos x} \quad \text{M1}$$

$$= \frac{\cos x}{\sin x} = \cot x \quad \text{A1}$$

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(b) Hence, deduce the value of $\cot 15^\circ$ in surd form. [2]

Let $x = 15^\circ$

$$\cot 15^o = \frac{1}{\sin 30^o} + \frac{1}{\tan 30^o} \quad \text{M1}$$

$$= 2 + \sqrt{3} \quad \text{A1}$$

(c) Solve the equation $\left(\operatorname{cosec} 2x + \cot 2x\right)^2 = 3$ for $0° \le x \le 360°$. [4]

$$\left(\operatorname{cosec} 2x + \cot 2x\right)^2 = 3$$

$$\left(\cot x\right)^2 = 3 \quad \text{M1}$$

$$\cot x = \pm\sqrt{3}$$

$$\tan x = \pm\frac{1}{\sqrt{3}} \quad \text{M1}$$

$$\text{Basic angle} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^o \quad \text{M1}$$

All 4 quadrants

$$x = 30^o, 150^o, 210^o, 330^o \quad \text{A1}$$

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10 **(a)** Given that $3x^2 - 2x + 4 = A(x-1)^2 + B(x-1) + C$ for all values of x, find the values of A, of B and of C. [3]

Comparing x^2 coefficient,

$A = 3$ B1

$3x^2 - 2x + 4 = 3(x-1)^2 + B(x-1) + C$

Let $x = 1$

$3(1)^2 - 2(1) + 4 = C$

$C = 5$ B1

Let $x = 0$

$4 = 3(-1)^2 + B(-1) + 5$

$4 = 8 - B$

$B = 4$ B1

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10 **(b)** A polynomial $f(x)$ can be written in the form $f(x) = Q(x)(x-1)(x-2) + Ax + B$ where

$Q(x)$ is a polynomial and A and B are constants.

When $f(x)$ is divided by $(x-1)$, the remainder is 3 and when $f(x)$ is divided by $(x-2)$, the remainder is 4.

Find the remainder when f(x) is divided by $(x-1)(x-2)$. [4]

$f(x) = Q(x)(x-1)(x-2) + Ax + B$

$f(1) = Q(1)(1-1)(1-2) + A(1) + B = 3$ M1

$A + B = 3$ ..... (1)

$f(2) = Q(2)(2-1)(2-2) + A(2) + B = 4$ M1

$2A + B = 4$ ..... (2)

Subtracting (1) from (2),

$(2A + B) - (A + B) = 4 - 3$

$A = 1$ M1

Substituting $A = 1$ into (1),

$1 + B = 3$

$B = 2$

Hence, the remainder is $x + 2$. A1

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10 **(c)** If $x + y = 5$ and $xy = 6$,

Find the value of

(i) $x^2 + y^2$ [2]

$(x+y)^2 = x^2 + y^2 + 2xy$ M1

$x^2 + y^2 = (x+y)^2 - 2xy$

$x^2 + y^2 = (5)^2 - 2(6)$

$x^2 + y^2 = (5)^2 - 2(6) = 13$ A1

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(ii) $x^3 + y^3$ [2]

$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$ M1

$x^3 + y^3 = (5)(13-6) = 35$ A1

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11 A particle moves in a straight line and passes a fixed point O with a velocity of $-8\ \text{m s}^{-1}$.

Its acceleration, $a\ \text{m s}^{-1}$, t seconds after passing O is given by $a = 6 - 2t$.

Find

(a) an expression for the velocity of the particle after t seconds, [2]

$a = 6 - 2t$

$v = \int (6 - 2t)\ \mathrm{d}t$ M1

$= 6t - t^2 + c$

When $t = 0$ and $v = -8$,

$c = -8$

Hence, the velocity of the particle after t seconds is $v = 6t - t^2 - 8$. A1

(b) the maximum velocity of the particle, [3]

$a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = 6 - 2t$

For stationary points, $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 0$. M1

$6 - 2t = 0$

$2t = 6$ M1

$t = 3$

When $t = 3$,

$v = 6(3) - (3)^2 - 8$ A1

$= 1 m/s$

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MARKING SCHEME

(c) the value of t when the particle is instantaneously at rest, [2]

When the particle is instantaneously at rest, $v = 0$.

$6t - t^2 - 8 = 0$ M1

$-\left(t^2 - 6t + 8\right) = 0$

$-(t-2)(t-4) = 0$

$t = 2,\ t = 4.$ A1

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(d) the distance travelled by the particle in the first 8 seconds of its motion. [4]

$v = 6t - t^2 - 8$

$s = \int \left(6t - t^2 - 8\right) \mathrm{d}t$ M1

$= 3t^2 - \frac{1}{3}t^3 - 8t + c_2$

When $t = 0$ and $s = 0$,

$c = 0$ M1

$s = 3t^2 - \frac{1}{3}t^3 - 8t$

$t = 0$, $s = 0$

$t = 2$, $s = -6\frac{2}{3}$

$t = 4$, $s = -5\frac{1}{3}$ M1

$t = 8$, $s = -42\frac{2}{3}$

Distance travelled by the particle in the first 10 seconds

$= 6\frac{2}{3} + 1\frac{1}{3} + 37\frac{1}{3} = 45\frac{1}{3}$ m A1

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- End of paper -

CHUNG CHENG HIGH SCHOOL (YISHUN)

O

2026 Preliminary Examination
Secondary Four Express / Five Normal Academic

CANDIDATE NAME

FORM CLASS / SUBJECT GROUP /

INDEX NUMBER

ADDITIONAL MATHEMATICS 4049/01

Paper 1

20 August 2026

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

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Question Number	Marks Possible	Marks Obtained
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13	7	
14	6	
Presentation Deduction		– 1 / – 2
TOTAL	90	

This document consists of **20** printed pages.

Mathematical Formulae

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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Identities

$$\sin^2 A + \cos^2 A = 1$$
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$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1 **(a)** Prove that $\dfrac{\cos\theta}{1-\sin\theta}+\dfrac{\cos\theta}{1+\sin\theta}=2\sec\theta$. [3]

(b) Hence, solve the equation $\dfrac{\cos\theta}{1-\sin\theta}+\dfrac{\cos\theta}{1+\sin\theta}=\tan^2\theta-1$ for $0<\theta<2\pi$. [4]

2 The equation of a curve is $y = kx^2 + 6x + 5 - k$, where k is a constant.

(a) Show that there is no real value of k for which the curve lies completely above the x-axis. [4]

(b) Find the value(s) of k for which the line $y = 2x$ is a tangent to the curve. [3]

3 The first three terms, in ascending powers of x, in the expansion of $(1+x)(1-5x)^n$ are $1-44x+px^2$, where p and n are constants.
Find the value of p and of n. [5]

4 In a laboratory, the temperature, T °C, of a chemical solution is being monitored t seconds after the cooling process has started. The rate of change of the temperature is given by $\frac{\mathrm{d}T}{\mathrm{d}t} = -4\mathrm{e}^{-0.04t}$. The initial temperature of the solution is $90\,°\mathrm{C}$.

(a) Find an expression for T, in terms of t. [3]

(b) A chemical indicator is to be added when the temperature reaches $40\,°\mathrm{C}$.
How many seconds after the cooling process has started should the indicator be added? [2]

(c) Will the temperature of the solution reach $-15\,°\mathrm{C}$? Explain your answer clearly. [1]

5 The curve C has equation $y = \ln\left[\dfrac{(2x+1)^3}{e^{3x}}\right]$, $x > -\dfrac{1}{2}$.

(a) Find $\dfrac{dy}{dx}$. [2]

(b) Find the y-coordinate of the stationary point of C, leaving your answer in the form $a + \ln b$, where a and b are rational numbers. [2]

6 Given that $\dfrac{\cos(2A+B)}{\cos(2A-B)}=\dfrac{3}{5}$, find the value of $\cot 2A \cot B$. [4]

7 **(a)** Express $\dfrac{6x^2-3x+4}{(x-3)(2x+1)^2}$ in partial fractions. [5]

(b) **Hence** show that $\int_4^{12} \frac{6x^2-3x+4}{(x-3)(2x+1)^2}\,\mathrm{d}x = a + \ln b$, where a and b are rational numbers.

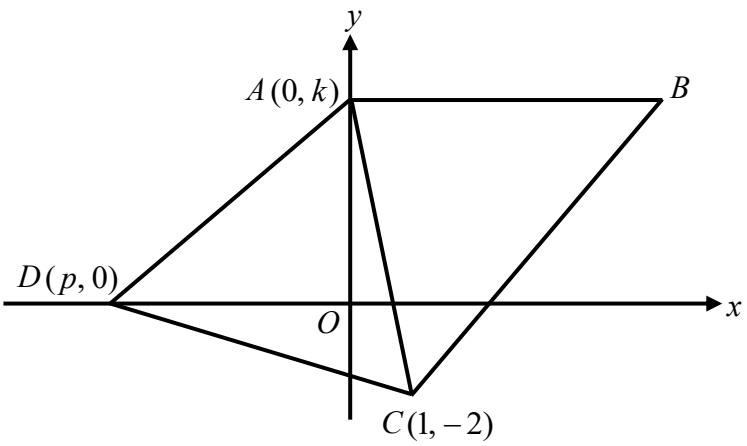
[4]

8 The polynomial $\mathrm{f}(x)$ is such that $\mathrm{f}(x) = ax^3 + bx^2 + cx - 4$, where a, b and c are integers. It is given that $\mathrm{f}'(0) = 5$. It is also given that $\mathrm{f}(x)$ has a factor of $x - 1$ and a remainder of 18 when divided by $x + 2$.

(a) Find the values of a, b and c. [5]

(b) Hence factorise $\mathrm{f}(x)$ completely. [2]

9 The diagram shows a quadrilateral $ABCD$ in which $AD = DC$.
Point A is $(0,\ k)$, point C is $(1, -2)$ and point D is $(p, 0)$, where k and p are constants.
AB is parallel to the x-axis and AC is perpendicular to the line $6y - x = 7$.



(a) Find the value of k and of p. [4]

(b) Given that the area of $ABCD$ is 33.5 units^2, find the coordinates of B. [2]

10 A particle moves along the curve $y = \frac{4x-10}{x+2}$.

Find the two possible values of y for which the rate of change of y is half the rate of change of x. [5]

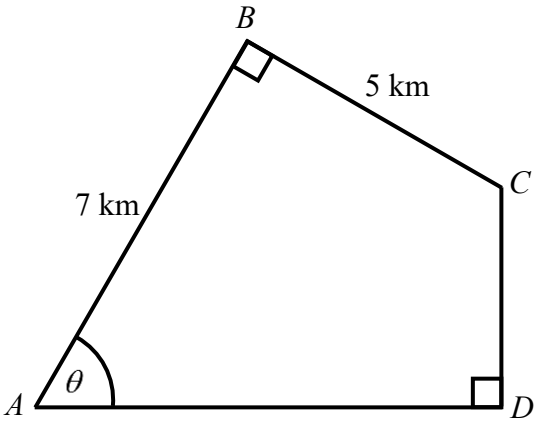
11 **(a)** **(i)** Factorise $x^3 - 8$. [1]

$x^3 - 8 = (x-2)(x^2+2x+4)$

(ii) Express $1+\lg(x^3-8)-2\lg(x-2)$, where $x>2$, in the form $\lg\frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are functions of x. Give your answer in its simplest form. [3]

(b) Solve the equation $4\log_7(x+3) = \log_{(x+3)} 7$, giving your answers in the form $a+b\sqrt{7}$, where a and b are constants. [4]

12 The diagram below shows a park with perimeter $ABCD$ where angle ABC = angle $ADC = 90°$ and angle $BAD = \theta$ where $0° < \theta < 90°$. AB = 7 km and BC = 5 km.



(a) Show that the perimeter of the park, P km, is given by $P = 12 + 12\sin\theta + 2\cos\theta$. [2]

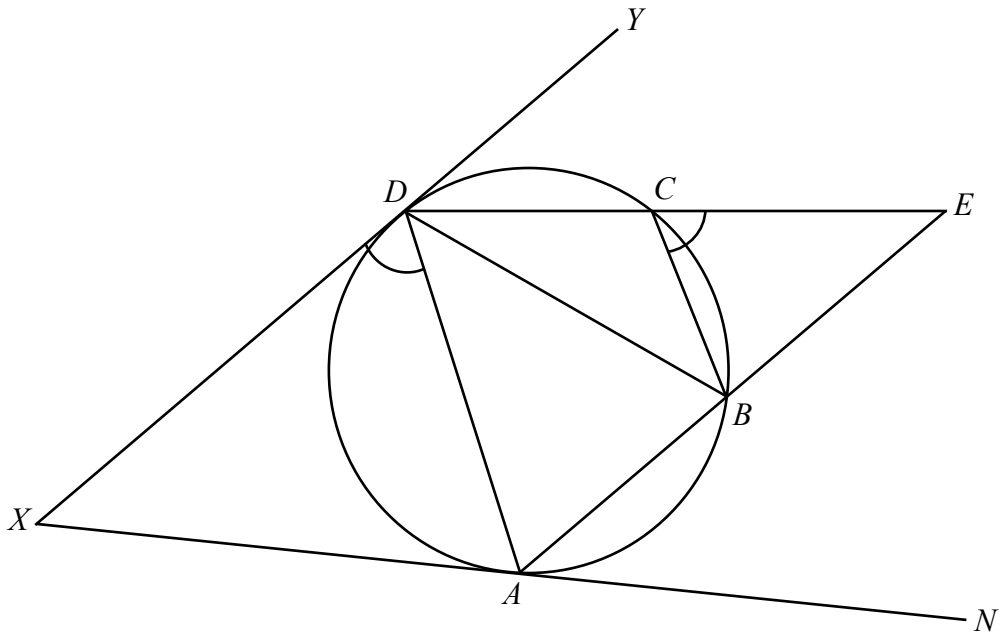
(b) Express P in the form $R\cos(\theta - \alpha) + 12$, where $R > 0$ and $0° < \alpha < 90°$. [3]

Hence find

(c) the maximum possible perimeter of the park and the corresponding value of θ. [2]

(d) the value of θ such that $P = 23.6$. [2]

13 In the diagram, A, B, C and D lie on a circle. The line XDY is the tangent to the circle at D and the line XAN is the tangent to the circle at A. The lines AB and DC are produced to meet at E and angle XDA = angle BCE.



(a) Prove that XY is parallel to AE. [2]

(b) Prove that triangle DAB is isosceles. [2]

(c) Prove that $DA^2 = XD \times AB$. [3]

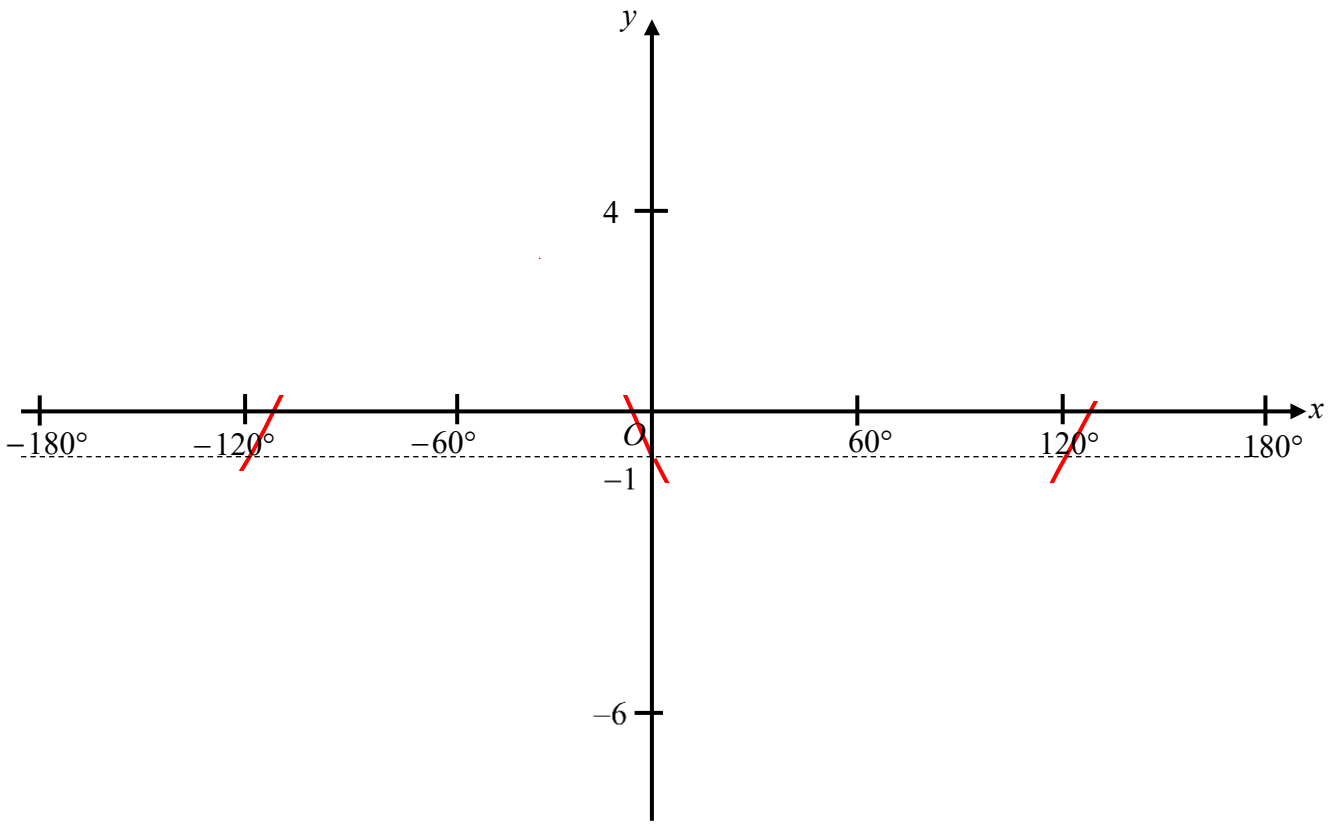
14 The function g is defined by $g(x) = -5\sin\frac{3x}{2} - 1$ for all values of x.

(a) Write down the amplitude and period of $g(x)$. [2]

Amplitude = 5

Period $= 360° \div \frac{3}{2} = 240°$

(b) On the axes, sketch the graph of $y = g(x)$, for $-180° \leq x \leq 180°$. [3]



(c) State the range of values of m such that the equation $-5\sin\frac{3x}{2} - 1 = m$ has no solution.

[1]

CHUNG CHENG HIGH SCHOOL (YISHUN)

2026 Preliminary Examination
Secondary Four Express / Five Normal Academic

CANDIDATE NAME			
FORM CLASS / SUBJECT GROUP	/	INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/02

Paper 2

28 August 2026

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
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6	7	
7	8	
8	9	
9	10	
10	11	
11	10	
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Presentation Deduction		– 1 / – 2
TOTAL	90	

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1 The sale, S, in dollars made by a student-run café h hours after opening is modelled by $S = 320 + 80h - 8h^2$, where $0 \le h \le 12$.

(a) By expressing S in the form $A - B(h + C)^2$, where A, B and C are constants, find the time at which the sale is greatest and state this greatest sale. [4]

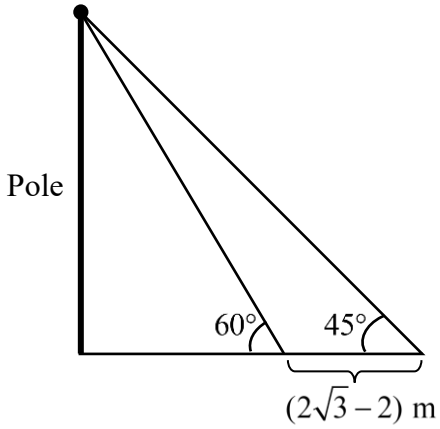
(b) After a promotional campaign, the sale at each time was found to be 25% higher than before. State the time at which the sale is greatest and the value of this biggest sale. [2]

2 Solve the equation $2e^{x}+3=9e^{-x}$, leaving your answer in the form $\ln\frac{b}{c}$. [4]

3 **(a)** Given that $y = \dfrac{(2x+1)\sin 2x}{4}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(2x+1)\cos 2x + \sin 2x}{2}$. [2]

(b) Hence find the exact value of $\int_0^{\frac{\pi}{4}} (2x+1)\cos 2x \ \mathrm{d}x$. [5]

4 A pole stands vertically on level ground. From point A, the angle of elevation of the top of the pole is $45°$. After walking directly towards the pole by $\left(2\sqrt{3}-2\right)$m to the point B, the angle of elevation becomes $60°$. Find the exact height of the pole, giving your answer in the form $a\sqrt{3}$, where a is an integer. [4]



5 The function is defined by $\mathrm{f}(x) = \sqrt{x^2 + 4}$, where $x > 0$.

(a) Explain whether $\mathrm{f}(x)$ is an increasing or decreasing function. [4]

(b) Hence explain why the equation $\sqrt{x^2 + 4}$ has at most one solution for every number *k*, where $x > 0$. [1]

6 Without using a calculator,

(a) Show that $\sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$ [2]

(b) Given that $\cos 15° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$, find the exact value of $\dfrac{\sin 15° + \cos 15°}{2-\sqrt{3}}$. Give your answer in the form $a\sqrt{6}+b\sqrt{2}$, where a and b are rational numbers. [4]

(c) Explain, without the use a calculator, why your answer is greater than 3. [1]

$\sqrt{6} > \sqrt{4}$

$\sqrt{6} > 2 \quad , \quad \sqrt{2} > 1$

$\sqrt{6} + \dfrac{3}{2}\sqrt{2} > 2 + 1$

> 3

7 Given that $\cos x = -\frac{4}{5}$ and $\tan x > 0$, find the exact value of

(a) $\sin 2x$, [2]

(b) $\cos \frac{x}{2}$, [3]

(c) $\cot \frac{x}{2}$ [3]

8 The circle C has centre $A(3,-2)$ and passes through the point $B(8,2)$.

(a) Find the equation of the circle C in standard form. [2]

(b) The line l has equation $3x+4y-32=0$. Show that the point B lies on the line l. [1]

(c) Show that the line is not a tangent to the point B. Clearly justify your answer. [3]

(d) Hence, find the equation of the tangent to circle C at B. [2]

(e) A circle D is a reflection of circle C along the line $x = 5$. Find the equation of the circle D. [1]

9 A particle moves along a straight line. Its velocity v m/s at a time t seconds is given by $v = t^3 - 9t^2 + 18t$. At $t = 0$, the particle is at the point O.

(a) Find the initial acceleration. [1]

(b) Find the time(s) when the particle is at instantaneous rest. [2]

(c) Find an expression for the displacement s metres from O at time t. [3]

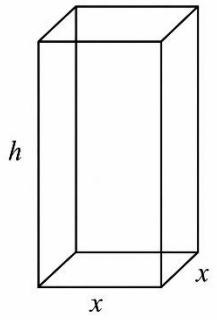
(d) Find the total distance travelled by the particle during the first 4 seconds. [3]

(e) Determine whether the particle is moving in the positive or negative direction when $t = 4$. [1]

10 A company designs an open-top rectangular water tanks with a square base of side x metres and height h metres for the four walls. The tank must hold exactly 256m^3 of water. The cost of materials is:

- \$90 per m^2 for the base,
- \$40 per m^2 for the four walls.

After construction, the outside of the four walls is painted. The paint cost \$15 per m^2.



(a) Show that the material cost C dollars can be written as

[2]

(b) Find the value of x for which the material cost is a minimum. [5]

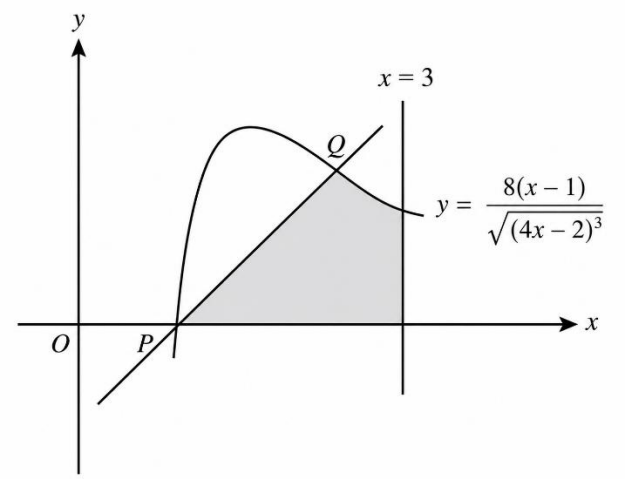
(c) Hence, find the minimum cost, giving your answer to the nearest dollar. [1]

(d) Show that the cost of paint, P, dollars is given by $P = \frac{15360}{x}$. [2]

(e) Given that the side length of the square base cannot exceed 8 m, determine the dimensions of the tank for which the cost of paint job is the least. [1]

11 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{4x}{\sqrt{4x-2}}\right)=\frac{8(x-1)}{\sqrt{(4x-2)^3}}$. [3]

The diagram shows part of the curve $y=\frac{8(x-1)}{\sqrt{(4x-2)^3}}$ and the line $x=3$. The curve intersects the x-axis at the point P. The line through P with gradient 1 intersects the curve again at the point Q.



(b) Show that the coordinates of Q is $\left(\frac{3}{2}, \frac{1}{2}\right)$. [5]

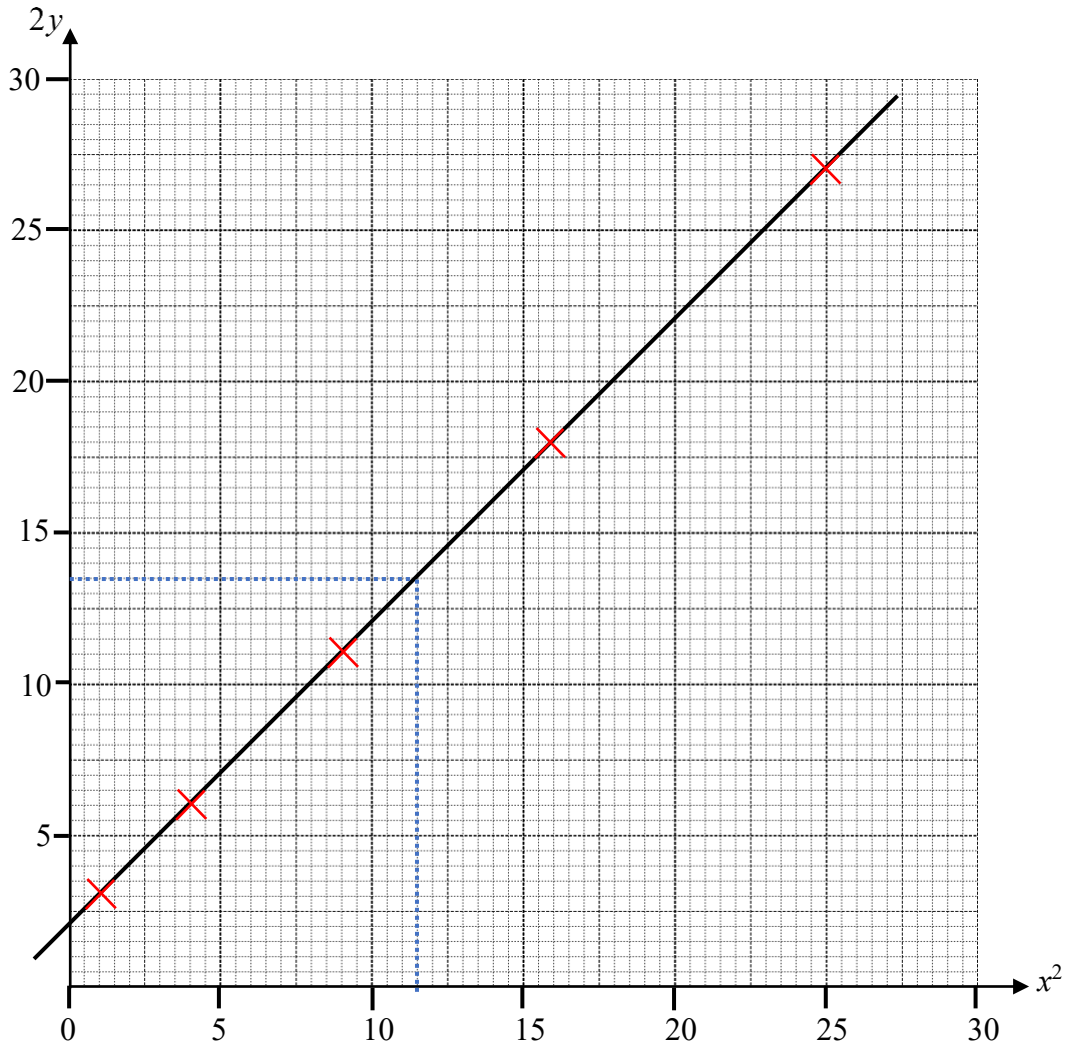
(c) Find the area of the shaded region bounded by the curve, the x-axis, the line PQ, and the line $x = 3$. [3]

12 A mathematics teacher designed an assessment consisting of x questions. The average time taken (y mins) by students is related to the number of questions and is related by $ae^{2y} = e^{bx^2+4}$, where a and b are constants. The table below shows the relationship between x and y.

x	1	2	3	4	5
y	1.5	3	5.5	9	13.5

(a) On the grid below, plot $2y$ against x^2 and draw a straight-line graph. [2]

x^2	**1**	**4**	**9**	**16**	**25**
y	**3**	**9**	**11**	**18**	**27**



(b) Use your graph to estimate the value of a and of b. [3]

(c) Use your graph to estimate the value of x when $e^y = 850$. [2]

(d) A student claims that there is a value of x when $e^y = 0.5$. Explain why the student's claim is impossible. [1]

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CHUNG CHENG HIGH SCHOOL (YISHUN)

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2026 Preliminary Examination
Secondary Four Express / Five Normal Academic

CANDIDATE NAME			
FORM CLASS / SUBJECT GROUP		INDEX NUMBER	

ADDITIONAL MATHEMATICS

4049/01

Paper 1

20 August 2026

2 hours 15 minutes

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Candidates answer on the Question Paper.

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$$\Delta = \frac{1}{2}bc\sin A$$

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1 **(a)** Prove that $\frac{\cos\theta}{1-\sin\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$. [3]

$$\text{LHS} = \frac{\cos\theta}{1-\sin\theta}+\frac{\cos\theta}{1+\sin\theta}$$

$$=\frac{\cos\theta(1+\sin\theta)+\cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$=\frac{2\cos\theta}{1-\sin^2\theta}$$

$$=\frac{2\cos\theta}{\cos^2\theta}$$

$$=\frac{2}{\cos\theta}$$

$$=2\sec\theta$$

$$= RHS \text{ (shown)}$$

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(b) Hence, solve the equation $\frac{\cos\theta}{1-\sin\theta}+\frac{\cos\theta}{1+\sin\theta}=\tan^2\theta-1$ for $0<\theta<2\pi$. [4]

$$2\sec\theta=\tan^2\theta-1$$

$$2\sec\theta=\sec^2\theta-1-1$$

$$\sec^2\theta-2\sec\theta-2=0$$

$$\sec\theta=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-2)}}{2(1)}$$

$\sec\theta=2.73205$ or $\sec\theta=-0.73205$

$\cos\theta=0.366025$ $\cos\theta=-1.366025$ (reject)

Basic angle = 1.19606

$\theta=1.19606$, 5.08712

$\theta=1.20$, 5.09 (3 s.f)

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2 The equation of a curve is $y = kx^2 + 6x + 5 - k$, where k is a constant.

(a) Show that there is no real value of k for which the curve lies completely above the x-axis. [4]

For the curve $y = kx^2 + 6x + 5 - k$ to lie **completely above** the x-axis, it must be an upward-opening parabola with no real roots.
$k > 0$ and Discriminant < 0

Discriminant
$= 6^2 - 4k(5-k)$
$= 4k^2 - 20k + 36$
$= 4\left[k^2 - 5k + 9\right]$
$= 4\left[(k-2.5)^2 + 2.75\right]$
$= 4(k-2.5)^2 + 11$

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$(k-2.5)^2 \geq 0$ for all real values of k.
$4(k-2.5)^2 + 11 \geq 11 > 0$

For all real values of k, Discriminant > 0

There is no real value of k for Discriminant < 0.

Thus, there is no real value of k for which the curve lies completely above the x-axis.

(b) Find the value(s) of k for which the line $y = 2x$ is a tangent to the curve. [3]

$kx^2 + 6x + 5 - k = 2x$

$kx^2 + 4x + 5 - k = 0$

Discriminant = 0

$4^2 - 4(k)(5-k) = 0$

$16 - 20k + 4k^2 = 0$

$k^2 - 5k + 4 = 0$

$(k-1)(k-4) = 0$

$k = 1$ or $k = 4$

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3 The first three terms, in ascending powers of x, in the expansion of $(1+x)(1-5x)^n$ are $1-44x+px^2$, where p and n are constants.
Find the value of p and of n. [5]

$$(1+x)\left[1-5nx+\frac{25n(n-1)}{2}x^2+...\right]$$

Compare x term:

$-5nx+x=-44x$

$-5n+1=-44$

$n=9$

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Compare x^2 term:

$\frac{(25)(9)(8)}{2}x^2-45x^2=px^2$

$900-45=p$

$p=855$

Solution: $n=9$, $p=855$

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4 In a laboratory, the temperature, T °C, of a chemical solution is being monitored t seconds after the cooling process has started. The rate of change of the temperature is given by $\frac{\mathrm{d}T}{\mathrm{d}t} = -4\mathrm{e}^{-0.04t}$. The initial temperature of the solution is 90 °C.

(a) Find an expression for T, in terms of t. [3]

$$T = \int -4\mathrm{e}^{-0.04t}\ \mathrm{d}t$$

$$= \frac{-4\mathrm{e}^{-0.04t}}{-0.04} + c$$

$$= 100\mathrm{e}^{-0.04t} + c$$

When $t = 0$, $T = 90$

$$90 = 100\mathrm{e}^{0} + c$$

$$c = -10$$

$$T = 100\mathrm{e}^{-0.04t} - 10$$

(b) A chemical indicator is to be added when the temperature reaches 40 °C.
How many seconds after the cooling process has started should the indicator be added? [2]

$$100\mathrm{e}^{-0.04t} - 10 = 40$$

$$100\mathrm{e}^{-0.04t} = 50$$

$$\mathrm{e}^{-0.04t} = 0.5$$

$$-0.04t = \ln 0.5$$

$t = 17.32867951 = 17.3$ (3 s.f.)

Time = 17.3 seconds

(c) Will the temperature of the solution reach −15 °C? Explain your answer clearly. [1]

$$T = 100\mathrm{e}^{-0.04t} - 10$$

As $t \to \infty$, $\mathrm{e}^{-0.04t} \to 0$

$$T \to -10$$

Since the temperature approaches −10 °C in the long run and will not fall lower, the temperature will not reach −15 °C

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5 The curve C has equation $y = \ln\left[\dfrac{(2x+1)^3}{e^{3x}}\right]$, $x > -\dfrac{1}{2}$.

(a) Find $\dfrac{dy}{dx}$. [2]

$$y = \ln\left[\frac{(2x+1)^3}{e^{3x}}\right]$$

$$y = \ln(2x+1)^3 - \ln e^{3x}$$

$$y = 3\ln(2x+1) - 3x$$

$$\frac{dy}{dx} = \frac{6}{2x+1} - 3$$

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(b) Find the y-coordinate of the stationary point of C, leaving your answer in the form $a + \ln b$, where a and b are rational numbers. [2]

$$\frac{6}{2x+1} - 3 = 0$$

$$\frac{6}{2x+1} = 3$$

$$6x + 3 = 6$$

$$x = \frac{1}{2}$$

$$y = 3\ln 2 - \frac{3}{2}$$

$$y = -\frac{3}{2} + \ln 8$$

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6 Given that $\dfrac{\cos(2A+B)}{\cos(2A-B)}=\dfrac{3}{5}$, find the value of $\cot 2A \cot B$. [4]

$5\cos(2A+B)=3\cos(2A-B)$

$5\left[\cos 2A\cos B-\sin 2A\sin B\right]=3\left[\cos 2A\cos B+\sin 2A\sin B\right]$

$2\cos 2A\cos B=8\sin 2A\sin B$

$\dfrac{\cos 2A\cos B}{\sin 2A\sin B}=\dfrac{8}{2}$

$\cot 2A\cot B=4$

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7 **(a)** Express $\dfrac{6x^2-3x+4}{(x-3)(2x+1)^2}$ in partial fractions. [5]

$$\frac{6x^2-3x+4}{(x-3)(2x+1)^2}=\frac{A}{x-3}+\frac{B}{2x+1}+\frac{C}{(2x+1)^2}$$

$$6x^2-3x+4=A(2x+1)^2+B(2x+1)(x-3)+C(x-3)$$

When $x=3$

$49=49A$

$A=1$

When $x=-0.5$

$7=-3.5C$

$C=-2$

When $x=0$

$4=A-3B-3C$

$4=1-3B+6$

$B=1$

$$\frac{6x^2-3x+4}{(x-3)(2x+1)^2}=\frac{1}{x-3}+\frac{1}{2x+1}-\frac{2}{(2x+1)^2}$$

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(b) **Hence** show that $\int_4^{12} \frac{6x^2-3x+4}{(x-3)(2x+1)^2}\,\mathrm{d}x = a+\ln b$, where a and b are rational numbers.

[4]

$$\int_4^{12} \frac{1}{x-3}+\frac{1}{2x+1}-\frac{2}{(2x+1)^2}\,\mathrm{d}x$$

$$=\left[\ln(x-3)+\frac{1}{2}\ln(2x+1)+\frac{1}{2x+1}\right]_4^{12}$$

$$=\left[\ln 9+\frac{1}{2}\ln 25+\frac{1}{25}\right]-\left[\ln 1+\frac{1}{2}\ln 9+\frac{1}{9}\right]$$

$$=\left[\ln 9+\ln 5+\frac{1}{25}\right]-\left[0+\ln 3+\frac{1}{9}\right]$$

$$=\left[\frac{1}{25}-\frac{1}{9}\right]+\left[\ln 9+\ln 5-\ln 3\right]$$

$$=-\frac{16}{225}+\ln\left(\frac{9\times 5}{3}\right)$$

$$=-\frac{16}{225}+\ln 15$$

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8 The polynomial $f(x)$ is such that $f(x) = ax^3 + bx^2 + cx - 4$, where a, b and c are integers. It is given that $f'(0) = 5$. It is also given that $f(x)$ has a factor of $x-1$ and a remainder of 18 when divided by $x+2$.

(a) Find the values of a, b and c. [5]

$f'(x) = 3ax^2 + 2bx + c$

$f'(0) = 5$

$c = 5$

$f(1) = 0$

$a(1)^3 + b(1)^2 + 5(1) - 4 = 0$

$a + b = -1$ ---------- [1]

$f(-2) = 18$

$a(-2)^3 + b(-2)^2 + 5(-2) - 4 = 18$

$-8a + 4b = 32$

$-2a + b = 8$ ---------- [2]

[1] – [2]: $3a = -9$

$a = -3$

Sub $a = -3$ into [1]:

$-3 + b = -1$

$b = 2$

Solution: $a = -3$, $b = 2$, $c = 5$

(b) Hence factorise $f(x)$ completely. [2]

$f(x) = -3x^3 + 2x^2 + 5x - 4$

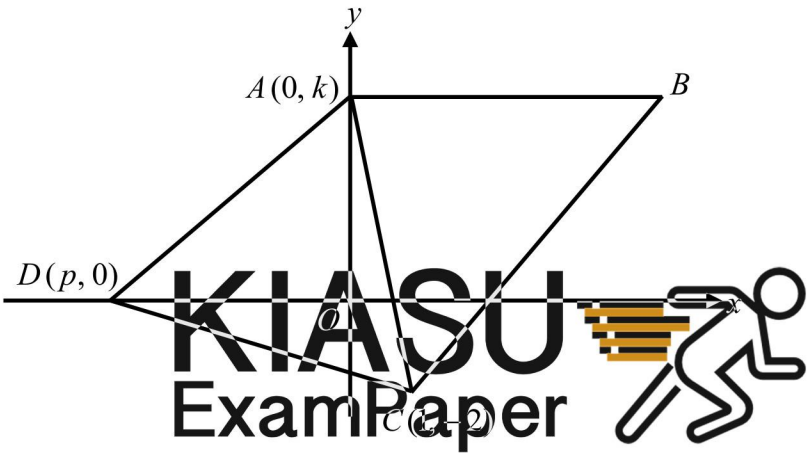
$= (x-1)(-3x^2 - x + 4)$

$= (x-1)(-3x-4)(x-1)$

$= (-3x-4)(x-1)^2$

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9 The diagram shows a quadrilateral $ABCD$ in which $AD = DC$.
Point A is $(0,\ k)$, point C is $(1, -2)$ and point D is $(p, 0)$, where k and p are constants.
AB is parallel to the x-axis and AC is perpendicular to the line $6y - x = 7$.



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(a) Find the value of k and of p. [4]

Rearrange the equation $6y - x = 7$

$$6y = x + 7$$

$$y = \frac{1}{6}x + \frac{7}{6}$$

Gradient of $AC = -6$

$$\frac{k-(-2)}{0-1} = -6$$

$$k + 2 = 6$$

$$k = 4$$

$$\sqrt{p^2 + 4^2} = \sqrt{(p-1)^2 + 2^2}$$

$$p^2 + 16 = p^2 - 2p + 1 + 4$$

$$2p = -11$$

$$p = -5.5$$

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(b) Given that the area of $ABCD$ is 33.5 units^2, find the coordinates of B. [2]

Let the coordinates of B be $(m, 4)$.

Area = 33.5

$$\frac{1}{2}\begin{vmatrix} 0 & -5.5 & 1 & m & 0 \\ 4 & 0 & -2 & 4 & 4 \end{vmatrix} = 33.5$$

$$0 + (-5.5)(-2) + 4 + 4m - (-2m) - (-5.5)(4) = 33.5 \times 2$$

$$37 + 6m = 67$$

$$6m = 30$$

$$m = 5$$

Coordinates of B = (5, 4)

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10 A particle moves along the curve $y = \dfrac{4x-10}{x+2}$.

Find the two possible values of y for which the rate of change of y is half the rate of change of x. [5]

$$\frac{dy}{dx} = \frac{(x+2)(4)-(4x-10)}{(x+2)^2}$$

$$= \frac{18}{(x+2)^2}$$

$$\frac{dy}{dt} = \frac{1}{2}\frac{dx}{dt}$$

$$\frac{dy}{dx} \times \frac{dx}{dt} = \frac{1}{2}\frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{1}{2}$$

$$\frac{18}{(x+2)^2} = \frac{1}{2}$$

$$(x+2)^2 = 36$$

$$x+2 = \pm 6$$

$x = 4$ or $x = -8$

Sub $x = 4$ into y: $y = \dfrac{4(4)-10}{4+2} = 1$

Sub $x = -8$ into y: $y = \dfrac{4(-8)-10}{-8+2} = 7$

$y = 1$ $y = 7$

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11 (a) (i) Factorise $x^3 - 8$. [1]

$x^3 - 8 = (x-2)(x^2+2x+4)$

(ii) Express $1+\lg(x^3-8)-2\lg(x-2)$, where $x>2$, in the form $\lg\frac{f(x)}{g(x)}$, where $f(x)$ and $g(x)$ are functions of x. Give your answer in its simplest form. [3]

$1+\lg(x^3-8)-2\lg(x-2)$

$=\lg 10+\lg(x^3-8)-\lg(x-2)^2$

$=\lg\frac{10(x^3-8)}{(x-2)^2}$

$=\lg\frac{10(x-2)(x^2+2x+4)}{(x-2)^2}$

$=\lg\frac{10(x^2+2x+4)}{x-2}$

(b) Solve the equation $4\log_7(x+3)=\log_{(x+3)}7$, giving your answers in the form $a+b\sqrt{7}$, where a and b are constants. [4]

$4\log_7(x+3)=\frac{1}{\log_7(x+3)}$

$\left[\log_7(x+3)\right]^2=\frac{1}{4}$

$\log_7(x+3)=\pm\frac{1}{2}$

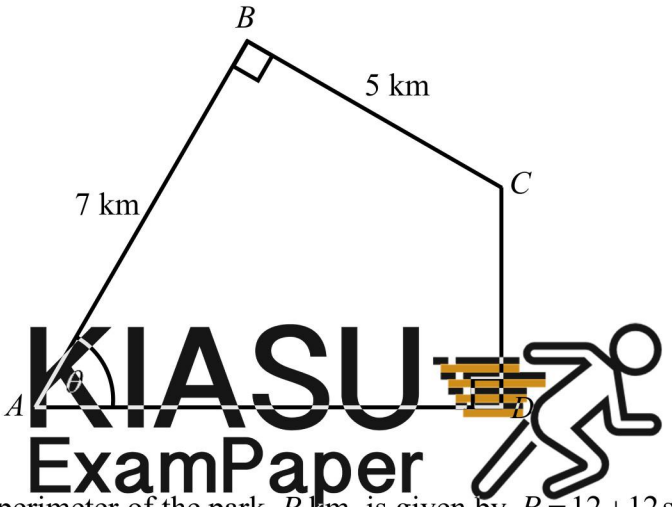
$x+3=7^{\frac{1}{2}}$ or $x+3=7^{-\frac{1}{2}}$

$x=-3+\sqrt{7}$ $x=-3+\frac{1}{\sqrt{7}}$

$x=-3+\frac{1}{7}\sqrt{7}$

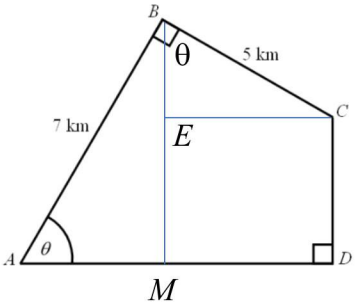
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12 The diagram below shows a park with perimeter $ABCD$ where angle ABC = angle $ADC = 90°$ and angle $BAD = \theta$ where $0° < \theta < 90°$. $AB = 7$ km and $BC = 5$ km.



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(a) Show that the perimeter of the park, P km, is given by $P = 12 + 12\sin\theta + 2\cos\theta$. [2]



$AM = 7\cos\theta$

$MD = EC = 5\sin\theta$

$CD = MB - BE$

$= 7\sin\theta - 5\cos\theta$

$P = AB + BC + CD + DM + MA$

$= 7 + 5 + (7\sin\theta - 5\cos\theta) + 5\sin\theta + 7\cos\theta$

$= 12 + 12\sin\theta + 2\cos\theta$

(b) Express P in the form $R\cos(\theta - \alpha) + 12$, where $R > 0$ and $0° < \alpha < 90°$. [3]

$P = 12 + 2\cos\theta + 12\sin\theta$

$R = \sqrt{12^2 + 2^2} = \sqrt{148} = 2\sqrt{37}$

$\tan\alpha = \dfrac{12}{2}$

$\alpha = 80.538°$

$= 80.5°$ (1 d.p)

$P = 2\sqrt{37}\cos(\theta - 80.5°) + 12$

Hence find

(c) the maximum possible perimeter of the park and the corresponding value of θ. [2]

$$P = 2\sqrt{37}\cos(\theta - 80.5°) + 12$$

Maximum perimeter

$= 2\sqrt{37} + 12$

$= 24.2$ km (3 s.f)

$\cos(\theta - 80.5°) = 1$

$(\theta - 80.5°) = 0°$

$\theta = 80.5°$

(d) the value of θ such that $P = 23.6$. [2]

$$23.6 = 2\sqrt{37}\cos(\theta - 80.538°) + 12$$

$$\cos(\theta - 80.538°) = 0.953514126$$

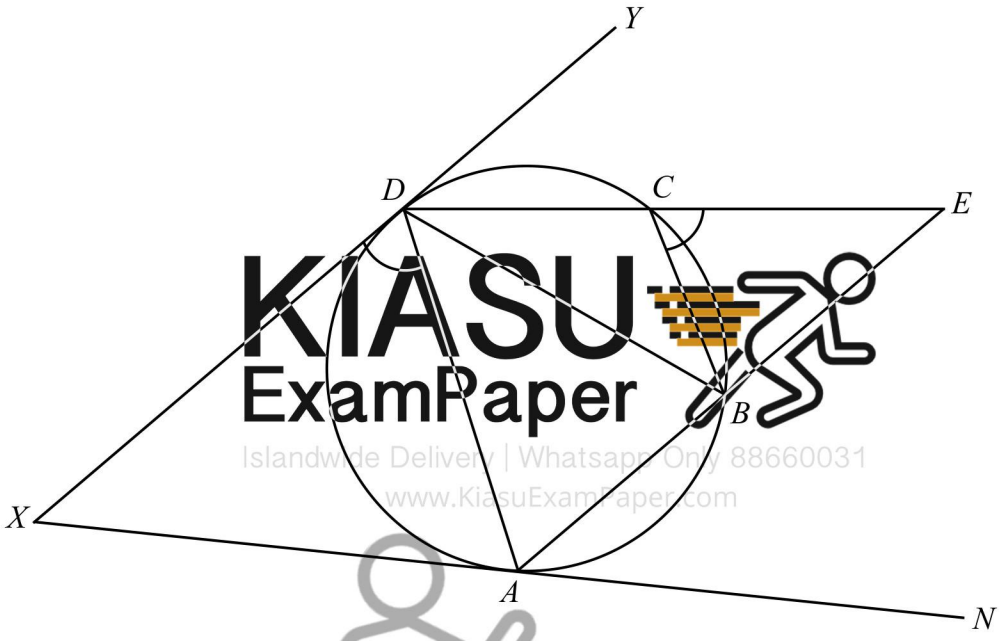
Basic angle = 17.539°

$(\theta - 80.538°) = -17.539°,\ 17.539°$

$\theta = 63.0°,\ 98.1°$ (reject)

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13 In the diagram, A, B, C and D lie on a circle. The line XDY is the tangent to the circle at D and the line XAN is the tangent to the circle at A. The lines AB and DC are produced to meet at E and angle XDA = angle BCE.



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(a) Prove that XY is parallel to AE. [2]

Let angle $BCE = x$

Angle $DCB = 180° - x$ (adjacent angles on a straight line)

Angle $DAB = x$ (angles in opposite segments)

Given that angle BCE = angle $XDA = x$

Therefore angle DAB = angle XDA

By the converse of the alternate angles property, XY// AE (shown)

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(b) Prove that triangle DAB is isosceles. [2]

Angle XDA = angle DBA (tangent-chord theorem)

From (a) Angle XDA = angle DAB

Therefore Angle DBA = angle DAB

Since the base angles *DBA* and *DAB* are equal,
by the converse of the base angles property of an isosceles triangle,
$DA = DB$ and triangle DAB is isosceles.

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(c) Prove that $DA^2 = XD \times AB$. [3]

Angle XDA = angle DAB (from (a))
Angle XAD = angle DBA (tangent chord theorem)

Triangle XDA is similar to triangle DAB (AA similarity).

$\frac{XD}{DA} = \frac{DA}{AB}$ [ratio of corresponding sides of similar triangles is constant]

$DA^2 = XD \times AB$

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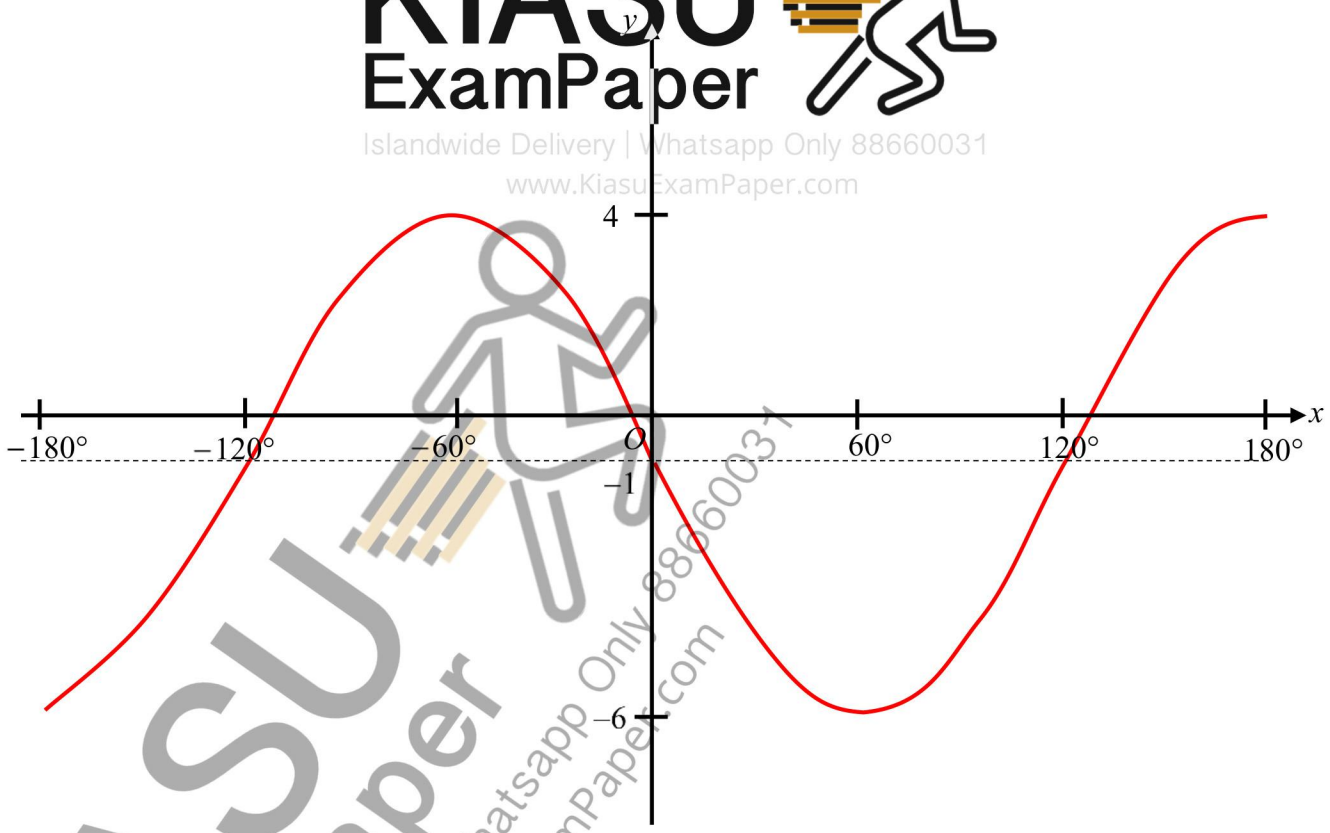
14 The function g is defined by $g(x) = -5\sin\frac{3x}{2} - 1$ for all values of x.

(a) Write down the amplitude and period of $g(x)$. [2]

Amplitude = 5

Period $= 360° \div \frac{3}{2} = 240°$

(b) On the axes, sketch the graph of $y = g(x)$, for $-180° \leq x \leq 180°$. [3]



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(c) State the range of values of m such that the equation $-5\sin\frac{3x}{2} - 1 = m$ has no solution. [1]

$m < -6$ or $m > 4$

CHUNG CHENG HIGH SCHOOL (YISHUN)

2026 Preliminary Examination

Secondary Four Express / Five Normal Academic

CANDIDATE NAME	
FORM CLASS / SUBJECT GROUP	/
INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/02

Paper 2

28 August 2026

2 hours 15 minutes

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Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers. Up to 2 marks may be deducted for improper presentation.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question Number	Marks Possible	Marks Obtained
1	6	
2	5	
3	7	
4	4	
5	5	
6	7	
7	8	
8	9	
9	10	
10	11	
11	10	
12	8	
Presentation Deduction		– 1 / – 2
TOTAL	90	

This document consists of **19** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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2. TRIGNOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

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1 The sale, S, in dollars made by a student-run café h hours after opening is modelled by $S = 320 + 80h - 8h^2$, where $0 \le h \le 12$.

(a) By expressing S in the form $A - B(h + C)^2$, where A, B and C are constants, find the time at which the sale is greatest and state this greatest sale. [4]

$$\begin{aligned} S &= 320 + 80h - 8h^2 \\ &= -8\left(h^2 - 10h - 40\right) \\ &= -8\left[(h-5)^2 - 25 - 40\right] \\ &= -8(h-5)^2 + 520 \\ &= 520 - 8(h-5)^2 \end{aligned}$$

Greatest sale = \$520

Time = 5 hours

(b) After a promotional campaign, the sale at each time was found to be 25% higher than before. State the time at which the sale is greatest and the value of this biggest sale. [2]

New greatest profit $= 1.25 \times \$520$
$= \$650$

Time = 5 hours

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2 Solve the equation $2e^{x}+3=9e^{-x}$, leaving your answer in the form $\ln\frac{b}{c}$. [4]

$$2e^{x}+3=9e^{-x}$$

$$\text{let } u=e^{x}$$

$$2u+3=\frac{9}{u}$$

$$2u^{2}+3u-9=0$$

$$(u+3)(2u-3)=0$$

$$u=-3 \quad \text{or} \quad u=\frac{3}{2}$$

$$e^{x}=-3 \quad \text{or} \quad e^{x}=\frac{3}{2}$$

(rejected) $\quad e^{x}=\frac{3}{2}$

$$x=\ln\frac{3}{2}$$

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3 **(a)** Given that $y = \dfrac{(2x+1)\sin 2x}{4}$, show that $\dfrac{dy}{dx} = \dfrac{(2x+1)\cos 2x + \sin 2x}{2}$. [2]

$$\frac{dy}{dx} = (2x+1)\left(\frac{1}{4}\right)(2\cos 2x) + \frac{1}{4}(\sin 2x)(2)$$

$$= \frac{1}{2}(2x+1)\cos 2x + \frac{1}{2}\sin 2x$$

$$= \frac{1}{2}\left[(2x+1)\cos 2x + \sin 2x\right]$$

$$= \frac{(2x+1)\cos 2x + \sin 2x}{2} \text{ (shown)}$$

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(b) Hence find the exact value of $\int_0^{\frac{\pi}{4}} (2x+1)\cos 2x \, dx$. [5]

$$\int_0^{\frac{\pi}{4}} \frac{(2x+1)\cos 2x + \sin 2x}{2} \, dx = \left[\frac{(2x+1)\sin 2x}{4}\right]_0^{\frac{\pi}{4}}$$

$$\int_0^{\frac{\pi}{4}} (2x+1)\cos 2x + \sin 2x \, dx = \left[\frac{(2x+1)\sin 2x}{2}\right]_0^{\frac{\pi}{4}}$$

$$\int_0^{\frac{\pi}{4}} (2x+1)\cos 2x \, dx = \left[\frac{(2x+1)\sin 2x}{2}\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \sin 2x \, dx$$

$$= \left[\frac{(2x+1)\sin 2x}{2}\right]_0^{\frac{\pi}{4}} + \left[\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{4}}$$

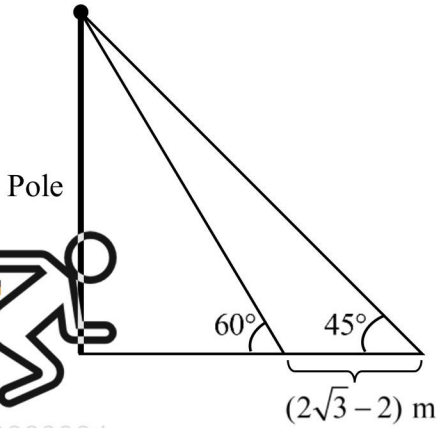
$$= \left[\frac{\left(\frac{\pi}{2}+1\right)\sin\frac{\pi}{2}}{2} - \frac{\sin 0}{2}\right] + \left[\frac{\cos\frac{\pi}{2}}{2} - \frac{\cos 0}{2}\right]$$

$$= \frac{\pi}{4} + \frac{1}{2} + 0 - \frac{1}{2}$$

$$= \frac{\pi}{4}$$

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4 A pole stands vertically on level ground. From point A, the angle of elevation of the top of the pole is $45°$. After walking directly towards the pole by $\left(2\sqrt{3}-2\right)$m to the point B, the angle of elevation becomes $60°$. Find the exact height of the pole, giving your answer in the form $a\sqrt{3}$, where a is an integer. [4]



$$\tan 60° = \frac{h}{h+2-2\sqrt{3}}$$

$$\sqrt{3} = \frac{h}{h+2-2\sqrt{3}}$$

$$\sqrt{3}h + 2\sqrt{3} - 6 = h$$

$$\sqrt{3}h - h = 6 - 2\sqrt{3}$$

$$h\left(\sqrt{3}-1\right) = 6 - 2\sqrt{3}$$

$$h = \frac{6-2\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{6\sqrt{3}-6+6-2\sqrt{3}}{3-1}$$

$$= \frac{4\sqrt{3}}{2}$$

$$= 2\sqrt{3}$$

Height $= 2\sqrt{3}$ m

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5 The function is defined by $\mathrm{f}(x) = \sqrt{x^2+4}$, where $x > 0$.

(a) Explain whether $\mathrm{f}(x)$ is an increasing or decreasing function. [4]

$$\mathrm{f}'(x) = \frac{1}{2}\left(x^2+4\right)^{-\frac{1}{2}}(2x)$$
$$= \frac{x}{\sqrt{x^2+4}}$$

$$x > 0$$
$$\sqrt{x^2+4} > 0$$
$$\frac{x}{\sqrt{x^2+4}} > 0$$

$\therefore$ since $\mathrm{f}'(x) > 0, \mathrm{f}(x)$ is an increasing function for $x > 0$.

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(b) Hence explain why the equation $\sqrt{x^2+4}$ has at most one solution for every number k, where $x > 0$. [1]

Since the function is an increasing for $x > 0$, any horizontal line will only cut f(x) at most once.

6 Without using a calculator,

(a) Show that $\sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$ [2]

$$\begin{aligned}\sin 15° &= \sin(45° - 30°) \\ &= \sin 45° \cos 30° - \cos 45° \sin 30° \\ &= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{6}-\sqrt{2}}{4} \quad \text{(shown)}\end{aligned}$$

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(b) Given that $\cos 15° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$, find the exact value of $\dfrac{\sin 15° + \cos 15°}{2-\sqrt{3}}$. Give your answer in the form $a\sqrt{6} + b\sqrt{2}$, where a and b are rational numbers. [4]

$$\begin{aligned}\frac{\sin 15° + \cos 15°}{2-\sqrt{3}} &= \frac{\dfrac{\sqrt{6}-\sqrt{2}}{4} + \dfrac{\sqrt{6}+\sqrt{2}}{4}}{2-\sqrt{3}} \\ &= \frac{\dfrac{2\sqrt{6}}{4}}{2-\sqrt{3}} \\ &= \frac{\sqrt{6}}{2(2-\sqrt{3})} \times \left(\frac{2+\sqrt{3}}{2+\sqrt{3}}\right) \\ &= \frac{2\sqrt{6}+\sqrt{18}}{2(4-3)} \\ &= \frac{2\sqrt{6}+3\sqrt{2}}{2} \\ &= \sqrt{6} + \frac{3}{2}\sqrt{2}\end{aligned}$$

(c) Explain, without the use a calculator, why your answer is greater than 3. [1]

$$\sqrt{6} > \sqrt{4}$$

$$\sqrt{6} > 2 \quad , \quad \sqrt{2} > 1$$

$$\begin{aligned}\sqrt{6} + \frac{3}{2}\sqrt{2} &> 2 + 1 \\ &> 3\end{aligned}$$

7 Given that $\cos x = -\frac{4}{5}$ and $\tan x > 0$, find the exact value of

(a) $\sin 2x$, [2]

$\sin 2x = 2\cos x \sin x$

$= 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right)$

$= \frac{24}{25}$

(b) $\cos\frac{x}{2}$, [3]

$\cos x = 2\cos^2\frac{x}{2} - 1$

$\left(-\frac{4}{5}\right) + 1 = 2\cos^2\frac{x}{2}$

$\frac{1}{10} = \cos^2\frac{x}{2}$

$\cos\frac{x}{2} = \sqrt{\frac{1}{10}}$ or $-\sqrt{\frac{1}{10}}$

(rejected)

(c) $\cot\frac{x}{2}$ [3]

$-\frac{4}{5} = 1 - 2\sin^2\frac{x}{2}$

$\sin\frac{x}{2} = \sqrt{\frac{9}{10}}$

$= \frac{3}{\sqrt{10}}$

$\cot\frac{x}{2} = \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}$

$= \frac{-\frac{1}{\sqrt{10}}}{\frac{3}{\sqrt{10}}}$

$= -\frac{1}{3}$

Alternate Answer

$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$

$\frac{3}{4} - \frac{3}{4}\tan^2\frac{x}{2} = 2\tan\frac{x}{2}$

$3\tan^2\frac{x}{2} + 8\tan\frac{x}{2} - 3 = 0$

$\left(3\tan\frac{x}{2} - 1\right)\left(\tan\frac{x}{2} + 3\right) = 0$

$\tan\frac{x}{2} = \frac{1}{3}$ (rejected) or $\tan\frac{x}{2} = -3$

$\cot\frac{x}{2} = -\frac{1}{3}$

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8 The circle C has centre $A(3,-2)$ and passes through the point $B(8,2)$.

(a) Find the equation of the circle C in standard form. [2]

$$\text{Radius} = \sqrt{(8-3)^2+(2+2)^2}$$
$$= \sqrt{41} \text{ units}$$

$$\therefore \text{Equation of circle : } (x-3)^2+(y+2)^2 = 41$$

(b) The line l has equation $3x+4y-32=0$. Show that the point B lies on the line l. [1]

$$3(8)+4(2)-32 = 24+8-32$$
$$= 0$$

Since B satisfies the equation, it lies on the line.

(c) Show that the line is not a tangent to the point B. Clearly justify your answer. [3]

$$\text{Gradient of } AB = \frac{2-(-2)}{8-3}$$
$$= \frac{4}{5}$$

$$\text{line } l : 4y = -3x + 25$$
$$y = -\frac{3}{4}x + \frac{25}{4}$$

$$-\frac{3}{4} \times \frac{4}{5} = -\frac{3}{5}$$
$$= -1$$

Since $M_{AB} \times M_l \neq -1$, l is not a tangent to the circle as it does not satisfy the property of tangent perpendicular to radius.

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(d) Hence, find the equation of the tangent to circle C at B. [2]

Gradient of $AB = \frac{4}{5}$

Gradient of tangent $= -\frac{5}{4}$

$y = -\frac{5}{4}x + c$

at $B(8, 2)$

$2 = -10 + c$

$c = 12$

Equation of tangent : $y = -\frac{5}{4}x + 12$

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(e) A circle D is a reflection of circle C along the line $x = 5$. Find the equation of the circle D. [1]

Equation of $D : (x-7)^2 + (y+2)^2 = 41$

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9 A particle moves along a straight line. Its velocity v m/s at a time t seconds is given by $v = t^3 - 9t^2 + 18t$. At $t = 0$, the particle is at the point O.

(a) Find the initial acceleration. [1]

$$a = 3t^2 - 18t + 18$$

when $t = 0$,

$$a = 18 \text{m/s}^2$$

(b) Find the time(s) when the particle is at instantaneous rest. [2]

$$t^3 - 9t^2 + 18t = 0$$
$$t(t^2 - 9t + 18) = 0$$
$$t(t-3)(t-6) = 0$$
$$t = 0 \text{ or } t = 3 \text{ or } t = 6$$

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(c) Find an expression for the displacement s metres from O at time t. [3]

$$s = \int v \, dt$$
$$= \int t^3 - 9t^2 + 18t \, dt$$
$$= \frac{t^4}{4} - 3t^3 + 9t^2 + c$$

when $t = 0, s = 0, c = 0$

$$\therefore s = \frac{t^4}{4} - 3t^3 + 9t^2$$

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(d) Find the total distance travelled by the particle during the first 4 seconds. [3]

when $t = 0, s = 0$

when $t = 3, s = 20.25 - 81 + 81$

$= 20.25$

when $t = 4, s = 64 - 192 + 144$

$= 16$

$s = 20.25 + (20.25 - 16)$

$= 24.5\text{m}$

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(e) Determine whether the particle is moving in the positive or negative direction when $t = 4$. [1]

when $t = 4$, $v = 4^3 - 9(4)^2 + 18(4)$

$= -8\text{m/s}$

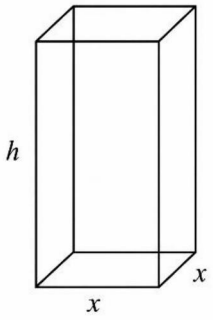
Since velocity is negative, the particle is moving in the negative direction.

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10 A company designs an open-top rectangular water tanks with a square base of side x metres and height h metres for the four walls. The tank must hold exactly 256m^3 of water. The cost of materials is:

- \$90 per m^2 for the base,
- \$40 per m^2 for the four walls.

After construction, the outside of the four walls is painted. The paint cost \$15 per m^2.



(a) Show that the material cost C dollars can be written as $C = 90x^2 + \frac{40960}{x}$. [2]

$$x^2h = 256$$

$$h = \frac{256}{x^2}$$

$$C = 90x^2 + 4(40)hx$$

$$= 90x^2 = 160\left(\frac{256}{x^2}\right)x$$

$$= 90x^2 + \frac{40960}{x} \text{ (shown)}$$

(b) Find the value of x for which the material cost is a minimum. [5]

$$\frac{dC}{dx} = 180x - \frac{40960}{x^2}$$

$$180x - \frac{40960}{x^2} = 0$$

$$180x^3 - 40960 = 0$$

$$x^3 = \frac{40960}{180}$$

$$x = \sqrt[3]{\frac{40960}{180}}$$

$$= 6.10514$$

$$= 6.11 \text{ (3s.f)}$$

$$\frac{d^2C}{dx^2} = 180 + \frac{81920}{x^3}$$

when $x = 6.10514$

$$\frac{d^2C}{dx^2} = 180 + \frac{81920}{(6.10514)^3}$$

$$= 540 (> 0)$$

since $\frac{d^2C}{dx^2} > 0$, the material cost is a minimum.

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(c) Hence, find the minimum cost, giving your answer to the nearest dollar. [1]

$$C = 90(6.10514)^2 + \frac{40960}{6.10514}$$
$$= 10063.64$$
$$= 10100$$

Hence, the minimum cost is \$10100

(d) Show that the cost of paint, P, dollars is given by $P = \frac{15360}{x}$. [2]

$$P = 4hx(15)$$
$$= 60\left(\frac{256}{x^2}\right)x$$
$$= \frac{15360}{x}$$

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(e) Given that the side length of the square base cannot exceed 8 m, determine the dimensions of the tank for which the cost of paint job is the least. [1]

$$x = 8, h = \frac{256}{8^2}$$
$$= 4$$

Dimension $= 8 \times 8 \times 4$ m

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11 **(a)** Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{4x}{\sqrt{4x-2}}\right)=\dfrac{8(x-1)}{\sqrt{(4x-2)^3}}$. [3]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{4x}{\sqrt{4x-2}}\right)=\frac{(4x-2)^{\frac{1}{2}}(4)-4x\left(\frac{1}{2}\right)(4x-2)^{-\frac{1}{2}}(4)}{4x-2}$$

$$=\frac{4(4x-2)^{\frac{1}{2}}-8x(4x-2)^{-\frac{1}{2}}}{4x-2}$$

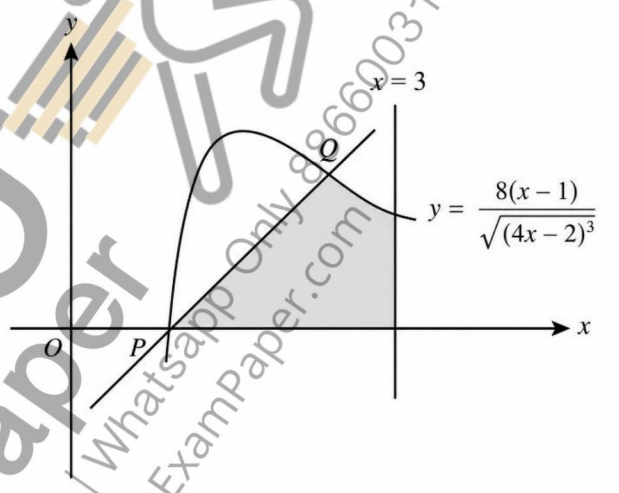
$$=\frac{4(4x-2)^{-\frac{1}{2}}[(4x-2)-2x]}{4x-2}$$

$$=\frac{4(2x-2)}{(4x-2)^{\frac{3}{2}}}$$

$$=\frac{8(x-1)}{(4x-2)^{\frac{3}{2}}}\quad\text{(shown)}$$

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The diagram shows part of the curve $y=\dfrac{8(x-1)}{\sqrt{(4x-2)^3}}$ and the line $x=3$. The curve intersects the x-axis at the point P. The line through P with gradient 1 intersects the curve again at the point Q.



(b) Show that the coordinates of Q is $\left(\dfrac{3}{2}, \dfrac{1}{2}\right)$. [5]

when $y=0,\ x=1$

$y=x+c$

at (1, 0)

$c=-1$

Eqn of PQ: $y=x-1$

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$$x-1=\frac{8(x-1)}{\sqrt{(4x-2)^3}}$$

$$(x-1)\sqrt{(4x-2)^3}=8(x-1)$$

$$(x-1)\sqrt{(4x-2)^3}-8(x-1)=0$$

$$(x-1)\left[\sqrt{(4x-2)^3}-8\right]=0$$

$x=1$ or $\sqrt{(4x-2)^3}-8=0$

(rej) $(4x-2)^3=64$

$$4x-2=4$$

$$x=\frac{3}{2}$$

$$y=\frac{1}{2}$$

$$Q=\left(\frac{3}{2},\frac{1}{2}\right)$$

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$$Q=\left(\frac{3}{2},\frac{1}{2}\right)$$

(c) Find the area of the shaded region bounded by the curve, the x-axis, the line PQ, and the line $x=3$. [3]

$$\text{Area}=\int_{\frac{3}{2}}^{3}\frac{8(x-1)}{\sqrt{(4x-2)^3}}\,dx+\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$=\left[\frac{4x}{\sqrt{4x-2}}\right]_{\frac{3}{2}}^{3}+\frac{1}{8}$$

$$=\frac{12}{\sqrt{10}}-\frac{6}{\sqrt{4}}+\frac{1}{8}$$

$$=0.91973$$

$$=0.920 \text{ units}^2 \text{ (3s.f)}$$

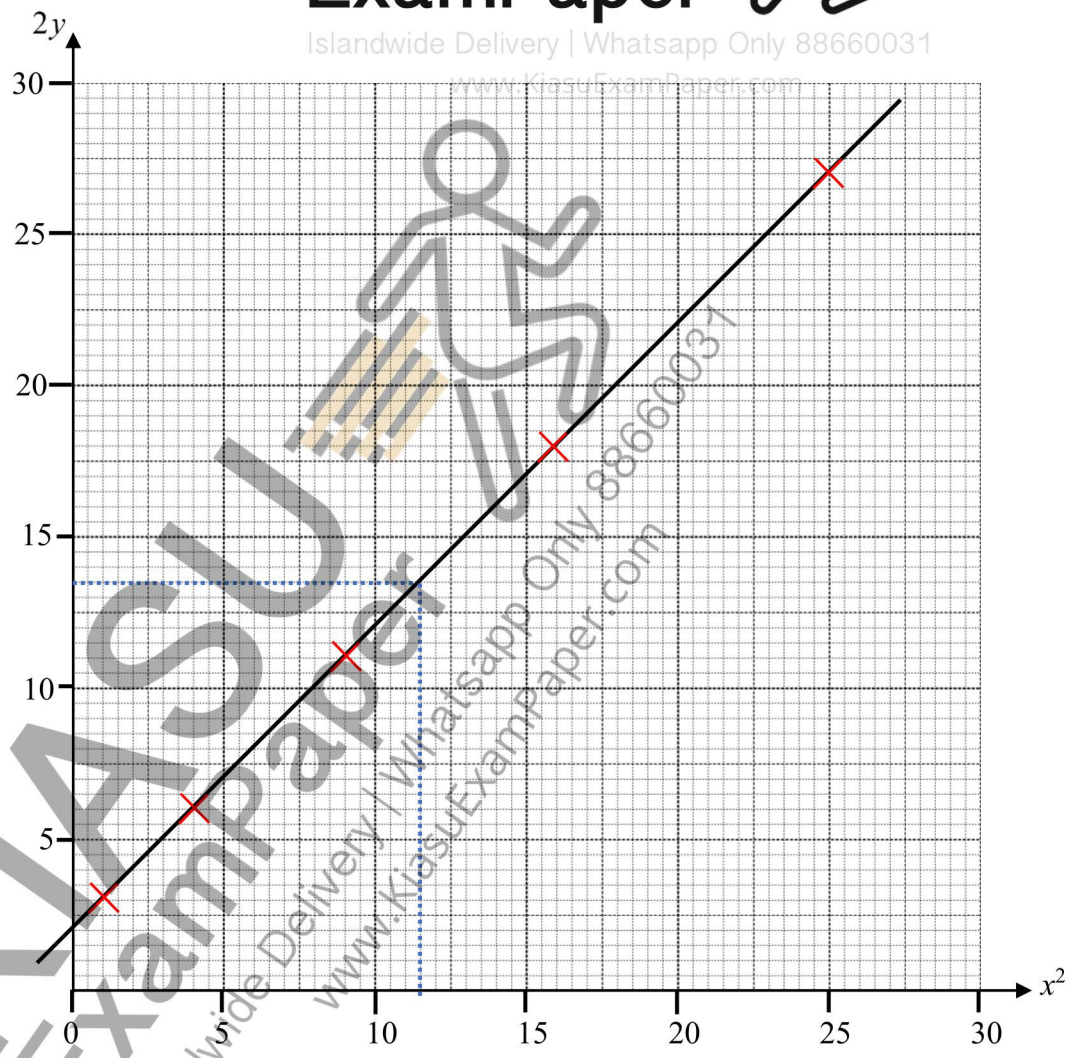
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12 A mathematics teacher designed an assessment consisting of x questions. The average time taken (y mins) by students is related to the number of questions and is related by $ae^{2y} = e^{bx^2+4}$, where a and b are constants. The table below shows the relationship between x and y.

x	1	2	3	4	5
y	1.5	3	5.5	9	13.5

(a) On the grid below, plot $2y$ against x^2 and draw a straight-line graph. [2]

x^2	1	4	9	16	25
y	3	9	11	18	27



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(b) Use your graph to estimate the value of a and of b. [3]

$$
\begin{aligned}
ae^{2y} &= e^{bx^2+4} \\
\ln ae^{2y} &= \ln e^{bx^2+4} \\
\ln a + \ln e^{2y} &= \ln e^{bx^2+4} \\
\ln a + 2y &= bx^2 + 4 \\
2y &= bx^2 + 4 - \ln a \\
4 - \ln a &= 2 \\
\ln a &= 2 \\
a &= e^2 \\
a &= 7.39 \\
b &= \frac{27-2}{25-0} \\
&= 1
\end{aligned}
$$

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(c) Use your graph to estimate the value of x when $e^y = 850$. [2]

$$
\begin{aligned}
e^y &= 850 \\
\ln e^y &= \ln 850 \\
y &= \ln 850 \\
2y &= 2\ln 850 \\
&= 13.490 \\
x^2 &= 11.5 \\
x &= \sqrt{11.5} \\
&= 3.39
\end{aligned}
$$

(d) A student claims that there is a value of x when $e^y = 0.5$. Explain why the student's claim is impossible. [1]

$$
\begin{aligned}
e^y &= 0.5 \\
\ln e^y &= \ln 0.5 \\
y &= \ln 0.5 \\
&= -0.693
\end{aligned}
$$

Since y refers to time, time cannot be negative.

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COMPASSVALE SECONDARY SCHOOL
Secondary Four Express
Secondary Five Normal (Academic)
Preliminary Examination 2026

NAME		MARKS	/90
CENTRE NUMBER	S	CLASS	INDEX NUMBER

ADDITIONAL MATHEMATICS 4049/01

Paper 1

18 August 2026

2 hour 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Class, index number, name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place on case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in the brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 90.

Set by: Mdm Cheng Edwina

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \pm \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer **all** the questions.

1 Solve $\sqrt{(85 + 42x)} + 7 = 2x$. [3]

2 Find the set of values of constant b such that the curve $y = x^2 + bx + 25$ intersects the line $y = -x - b$ at two distinct points. [4]

3 **(a)** Find, in exact value, the equation of the tangent of the curve $y = \ln\left(\frac{12x}{(2x-7)^2}\right)$ at the point $x = 4$. [6]

(b) The tangent intersects the line $y = -\frac{7}{4}x + 17$ at Q. Show that the x-coordinate of Q is $-1 + \ln 4\sqrt{3}$. [3]

(c) Two points are given by $P(-1, -10)$ and $R(5, 4)$. Calculate the area of triangle QPR to 1 decimal place. [3]

4 **(a)** Find the derivative of the curve $f(x) = 0.75x^4 - 10x^3 + 49.5x^2 - 108x + 11$ and factorise the derivative to the form $3(x + a)(x + b)^2$, where a and b are integers. [5]

(b) Hence find the x-coordinates of each of the stationary points of the curve $f(x) = 0.75x^4 - 10x^3 + 49.5x^2 - 108x + 11$. [2]

(c) Find the nature of each stationary point. [4]

5 **(a)** By considering the general term of $\left(6x - \frac{a}{x}\right)^5$, where a is a constant, explain why all the terms in the expansion are dependent on x. [3]

(b) Given that the coefficient of x is 18 times larger than the coefficient of x^{-3}, find the possible values of a. [5]

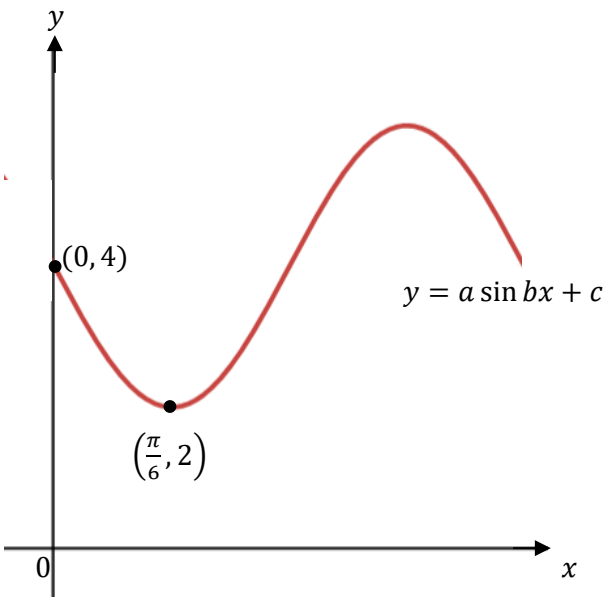
6 The number of bacteria, N, in a culture after t hours is modelled by the formula $N = 500(2^{1.0566t})$.

(a) Calculate the number of bacteria after 200 minutes. [1]

(b) An antibiotic is effective only if it is administered before the number of bacteria reaches 25 000. If the bacteria is introduced to the culture at 9.30 am, explain if the antibiotic is effective if it is administered at 2.45 pm. [4]

(c) Explain whether this formula is still valid in the long run. [1]

7



The diagram shows the curve $y = a\sin bx + c$ for $0 \leq x \leq \frac{2\pi}{3}$ radians. The curve has a minimum point at $\left(\frac{\pi}{6}, 2\right)$ and it meets the y-axis at $(0, 4)$.

(a) Explain why $b = 3$. [2]

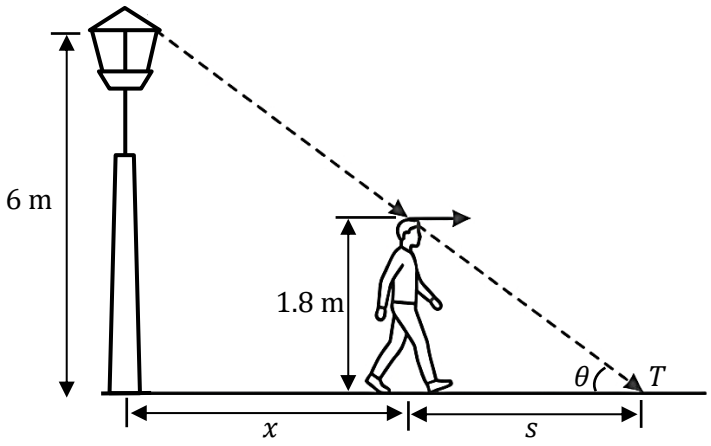
(b) Hence, find the equation of the curve. [2]

8 A circle, with centre C, has equation $x^2 + y^2 - 12x + 14y + 4 = 0$.

(a) Find the coordinates of C and the radius of the circle. [4]

(b) Find the shortest distance of C from the line $y = x + 1$ and hence determine if the circle will intersect the line $y = x + 1$. [7]

9



A vertical lamp post of height, 6 meters, cast a shadow of length, s meters, of a man of height 1.8 meters walking away from the lamp post at a uniform speed of 1.5 meters per second. x meters is the distance of the man from the lamp post.

(a) Show that $s = \frac{3}{7}x$. [2]

(b) Find the rate at which the length of his shadow is increasing. [2]

(c) Given that θ is the angle of elevation from tip of shadow, T to the top of the lamp post.

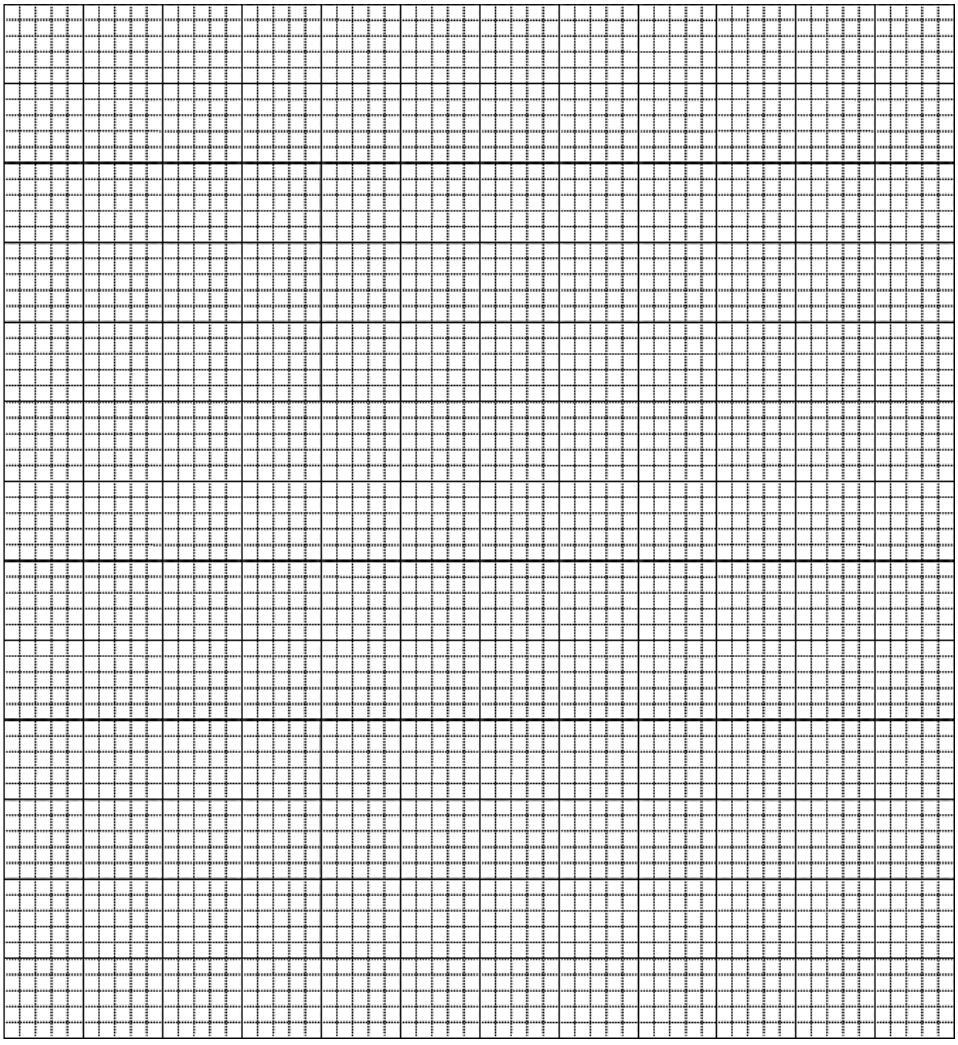
(i) Express $\tan\theta$ in terms of x. [2]

(ii) Find the rate of change of angle θ, in radians per second, at the instant when the man is 3 meters away from the lamp post. [5]

10 A student measures the length of a pendulum, l centimetre, and the time, T seconds, it takes to swing one oscillation. The relationship can be modelled by the formula $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. The table below shows corresponding values of l and T.

l	0.5	1.2	1.9	2.25	2.5
T	1.42	2.20	2.77	3.01	3.17

(a) Plot T^2 against l and draw a straight line graph to illustrate the information. [2]



(b) Use your graph to estimate the value of g. [3]

(c) Use your graph to estimate the time taken when the length of the pendulum is 2 cm. [2]

11 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{3x}\sqrt{x}\right) = \frac{e^{3x}(6x+1)}{2\sqrt{x}}$. [3]

(b) Hence find $\int \frac{24xe^{3x}}{\sqrt{x}} + \frac{4e^{3x}}{\sqrt{x}}\,\mathrm{d}x$. [2]

12 **(a)** Factorise $128x^3 - 2(x+2)^3$. [2]

(b) Given that $128x^3 - 2(x+2)^3$ is exactly divisible by $ax + 4b$, where a and b are rational numbers. Use **part (a)**, state the value of a and b. [3]

13 Given that θ is obtuse, $\sin\theta = p$, express, in terms of p, $\cos(\theta + 60°)$. [3]

[End of Paper]

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COMPASSVALE SECONDARY SCHOOL
Secondary Four Express
Preliminary Examination 2026

NAME		MARKS	/90
CENTRE NUMBER	S	CLASS	INDEX NUMBER

ADDITIONAL MATHEMATICS 4049/02

PAPER 2 **20 August 2026**

2 hour 15 minutes

Candidates answer on the Question Paper.

No Additional Materials

READ THESE INSTRUCTIONS FIRST

Write your Class, index number, name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place on case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in the brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 90.

Set by: Mr Ng HP

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **Do not use a calculator in this question.**

A cylinder whose base radius is $\left(\sqrt{5}-1\right)$ cm has a volume of $\left(6\sqrt{5}-2\right)\pi$ cm^3. Find the exact height of the cylinder, giving your answer in the form $a+b\sqrt{5}$, where a and b are integers. [6]

2 A shop sells x cups of bubble tea per day. Its daily profit, $P(x)$ is calculated by subtracting its cost $C(x)$ from its revenue $R(x)$. It is given that its cost, $C(x) = \$(140 + 0.5x)$ and its revenue, $R(x) = \$(4.5x - 0.01x^2)$.

(a) Express its profit, $P(x)$ in the form $A - B(x + c)^2$, where A, B and C are constants. [3]

(b) State the maximum daily profit and the corresponding number of cups of bubble tea that should be sold on that day. [2]

(c) Find the total lost incurred if the shop closes for 5 days due to manpower shortage. [2]

3 The gradient of the tangent at a point (x, y) on a curve is given by $\frac{3}{(2+3x)^2} + \frac{3}{\cos^2 x}$. The curve has y-intercept 1. Find y in terms of x. [5]

4 **Do not use a calculator in this question.**

A polynomial is given by $f(x) = 4x^3 - 2px^2 + p(p-4)x - 8$, where p is a constant.

(a) Find the remainder when $f(x)$ is divided by x. [2]

(b) Given that $x - p$ is a factor of $f(x)$, show that $3p^3 - 4p^2 - 8 = 0$. [2]

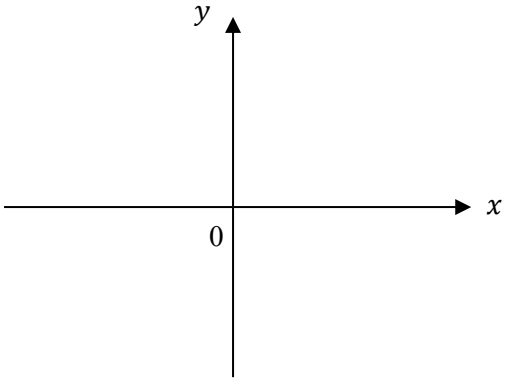
(c) Find the value(s) of p. [5]

5 **(a)** Given that $y = \frac{1+\ln x}{1-\ln x}$, where $x > 0$ and $x \neq a$.

(i) Find the value of a. [1]

(ii) Prove that y is an increasing function. [3]

(b) **(i)** Sketch the graph of $y = 1 + \ln x$, labelling the value of x where the graph intersects the x-axis. [2]



(ii) By sketching a line on your graph in part **(i)**, find the number of solutions to the equation

$$x + \ln x = 0.$$ [2]

6 **(a)** Prove that $\cot\theta = \cot 2\theta + \operatorname{cosec} 2\theta$. [3]

(b) Hence, find the values of θ such that $\cot 2\theta + \operatorname{cosec} 2\theta = 2\cos\theta$, for $0 \leq \theta \leq \pi$. [4]

7 A particle is travelling in a straight line. Its velocity, v m/s, at time t seconds is given by

$$v = e^{2t-1} - e^{-2t+1}.$$

(a) Determine whether the particle is stationary when $t = 0.5$. [1]

(b) Express the acceleration, a m/s^2, of the particle in terms of t and show that $a \neq 0$ for all values of t. [3]

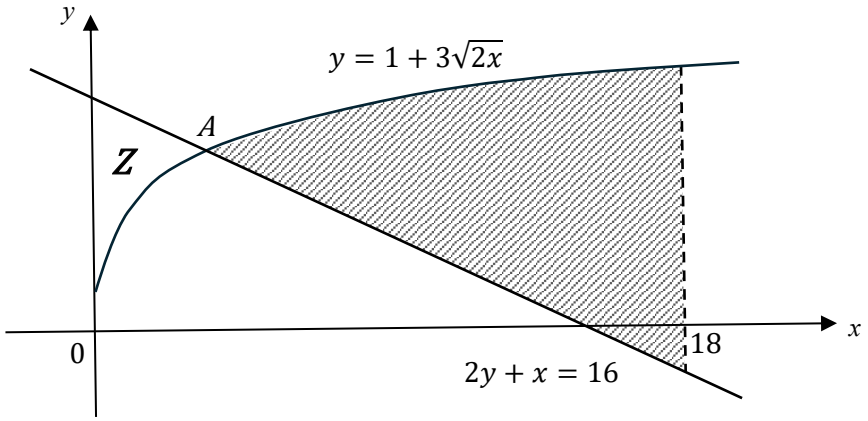
(c) Find the total distance travelled by the particle in the first 2 seconds. [4]

8 (a) (i) Express $\frac{9^{x+2}-3^{2x+1}}{27^x(9^{1-x})}$ in the form ab^{x-1}, where a and b are integers. [4]

(ii) If $\frac{9^{x+2}-3^{2x+1}}{27^x(9^{1-x})} + 3^{x-1} = 3^y$, find y in terms of x. [2]

(b) Solve the equation $10^{5k-8} = 4(5^k)$. [4]

9



The diagram shows the curve $y = 1 + 3\sqrt{2x}$ and the line $2y + x = 16$.
The curve and the line intersect at the point A.

(a) Show that the coordinates of $A = (2, 7)$. [3]

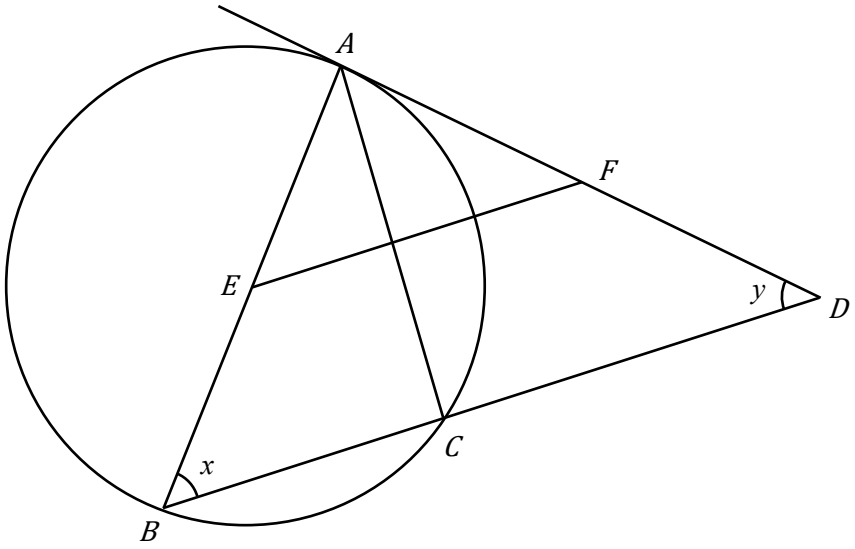
(b) Find the shaded area bounded by the curve $y = 1 + 3\sqrt{2x}$, the line $2y + x = 16$ and the line $x = 18$. [4]

(c) Area $\boldsymbol{Z}$ is bounded by the curve $y = 1 + 3\sqrt{2x}$, the line $2y + x = 16$ and the y-axis. To find the area $\boldsymbol{Z}$, a student writes down the formula

$$\text{Area } \boldsymbol{Z} = \int_a^7 g(y)\, dy + B.$$

Find an expression for $g(y)$ and the values of a and B. [3]

10



The diagram shows a triangle ABC whose vertices lie on the circumference of a circle. The tangent to the circle at point A meets the line BC extended at point D. Points E and F are midpoints of AB and AD respectively. Angle $ABC = x$ and angle $ADC = y$.

(a) Prove that triangle EAF is similar to triangle ACD. [4]

(b) Prove that $CD \times EF = 2(AF)^2$. [3]

(c) If E is the centre of the circle, prove that angle $BAC = y$. [2]

11 A water dispenser contains 10 litres of water initially. Water is poured into the dispenser at a constant rate of $4\cos\theta$ litres per minute, for some positive real value of θ. At the same time, water is dispensed out from the dispenser at a constant rate of $6\sin\theta$ litres per minute.

(a) Explain why the value of θ cannot fall in the range between $\frac{\pi}{2}$ to 2π. [1]

(b) Find the value of θ, where $6 < \theta < 8$, for which the volume of water in the dispenser remains constant. [3]

(c) **(i)** If, instead, the water in the dispenser is emptied after 4 minutes, show that

$$3\sin\theta - 2\cos\theta = 1.25.$$ [2]

(ii) By writing $3\sin\theta - 2\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and α is acute, find the volume of water poured into the dispenser during the 4 minutes. [5]

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COMPASSVALE SECONDARY SCHOOL
Secondary Four Express
Secondary Five Normal (Academic)
Preliminary Examination 2026

NAME	**MARK SCHEME**	MARKS	/90
CENTRE NUMBER	S	CLASS	INDEX NUMBER

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ADDITIONAL MATHEMATICS **4049/01**

PAPER 1 **18 August 2026**

2 hour 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

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You may use an HB pencil for any diagrams or graphs.
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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place on case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in the brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 90.

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Set by: Mdm Cheng Edwina

1 Solve $\sqrt{(85+42x)}+7=2x$. [3]

$\sqrt{(85+42x)}+7=2x$
$\sqrt{(85+42x)}=2x-7$
$85+42x=(2x-7)^2$ [M1]
$4x^2-70x-36=0$
$(4x+2)(x-18)=0$
$x=-0.5$ (reject) *or* $x=18$ [A1 each, must reject $x=-0.5$]

2 Find the set of values of constant b such that the curve $y=x^2+bx+25$ intersects the line $y=-x-b$ at two distinct points. [4]

$y=x^2+bx+25$ ---(1)
$y=-x-b$ ---(2)

Sub (1) into (2),
$x^2+bx+25=-x-b$
$x^2+bx+x+25+b=0$ [M1]

Since line and curve intersects at two distinct points,
Discriminant >0
$(b+1)^2-4(1)(25+b)>0$ [M1]
$b^2-2b-99>0$
$(b+9)(b-11)>0$ [M1]
$b<-9$ or $b>11$ [A1]

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3 (a) Find, in exact value, the equation of the tangent of the curve $y = \ln\left(\frac{12x}{(2x-7)^2}\right)$ at the point $x = 4$. [6]

$y = \ln\left(\frac{12x}{(2x-7)^2}\right)$

$y = \ln(12x) - \ln(2x-7)^2$

$y = \ln(12x) - 2\ln(2x-7)$ [M1: split ln into 2 simpler terms]

$\frac{dy}{dx} = \frac{12}{12x} - \frac{2(2)}{2x-7}$

$\frac{dy}{dx} = \frac{1}{x} - \frac{4}{2x-7}$ [A1 for $\frac{1}{x}$; A1 for $-\frac{4}{2x-7}$]

Alternative method for differentiating:

$y = \ln\left(\frac{12x}{(2x-7)^2}\right)$

$\frac{dy}{dx} = \frac{1}{\frac{12x}{(2x-7)^2}}\left(\frac{(2x-7)^2(12)-(12x)(2)(2x-7)(2)}{((2x-7)^2)^2}\right)$ [M1 differentiate ln correctly, M1 differentiate $\left(\frac{12x}{(2x-7)^2}\right)$ correctly either using product or quotient law. −M1 for every error]

$\frac{dy}{dx} = \frac{(2x-7)^2}{12x}\left(\frac{(2x-7)^2(12)-(12x)(2)(2x-7)(2)}{((2x-7)^2)^2}\right)$

$\frac{dy}{dx} = \frac{1}{x}\left(\frac{2x-7-(4x)}{2x-7}\right)$

$\frac{dy}{dx} = \frac{-2x-7}{x(2x-7)}$ [M1 o.e. simplify into a fraction]

When $x = 4$,

Gradient of tangent $= \frac{1}{4} - \frac{4}{2(4)-7} = -\frac{15}{4}$ [M1 f.t. sub $x = 4$ into their derivative]

And $y = \ln\left(\frac{12(4)}{(2(4)-7)^2}\right) = \ln 48$ [A1 – strictly exact]

Equation of tangent is $y = -\frac{15}{4}x + c$

When $x = 4$ and $y = \ln 48$,

$\ln 48 = -\frac{15}{4}(4) + c$

$c = 15 + \ln 48$

Equation of tangent is $y = -\frac{15}{4}x + 15 + \ln 48$ [A1]

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(b) The tangent intersects the line $y = -\frac{7}{4}x + 17$ at Q. Show that the x-coordinate of Q is $-1 + \ln 4\sqrt{3}$. [3]

$y = -\frac{7}{4}x + 17$ ---(1)
$y = -\frac{15}{4}x + 15 + \ln 48$ ---(2)
Sub (1) into (2),
$-\frac{7}{4}x + 17 = -\frac{15}{4}x + 15 + \ln 48$ [M1 f.t.: sub line into their tangent]
$2x = -2 + \ln 48$
$x = -1 + \frac{1}{2}\ln 48$ [A1]
$x = -1 + \ln \sqrt{16 \times 3}$
$x = -1 + \ln 4\sqrt{3}$ (shown) [A1]

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(c) Two points are given by $P(-1, -10)$ and $R(5, 4)$. Calculate the area of triangle QPR to 1 decimal place. [3]

When $x = -1 + \ln 4\sqrt{3}$, $y = 18\frac{3}{4} - \frac{7}{4}\ln 4\sqrt{3}$
Q (0.9356, 15.3626) [M1]

Area of triangle $QPR = \frac{1}{2}\begin{vmatrix} 5 & 0.9356 & -1 & 5 \\ 4 & 15.3626 & -10 & 4 \end{vmatrix}$ [M1 f.t. with their Q]

$= \frac{1}{2}[63.457 - (-61.6202)]$
$= 62.5\ units^2$ [A1]

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4 **(a)** Find the derivative of the curve $f(x) = 0.75x^4 - 10x^3 + 49.5x^2 - 108x + 11$ and factorise the derivative to the form $3(x + a)(x + b)^2$, where a and b are integers. [5]

$f(x) = 0.75x^4 - 10x^3 + 49.5x^2 - 108x + 11$
$f'(x) = 3x^3 - 30x^2 + 99x - 108$ [A1]

$f'(3) = 3(3)^3 - 30(3)^2 + 99(3) - 108 = 0$
By factor theorem, $(x - 3)$ is a factor of $f'(x)$. [B1]

By long division or comparing coefficients,

$3x^3 - 30x^2 + 99x - 108 = (x - 3)(3x^2 + bx + 36)$ [M1 for long division or comparing coefficients]

Comparing coefficients of x^2,
$-30 = b - 9$
$b = -21$

$f'(x) = 3x^3 - 30x^2 + 99x - 108$
$= (x - 3)(3x^2 - 21x + 36)$ [M1 for $(3x^2 - 21x + 36)$ *o.e.*]
$= (x - 3)(x - 3)(3x - 12)$
$= 3(x - 4)(x - 3)^2$ [A1]

OR $3x^3 - 30x^2 + 99x - 108 = 3(x + a)(x + b)^2$ [M1 f.t. with their derivative]
$3x^3 - 30x^2 + 99x - 108 = 3(x + a)(x^2 + 2bx + b^2)$

Compare coefficient of x, [M1 attempt to compare coefficients/constant]
$99 = 3b^2 + 6ab$
$33 = b^2 + 2ab \quad ---(1)$

Compare coefficients of x^2,
$-30 = 6b + 3a$
$a = -10 - 2b \quad ---(2)$

Sub (2) into (1),
$33 = b^2 + 2b(-10 - 2b)$
$33 = b^2 - 20b - 4b^2$
$3b^2 + 20b + 33 = 0$
$(b + 3)(3b + 11) = 0$
$b = -3 \text{ or } b = -\frac{11}{3} (reject)$ [A1. Must reject $-\frac{11}{3}$]

When $b = -3$,
$a = -10 - 2(-3) = -4$ [A1]

Hence $3x^3 - 30x^2 + 99x - 108 = 3(x - 4)(x - 3)^2$ [A1]

(b) Hence find the x-coordinates of each of the stationary points of the curve $f(x) = 0.75x^4 - 10x^3 + 49.5x^2 - 108x + 11$. [2]

Let $f'(x) = 0$,
$3(x - 4)(x - 3)^2 = 0$ [M1 f.t.: using their 'part (a) answer' = 0]
$x = 4$ or $x = 3$ [A1 for both]

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(c) Find the nature of each stationary point. [4]

Either by 1st derivative test or 2nd derivative test.

[For $x = 3$, Students may use 2nd derivative test. But it would fail. Hence, students must show 1st derivative test.]
When $x = 3$,
$f''(x) = 9(3)^2 - 60(3) + 99 = 0$
Second derivative test fail.

By first derivative test, [M1 for attempt to use 1st derivative test on their $x = 3$]

x	2.99	3	3.001
$f'(x)$	-0.000303 (<0)	0	-0.000002997 (<0)
Sketch of derivative	\	———	\

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$x = 3$ is a point of inflexion. [A1: no follow through]

For $x = 4$,
Using 1st derivative test: [M1 for attempt to use 1st derivative test on their $x = 4$]

x	3.99	4	4.001
$f'(x)$	-0.029403 (<0)	0	0.003006 (>0)
Sketch of derivative	\	———	/

$x = 4$ is a minimum point. [A1: no follow through]

OR By using 2nd derivative test,

$f'(x) = 3x^3 - 30x^2 + 99x - 108$
$f''(x) = 9x^2 - 60x + 99$ [M1 for correct second derivative using *'their first derivative in part (a)'*]

When $x = 4$,
$f''(x) = 9(4)^2 - 60(4) + 99 = 3$

Since $f''(x) > 0$, $x = 4$ is a minimum point. [A1: no follow through]

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5 **(a)** By considering the general term of $\left(6x - \frac{a}{x}\right)^5$, where a is a constant, explain why all the terms in the expansion are dependent on x. [3]

For $\left(6x - \frac{a}{x}\right)^5$,
$T_{r+1} = \binom{5}{r}(6x)^{5-r}\left(-\frac{a}{x}\right)^r$
$= \binom{5}{r}(6)^{5-r}(-a)^r(x)^{5-r}(x)^{-r}$
$= \binom{5}{r}(6)^{5-r}(-a)^r(x)^{5-2r}$ [M1]

Let $5 - 2r = 0$
$r = 2.5$ [M1]

Given that $r = 2.5$ is not a positive integer [A1], hence, there is no term independent of x. All the terms in the expansion are dependent on x.

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(b) Given that the coefficient of x is 18 times larger than the coefficient of x^{-3}, find the possible values of a. [5]

Let $5 - 2r = 1$
$r = 2$

Coefficient of $x = \binom{5}{2}(6)^3(-a)^2$
$= 2160a^2$ [M1]

Let $5 - 2r = -3$
$r = 4$

Coefficient of $x = \binom{5}{4}(6)(-a)^4$
$= 30a^4$ [M1]

Since coefficient of $x = 18 \times$ coefficient of x^{-3},
$2160a^2 = 18(30a^4)$ [M1 f.t. using their $2160a^2$ and $30a^4$]
$540a^4 - 2160a^2 = 0$
$a^2(540a^2 - 2160) = 0$
$a = 0$ (reject) or $a = \pm 2$ [A2 for all 3 values of a or A1 for 2 correct answers including rejecting $a = 0$]

6 The number of bacteria, N, in a culture after t hours is modelled by the formula $N = 500(2^{1.0566t})$.

(a) Calculate the number of bacteria after 200 minutes. [1]

When $k = 1.0566, t = 200$ mins $= 3\frac{1}{3}$ hrs,
$N = 500\left(2^{1.0566 \times 3\frac{1}{3}}\right)$
$N = 5743$ *or* 5744 [B1]

(b) An antibiotic is effective only if it is administered before the number of bacteria reaches 25 000. If the bacteria is introduced to the culture at 9.30 am, explain if the antibiotic is effective if it is administered at 2.45 pm. [4]

When $N = 25000$,
$25000 = 500(2^{1.0566t})$
$2^{1.0566t} = 50$
$\lg(2^{1.0566t}) = \lg 50$ [M1 f.t. – lg both sides]
$1.0566t = \frac{\lg 50}{\lg 2}$
$t = \frac{\frac{\lg 50}{\lg 2}}{1.0566}$
$t = 5.341$ hours = 5 hours 20 min [A1]

To be considered effective, the antibiotic must be administered before 2.50 pm. Since **2.45 pm is before 2.50 pm** [A1 f.t. for comparing 2.45 pm with 2.50 pm], the antibiotic is **effective**. [A1 f.t.]

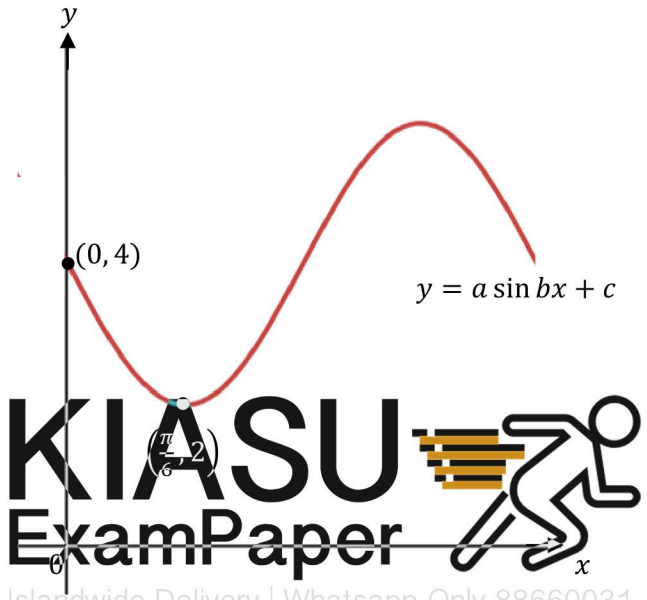
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(c) Explain whether this formula is still valid in the long run. [1]

This formula is **not valid** in the long run as it does not account for **decreasing of bacteria due to environmental factors** [B1: must include 'not valid' and reason]

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7



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The diagram shows the curve $y = a\sin bx + c$ for $0 \le x \le \frac{2\pi}{3}$ radians. The curve has a minimum point at $\left(\frac{\pi}{6}, 2\right)$ and it meets the y-axis at $(0, 4)$.

(a) Explain why $b = 3$. [2]

Method 1	Method 2
As given by question, period of 1 full cycle $= \frac{2\pi}{3}$ [A1: state period] $\frac{2\pi}{b} = \frac{2\pi}{3}$ $b = 3$. [A1]	$\frac{1}{4}$ *cycle* $= \frac{\pi}{6}$ 1 full cycle $= \frac{4\pi}{6} = \frac{2\pi}{3}$ [A1: state period] $\frac{2\pi}{b} = \frac{2\pi}{3}$ $b = 3$. [M1]

(b) Hence, find the equation of the curve. [2]

$c = 4$
amplitude $= 4 - 2 = 2$
$a = -2$

Equation of curve is $y = -2\sin 3x + 4$ [B1 for $-2\sin 3x$; B1 for 4]

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8 A circle, with centre C, has equation $x^2 + y^2 - 12x + 14y + 4 = 0$.

(a) Find the coordinates of C and the radius of the circle. [4]

$x^2 + y^2 - 12x + 14y + 4 = 0$

$(x-6)^2 + (y+7)^2 - (-6)^2 - 7^2 + 4 = 0$ [M1 for correct completing the square for x ; M1 for correct completing the square for y]

$(x-6)^2 + (y+7)^2 = 81$

$C(6, -7)$ and radius $= 9$ [A1 each]

Or by general method

$2g = -12 \qquad 2f = 14$

$g = -6 \qquad f = 7$ [M1: for finding g and f values]

$C = (-g, -f) = (6, -7)$ [A1]

Radius

$= \sqrt{f^2 + g^2 - c}$

$= \sqrt{7^2 + (-6)^2 - 4}$ [M1]

$= 9$ [A1]

(b) Find the shortest distance of C from the line $y = x + 1$ and hence determine if the circle will intersect the line $y = x + 1$. [7]

Perpendicular distance, l, from C to the line is the shortest distance of C from the line.

Gradient of line $l = -1$ [M1]

Equation of line l is $y = -x + c$

When $x = 6, y = -7$,

$-7 = -6 + c$

$c = -1$

Equation of line l is $y = -x - 1$ ---(1) [M1]

Sub (1) into $y = x + 1$,

$x + 1 = -x - 1$ [M1 f.t.]

$-2x = 2$

$x = -1$

When $x = -1, y = 0$. Hence the two lines intersect at point $(-1, 0)$

Shortest distance $= \sqrt{(6 - (-1))^2 + (-7 - 0)^2}$ [M1 f.t.]

$= \sqrt{98}$

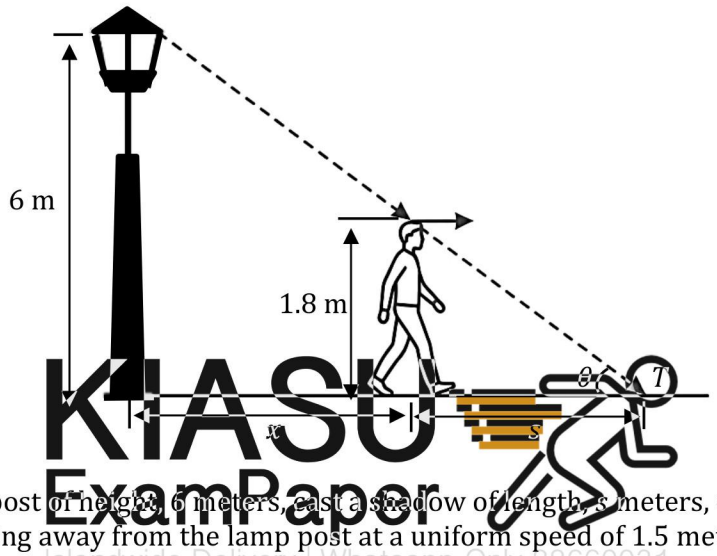
$= 7\sqrt{2}$ or 9.899 [A1]

Since **shortest distance from C to line is > radius of circle** [A1]. The circle **will not intersect** [A1] the line

[Turn over

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9



A vertical lamp post of height 6 meters, cast a shadow of length, s meters, of a man of height 1.8 meters walking away from the lamp post at a uniform speed of 1.5 meters per second. x meters is the distance of the man from the lamp post.

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(a) Show that $s = \frac{3}{7}x$. [2]

By similar triangle,

$\frac{1.8}{6} = \frac{s}{x+s}$ [M1]

$1.8x + 1.8s = 6s$

$1.8x = 4.2s$

$s = \frac{1.8x}{4.2} = \frac{3}{7}x$ [A1]

(b) Find the rate at which the length of his shadow is increasing. [2]

Given $\frac{dx}{dt} = 1.5$ *and* $\frac{ds}{dt} = ?$

$s = \frac{3}{7}x$

$\frac{ds}{dx} = \frac{3}{7}$ [M1]

$\frac{ds}{dt} = \frac{ds}{dx} \times \frac{dx}{dt}$

$\frac{ds}{dt} = \frac{3}{7} \times 1.5$

$\frac{ds}{dt} = \frac{9}{14}$ or 0.643 m/s [A1]

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(c) Given that θ is the angle of elevation from tip of shadow, T to the top of the lamp post.

(i) Express $\tan\theta$ in terms of x. [2]

$\tan\theta = \frac{6}{x+s}$ or $\tan\theta = \frac{1.8}{s}$ [M1]

$\tan\theta = \frac{6}{x+\frac{3}{7}x}$ $\tan\theta = \frac{1.8}{\frac{3}{7}x}$

$\tan\theta = \frac{21}{5x}$ *or* $\frac{4.2}{x}$ [A1]

(ii) Find the rate of change of angle θ, in radians per second, at the instant when the man is 3 meters away from the lamp post. [5]

When $x = 3$, $\frac{d\theta}{dt} = ?$

Using $\tan\theta = \frac{4.2}{x}$

$x = \frac{4.2}{\tan\theta} = 4.2(\tan\theta)^{-1}$

$\frac{dx}{d\theta} = 4.2(-1)(\tan\theta)^{-2}(\sec^2\theta)$ [M1 f.t. using their $\tan\theta$]

$\frac{dx}{d\theta} = \frac{-4.2\sec^2\theta}{(\tan\theta)^2}$

$\frac{dx}{d\theta} = \frac{-4.2(1+\tan^2\theta)}{(\tan\theta)^2}$ *or* $\frac{-4.2}{(\cos\theta)^2(\tan\theta)^2}$ *or* $\frac{-4.2}{(\sin\theta)^2}$ [A1]

When $\frac{dx}{dt} = 1.5, x = 3,$

$\tan\theta = \frac{4.2}{3}$

$\tan\theta = 1.4$ [M1 f.t. using their $\tan\theta$]

$\frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$

$1.5 = \frac{-4.2(1+\tan^2\theta)}{(\tan\theta)^2} \times \frac{d\theta}{dt}$ [M1 f.t. using their $\frac{dx}{d\theta}$]

$\frac{d\theta}{dt} = \frac{1.5}{\frac{-4.2(1+1.4^2)}{(1.4)^2}} = -0.236$ radian/seconds [A1]

Alternative solution: Students are also allowed to find $\theta = 0.8097$ rad and sub this value into their connected rate of change to find $\frac{d\theta}{dt}$.

When $\frac{dx}{dt} = 1.5, x = 3,$

$\tan\theta = \frac{4.2}{3}$

$\tan\theta = 1.4$

$\theta = 0.9505$ [M1]

$\frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$

$1.5 = \frac{-4.2(1+\tan^2\theta)}{(\tan\theta)^2} \times \frac{d\theta}{dt}$ [M1 f.t. using their $\frac{dx}{d\theta}$]

$\frac{d\theta}{dt} = \frac{1.5}{\frac{-4.2(1+\tan^2 0.9505)}{(\tan 0.9505)^2}} = -0.236$ rad/s [A1]

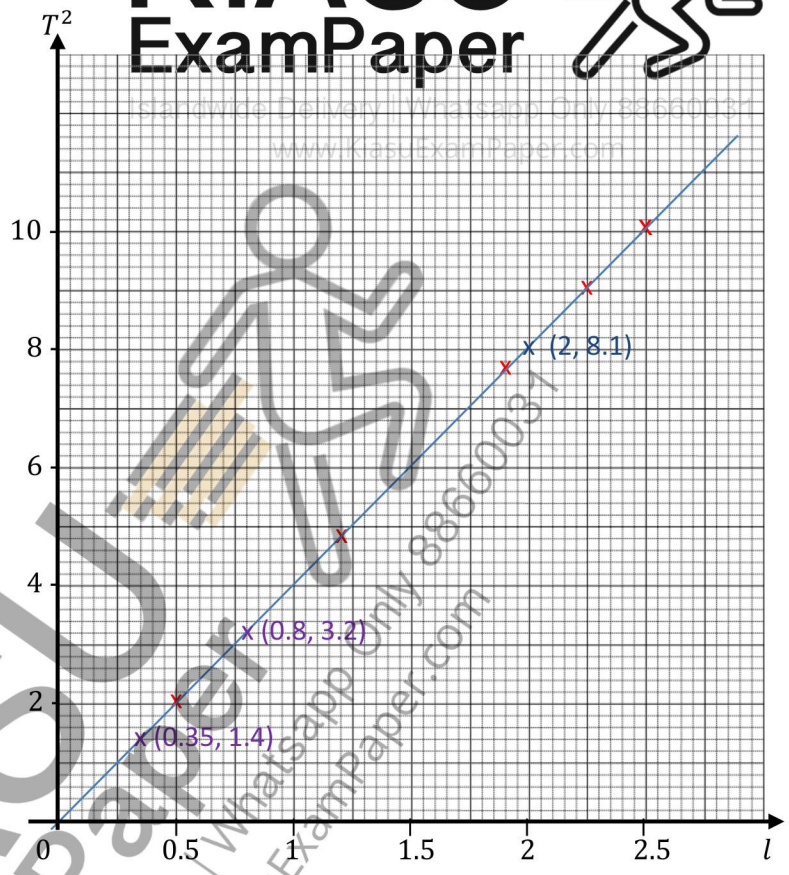
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10 A student measures the length of a pendulum, l centimetre, and the time, T seconds, it takes to swing one oscillation. The relationship can be modelled by the formula $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. The table below shows corresponding values of l and T.

l	0.5	1.2	1.9	2.25	2.5
T	1.42	2.20	2.77	3.01	3.17

(a) Plot T^2 against l and draw a straight line graph to illustrate the information. [2]

l	0.5	1.2	1.9	2.25	2.5
T^2	2.0164	4.84	7.6729	9.0601	10.0489



[M1: uses square correctly, correct table of values to at least 1 dp+ plot T^2 against l correctly]
[A1: everything else is correct e.g.

- cut vertical axis,
- correct scale of 2 cm for every 0.5 units on the l-axis and 2 cm for every2 units on the T^2-axis;
- All labels correctly marked out and labelled]

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(b) Use your graph to estimate the value of g. [3]

$T = 2\pi\sqrt{\frac{l}{g}}$

$T^2 = \frac{4\pi^2}{g}(l)$

$Y = T^2; X = l; m = \frac{4\pi^2}{g}$ [B1 for $m = \frac{4\pi^2}{g}$]

$\frac{4\pi^2}{g} = \frac{3.2-1.4}{0.8-0.35}$ [M1: method for gradient from graph, which must include good choice of points. If point is not on the line, no M1]

$g = \frac{4\pi^2}{4}$

$g = 9.87 (to\ 3sf)$ [A1: allow 9.77 to 9.97]

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(c) Use your graph to estimate the time taken when the length of the pendulum is 2 cm. [2]

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When $l = 2, T^2 = 8.1$ [M1 f.t.: Read off at $l = 2$ correctly]

$T = 2.85\ s\ (to\ 3sf)$ [A1: allow 2.83, 2.85, 2.86]

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11 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{3x}\sqrt{x}\right) = \frac{e^{3x}(6x+1)}{2\sqrt{x}}$ [3]

$\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{3x}\sqrt{x}\right)$

$= 3e^{3x}\sqrt{x} + e^{3x}\left(\frac{1}{2\sqrt{x}}\right)$ [M1 for $3e^{3x}$; M1 for $\left(\frac{1}{2\sqrt{x}}\right)$]

$= \frac{3e^{3x}\sqrt{x}(2\sqrt{x}) + e^{3x}}{2\sqrt{x}}$

$= \frac{6xe^{3x} + e^{3x}}{2\sqrt{x}}$

$= \frac{e^{3x}(6x+1)}{2\sqrt{x}}$ [A1]

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(b) Hence find $\int \frac{24xe^{3x}}{\sqrt{x}} + \frac{4e^{3x}}{\sqrt{x}}\,\mathrm{d}x$ [2]

$\int \frac{24xe^{3x}}{\sqrt{x}} + \frac{4e^{3x}}{\sqrt{x}}\,\mathrm{d}x = \int \frac{4e^{3x}(6x+1)}{\sqrt{x}}\,\mathrm{d}x = 4\int \frac{e^{3x}(6x+1)}{\sqrt{x}}\,\mathrm{d}x \quad a^3 - b^3 = (a - b)\left(a^2 + ab + b^{(2)}\right)$

From part (a), $\int \frac{e^{3x}(6x+1)}{2\sqrt{x}}\,\mathrm{d}x = e^{3x}\sqrt{x} + c$

Hence $\int \frac{e^{3x}(6x+1)}{\sqrt{x}}\,\mathrm{d}x = 2e^{3x}\sqrt{x} + c$ [M1]

Therefore, $\int \frac{24xe^{3x}}{\sqrt{x}} + \frac{4e^{3x}}{\sqrt{x}}\,\mathrm{d}x$

$= 4\int \frac{e^{3x}(6x+1)}{\sqrt{x}}\,\mathrm{d}x$

$= 4\left(2e^{3x}\sqrt{x}\right) + c$

$= 8e^{3x}\sqrt{x} + c$ [A1]

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12 **(a)** Factorise $128x^3 - 2(x+2)^3$. [2]

$128x^3 - 2(x+2)^3$
$= 2(64x^3 - (x+2)^3)$
$= 2((4x)^3 - (x+2)^3)$
$= 2(4x - (x+2))(16x^2 + 4x(x+2) + (x+2)^2)$ [M1]
$= 2(3x-2)(21x^2 + 12x + 4)$ [A1]

(b) Given that $128x^3 - 2(x+2)^3$ is exactly divisible by $ax + 4b$, where a and b are rational numbers, use **part (a)**, state the value of a and b. [3]

$128x^3 - 2(x+2)^3 = 2(3x-2)(21x^2 + 12x + 4)$

$ax + 4b = 2(3x-2)$ [M1 f.t. – Compare with their linear factor in part (a)]

$ax + 4b = 6x - 4$
Comparing,
$a = 6, \ b = -1$ [B1 each]

Alternative:

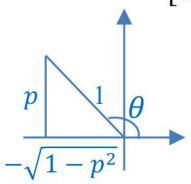
$$ax + 4b = 3x - 2$$

Comparing

$$a = 3, \ b = -\frac{1}{2}$$

13 Given that θ is obtuse, $\sin\theta = p$, express, in terms of p, $\cos(\theta + 60°)$. [3]

θ is obtuse, therefore θ is in the 2nd quadrant.



$\cos(\theta + 60°)$
$= \cos\theta\cos 60° - \sin\theta\sin 60°$
$= \left(-\sqrt{1-p^2}\right)\left(\frac{1}{2}\right) - p\left(\frac{\sqrt{3}}{2}\right)$ [M1 for both $\left(\frac{1}{2}\right)$ and $\left(\frac{\sqrt{3}}{2}\right)$]
[M1 for $-\sqrt{1-p^2}$]
$= -\frac{\sqrt{3}p + \sqrt{1-p^2}}{2}$ [A1 for giving as single fraction. Accept $\frac{-\sqrt{3}p - \sqrt{1-p^2}}{2}$ or equivalent]

[End of Paper]

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COMPASSVALE SECONDARY SCHOOL
Secondary Four Express
Preliminary Examination 2026

NAME	Mark Scheme	MARKS	/90
CENTRE NUMBER	S	CLASS	INDEX NUMBER

ADDITIONAL MATHEMATICS **4049/02**

PAPER 2 **20 August 2026**

2 hour 15 minutes

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Candidates answer on the Question Paper.

No Additional Materials

READ THESE INSTRUCTIONS FIRST

Write your Class, index number, name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place on case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in the brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 90.

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This document consists of **20** printed pages including **2** blank pages.

Set by: Mr Ng HP

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

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1 **Do not use a calculator in this question.**

A cylinder whose base radius is $(\sqrt{5}-1)$ cm has a volume of $(6\sqrt{5}-2)\pi$ cm^3. Find the exact height of the cylinder, giving your answer in the form $a+b\sqrt{5}$, where a and b are integers. [6]

$$V = \pi r^2 h$$

$$(6\sqrt{5}-2)\pi = \pi(\sqrt{5}-1)^2 h \quad \text{[M1]}$$

$$h = \frac{6\sqrt{5}-2}{(\sqrt{5}-1)^2}$$

$$= \frac{6\sqrt{5}-2}{6-2\sqrt{5}} \quad \text{[M1] – correct expansion of } (\sqrt{5}-1)^2$$

$$= \frac{6\sqrt{5}-2}{6-2\sqrt{5}} \times \frac{6+2\sqrt{5}}{6+2\sqrt{5}} \quad \text{[M1] – correct conjugate when rationalising}$$

$$= \frac{48+32\sqrt{5}}{16} \quad \text{[M1,M1] – numerator, denominator}$$

$$= 3+2\sqrt{5} \quad \text{[A1]}$$

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2 A shop sells x cups of bubble tea per day. Its daily profit, $P(x)$ is calculated by subtracting its cost $C(x)$ from its revenue $R(x)$. It is given that its cost, $C(x) = \$(140 + 0.5x)$ and its revenue, $R(x) = \$(4.5x - 0.01x^2)$.

(a) Express its profit, $P(x)$ in the form $A - B(x + c)^2$, where A, B and C are constants. [3]

$P(x) = (4.5x - 0.01x^2) - (140 + 0.5x)$ [B1]
$= -0.01x^2 + 4x - 140$
$= -0.01(x^2 - 400x) - 140$ [M1] – attempt to complete the sq.
$= -0.01[(x - 200)^2 - 200^2] - 140$
$= -0.01(x - 200)^2 + 260$ [A1]

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(b) State the maximum daily profit and the corresponding number of cups of bubble tea that should be sold on that day. [2]

Maximum profit = \$260 [B1]
Corr. number of cups of bubble tea = 200 [B1]

(c) Find the total lost incurred if the shop closes for 5 days due to manpower shortage. [2]

Sub $x = 0$, $P(0) = -140$ [M1√] – sub $x = 0$ into their $P(x)$
Lost for 5 days = \$140 x 5 = \$700 [A1]

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3 The gradient of the tangent at a point (x, y) on a curve is given by $\frac{3}{(2+3x)^2} + \frac{3}{\cos^2 x}$. The curve has y-intercept 1. Find y in terms of x. [5]

$\frac{dy}{dx} = 3(2+3x)^{-2} + 3\sec^2 x$

$y = \int 3(2+3x)^{-2} + 3\sec^2 x \ dx$ [M1] – writing y as $\int \frac{dy}{dx} \ dx$

$= \frac{3(2+3x)^{-1}}{-3} + 3\tan x + C$ [M1, M1] – correct integrations

Sub $x = 0, y = 1, \ C = \frac{3}{2}$ [M1] – correct substitution of x & y

$y = -\frac{1}{2+3x} + 3\tan x + \frac{3}{2}$ [A1]

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4 **Do not use a calculator in this question.**

A polynomial is given by $f(x) = 4x^3 - 2px^2 + p(p-4)x - 8$, where p is a constant.

(a) Find the remainder when $f(x)$ is divided by x. [2]

By remainder theorem,
$R = f(0) = -8$ [M1, A1] – need to mention $R = f(0)$

(b) Given that $x - p$ is a factor of $f(x)$, show that $3p^3 - 4p^2 - 8 = 0$. [2]

By factor theorem,
$f(p) = 0$ [M1]
$4(p)^3 - 2p(p)^2 + p(p-4)(p) - 8 = 0$
$3p^3 - 4p^2 - 8 = 0$ (shown) [A1]

(c) Find the value(s) of p. [5]

Let $g(p) = 3p^3 - 4p^2 - 8 = 0$
By trial and error, $g(2) = 0$
By factor theorem, $(p-2)$ is a factor of $g(p)$ [B1] – with working & not writing $g(p)$ as $f(x)$

By long division / comparing coefficients, [M1] – valid method to factorise $g(p)$
$g(p) = (p-2)(3p^2 + 2p + 4)$ [M1] – correct quotient

$g(p) = (p-2)(3p^2 + 2p + 4) = 0$
$p = 2$, $p = \frac{-2 \pm \sqrt{2^2 - 4(3)(4)}}{6}$ (or $b^2 - 4ac < 0$) (no soln) [A1, A1] – should show the quadratic factor has no solution

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5 **(a)** Given that $y = \frac{1+\ln x}{1-\ln x}$, where $x > 0$ and $x \neq a$.

(i) Find the value of a. [1]

$1 - \ln a = 0$

$a = e$ [B1]

(ii) Prove that y is an increasing function. [3]

$\frac{dy}{dx} = \frac{\left(1-\ln x\right)\left(\frac{1}{x}\right) - (1+\ln x)\left(\frac{1}{x}\right)}{(1-\ln x)^2}$ [M1] – attempted product/quotient rule

$= \frac{2}{x(1-\ln x)^2}$ [M1] – correct answer for dy/dx

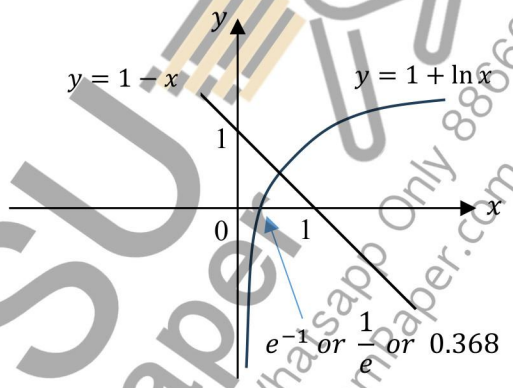
Given $x > 0$ and the perfect square $(1-\ln x)^2 > 0$,

$\frac{2}{x(1-\ln x)^2} > 0,$

$\frac{dy}{dx} > 0$

Hence, y is an increasing function. [A1] – with reasoning

(b) **(i)** Sketch the graph of $y = 1 + \ln x$, labelling the value of x where the graph intersects the x-axis. [2]



[G1] – correct shape & asymptote

[B1] – x-intercept ($\frac{1}{e}, e^{-1}$ or 0.368)

(ii) By sketching a line on your graph in part **(i)**, find the number of solutions to the equation $x + \ln x = 0$. [2]

$1 + \ln x = 1 - x$

Sketch of $y = 1 - x$ on the graph [M1] – correct shape, label y or x-intercept

1 solution [A1]

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6 **(a)** Prove that $\cot\theta = \cot 2\theta + \operatorname{cosec} 2\theta$. [3]

$$RHS = \cot 2\theta + \operatorname{cosec} 2\theta$$

$$= \frac{\cos 2\theta}{\sin 2\theta} + \frac{1}{\sin 2\theta}$$ [M1] – change $\cot 2\theta$ & $\operatorname{cosec} 2\theta$

$$= \frac{(2\cos^2\theta - 1) + 1}{2\sin\theta\cos\theta}$$ [M1] – correct double angle formulas

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta = LHS$$ [A1]

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(b) Hence, find the values of θ such that $\cot 2\theta + \operatorname{cosec} 2\theta = 2\cos\theta$, for $0 \leq \theta \leq \pi$. [4]

$$\cot\theta = 2\cos\theta$$

$$\frac{\cos\theta}{\sin\theta} = 2\cos\theta$$ [M1]

$$\cos\theta - 2\cos\theta\sin\theta = 0$$

$$\cos\theta\,(1 - 2\sin\theta) = 0$$ [M1]

$$\cos\theta = 0 \quad \text{or} \quad \sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}\ (\text{rej}) \quad \text{or} \quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$ [A1, A1]

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7 A particle is travelling in a straight line. Its velocity, v m/s, at time t seconds is given by

$$v = e^{2t-1} - e^{-2t+1}.$$

(a) Determine whether the particle is stationary when $t = 0.5$. [1]

$v = e^{2(0.5)-1} - e^{-2(0.5)+1} = 0$

Hence, the particle is stationary when $t = 0.5$. [B1] – with calculation of $v = 0$

(b) Express the acceleration, a m/s^2, of the particle in terms of t and show that $a \neq 0$ for all values of t. [3]

$a = 2e^{2t-1} + 2e^{-2t+1}$ [B1]

Let $2e^{2t-1} + 2e^{-2t+1} = 0$

$2e^{2t-1} = -2e^{-2t+1}$

$\frac{e^{2t-1}}{e^{-2t+1}} = -1$ [M1] – any reasonable working

$e^{4t} = -1$

Since $e^{4t} \geq 0$, there is no solution, hence $a \neq 0$. [A1]

(c) Find the total distance travelled by the particle in the first 2 seconds. [4]

Total distance

$= -\int_0^{0.5} e^{2t-1} - e^{-2t+1}\,dx + \int_{0.5}^{2} e^{2t-1} - e^{-2t+1}\,dx$ [M1] – correctly split integrals + limits

$= -\frac{1}{2}[e^{2t-1} + e^{-2t+1}]_0^{0.5} + \frac{1}{2}[e^{2t-1} + e^{-2t+1}]_{0.5}^{2}$ [M1√] – correctly integrate their integral

$= -\frac{1}{2}[(e^0 + e^0) - (e^{-1} + e^1)] + \frac{1}{2}[(e^3 + e^{-3}) - (e^0 + e^0)]$ [M1] – correct substitution

$= 0.543 + 9.068 = 9.61$ (3 sf) m [A1]

* Award 2 marks if use $s = \int_0^2 v\ dx$ to get 11.2

Alternative method:

$s = \int e^{2t-1} - e^{-2t+1}\,dx$

$= \frac{1}{2}e^{2t-1} + \frac{1}{2}e^{-2t+1} + C$ [M1] – correctly integrate with $+C$

At t = 0, $s = \frac{1}{2}(e^{-1} + e^1) + C = 1.5431 + C$ (5 sf) [M1√] – attempt to find s for $t = 0, 0.5, 2$

At t = 0.5, $s = \frac{1}{2}(e^0 + e^0) + C = 1 + C$ [M1√] -

At t = 2, $s = \frac{1}{2}(e^3 + e^{-3}) + C = 10.068 + C$ (5 sf)

Total distance $= (1.5431 + C - 1 - C) + (10.068 + C - 1 - C)$

$= 9.61$ (3 s.f.) m [A1]

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8 (a) (i) Express $\frac{9^{x+2}-3^{2x+1}}{27^x(9^{1-x})}$ in the form ab^{x-1}, where a and b are integers. [4]

$= \frac{3^{2x+4}-3^{2x+1}}{3^{3x}(3^{2-2x})}$ [M1] – change to base 3

$= \frac{3^{2x}.3^4-3^{2x}.3}{3^{3x+(2-2x)}}$ [M1] – attempt to split numerator or simplify denominator using $a^m + a^n = a^{m+n}$

$= \frac{3^{2x}(3^4-3)}{3^{x+2}}$

$= 3^{2x-(x+2)}(3^4-3)$ [M1] – simplify $\frac{3^{2x}}{3^{x+2}}$ using $\frac{a^m}{a^m} = a^{m-n}$

$= 3^{x-2}(3 \times 26)$

$= 26(3^{x-1})$ [A1]

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(ii) If $\frac{9^{x+2}-3^{2x+1}}{27^x(9^{1-x})} + 3^{x-1} = 3^y$, find y in terms of x. [2]

$26(3^{x-1}) + 3^{x-1} = 3^y$

$3^{x-1}(27) = 3^y$ [M1] – attempt to factorise 3^{x-1}

$3^{x-1} \cdot 3^3 = 3^y$

$y = x + 2$ [A1]

(b) Solve the equation $10^{5k-8} = 4(5^k)$. [4]

$\lg 10^{5k-8} = \lg 4(5^k)$ [M1] – correctly taking log on both sides

$(5k-8)\lg 10 = \lg 4 + \lg 5^k$ [M1] – attempt to simplify by applying product law or power law

$5k - 8 = \lg 4 + k\lg 5$

$5k - k\lg 5 = \lg 4 + 8$

$k = \frac{\lg 4+8}{5-\lg 5}$ [M1] – attempt to making k the subject

$= 2$ [A1]

or $(2 \times 5)^{5k-8} = 2^2(5^k)$ [M1] – split to base 2 & 5

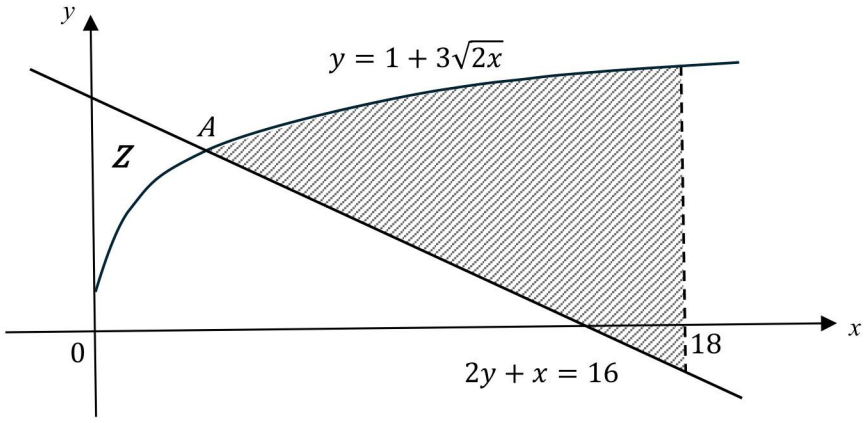
$2^{5k-8} \cdot 5^{5k-8} = 2^2 \cdot 5^k$ [M1] – attempt to simplify using indices law

$5k - 8 = 2$ and $5k - 8 = k$ [M1] – compare exponents of 2 and of 5

$k = 2$ [A1]

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9



The diagram shows the curve $y = 1 + 3\sqrt{2x}$ and the line $2y + x = 16$.
The curve and the line intersect at the point A.

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(a) Show that the coordinates of $A = (2, 7)$. [3]

$y = 1 + 3\sqrt{2x}$ ..... (1)
$2y + x = 16$ ..... (2)

Sub (1) into (2)
$2\left(1 + 3\sqrt{2x}\right) + x = 16$ [M1] – attempt to solve simultaneous eqn
$72x = (14 - x)^2$
$72x = 196 - 28x + x^2$
$x^2 - 100x + 196 = 0$ [M1] – correct quadratic eqn
$(x - 2)(x - 98) = 0$
$x = 2$ or $x = 98$ (rej)
Sub $x = 2, y = 7$
Hence, $A = (2, 7)$ (shown) [A1] – with working

(b) Find the shaded area bounded by the curve $y = 1 + 3\sqrt{2x}$, the line $2y + x = 16$ and the line $x = 18$. [4]

Shaded Area
$= \int_2^{18} upper\ curve - lower\ curve\ dx$
$= \int_2^{18} \left(1 + 3\sqrt{2x}\right) - \frac{1}{2}(-x + 16)\ dx$ [M1,M1] – or equivalent
$= \left[\frac{1}{4}x^2 + (2x)^{\frac{3}{2}} - 7x\right]_2^{18}$ [M1] – correct integration
$= \left(\frac{1}{4}(18)^2 + (36)^{\frac{3}{2}} - 7(18)\right) - \left(\frac{1}{4}(2)^2 + (4)^{\frac{3}{2}} - 7(2)\right)$
$= 176$ unit^2 [A1]

Accept: Area under curve – big triangle + small triangle below x-axis
$= \int_2^{18} \left(1 + 3\sqrt{2x}\right) dx - \frac{1}{2}(16 - 2)(7) + \frac{1}{2}(18 - 16)(1)$
$= \left[x + (2x)^{\frac{3}{2}}\right]_2^{18} - 49 + 1$ [M1, M1, M1
$= 176$ unit^2 [A1]

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(c) Area Z is bounded by the curve $y = 1 + 3\sqrt{2x}$, the line $2y + x = 16$ and the y-axis. To find the area Z, a student writes down the formula

$$\text{Area } Z = \int_a^7 g(y)\,dy + B.$$

Find an expression for $g(y)$ and the values of a and B. [3]

$$y = 1 + 3\sqrt{2x}$$
$$3\sqrt{2x} = y - 1$$
$$9(2x) = y^2 - 2y + 1$$
$$x = \frac{1}{18}(y^2 - 2y + 1) \text{ or } \frac{1}{18}(y-1)^2$$

Sub $x = 0$ into eqn of curve, $y = 1$

Sub $x = 0$ into eqn of line, $y = 8$

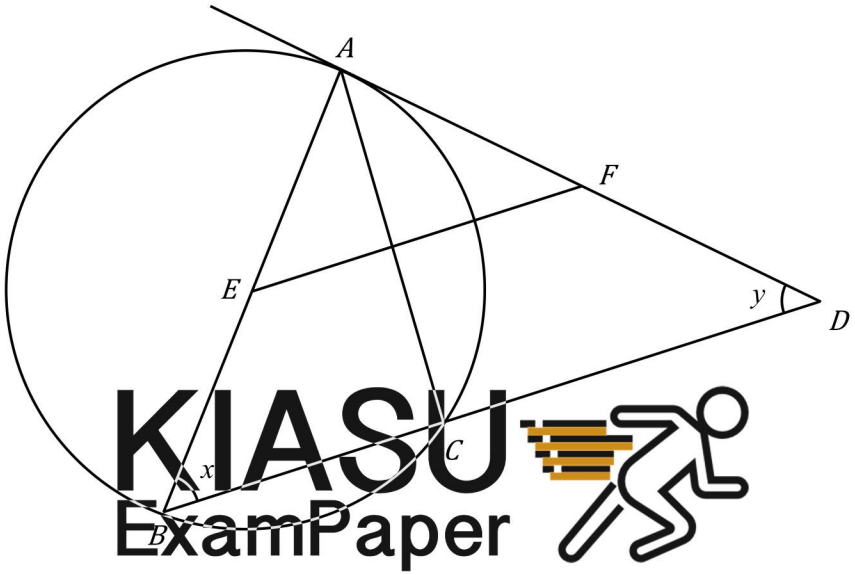
Area of triangle, $B = \frac{1}{2}(2-0)(8-7) = 1$

$g(y) = \frac{1}{18}(y-1)^2$, $fa = 1$, $B = 1$ [B1, B1, B1] – with workings for $g(y)$, a & h

or $\frac{1}{18}(y^2 - 2y + 1)$

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10



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The diagram shows a triangle ABC whose vertices lie on the circumference of a circle. The tangent to the circle at point A meets the line BC extended at point D. Points E and F are midpoints of AB and AD respectively. Angle $ABC = x$ and angle $ADC = y$.

(a) Prove that triangle EAF is similar to triangle ACD. [4]

By midpoint theorem, $EF \;//\; BD$ [B1]

$\angle AFE = y$ (corr. angles, $EF//BD$) [M1] –either x or y, with mention of midpoint theorem

$\angle AEF = x$ (corr. angles, $EF//BD$)

$\angle CAD = x$ (alt. segment thm.) [B1]

Hence, $\angle AFE = \angle CDA = y$

$\angle AEF = \angle CAD = x$

$\angle EAF = \angle ACD$ (Sum of angles of a triangle)

$\therefore\ \Delta EAF$ is similar to ΔACD by AAA similarity test [A1]

(b) Prove that $CD \times EF = 2(AF)^2$. [3]

$\frac{CD}{AF} = \frac{AD}{EF}$ [M1]

$CD \times EF = AD \times AF$

$CD \times EF = 2(AF) \times AF$ [M1] – $AD = 2AF$

$CD \times EF = 2(AF)^2$ (shown) [A1]

(c) If E is the centre of the circle, prove that angle $BAC = y$. [2]

$\angle ACB = 90^o$ ($\angle$ in a semicircle) [B1]

In ΔBAC, $\angle BAC = 180^o - 90^o - x$

$= 90^o - x$

In ΔADC, $y = 180^o - 90^o - x = 90^o - x$

Hence, $\angle BAC = y$ (shown) [A1] – with correct reasoning

11 A water dispenser contains 10 litres of water initially. Water is poured into the dispenser at a constant rate of $4\cos\theta$ litres per minute, for some positive real value of θ. At the same time, water is dispensed out at a constant rate of $6\sin\theta$ litres per minute.

(a) Explain why the value of θ cannot fall in the range between $\frac{\pi}{2}$ to 2π. [1]

The rates $4\cos\theta$ and $6\sin\theta$ should be positive in real life. But when $\frac{\pi}{2} < \theta < 2\pi$, either $\cos\theta < 0$ or $\sin\theta < 0$. [B1]

(b) Find the value of θ, where $6 < \theta < 8$, for which the volume of water in the dispenser remains constant. [3]

(volume of water in = volume of water out) (accept rate of water in = rate of water out)

$(4\cos\theta) \times t = (6\sin\theta) \times t$ [M1] – accept $4\cos\theta = 6\sin\theta$

$\tan\theta = \frac{4}{6}$

Ref. angle, $\alpha = \tan^{-1}\frac{4}{6} = 0.588$ [M1]

$\theta = 2\pi + \alpha = 6.87$ (3s.f.) [A1]

(c) **(i)** If, instead, the water in the dispenser is emptied after four minutes, show that

$$3\sin\theta - 2\cos\theta = 1.25.$$ [2]

(volume of water out – volume of water in = 10 litres)

$(6\sin\theta) \times 4 - (4\cos\theta) \times 4 = 10$ [M1]

$24\sin\theta - 16\cos\theta = 10$

$3\sin\theta - 2\cos\theta = 1.25$ [A1]

(ii) By writing $3\sin\theta - 2\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and α is acute, find the volume of water poured into the dispenser during the 4 minutes. [5]

Let $3\sin\theta - 2\cos\theta = R\sin(\theta - \alpha)$, where $R > 0$ and α is acute

$R = \sqrt{2^2 + 3^2} = \sqrt{13}$ [B1]

$\alpha = \tan^{-1}\frac{2}{3}$

$= 0.588$ [B1]

$3\sin\theta - 2\cos\theta = 1.25$

$\sqrt{13}\sin(\theta - 0.588) = 1.25$ [M1]

$\sin(\theta - 0.588) = \frac{1.25}{\sqrt{13}}$

Ref. angle $= \sin^{-1}\frac{1.25}{\sqrt{13}}$

$= 0.354$

$\theta - 0.588 = 0.354$ [M1]

$\theta = 0.942$

Volume of water poured in = $4\cos 0.942 \times 4 = 9.411$ litres (3 s.f.) [A1]

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GAN ENG SENG SCHOOL
Preliminary Examination 2026

CANDIDATE NAME	
CLASS	
INDEX NUMBER	

ADDITIONAL MATHEMATICS
Paper 1

4049/01

25 August 2026
2 hours 15 minutes

Sec 4 Express/ 5 Normal (Academic)

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

	For Examiner's Use
Total	/90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Water is poured at a constant rate of 27π cm^3/s into a hemispherical container.
When the depth of water directly below the centre is x cm, the volume V cm^3 of water in the container is given by $V = \frac{1}{6}\pi x^2(54 - x)$.

Find the rate of change of the depth of the water directly below the centre when $x = 9$ cm. [3]

2 The curve $y = f(x)$ is such that $f'(x) = e^x + e^{-2x}$.

(a) Explain why the curve $y = f(x)$ has no stationary points. [2]

(b) Given that the curve passes through the point $\left(0, \frac{3}{2}\right)$, find an expression for $f(x)$. [4]

[2]

3 (a) Factorise completely the cubic polynomial $2x^3 - 128$. [2]

(b) Express $\dfrac{5x^2 - 18x + 40}{2x^3 - 128}$ as the sum of 2 partial fractions. [5]

4 The function f is defined, for all real values of x, by $f(x) = (x-2)x^2$.

(a) Find the values of x for which f is an increasing function. [3]

(b) Find the coordinates of the stationary points of f and determine the nature of these stationary points. [6]

5 **(a)** Show that $\sin 3x$ can be expressed as $3\sin x - 4\sin^3 x$. [3]

(b) Hence find all the values of x for which $\sin 3x = \sin^2 x$, where $0° \leq x \leq 360°$. [5]

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6 The equation of a circle C_1, with centre A, is $x^2 + y^2 - 4x + 6y = 12$.

(a) Find the coordinates of A and the radius of C_1. [4]

(b) Show that the point $P(6, 0)$ lies on C_1. [1]

(c) Find the equation of the tangent to C_1 at P. [3]

A second circle C_2, has diameter AP.

(d) Find the equation of C_2. [3]

(e) State the equation of the tangent to C_2 at P. [1]

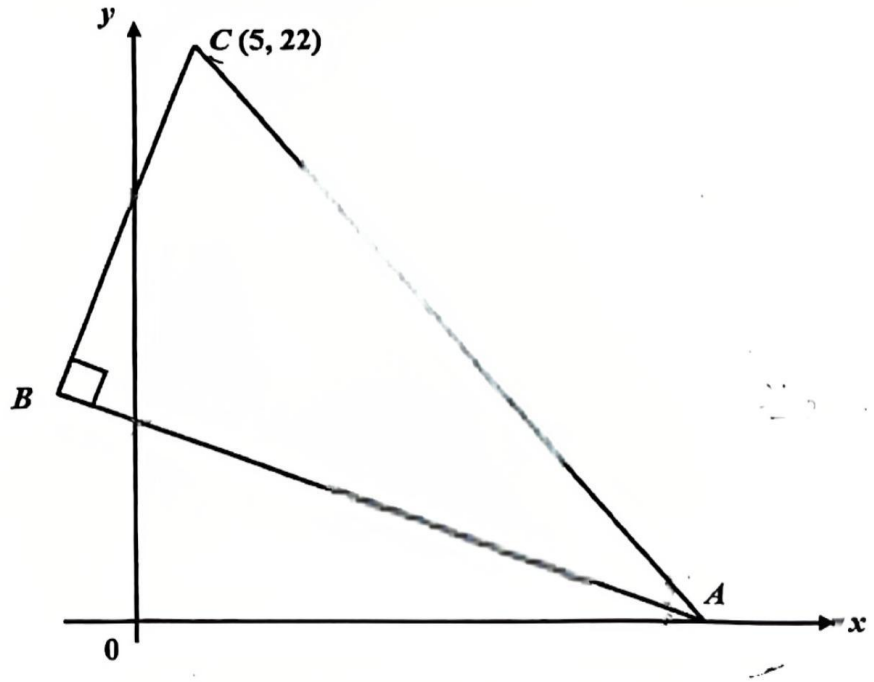
(c) Giv

7 **(a)** Show that $\frac{d}{dx}\left(\frac{\ln x^2}{3x}\right)=\frac{2}{3x^2}-\frac{2\ln x}{3x^2}$. [3]

(b) Hence integrate $\frac{\ln x}{x^2}$ with respect to x. [3]

(c) Given that the curve $y = f(x)$ passes through the point $(1,3)$ and is such that $f'(x) = \frac{\ln x}{x^2}$, find $f(x)$. [2]

8 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a triangle ABC in which the point C is (5, 22), the point A lies on the x-axis and angle ABC is $90°$. The equation of AB is $3y+4x=36$.

(a) Find the coordinates of B. [5]

(b) Given that $ABCD$ is a rectangle, find the coordinates of D and the area of the rectangle. [4]

9 **(a)** Given that $\ln(x^4y^2) = 5 + \ln y - \ln x$, where x and y are positive, express y in terms of x. [3]

(b) Solve the equation $\log_3\left(\frac{x+1}{x-1}\right) + 2\log_9(x-1) = \frac{4}{\log_3(x+1)}$. [5]

10 **(a)** Given that $\frac{8}{3-\sqrt{5}} = \sqrt{a+b\sqrt{5}}$, where a and b are integers, find, **without using a calculator**, the value of a and of b. [4]

(b) Given that $4-\sqrt{7}$ is a root of the equation $-x^2 + px + q = 0$, where p and q are integers, find the value of p and of q. [4]

Find the se
entirely

11 **(a)** Show that $\sin^2 x \cos^2 x = k(\sin^2 2x)$, where k is a constant. State the value of k. [2]

(b) Hence use the result in **(a)**, differentiate $\sin^2 x \cos^2 x$ with respect to x. [2]

2 Find the set of values of the constant k for which the curve $y = x^2 + (3k+2)x - 1$ lies entirely above the line $y = 2x - 2$. [4]

13 (a) Express $3y = 2x^2 - 4x + c$ in the form of $y = a(x-h)^2 + k$, where c, a, h and k are constants. [2]

(b) Find the value of c for which $3y = 2x^2 - 4x + c$ has repeated roots. [2]

End of paper

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GAN ENG SENG SCHOOL
Preliminary Examination 2026

CANDIDATE NAME			
CLASS		**INDEX NUMBER**	

ADDITIONAL MATHEMATICS
Paper 2

4049/02

1 September 2026
2 hours 15 minutes

Sec 4 Express/ 5 Normal (Academic)

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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	For Examiner's Use
Total	/90

Mathematical Formulae

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$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

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$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

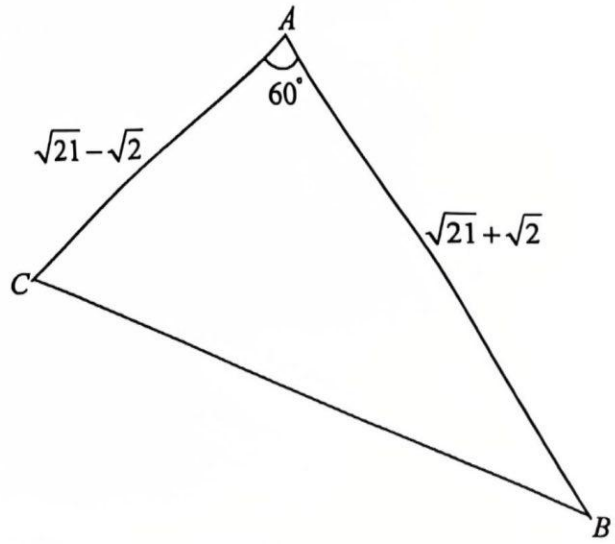
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 **Do not use a calculator in this question.**

All lengths in this question are in centimetres.



The diagram shows triangle ABC with $AC = \sqrt{21} - \sqrt{2}$, $AB = \sqrt{21} + \sqrt{2}$ and $CAB = 60^\circ$.

Find the exact length of BC. [3]

2 The term independent of x in the expansion of $\left(x^2-\frac{k}{4x}\right)^9$, where k is a constant, is 5376.

(a) Find the value of k. [4]

(b) Using your value of k, find the coefficient of x^9 in the expansion of $\left(1-9x^9\right)\left(x^2-\frac{k}{4x}\right)^9$. [3]

3 A particle travels in a straight line so that t seconds after leaving a fixed point O, its velocity, v m/s, is given by $v = 3\cos 2t + 4\sin 2t$.

(a) Find the value of t when the particle first attains its greatest distance from O. [3]

(b) Find the total distance travelled by the particle in the interval $t = 0$ to $t = 4$. [6]

4 A rainforest started with a population of 27 deers.

The population of the deers is predicted to increase so that after a period of t years, the population, P_f, is given by the equation $P_f = \frac{a}{3+2e^{-0.18t}}$, where a is a constant to be found.

(a) Show that $a = 135$. [1]

(b) Find the minimum number of years required for the population of the deers to increase to 35 deers. [3]

(c) Another population of deers lives in a national park.

The population, P_n, is given by the equation $P_n = \frac{135}{3+2e^{\lambda t}}$.

(i) The population of deers increase to 40 after 5 years. Find λ. [3]

(ii) Find the value of P_n as t becomes very large. [1]

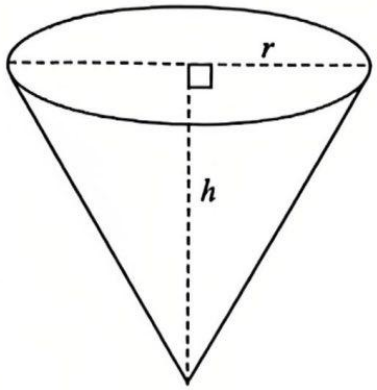
5 (a) (i) Show that $3x+2$ is a factor of $f(x)=6x^3-8x^2+x+6$ and hence factorise $6x^3-8x^2+x+6$ completely. [5]

(ii) Determine the number of real roots of the equation $f(x)=0$, justifying your answer. [2]

(b) **(i)** Solve the following equation $4(3)^{2p}+(3)^{p}+5=8$. [3]

(ii) Show that the real solution from part **(b)(i)** may be written in the form $1-\log_a b$ where a and b are integers to be determined. [2]

6 The diagram shows a open circular cone with radius r cm and vertical height h cm.
The cone is made from a piece of thin metal sheet and it can contain 110π cm^3 of liquid.
The cost of the metal sheet is \$1.15 per square centimetre.
Assume that the thickness of the metal sheet is negligible.



(a) Express h in terms of r. [1]

(b) Show that the cost needed to make the cone, \$$C$ is given by

$$C = \frac{1.15\pi\sqrt{r^6 + 108\,900}}{}.$$

[3]

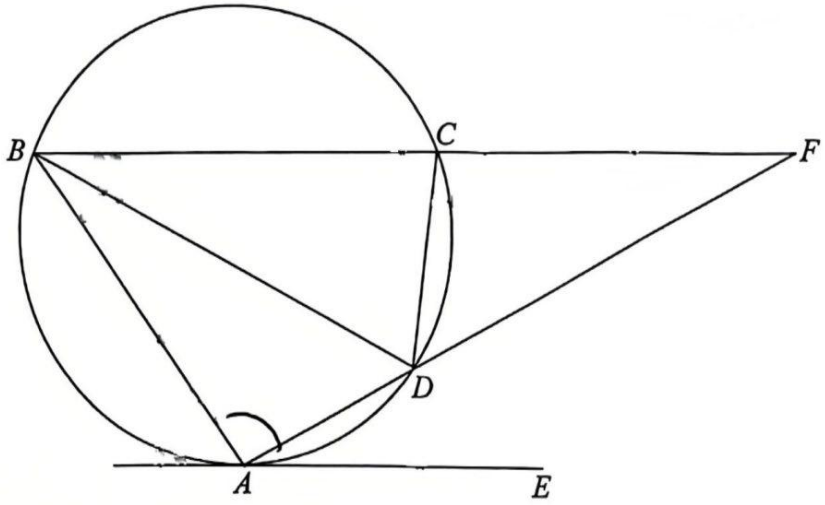
Continuation of working space for question **6(b)**.

(c) Given that r varies, find the value of r for which C has a minimum value. [4]

Continuation of working space for question 6(c).

(d) Explain why this value of r gives the minimum cost needed to make the cone. [1]

7



In the diagram, AE is a tangent to the circle at A.
ADF and BCF are straight lines, AE is parallel to BF and $AB = CF$.

(a) Prove that $\angle BAD = \angle DCF$. [1]

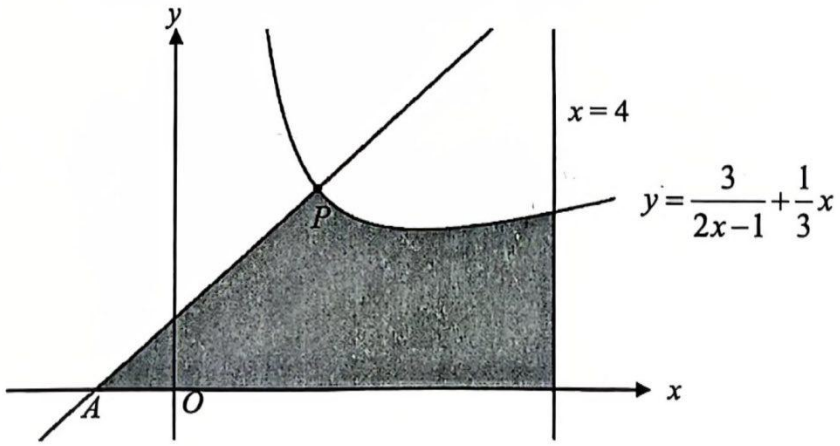
(b) Prove that triangle ADB is congruent to triangle CDF. [3]

Continuation of working space for question **7(b).**

(c) Prove that $\angle ABC = 2\angle EAD$. [2]

8 The diagram shows part of the curve $y = \frac{3}{2x-1} + \frac{1}{3}x$ where $x \neq \frac{1}{2}$ and the line $x = 4$.

The normal to the curve at the point $P(a, b)$ cuts the x-axis at A.



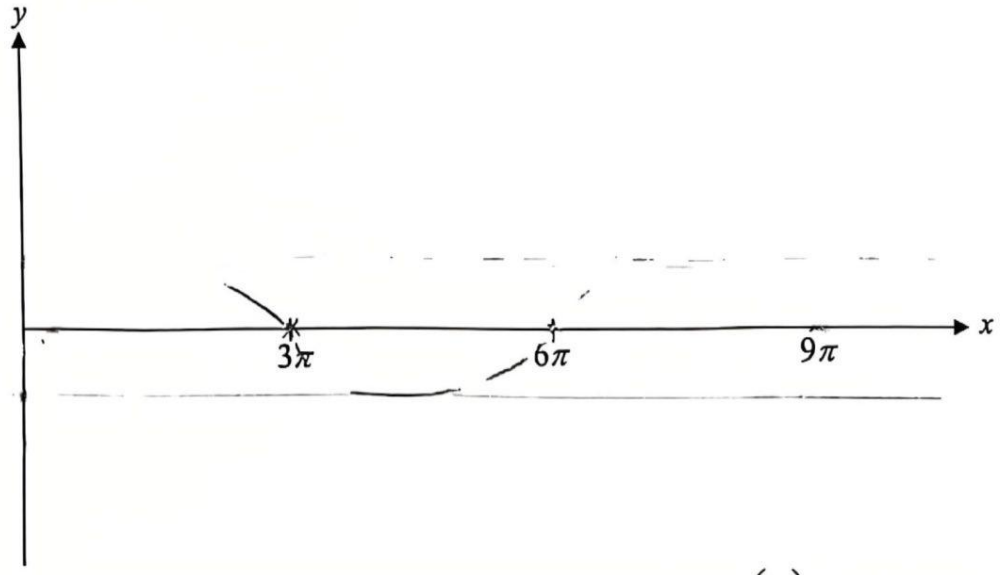
Given that the gradient of this normal is $\frac{6}{7}$, calculate

(a) the value of a and of b, [5]

(b) the area of the shaded region. [5]

9 **(a)** State the amplitude and period of $y = 2\sin\left(\frac{x}{3}\right) - 1$. [2]

(b) Sketch the graph of $y = 2\sin\left(\frac{x}{3}\right) - 1$ for $0 \le x \le 9\pi$. [3]

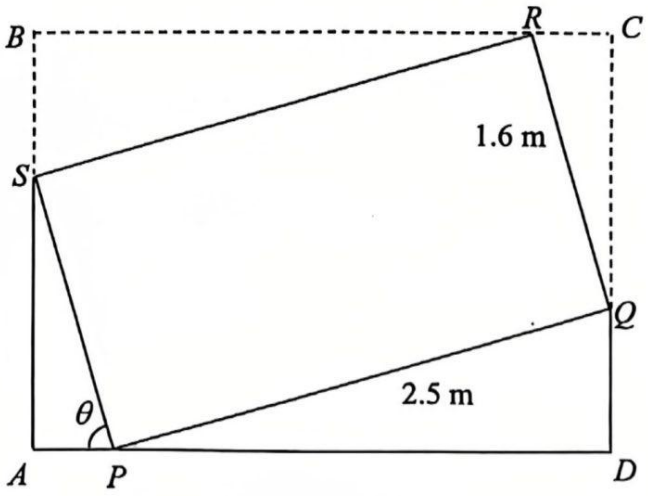


(c) It is known that α and β are the roots of the equation $2\sin\left(\frac{x}{3}\right) - 1 = k$, where $0 < k < 1$ and $\alpha < \beta < 3\pi$.

Find β in terms of α. [1]

(d) Given that $2\sin\left(\frac{x}{3}\right) = p + 1$ where p is a constant, write down the range of values of p for which the equation has 4 solutions for $0 \le x \le 9\pi$. [1]

10 The diagram below shows the plan view of a 2.5 m by 1.6 m rectangular fish pond $PQRS$ that Mr Phua is planning to build in his rectangular garden $ABCD$.
The fish pond is positioned at a corner of the garden in such a way that angle $APS = \theta^\circ$, where θ varies.



(a) Show that the length of AB, l metres can be expressed in the form $a\cos\theta + b\sin\theta$, where a and b are constants to be found. [2]

(b) Express l in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is acute. [4]

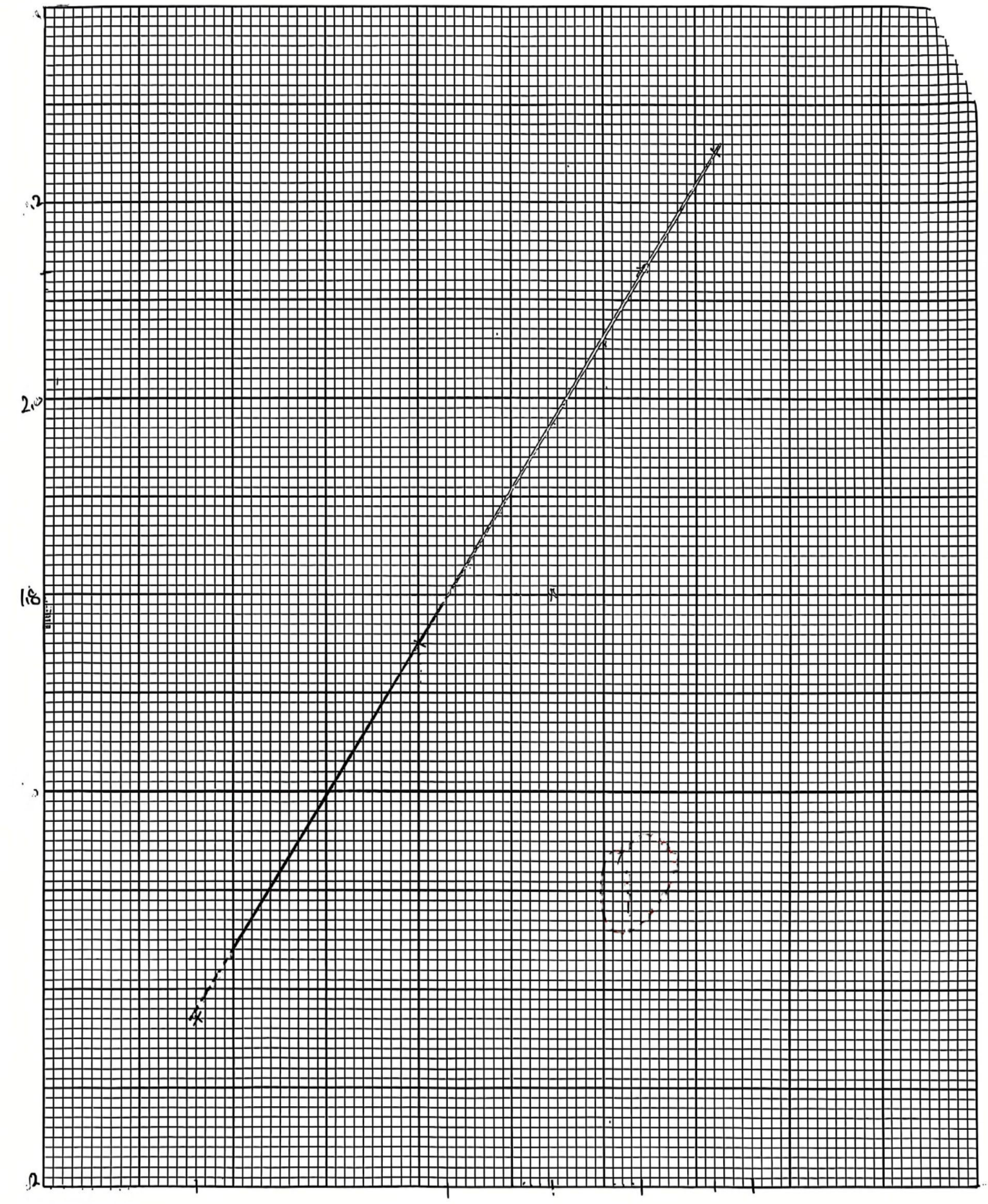
(c) Given that the length of AB is 2.4 m long, find the value of θ. [3]

11 Recorded values of the population P, in millions, of a bacteria colony, x minutes after observations began, are shown in the table below.

x	5	10	15	20	25
P	35.6	68.2	75.1	145.7	188.6

It is known that $P = ax^n + 12$, where a and n are constants.
It is believed that an error was made in recording one of the values of P.

(a) Plot $\lg(P-12)$ against $\lg x$ and draw a straight line graph on page 20 to illustrate the information. [3]



Use the graph to estimate

(b) a value of P to replace the incorrect reading, [2]

(c) the value of each of the constants a and n. [5]

GESS EYE 2026 4EXP/5NA A-Math P1 Marking Scheme

Question 1

GESS EYE 2026 4EXP5NA AM P1 Marking Scheme

Some failed to see the use of chain rule.

1 $V = \frac{1}{6}\pi x^2(54 - x)$

$V = 9\pi x^2 - \frac{1}{6}\pi x^3$

$\frac{dV}{dx} = 18\pi x - \frac{1}{2}\pi x^2$ (M1)

$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$

$27\pi = \left(18\pi(9) - \frac{1}{2}\pi(9)^2\right) \times \frac{dx}{dt}$ (M1)

$27\pi = \left(162\pi - \frac{81}{2}\pi\right) \times \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{27\pi}{1} \times \frac{2}{243\pi} = \frac{2}{9}$ cm/s (A1)

Marker's comments:

$\left(\frac{P1}{0.9} + \frac{P2}{0.9}\right) x$.

$=$ _____

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Question 2

2 **(a)** $f'(x) = 0$

$e^x + e^{-2x} = 0$ (M1)

$e^x = -e^{-2x}$

$e^x = -\frac{1}{e^{2x}}$

$e^{3x} = -1$

As e^{3x} is always > 0, there is no solution to the above equation,

therefore there is no stationary point. (A1)

Otherwise need to write: $e^x > 0,\ e^{-2x} > 0$

$\Rightarrow e^x + e^{-2x} > 0$.

(b) $f'(x) = e^x + e^{-2x}$

Integrate wrt x:

$f(x) = e^x - \frac{1}{2}e^{-2x} + c$ (M2,1,0)

(c is an arbitary constant)

Sub. $\left(0, \frac{3}{2}\right)$: $\frac{3}{2} = e^0 - \frac{1}{2}e^0 + c$ (M1)

$c = 1$

$\Rightarrow f(x) = e^x - \frac{1}{2}e^{-2x} + 1$ (A1)

Marker's comments:

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Question 3

3 **(a)** $2x^3 - 128 = 2(x^3 - 64)$ (M1)

$= 2(x^3 - 4^3)$

$= 2(x-4)(x^2 + 4x + 16)$ (A1)

(b) Let $\frac{5x^2 - 18x + 40}{2x^3 - 128} = \frac{A}{2(x-4)} + \frac{Bx + C}{x^2 + 4x + 16}$ (M1)

$\Rightarrow 5x^2 - 18x + 40 = A(x^2 + 4x + 16) + 2(Bx + C)(x - 4)$

Put $x = 4$: $80 - 72 + 40 = A(16 + 16 + 16)$ (M1)

$A = 1$

Put $x = 0$: $40 = (1)(16) + 2(C)(-4)$ (M1)

$24 = -8C$

$C = -3$

Put $x = 1$: $5 - 18 + 40 = (1 + 4 + 16) + 2(B - 3)(1 - 4)$ (M1)

$27 - 21 = -6B + 18$

$B = 2$

$\therefore \frac{5x^2 - 18x + 40}{2x^3 - 128} = \frac{1}{2(x-4)} + \frac{2x - 3}{x^2 + 4x + 16}$ (A1)

(If students do not get (a) correct, award max. 3 working marks for showing correct working to solve (b)).

Marker's comments:

Some still not sure about the 3 type partial fracti

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Question 4

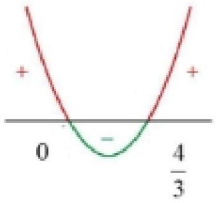
4 **(a)** $f(x) = (x-2)x^2 = x^3 - 2x^2$

Diff. wrt. x:

$f'(x) = 3x^2 - 4x$ (M1)

$f'(x) > 0 \Rightarrow \quad 3x^2 - 4x > 0$ (M1)

$x(3x-4) > 0$



The values of x: $x < 0$ or $x > \frac{4}{3}$ (A1)

(b) At stationary points, $f'(x) = 0$

$3x^2 - 4x = 0$

$x(3x-4) = 0$

$x = 0 \quad 3x - 4 = 0$

$x = 0 \quad x = \frac{4}{3}$ (M1)

$f''(x) = 6x - 4$ (M1)

At $x = 0$, $y = (0-2)(0)^2 = 0$

$f''(0) = -4\ (< 0)$ (M1) – value must be shown

$(0,0)$ is a max. point. (A1)

At $x = \frac{4}{3}$, $y = \left(\frac{4}{3} - 2\right)\left(\frac{4}{3}\right)^2 = -\frac{32}{27}$

$f''\left(\frac{4}{3}\right) = \frac{4}{3}(6) - 4 = 4\ (> 0)$ (M1) – value must be shown

$\left(\frac{4}{3}, -\frac{32}{27}\right)$ is a min. point. (A1)

Marker's comments:

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Question 5

5 **(a)** $\sin 3x = \sin(x+2x)$

$= \sin x \cos 2x + \cos x \sin 2x$ (M1)

$= \sin x(1-2\sin^2 x) + \cos x(2\sin x \cos x)$ (M1)

$= \sin x - 2\sin^3 x + 2\sin x \cos^2 x$

$= \sin x - 2\sin^3 x + 2\sin x(1-\sin^2 x)$ (M1)

$= \sin x - 2\sin^3 x + 2\sin x - 2\sin^3 x$

$= 3\sin x - 4\sin^3 x$

(b) $\sin 3x = \sin^2 x$

$3\sin x - 4\sin^3 x = \sin^2 x$ (M1)

$4\sin^3 x + \sin^2 x - 3\sin x = 0$

$\sin x(4\sin^2 x + \sin x - 3) = 0$

$\sin x = 0 \qquad 4\sin^2 x + \sin x - 3 = 0$

$x = 0°, 180°, 360° \qquad (4\sin x - 3)(\sin x + 1) = 0$ (M1)

$\sin x = \frac{3}{4} \qquad \sin x = -1$

$\alpha = 48.590° \qquad x = 270°$

$x = 48.6°, 131.4°$

$\therefore \quad x = 0°, 48.6°, 131.4°, 180°, 270°, 360°$ (A3,2,1)

Marker's comments:

Do not cancel away sin x !!

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Question 6

6 **(a)** $x^2+y^2-4x+6y=12$

$x^2-4x+(-2)^2+y^2+6y+(3)^2=12+(-2)^2+(3)^2$ (M1)

$(x-2)^2+(y+3)^2=25$ (M1)

A is $(2,-3)$. (A1)

The radius is 5 units. (A1)

(b) Sub. $(6,0)$:

LHS $=(6-2)^2+(0+3)^2=16+9=25$ (B1)

$(6,0)$ lies on the circle.

(c) Gradient of $AP=\frac{0-(-3)}{6-2}=\frac{3}{4}$ (M1)

Gradient of the tangent at $P=-\frac{4}{3}$ (M1)

Equation of the tangent: $(y-0)=-\frac{4}{3}(x-6)$

$y=-\frac{4}{3}x+8$ (A1)

(d) Mid pt. of $AP=\left(\frac{2+6}{2},\frac{-3+0}{2}\right)=\left(4,\frac{-3}{2}\right)$ (M1)

Radius $=\frac{5}{2}$ units (M1)

Equation of C_2: $(x-4)^2+\left(y+\frac{3}{2}\right)^2=\left(\frac{5}{2}\right)^2$ (A1)

(e) Equation of the tangent to C_2 at P: $y=-\frac{4}{3}x+8$ (B1)

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Question 7

7 **(a)** $\dfrac{d}{dx}\left(\dfrac{\ln x^2}{3x}\right) = \dfrac{d}{dx}\left(\dfrac{2\ln x}{3x}\right)$

$= \left(\dfrac{2}{3}\right)\dfrac{d}{dx}\left(\dfrac{\ln x}{x}\right)$

$= \left(\dfrac{2}{3}\right)\left(\dfrac{x\left(\frac{1}{x}\right) - \ln x(1)}{x^2}\right)$ (M2,1,0)

$= \left(\dfrac{2}{3}\right)\left(\dfrac{1}{x^2} - \dfrac{\ln x}{x^2}\right)$ (M1)

$= \dfrac{2}{3x^2} - \dfrac{2\ln x}{3x^2}$ (Shown)

(b) $\dfrac{d}{dx}\left(\dfrac{\ln x^2}{3x}\right) = \dfrac{2}{3x^2} - \dfrac{2\ln x}{3x^2}$

$\left(\dfrac{\ln x^2}{3x}\right) = \int \dfrac{2}{3x^2} - \dfrac{2\ln x}{3x^2}\,dx$ (M1)

$\dfrac{2}{3}\int \dfrac{\ln x}{x^2}\,dx = \int \dfrac{2}{3}x^{-2}\,dx - \dfrac{\ln x^2}{3x}$

$\dfrac{2}{3}\int \dfrac{\ln x}{x^2}\,dx = \dfrac{2x^{-1}}{3(-1)} + c - \dfrac{2\ln x}{3x}$ (M1)

(c : constant)

$\int \dfrac{\ln x}{x^2}\,dx = \dfrac{-1}{x} - \dfrac{\ln x}{x} + d$ (A1)

(d : constant)

(c) $f(x) = \int \dfrac{\ln x}{x^2}\,dx$

$f(x) = \dfrac{-1}{x} - \dfrac{\ln x}{x} + d$ (M1)

Sub. (1, 3): $3 = -1 - 0 + d$

$d = 4$

$f(x) = \dfrac{-1}{x} - \dfrac{\ln x}{x} + 4$ (A1)

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Question 8

8 **(a)** $3y+4x=36$

$y=-\frac{4}{3}x+12 \quad ...(1)$

Grad. of $AB=-\frac{4}{3}$

Grad. of $BC=\frac{3}{4}$ (M1)

Equation of BC: $(y-22)=\frac{3}{4}(x-5)$

$y=\frac{3}{4}x+\frac{73}{4} \quad ...(2)$ (A1)

Sub into (1): $\frac{3}{4}x+\frac{73}{4}=-\frac{4}{3}x+12$ (M1)

$x=-3$

$y=\frac{3}{4}(-3)+\frac{73}{4}=16$ (A1,0)

B is $(-3,16)$. (A1)

(b) Mid pt. of AC = Mid pt. of BD

$\left(\frac{9+5}{2},\frac{0+22}{2}\right)=\left(\frac{x-3}{2},\frac{y+16}{2}\right)$ (M1)

$(7,11)=\left(\frac{x-3}{2},\frac{y+16}{2}\right)$

$x=17, \quad y=6$

D is $(17,6)$. (A1)

Area $=\frac{1}{2}\begin{vmatrix}9 & 17 & 5 & -3 & 9\\ 0 & 6 & 22 & 16 & 0\end{vmatrix}$

$=\frac{1}{2}[54+374+80+0-(0+30-66+144)]$ (M1)

$=200$ units^2 (A1)

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Question 9

9 **(a)** $\ln(x^4y^2) = 5 + \ln y - \ln x$

$\ln(x^4y^2) = 5 + \ln\left(\frac{y}{x}\right)$ (M1)

$\ln(x^4y^2) - \ln\left(\frac{y}{x}\right) = 5$

$\ln\left(x^4y^2 \times \frac{x}{y}\right) = 5$

$\ln(x^5y) = 5$ (M1)

$x^5y = e^5$

$y = \frac{e^5}{x^5}$ (A1)

(b) $\log_3\left(\frac{x+1}{x-1}\right) + 2\log_9(x-1) = \frac{4}{\log_3(x+1)}$

$\log_3(x+1) - \log_3(x-1) + \frac{2\log_3(x-1)}{\log_3 3^2} = \frac{4}{\log_3(x+1)}$

(M1) (M1)

$\log_3(x+1) - \log_3(x-1) + \log_3(x-1) = \frac{4}{\log_3(x+1)}$

$(\log_3(x+1))^2 = 4$ (M1)

$\log_3(x+1) = \pm 2$

$\Rightarrow x+1 = 3^2$ $x+1 = 3^{-2}$

$x = 8$ (A1) $x = \frac{1}{9} - 1 = \frac{-8}{9}$ (Rejected) (A1)

Marker's comments:

Common mistakes:

$\log_3(x+1) \times \log_3(x+1) = \log_3(x+1)^2$

Actual: $[\log_3(x+1)]^2$!!

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Question 10

10 (a)

$$\frac{8}{3-\sqrt{5}} = \sqrt{a+b\sqrt{5}}$$

$$\frac{8(3+\sqrt{5})}{9-5} = \sqrt{a+b\sqrt{5}} \qquad \text{(M1)}$$

$$6+2\sqrt{5} = \sqrt{a+b\sqrt{5}}$$

$$\left(6+2\sqrt{5}\right)^2 = a+b\sqrt{5} \qquad \text{(M1)}$$

$$36+24\sqrt{5}+20 = a+b\sqrt{5}$$

$$56+24\sqrt{5} = a+b\sqrt{5}$$

$$\Rightarrow a = 56 \quad \text{(A1)} \qquad b = 24 \quad \text{(A1)}$$

(b) $x^2 - px + q = 0$

$$-(4-\sqrt{7})^2 + p(4-\sqrt{7}) + q = 0 \qquad \text{(M1)}$$

$$-\left(16-8\sqrt{7}+7\right)+4p-\sqrt{7}p+q=0$$

$$-23+8\sqrt{7} = -(4p+q)+\sqrt{7}p \qquad \text{(M1)}$$

$$\Rightarrow p = 8 \qquad \text{(A1)}$$

$$\Rightarrow -23 = -(4(8)+q)$$

$$q = -9 \qquad \text{(A1)}$$

Marker's comments:

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Question 11

11 **(a)** $\sin^2 x\cos^2 x = (\sin x\cos x)^2$

$$= \left(\frac{1}{2}\times 2\sin x\cos x\right)^2$$

$$= \left(\frac{1}{2}\sin 2x\right)^2 \qquad \text{(M1)}$$

$$= \frac{1}{4}\sin^2 2x$$

$k = \frac{1}{4}$ (A1)

(b) $\frac{d}{dx}\left(\sin^2 x\cos^2 x\right) = \frac{d}{dx}\left(\frac{1}{4}\sin^2 2x\right)$

$= \frac{1}{4}(2)(\sin 2x)(\cos 2x)(2)$ (M1)

$= \sin 2x\cos 2x$ or $\frac{1}{2}\sin 4x$ (A1)

Marker's comments:

Question 12

12 $2x-2 = x^2+(3k+2)x-1$

$x^2+3kx+1=0$ (M1)

As there is no solution to the equation,

$\Rightarrow b^2-4ac<0$

$(3k)^2-4(1)(1)<0$ (M1)

$9k^2-4<0$

$(3k-2)(3k+2)<0$ (M1)

$\Rightarrow -\frac{2}{3}<k<\frac{2}{3}$ (A1)

Marker's comments:

always factorise for quadratic inequalities!

Some tend to:

$9k^2-4<0$

$9k^2<4$

$k^2<\frac{4}{9}$

$k<\pm\frac{2}{3}$

which is wrong!!

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Question 13

13 **(a)** $3y = 2x^2 - 4x + c$

$$y = \frac{2}{3}\left(x^2 - 2x\right) + \frac{c}{3} \qquad \text{(M1)}$$

$$y = \frac{2}{3}\left[(x-1)^2 - (-1)^2\right] + \frac{c}{3}$$

$$y = \frac{2}{3}(x-1)^2 - \frac{2}{3} + \frac{c}{3} \qquad \text{(A1)}$$

(b) $3y = 2x^2 - 4x + c$

$$y = \frac{2}{3}x^2 - \frac{4}{3}x + \frac{c}{3}$$

For repeated roots,

$\Rightarrow b^2 - 4ac = 0$

$$\left(-\frac{4}{3}\right)^2 - 4\left(\frac{2}{3}\right)\left(\frac{c}{3}\right) = 0 \qquad \text{(M1)}$$

$$\frac{16}{9} - \frac{8c}{9} = 0$$

$$c = 2 \qquad \text{(A1)}$$

Marker's comments:

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2026 Sec 4 Express Prelim A-Math Paper 2 Marking Scheme (GESS 4EXP_5NA AM P2 PRELIM 2026 OLP)

Marking Scheme
2026 Sec 4 Express PRELIM AM Paper 2

		Marker's Comments	
1	$BC^2 = AC^2 + AB^2 - 2(AC)(AB)\cos 60^\circ$ $= \left(\sqrt{21}-\sqrt{2}\right)^2 + \left(\sqrt{21}+\sqrt{2}\right)^2 - 2\left(\sqrt{21}-\sqrt{2}\right)\left(\sqrt{21}+\sqrt{2}\right)\left(\frac{1}{2}\right)$ ---M1 $= \left(21-2\sqrt{42}+2\right)+\left(21+2\sqrt{42}+2\right)-2(21-2)\left(\frac{1}{2}\right)$ $= \left(23-2\sqrt{42}\right)+\left(23+2\sqrt{42}\right)-19$ ---M1 $= 27$ $BC = \sqrt{27}$ ← a few forgot!! $= 3\sqrt{3}$ ---A1	3 marks	
2(a)	$\left(x^2 - \frac{k}{4x}\right)^9$ $T_{r+1} = \binom{9}{r}\left(x^2\right)^{9-r}\left(\frac{-k}{4x}\right)^r$ → Mistake: $(-k)^r\left(\frac{1}{4}x^{-1}\right)^r$ Actual: $\left(\frac{1}{4}x^{-1}\right)^r$ $= \binom{9}{r}\left(x^{18-2r}\right)\left(\frac{-k}{4}\right)^r\left(x\right)^{-r}$ and $(-k)^r = (-1)^r k^r$ Actual: $(-1)^r(k)^r$ $= \binom{9}{r}\left(x^{18-3r}\right)\left(\frac{-k}{4}\right)^r$ ---M1(x^{18-3r} seen) Term independent of x $18-3r = 0$ ---M1 $r = 6$ $\binom{9}{6}\left(\frac{-k}{4}\right)^6 = 5376$ $84 \times \frac{1}{4096} \times k^6 = 5376$ ---M1 $k^6 = 5376 \div \frac{84}{4096}$ $k^6 = 262144$ $k = \sqrt[6]{262144}$ $= \pm 8$ ---A1	4 marks	

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(b)	$\left(1-9x^9\right)\left(x^2-\frac{2}{x}\right)^9$ Term in x^9 in $\left(x^2-\frac{2}{x}\right)^9$ $18-3r=9---\text{M1}$ $r=3$ The term in x^9 is $\binom{9}{3}x^9(-2)^3$ $=84\times x^9\times -8$ $=-672x^9---\text{A1}$ $\left(1-9x^9\right)\left(...-672x^9+5376+...\right)$ Coeff of $x^3=-9(5376)+1(-672)$ $=-49056---\text{A1}$ Also accept The term in x^9 is $\binom{9}{3}x^9(2)^3$ $=84\times x^9\times 8$ $=672x^9---\text{A1}$ $\left(1-9x^9\right)\left(...672x^9+5376+...\right)$ Coeff of $x^3=-9(5376)+1(672)$ $=-47712$	**3 marks**	
3(a)	$v=3\cos 2t+4\sin 2t$ Let $v=0---\text{M1}$ $4\sin 2t=-3\cos 2t$ $\tan 2t=-\frac{3}{4}$ basic angle $2t=\tan^{-1}\left(\frac{3}{4}\right)---\text{M1}$ $=0.643501$ radian $2t=\pi-0.643501$ $t=1.249046$ sec $=1.25$ sec $---\text{A1}$ Some gave "degrees"	**3 marks**	

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(a few forgot to state).

Questions 3(b)–4

(b)	$2t = \pi - 0.643501, 2\pi - 0.643501, 3\pi - 0.643501$ ---M1 $t = 1.24905, 2.81984, 4.39064$ $v = 3\cos 2t + 4\sin 2t$ $s = \int (3\cos 2t + 4\sin 2t)dt$ $= \frac{3\sin 2t}{2} - \frac{4\cos 2t}{2} + c$ ---M1 At $t = 0, s = 0$ $0 = \frac{3}{2}(0) - 2(1) + c$ $c = 2$ ---A1 $s = \frac{3\sin 2t}{2} - 2\cos 2t + 2$ At $t = 1.24905$s, $s = 4.5$ m At $t = 2.81984$s, $s = -0.5$ m At $t = 4$s, $s = 3.775037$ m } ---M1 (All 3 parts to be accurate) Total distance travelled $= 4.5 + (4.5+0.5) + (0.5 + 3.775037)$ ---M1 $= 13.8$ m ---A1	**6 marks**	
4(a)	$P = \frac{a}{3 + 2e^{-0.18t}}$ At $t = 0, P = 27$ $27 = \frac{a}{3+2}$ ---A1 $a = 135$	**1 mark**	
(b)	$35 = \frac{135}{3 + 2e^{-0.18t}}$ $35(3 + 2e^{-0.18t}) = 135$ $3 + 2e^{-0.18t} = \frac{135}{35}$ $2e^{-0.18t} = \frac{135}{35} - 3$ $e^{-0.18t} = \frac{3}{7}$ ---M1 $\ln e^{-0.18t} = \ln \frac{3}{7}$ ---M1 $-0.18t = \ln \frac{3}{7}$ $t = 4.71$ After 5 years ---A1	**3 marks**	

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Questions 4(c)–5(a)(i)

(c)(i)	$P = \frac{135}{3+2e^{\lambda t}}$ At $t=5, P=40$ $40 = \frac{135}{3+2e^{5\lambda}}$ ---M1 $40(3+2e^{5\lambda}) = 135$ $3+2e^{5\lambda} = \frac{135}{40}$ $2e^{5\lambda} = \frac{135}{40} - 3$ $e^{5\lambda} = \frac{1}{2}\left(\frac{135}{40} - 3\right)$ $e^{5\lambda} = \frac{3}{16}$ ---M1 $\ln e^{5\lambda} = \ln\frac{3}{16}$ $5\lambda = \ln\frac{3}{16}$ $\lambda = -0.335$ ---A1	**3 marks**	
(ii)	$P_n = \frac{135}{3+2e^{\lambda t}}$ At $t \to \infty, e^{\lambda t} \to 0$ $2e^{\lambda t} \to 0$ $3+2e^{\lambda t} \to 3$ $P_n \to \frac{135}{3}$ ---B1 P_n approaches 45 as t becomes very large.	**1 mark**	
5(a)(i)	$f(-\frac{2}{3}) = 6\left(-\frac{2}{3}\right)^3 - 8\left(-\frac{2}{3}\right)^2 - \frac{2}{3} + 6$ $= -\frac{16}{9} - \frac{32}{9} - \frac{2}{3} + 6$ $= 0$ ---M1 By factor theorem, $(3x+2)$ is a factor of $f(x)$. Method 1 $6x^3 - 8x^2 + x + 6 = (3x+2)(2x^2 + bx + 3)$ ---M1 Compare term in x^2: $3bx^2 + 4x^2 = -8x^2$ ---M1 $3b + 4 = -8$ $3b = -12$ $b = -4$ ---A1 $f(x) = (3x+2)(2x^2 - 4x + 3)$ ---A1	**5 marks**	

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Question 5(a) Method 2 – 5(b)(i)

Method 2

$$3x+2\overline{)6x^3-8x^2+x+6}$$

$$\begin{array}{r} 2x^2-4x-3 \\ 3x+2\overline{)6x^3-8x^2+x+6} \\ -(6x^3+4x^2) \quad \text{---M1} \\ \hline -12x^2+x+6 \\ -(-12x^2-8x) \quad \text{---M1} \\ \hline 9x+6 \\ -(9x+6) \quad \text{---A1} \\ \hline 0 \end{array}$$

$f(x)=(3x+2)(2x^2-4x+3)$ ---A1

(ii)

$f(x)=(3x+2)(2x^2-4x+3)$

$(2x^2-4x+3)=0$

Discriminant $=(-4)^2-4(2)(3)$

$=16-24$

$=-8<0$ (No real roots) ---M1

$3x+2=0$

$x=-\frac{2}{3}$ ---A1

Number of real roots = 1

2 marks

(b)(i)

$4(3)^{2p}+(3)^{p}+5=8$ Some cannot see the quadratic equation!

Let $(3)^p=y$

$4y^2+y+5=8$ ---M1

$4y^2+y-3=0$

$(4y-3)(y+1)=0$

$y=\frac{3}{4}$ or $y=-1$

$(3)^p=\frac{3}{4}$ or $(3)^p=-1$ (undefined)

$\lg(3)^p=\lg\frac{3}{4}$ ---M1

$p\lg 3=\lg\frac{3}{4}$

$p=\lg\frac{3}{4}\div\lg 3$

$=-0.262$ ---A1

3 marks

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Questions 5(b)(ii)–6(b)

(b) (ii)	$p\lg 3 = \lg\frac{3}{4}$ $p\lg 3 = \lg 3 - \lg 4$ $p = \frac{\lg 3 - \lg 4}{\lg 3}$ ---M1 $= 1 - \frac{\lg 4}{\lg 3}$ $= 1 - \log_3 4$ $a = 3, b = 4$ ---A1,0	**2 marks**	
6(a)	$\frac{1}{3}\pi r^2 h = 110\pi$ $\frac{1}{3} r^2 h = 110$ $r^2 h = 330$ $h = \frac{330}{r^2}$ ---B1	**1 mark**	
6(b)	$l^2 = r^2 + h^2$ $l = \sqrt{r^2 + \frac{108900}{r^4}}$ ---M1 $= \sqrt{\frac{r^6 + 108900}{r^4}}$ $= \frac{\sqrt{r^6 + 108900}}{r^2}$ $A = \pi r l$ $= \frac{\pi r\sqrt{r^6 + 108900}}{r^2}$ ---M1 $= \frac{\pi\sqrt{r^6 + 108900}}{r}$ $C = \frac{1.15\pi\sqrt{r^6 + 108900}}{r}$ ---A1	**3 marks**	Some forgot the word "open" and included area of πr^2 !!

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Questions 6(c)–7(a)

(c)

$$C = \frac{1.15\pi\sqrt{r^6+108900}}{r}$$

$$\frac{dC}{dr} = \frac{r\frac{d}{dr}\left(1.15\pi\sqrt{r^6+108900}\right)-\left(1.15\pi\sqrt{r^6+108900}\right)\frac{d}{dr}r}{r^2}$$

$$= \frac{r\times 1.15\pi\times\frac{1}{2}\left(r^6+108900\right)^{-\frac{1}{2}}\left(6r^5\right)-\left(1.15\pi\sqrt{r^6+108900}\right)}{r^2} \quad ---\text{M1}$$

$$= \frac{3.45\pi r^6\left(r^6+108900\right)^{-\frac{1}{2}}-1.15\pi\left(r^6+108900\right)^{\frac{1}{2}}}{r^2}$$

$$= \frac{\left(r^6+108900\right)^{-\frac{1}{2}}\left[3.45\pi r^6-1.15\pi\left(r^6+108900\right)\right]}{r^2}$$

$$= \frac{\left[2.3\pi r^6-125235\pi\right]}{r^2\sqrt{r^6+108900}} \quad ---\text{M1}$$

Let $\frac{dC}{dr}=0$

$$2.3\pi r^6-125235\pi=0 \quad ---\text{M1}$$

$$r^6 = \frac{125235\pi}{2.3\pi}$$

$$= 54450$$

$$r = \sqrt[6]{54450}$$

$$= 6.16 \text{ cm} \quad ---\text{A1}$$

4 marks

(d)

r	6.10	6.16	6.2
$\frac{dC}{dr}$	−1.42	0	1.09
Shape of tangent	\	—	/

values must be shown !

—B1(calculated values to be reflected)

Therefore, the cost is minimum.

1 mark

7(a)

Let $\angle BAD = x$

$\angle BCD = 180^\circ - x$ (Angles in opposite segments)

$\angle DCF = 180^\circ - \angle BCD$

$= 180^\circ - (180^\circ - x)$ (Adjacent angles in a straight line)

$= x$

} B1

1 mark

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Questions 7(b)(c)–8(a)

(b)

$AB = CF$ (Given)
$\angle BAD = FCD$ (From (i)) ----M1

Let $\angle DAE = y$
$= \angle DBA$ (Tangent-chord theorem) ----M1

$\angle DFC = \angle DAE = y$ (Alternating angles, BF is parallel to AE)

$\angle DFC = \angle DBA = y$

ΔADB is congruent to ΔCDF (ASA congruency) ----A1

3 marks

(c)

From (i)

ΔADB is congruent to ΔCDF (ASA congruency)

$DB = DF$ ----M1

ΔDFB is an isosceles triangle

$\angle DFB = \angle DFB = y$

$\therefore \angle ABC = \angle ABD + \angle DBC$ ----A1

$= 2y$

2 marks

8(a)

$$y = 3(2x-1)^{-1} + \frac{1}{3}x$$

$$\frac{dy}{dx} = -3(2x-1)^{-2}(2) + \frac{1}{3}$$

$$= \frac{-6}{(2x-1)^2} + \frac{1}{3} \text{ ---M1}$$

At $x = a$, gradient of tangent $= -\frac{7}{6}$

$$\frac{-6}{(2a-1)^2} + \frac{1}{3} = -\frac{7}{6} \text{ ---M1}$$

$$\frac{-6}{(2a-1)^2} = -\frac{7}{6} - \frac{1}{3}$$

$$\frac{-6}{(2a-1)^2} = -\frac{9}{6}$$

$$\frac{1}{(2a-1)^2} = \frac{9}{36}$$

$$(2a-1)^2 = 4$$

$(2a-1) = 2$ or $(2a-1) = -2$ ---M1

$a = \frac{3}{2}$ ---A1 or $a = -\frac{1}{2}$ (rej as $a > 0$)

$$y = \frac{3}{2\left(\frac{3}{2}\right)-1} + \frac{1}{3}\left(\frac{3}{2}\right)$$

$$= \frac{3}{2} + \frac{1}{2}$$

$$= 2$$

$\therefore b = 2$ ---A1

5 marks

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Question 8(b)

(b)

Equation of normal at P

$y = \frac{6}{7}x + c$

Sub $(\frac{3}{2}, 2)$

$2 = \frac{6}{7}\left(\frac{3}{2}\right) + c$ ---M1

$2 = \frac{18}{14} + c$

$c = 2 - \frac{9}{7}$

$= \frac{5}{7}$

$y = \frac{6}{7}x + \frac{5}{7}$

$7y = 6x + 5$

To find x – coordinate of A

Let $y = 0$

$\frac{6}{7}x + \frac{5}{7} = 0$

$\frac{6}{7}x = -\frac{5}{7}$

$x = -\frac{5}{6}$ ---A1

Area of triangle

$= \frac{1}{2} \times \left(\frac{3}{2} + \frac{5}{6}\right) \times 2$

$= \frac{7}{3}$ units^2

$\int_{1.5}^{4} \left(\frac{3}{2x-1} + \frac{1}{3}x\right) dx$

$= \left[\frac{3}{2}\ln(2x-1) + \frac{x^2}{6}\right]_{1.5}^{4}$ ---M1

$= \left[\frac{3}{2}\ln 7 + \frac{4^2}{6}\right] - \left[\frac{3}{2}\ln 2 + \frac{1.5^2}{6}\right]$ ---M1

$= \frac{3}{2}\ln 7 - \frac{3}{2}\ln 2 + \frac{16}{6} - \frac{1.5^2}{6}$

$= 4.17$ units^2

Total area

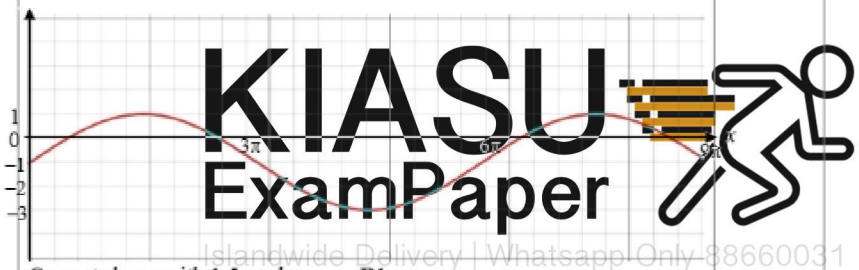
$= \frac{7}{3} + 4.17$

$= 6.50$ units^2 ---A1

5 marks

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Questions 9–10

9(a)	Amplitude $= 2$ ---B1 Period $= 2\pi \div \frac{1}{3}$ $= 6\pi$ or 1080° ---B1		
(b)	y  Correct shape with 1.5 cycles ---B1 Points $(0,-1),(3\pi,-1),(6\pi,-1),(9\pi,-1)$ should be seen---B1 Max/Min points $(1.5\pi,1),(4.5\pi,-3),(7.5\pi,1)$ should be seen---B1		
(c)	$\beta = 3\pi - \alpha$ ---B1		
(d)	$-1 \le p < 1$ ---B1		
10(a)	In ΔPAS, $\sin\theta = \frac{AS}{1.6}$ --- $AS = 1.6\sin\theta$ } M1 In ΔRBS, $\cos\theta = \frac{BS}{2.5}$ --- $BS = 2.5\cos\theta$ $AB = 2.5\cos\theta + 1.6\sin\theta$ ---A1	**2 marks**	
(b)	$R = \sqrt{2.5^2 + 1.6^2}$ ---M1 $= \sqrt{\frac{881}{100}}$ $= \frac{\sqrt{881}}{10}$ $\tan\alpha = \frac{1.6}{2.5}$ ---M1 $\alpha = 32.6192^\circ$ $l = \frac{\sqrt{881}}{10}\cos(\theta - 32.6^\circ)$ ---A1,A1	**4 marks**	
(c)	$\frac{\sqrt{881}}{10}\cos(\theta - 32.6192^\circ) = 2.4$ $\cos(\theta - 32.6192^\circ) = 2.4 \div \frac{\sqrt{881}}{10}$ ---M1 basic angle $(\theta - 32.6192^\circ) = 36.0425^\circ$ ---M1 $\theta = 68.7^\circ$ ---A1 Some cal. 68.6 due to not using working values.	**3 marks**	

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Question 11

11(a)

x	5	10	15	20	25
lgx	0.70	1	1.18	1.30	1.40
$\lg(P-12)$	1.37	1.75	1.80	2.13	2.25

$---$A2,1,0

Correct points plotted with a straight line graph drawn $---$A1

3 marks

(b)

$\lg(P-12)=2\pm0.05---$M1

$P-12=10^2$

$P=112---$A1

2 marks

(c)

$P=ax^n+12$

$P-12=ax^n$

$\lg(P-12)=\lg ax^n---$M1

$\lg(P-12)=\lg a+\lg x^n$

$\lg(P-12)=\lg a+n\lg x$

$n=$ gradient

$=\dfrac{1.75-0.48}{1}---$M1

$=1.27\pm0.1---$A1

$\lg a=0.48\pm0.07---$M1

$a=10^{0.48}$

$=3.02---$A1

5 marks

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O

PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS 4049/01

Paper 1

28 August 2026

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

Total	**/90**

Checked by student: …… Date: ……

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}\, b + \binom{n}{2} a^{n-2}\, b^2 + \ldots + \binom{n}{r} a^{n-r}\, b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n\,(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The percentage Gross Domestic Product (GDP) growth of a country from 2014 to 2024 can be modelled by the quadratic equation $G(t) = 0.7t^2 - 7t + 11.2$, where t is the number of years after 2014.

(a) By completing the square, find the year in which the percentage growth is the lowest, and state this minimum percentage GDP growth. [4]

(b) Explain why this model may not be suitable for predicting the country's percentage GDP growth in the long run. [1]

2 **Do not use a calculator in this question.**

It is given that $\sqrt{2a} = \sqrt{b} + 12$, where a and b are positive real numbers.

(a) Show that $2a - 3b = 216 - 2\left(\sqrt{b} - 6\right)^2$. [4]

(b) Hence state the least possible positive value of $\dfrac{1}{2a - 3b}$. [1]

3 **(a)** Express $\frac{1}{n(n+1)}$ in partial fractions. [3]

(b) Hence show that $\frac{1}{3(4)}+\frac{1}{4(5)}+\frac{1}{5(6)}+...+\frac{1}{(n-1)n}+\frac{1}{n(n+1)}=\frac{n-2}{3(n+1)}$. [3]

4 **(a)** Find the coefficient of x^3 in the binomial expansion of $\left(\frac{1}{x}-\frac{x^2}{2}\right)^{12}$. [3]

(b) The coefficients of x and x^2 in the binomial expansion of $(3+2x)^n$ are equal. Find the value of n. [4]

5 **(a)** The graph of $y = a + \log_b x$, where a and b are constants, passes through the points with coordinates $(9,0)$ and $(243,3)$.

Find the values of a and b, and hence express x in terms of y. [5]

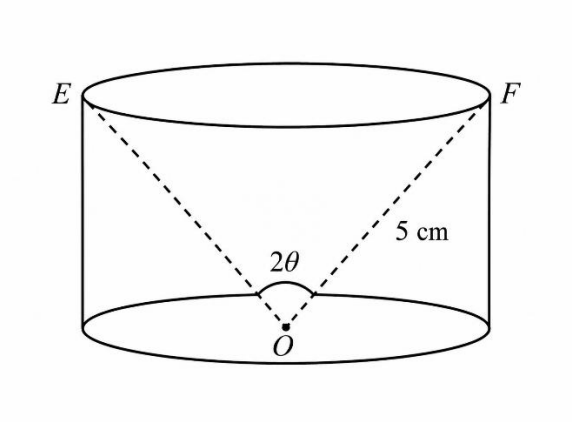
(b) One student was infected with HFMD in a school of 2900 students.
The spread of the disease through the school population was given by $N = \frac{2900}{1+2899e^{-0.65t}}$, where N was the total number of students infected after t days. The school would suspend lessons for a week when more than 40% of the students were affected.

Find, correct to the nearest whole number, the number of days after the first student was infected with HFMD that the school would suspend lessons. [4]

6 **(a)** Prove that $\tan x + \dfrac{1}{\tan x} = \dfrac{1}{\sin x \cos x}$. [2]

(b) Hence solve $\dfrac{2}{\sin x \cos x} = 1 + 3\tan x$ for $0 \leq x \leq \pi$. [4]

7



The diagram shows an open cylinder tank with O as the centre of the base and $0° < \theta < 90°$. It is given that angle $EOF = 2\theta$ and $OF = 5$ cm.

(a) Prove that the total surface area of the cylinder is

$$S = \frac{25\pi}{2}\left(2\sin 2\theta - \cos 2\theta + 1\right).$$ [4]

(b) Express S in the form $\frac{25\pi}{2}\left(R\sin\left(2\theta-\alpha\right)+1\right)$, where $R > 0$ and α is an acute angle. [3]

(c) Hence find the maximum total surface area of the cylindrical tank and the corresponding value of θ. [2]

8 The chickens in a farm are infected by a certain bird flu. The number of chickens (in thousands) in the farm is modelled by $N = \dfrac{27}{2 + \alpha t e^{\beta t}}$, where t is the number of days elapsed since the start of the spread of the bird flu and α and β are constants.

(a) Express $\ln\left(\dfrac{27 - 2N}{Nt}\right)$ as a linear function of t in terms of α and β. [4]

(b) It is given that the slope and the intercept on the horizontal axis of the graph of the linear function obtained in **(a)** are -0.1 and $10\ln 0.03$ respectively.

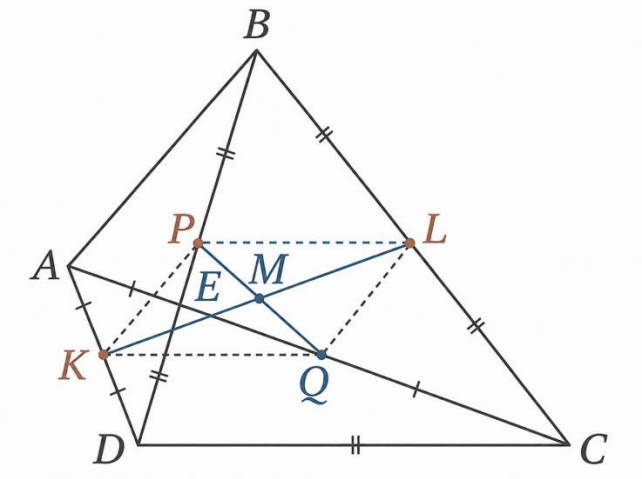
Find α and β. [3]

9 The points A and B have coordinates $(5,7)$ and $(13,1)$ respectively. A point P moves such that $PA = PB$. The set of all possible positions of P satisfying $PA = PB$ forms a straight line, L.

(a) Show that the equation of L is given by $y = \frac{4}{3}x - 8$. [3]

(b) L intersects the x-axis at H and the y-axis at K.
A circle C passes through the three points O, H and K, where O is the origin.
Linabell claims that the circumference of C is greater than 30 units.
Determine if Linabell is correct. [4]

10



In the diagram, AB, BC, CD, DA, AC, and DB are walkways connecting four blocks of apartments. An urban planner plans to build some facilities at K, L, P and Q, which are midpoints of AD, BC, BD and AC respectively.

Show that

(a) $PK = LQ$, [3]

(b) KL bisects PQ. [4]

11 A curve, C, has equation $y = (2x+8)^{\frac{3}{2}} + 3x^2$, where $x > -4$.

(a) Find $\frac{dy}{dx}$. [1]

(b) Duffy claims that two of the tangents to C are parallel to the straight line $6x + y + 4 = 0$. Determine if Duffy is correct. Show your working clearly. [6]

12 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^3 2x\right) = 6\sin^2 2x\cos 2x.$ [1]

(b) Hence find $\int \cos^3 2x \, \mathrm{d}x.$ [5]

13 A particle is travelling in a straight line. It passes a fixed point A with a velocity of -7 m/s. Its acceleration, after passing A is given by $a = 8 - 2t$ for $0 \leq t \leq T$.
On reaching its maximum speed at time T seconds, the particle begins to decelerate at a constant rate of 1.5 m/s^2 until it comes to a rest at point B.

(a) Show that the particle first came to instantaneous rest when $t = 1$. [3]

(b) Find the maximum velocity attained by the particle and the value of T. [2]

(c) Find the total distance travelled by the particle from A to B. [4]

END OF PAPER

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PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS 4049/02

Paper 2

31 August 2026

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

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You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

Total	**/90**

Checked by student: …… Date: ……

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}\, b + \binom{n}{2} a^{n-2}\, b^2 + \ldots + \binom{n}{r} a^{n-r}\, b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n\,(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The simultaneous equations $y = x^2 + px - 6$ and $y = 2x + \frac{3}{2}$ have a common solution for which $x = 3$. Find the value of p, and hence solve the simultaneous equations. [5]

2 **(a)** **(i)** In an architectural design, a support beam is modelled by the line

$$y = 2x + k$$

and a parabolic arch is modelled by

$$y = x^2 - 4x + 7.$$

The beam is to touch, but not cut, the arch. Find the value of k. [3]

(ii) The beam is then raised vertically by 3 units from this tangent position. Without drawing or referring to a graph, determine the number of points at which the beam intersects the arch. [2]

(b) Represent the solution set of $(2x+1)^2 \geq 4x+8$ on a number line. [3]

3 A cubic polynomial is given as $\mathrm{P}(x) = x^3 + ax^2 + bx + c$. The graph of $y = \mathrm{P}(x)$ cuts the y-axis at $(0, 2)$. The remainder when $\mathrm{P}(x)$ is divided by $x + 1$ is -3, and $x - 2$ is a factor of $\mathrm{P}(x)$.

(a) Find the values of a, b and c. [4]

(b) Hence solve the equation $\mathrm{P}(x) = 0$, leaving your answers in exact form. [3]

4 The amount of medicine remaining in a patient's bloodstream t hours after it is taken is M mg. It is modelled by the equation

$$\ln\left(\frac{M}{5}\right) = \ln 16 - \frac{t}{3}\ln 2 .$$

(a) Show that the model can be written in the form

$$M = 80e^{-\frac{t}{3}\ln 2} .$$ [2]

(b) Find the time taken for the medicine to decrease to 20 mg. [3]

(c) A second dose should be taken only when the amount of medicine remaining in the bloodstream has fallen to less than 15 mg.

A patient takes the medicine at 07 00. The patient asks whether the second dose can be taken at 14 20.

Use the model to determine whether the second dose can be taken at 14 20.
Give a reason for your answer. [3]

5 The graph of $y = a\cos(bx) + c$ has adjacent minimum points at $(0, -1)$ and $(4\pi, -1)$. It also has a maximum point at $(2\pi, 5)$.

(a) Find the values of a, b and c. [3]

(b) Hence sketch the graph of $y = a\cos(bx) + c$ for $0 \leq x \leq \pi$. [2]

6 **(a)** Without using a calculator, show that $\tan\frac{\pi}{12} = 2 - \sqrt{3}$. [2]

(b) Show that $\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan\theta}{1 + \tan\theta}$. [2]

A rotating stage turns through an angle θ radians, where $0 \le \theta \le \frac{\pi}{4}$.

A sensor signal S is given by $S = \frac{1 - \tan\theta}{1 + \tan\theta}$. The sensor is aligned when $S \le 2 - \sqrt{3}$.

(c) Using the results in **(a)** and **(b)**, find the range of values of θ for which the sensor is aligned. [3]

(d) The rotating stage moves uniformly from $\theta = 0$ to $\theta = \frac{\pi}{4}$ in 24 seconds.

The locking mechanism is activated only if the sensor remains aligned continuously for more than 7.5 seconds.

Determine whether the locking mechanism is activated. Justify your answer. [2]

7 The points $A(-3,4)$, $B(5,0)$ and $C(1,-2)$ are joined to form triangle ABC. The perpendicular from C to AB meets AB at D. The point E lies on AB produced, such that $CE = CA$.

(a) Find the coordinates of point D. [4]

(b) Find the coordinates of point E. [3]

(c) Find the area of triangle ACE. [2]

8 The curve C has equation $y = x^3 + ax^2 + bx + 4$, where a and b are constants. It is given that C has a stationary point when $x = 1$. The tangent to C at $x = 2$ is parallel to the line $y = -3x + 7$.

(a) Find the values of a and b. [4]

(b) Find the coordinates of all the stationary points of C, and determine their nature. [5]

9 The radius, r cm, of a circular chemical stain t seconds after it starts spreading is modelled by $r = 0.04t^2 + 0.6t$ and $0 \leq t \leq 15$.

(a) Find $\frac{\mathrm{d}A}{\mathrm{d}t}$ in terms of t, where A is the area of the stain. [3]

(b) Find the rate of increase of the area of the stain when $t = 10$. [2]

(c) For safety reasons, the laboratory has an automatic alert system. The system is activated if the area of the chemical stain is increasing at a rate greater than 100 cm^2/s.

This is because the cleaning team needs to respond immediately before the stain spreads beyond the containment mat.

Determine whether the alert system would already have been activated by the time $t = 14$.
Justify your answer clearly. [3]

10 A clinic uses an e-bike courier to deliver temperature-sensitive medicine to a care centre 12 km away.

The courier travels against a steady headwind. When riding at a speed of v km/h relative to the air, where $v > 5$, the actual ground speed is $v - 5$ km/h.

The total energy used for the journey, E watt-hours (Wh) consists of two parts.

- Motor energy: $\frac{v^2}{80}$ Wh for every kilometre travelled
- Cooling box energy: 112.5 Wh for every hour of the journey

(a) Show that $E = \frac{3v^2}{20} + \frac{1350}{v-5}$. [3]

(b) Find the value of v for which E is stationary. [4]

(c) Determine whether this stationary value of E is a minimum. [3]

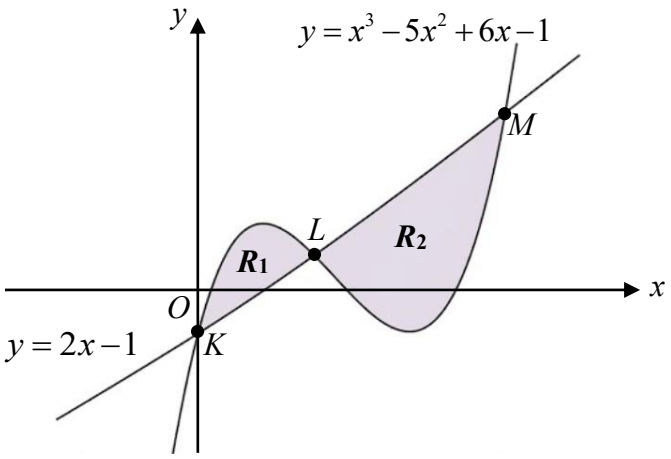
(d) For safety reasons, the courier is only allowed to choose a speed in the range

$$12 \le v \le 18 .$$

The battery can supply at most 155 Wh for this journey.

Determine the speed the courier should choose, and whether the battery limit is satisfied. Justify your answer. [2]

11 The curve C has equation $y = x^3 - 5x^2 + 6x - 1$. Line l has equation $y = 2x - 1$. The curve C and the line l intersect at the points K, L and M, as shown in the diagram. The shaded region consists of the two regions R_1 and R_2.



(a) Find the coordinates of K, L and M. [4]

(b) Find the total shaded area. [6]

END OF PAPER

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PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4049/01**

Paper 1 **28 August 2026**

2 hours 15 minutes

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The total number of marks for this paper is 90.

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Total	**/90**

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1 The percentage Gross Domestic Product (GDP) growth of a country from 2014 to 2024 can be modelled by the quadratic equation $G(t) = 0.7t^2 - 7t + 11.2$, where t is the number of years after 2014.

(a) By completing the square, find the year in which the percentage growth is the lowest, and state this minimum percentage GDP growth. [4]

$0.7t^2 - 7t + 11.2 = 0.7\left(t^2 - 10t\right) + 11.2$

$= 0.7\left(t - 5\right)^2 - 6.3$ ---- B1 for $0.7(t-5)^2$ and B1 for -6.3

Hence, the year in which the percentage growth is the lowest is **2019** ----- A1
and the **minimum percentage GDP growth** is -6.3% ----- A1

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(b) Explain why this model may not be suitable for predicting the country's percentage GDP growth in the long run. [1]

The model is **quadratic,** hence it will **eventually increase without bound as *t* becomes large.** This is unlikely to reflect real GDP growth in the long run as economic growth is affected by many changing factors. -----B1

2 **Do not use a calculator in this question.**

It is given that $\sqrt{2a} = \sqrt{b} + 12$, where a and b are positive real numbers.

(a) Show that $2a - 3b = 216 - 2\left(\sqrt{b} - 6\right)^2$. [4]

$2a = \left(\sqrt{b} + 12\right)^2 = b + 24\sqrt{b} + 144$ ----- M1 for squaring both sides

$2a - 3b = -2b + 24\sqrt{b} + 144$

$= 144 - 2(b - 12\sqrt{b})$ ----- M1 for factorising

$= 144 - 2(b - 12\sqrt{b} + (-6)^2 - (-6)^2)$ ----- M1 for completing the square

$= 144 - 2[(\sqrt{b} - 6)^2 - 36]$ ----- M1 for simplifying

$= 144 + 72 - 2(\sqrt{b} - 6)^2$

$= 216 - 2(\sqrt{b} - 6)^2$ ----- AG

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Alternative Method:

$2a = \left(\sqrt{b} + 12\right)^2 = b + 24\sqrt{b} + 144$ ----- M1 for expansion

$2a - 3b = -2b + 24\sqrt{b} + 144$

$216 - 2\left(\sqrt{b} - 6\right)^2 = 216 - 2(b - 12\sqrt{b} + 36)$ ----- M1 for expansion

$= 216 - 2b + 24\sqrt{b} - 72$

$= 144 - 2b + 24\sqrt{b}$ ----- (2) ----- M1 for simplifying

Since (1) = (2), ----- M1 for comparing

$2a - 3b = 216 - 2\left(\sqrt{b} - 6\right)^2$ ----- AG

(b) Hence state the least possible positive value of $\dfrac{1}{2a - 3b}$. [1]

Maximum value of $2a - 3b = 216$

Least value of $\dfrac{1}{2a - 3b} = \dfrac{1}{216}$ ----- B1

3 (a) Express $\frac{1}{n(n+1)}$ in partial fractions. [3]

Let $\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$ ------ M1

$1 = A(n+1) + Bn$

When $n = 0, A = 1$ ------ A1

When $n = -1, B = -1$ ------ A1

$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$ ------ minus 1 mark without final statement

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(b) Hence show that $\frac{1}{3(4)} + \frac{1}{4(5)} + \frac{1}{5(6)} + ... + \frac{1}{(n-1)n} + \frac{1}{n(n+1)} = \frac{n-2}{3(n+1)}$. [3]

$\frac{1}{3(4)} + \frac{1}{4(5)} + \frac{1}{5(6)} + ... + \frac{1}{n(n-1)} + \frac{1}{n(n+1)}$

$= \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \frac{1}{5} - \frac{1}{6} + ... + \frac{1}{n-1} - \frac{1}{n} + \frac{1}{n} - \frac{1}{n+1}$ ------ M1 for writing specific terms

$= \frac{1}{3} - \frac{1}{n+1}$ ------ M1 for simplifying

$= \frac{n+1-3}{3(n+1)}$ ------ M1 for same denominator

$= \frac{n-2}{3(n+1)}$ (shown) ------ AG

4 **(a)** Find the coefficient of x^3 in the binomial expansion of $\left(\frac{1}{x}-\frac{x^2}{2}\right)^{12}$. [3]

T_{r+1} in $\left(\frac{1}{x}-\frac{x^2}{2}\right)^{12}=\binom{12}{r}\left(x^{-1}\right)^{12-r}\left(-\frac{x^2}{2}\right)^r$ ------ M1 for general term

$=\binom{12}{r}(x)^{-12+r}(-2)^{-r}(x)^{2r}$

$=\binom{12}{r}(-2)^{-r}x^{-12+3r}$

$-12+3r=3$ ------ M1 for equating power of x to 3

$r=5$

Coefficient of $x^3=\binom{12}{5}(-2)^{-5}=-24\frac{3}{4}$ ------ A1

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(b) The coefficients of x and x^2 in the binomial expansion of $(3+2x)^n$ are equal. Find the value of n. [4]

$(3+2x)^n=3^n+\binom{n}{1}3^{n-1}(2x)+\binom{n}{2}3^{n-2}(2x)^2+\ldots$ ------ M1 for general term

$\binom{n}{1}3^{n-1}(2)=\binom{n}{2}3^{n-2}(4)$ ------ M1 for equating

$n3^{n-1}(2)=\frac{n(n-1)}{2}3^{n-2}(4)$ ------ M1 for simplifying both sides

$\frac{3^n}{3}=\frac{3^n}{9}(n-1)$

$1=\frac{n-1}{3}$

$n=4$ ------ A1

5 **(a)** The graph of $y = a + \log_b x$, where a and b are constants, passes through the points with coordinates $(9,0)$ and $(243,3)$.

Find the values of a and b, and hence express x in terms of y. [5]

$0 = a + \log_b 9$

$a = -\log_b 9$

$3 = a + \log_b 243$ ––––– M1 for substituting coordinates for either pair

$\log_b 243 - \log_b 9 = 3$ ––––– M1 for reducing to one unknown

$\log_b 27 = 3$

$b^3 = 27$

$b = 3$

$a = -2\log_b 3 = -2\log_3 3 = -2$

$b = 3, a = -2$ ––––– A1 for both

$y = -2 + \log_3 x$ ––––– M1

$y + 2 = \log_3 x$

$x = 3^{y+2}$ ––––– A1

Alternative Method:

$0 = a + \log_b 9$

$a = -2\log_b 3$ ––––– (1)

$3 = a + \log_b 243$

$a = 3 - 5\log_b 3$ ––––– (2) ––––– either (1) or (2) ––––– M1

$-2\log_b 3 = 3 - 5\log_b 3$ ––––– M1 for reducing to one unknown

$3\log_b 3 = 3$

$\log_b 3 = 1$

$b = 3, a = -2$ ––––– A1

$y = -2 + \log_3 x$ ––––– M1

$y + 2 = \log_3 x$

$x = 3^{y+2}$ ––––– A1

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(b) One student was infected with HFMD in a school of 2900 students.
The spread of the disease through the school population was given by $N = \frac{2900}{1+2899e^{-0.65t}}$, where N was the total number of students infected after t days. The school would suspend lessons for a week when more than 40% of the students were affected.

Find, correct to the nearest whole number, the number of days after the first student was infected with HFMD that the school would suspend lessons. [4]

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$40\% \text{ of } 2900 = 1160 \text{ ------M1}$

$\frac{2900}{1+2899e^{-0.65t}} = 1160 \text{ ------M1}$

$1+2899e^{-0.65t} = \frac{5}{2}$

$e^{-0.65t} = \frac{3}{5798}$

$\ln e^{-0.65t} = \ln\left(\frac{3}{5798}\right)$

$-0.65t = \ln\left(\frac{3}{5798}\right) \text{ ------M1}$

$t = \ln\left(\frac{3}{5798}\right) \div -0.65t$

$t = 11.641$

The number of days is estimated to be 12. ------A1

6 **(a)** Prove that $\tan x + \dfrac{1}{\tan x} = \dfrac{1}{\sin x \cos x}$. [2]

$$\tan x + \frac{1}{\tan x}$$

$$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \text{ ----- M1}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \text{ ----- M1}$$

$$= \frac{1}{\sin x \cos x} \text{ ----- AG}$$

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(b) Hence solve $\dfrac{2}{\sin x \cos x} = 1 + 3\tan x$ for $0 \le x \le \pi$. [4]

$$2\tan x + \frac{2}{\tan x} = 1 + 3\tan x \text{ ----- M1}$$

$$-\tan x + \frac{2}{\tan x} = 1$$

$$-\tan^2 x - \tan x + 2 = 0$$

$$\tan^2 x + \tan x - 2 = 0$$

$$(\tan x + 2)(\tan x - 1) = 0 \text{ ----- M1}$$

$$\tan x = -2 \text{ or } \tan x = 1$$

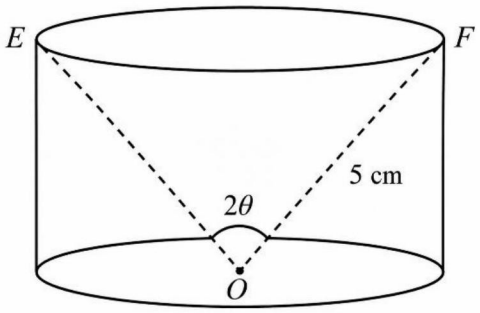
Basic angle = 1.1071 or $x = \dfrac{\pi}{4}$ ----- A1

$$x = \pi - 1.1071$$

$$= 2.03 \text{ ----- A1}$$

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7



The diagram shows an open cylinder tank with O as the centre of the base and $0° < \theta < 90°$. It is given that angle $EOF = 2\theta$ and $OF = 5$ cm.

(a) Prove that the total surface area of the cylinder is

$$S = \frac{25\pi}{2}(2\sin 2\theta - \cos 2\theta + 1).$$ [4]

Let r be the radius of the cylinder and h be the height of the cylinder.

$\sin\theta = \frac{r}{5}$ ------(1)------ M1

$r = 5\sin\theta$

$\cos\theta = \frac{h}{5}$ ------(2) or (1)------ M1

$h = 5\cos\theta$

$S = \pi(5\sin\theta)^2 + 2\pi(5\sin\theta)(5\cos\theta)$ ----- M1

$= 25\pi\sin^2\theta + 50\pi\sin\theta\cos\theta$

$= 25\pi\sin^2\theta + 25\pi\sin 2\theta$

$= 25\pi\left(\frac{1-\cos 2\theta}{2}\right) + 25\pi\sin 2\theta$ ----- M1

$= \frac{25\pi}{2}(2\sin 2\theta - \cos 2\theta + 1)$ ----- AG

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(b) Express S in the form $\frac{25\pi}{2}\left(R\sin\left(2\theta-\alpha\right)+1\right)$, where $R > 0$ and α is an acute angle. [3]

Let $R\sin\left(2\theta-\alpha\right)=2\sin 2\theta-\cos 2\theta$

$R=\sqrt{2^2+1^2}=\sqrt{5}$ ------M1

$\tan\alpha=\frac{1}{2}$

$\alpha=\tan^{-1}\frac{1}{2}=26.565°$ ------M1

$S=\frac{25\pi}{2}\left(\sqrt{5}\sin\left(2\theta-26.6°\right)+1\right)$ ------A1

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(c) Hence find the maximum total surface area of the cylindrical tank and the corresponding value of θ. [2]

Maximum $S=\frac{25\pi}{2}\left(\sqrt{5}+1\right)=127$ (3 s.f.) ------B1

$2\theta-26.565°=90°$

$\theta=58.3°$ (1d.p.) ------B1

8 The chickens in a farm are infected by a certain bird flu. The number of chickens (in thousands) in the farm is modelled by $N = \dfrac{27}{2+\alpha te^{\beta t}}$, where t is the number of days elapsed since the start of the spread of the bird flu and α and β are constants.

(a) Express $\ln\left(\dfrac{27-2N}{Nt}\right)$ as a linear function of t in terms of α and β. [4]

$N = \dfrac{27}{2+\alpha te^{\beta t}}$

$2N + N\alpha te^{\beta t} = 27$ ------ M1

$27 - 2N = N\alpha te^{\beta t}$

$\dfrac{27-2N}{Nt} = \alpha e^{\beta t}$ ------ M1

$\ln\left(\dfrac{27-2N}{Nt}\right) = \ln \alpha e^{\beta t}$ ------ M1

$= \ln\alpha + \beta t$ ------ A1

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(b) It is given that the slope and the intercept on the horizontal axis of the graph of the linear function obtained in **(a)** are -0.1 and $10\ln 0.03$ respectively.

Find α and β. [3]

$\beta = -0.1$ ----- B1

$0 = -0.1(10\ln 0.3) + \ln\alpha$ ----- M1

$\ln\alpha = \ln 0.03$

$\alpha = 0.03$ ----- A1

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9 The points A and B have coordinates $(5,7)$ and $(13,1)$ respectively. A point P moves such that $PA = PB$. The set of all possible positions of P satisfying $PA = PB$ forms a straight line, L.

(a) Show that the equation of L is given by $y = \frac{4}{3}x - 8$. [3]

Since $PA = PB$, P lies on the perpendicular bisector of AB.

Midpoint of $AB = \left(\frac{5+13}{2}, \frac{7+1}{2}\right) = (9,4)$ ----M1

Gradient of $AB = \frac{1-7}{13-5} = -\frac{3}{4}$ ----M1

Equation of L:

$y - 4 = \frac{4}{3}(x-9)$ -----M1

$y = \frac{4}{3}x - 8$ -----AG

Alternative Method:

Let $P(x, y)$

$\sqrt{(x-5)^2 + (y-7)^2} = \sqrt{(x-13)^2 + (y-1)^2}$ -----M1

$(x-5)^2 + (y-7)^2 = (x-13)^2 + (y-1)^2$

$x^2 - 10x + 25 + y^2 - 14y + 49 = x^2 - 26x + 169 + y^2 - 2y + 1$ -----M1

$-10x - 14y + 74 = -26x - 2y + 170$

$-16x + 12y + 96 = 0$

$-4x + 3y + 24 = 0$

$3y = 4x - 24$ -----M1

$y = \frac{4}{3}x - 8$ -----AG1

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(b) L intersects the x-axis at H and the y-axis at K.
A circle C passes through the three points O, H and K, where O is the origin.
Linabell claims that the circumference of C is greater than 30 units.
Determine if Linabell is correct. [4]

$H(6,0)$ – – – – – B1

$K(0,-8)$ – – – – – B1

Diameter of $C = HK$

$= \sqrt{(6-0)^2+(0-(-8))^2}$ – – – – – M1

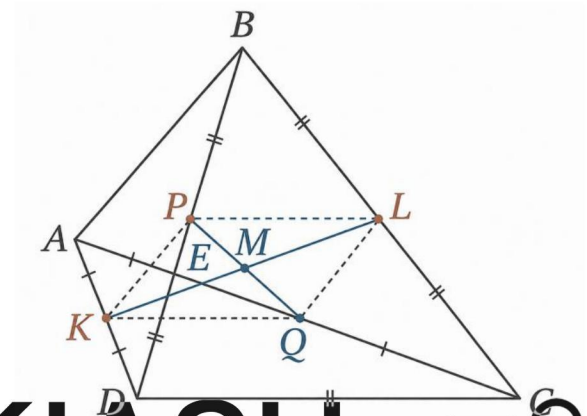
$= 10$

Circumference of $C = \pi d = 10\pi = 31.416 > 30$

Hence, Linabell is correct. – – – – – A1 with working

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10



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In the diagram, AB, BC, CD, DA, AC, and DB are walkways connecting four blocks of apartments. An urban planner plans to build some facilities at K, L, P and Q, which are midpoints of AD, BC, BD and AC respectively.

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Show that

(a) $PK = LQ$, [3]

In ΔADB,

$AK = KD$ and $BP = PD$ – – – – – M1

By Midpoint Theorem, $KP = \frac{1}{2} AB$ and $KP // AB$. – – – – – M1

In ΔACB,

$AQ = QC$

$BL = LC$

By Midpoint Theorem, $QL = \frac{1}{2} AB$ and $QL // AB$. – – – – – M1

$KP = QL$ (proven) – – – – – AG

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(b) KL bisects PQ. [4]

From (a), $KP = QL$

$KP // AB$ and $QL // AB$. Hence, $KP // QL$ – – – – – M1

Hence, $KPLQ$ is a parallelogram – – – – – M1

(a pair of opp. sides equal and parallel) – – – – – M1

Hence, KL bisects PQ. (diagonal bisects in parallelogram) – – – – – A1 with property

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11 A curve, C, has equation $y = (2x+8)^{\frac{3}{2}} + 3x^2$, where $x > -4$.

(a) Find $\frac{dy}{dx}$. [1]

$\frac{dy}{dx} = 3(2x+8)^{\frac{1}{2}} + 6x$ ----- B1

(b) Duffy claims that two of the tangents to C are parallel to the straight line $6x + y + 4 = 0$. Determine if Duffy is correct. Show your working clearly. [6]

$6x + y + 4 = 0$

$y = -6x - 4$

$\frac{dy}{dx} = -6$ ----- M1

$3\sqrt{2x+8} + 6x = -6$

$\sqrt{2x+8} + 2x = -2$

$\sqrt{2x+8} = -2 - 2x, x \le -1$ ----- M1

$2x + 8 = 4x^2 + 8x + 4$ ----- M1

$2x^2 + 3x - 2 = 0$

$(2x-1)(x+2) = 0$ ----- M1

$x = \frac{1}{2}$ or $x = -2$

When $x = \frac{1}{2}$, $3\sqrt{2x+8} + 6x = 6 \neq -6$.

When $x = -2$, $3\sqrt{2x+8} + 6x = -6$.

Hence, reject $x = \frac{1}{2}$ ----- A1 with rejection

Hence, Duffy is incorrect as there is only one solution. ----- A1

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12 (a) Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^3 2x\right) = 6\sin^2 2x\cos 2x.$ [1]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^3 2x\right) = 3\sin^2 2x\cos 2x(2) \text{ ----- M1}$$
$$= 6\sin^2 2x\cos 2x \text{ ----- AG}$$

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(b) Hence find $\int \cos^3 2x\ \mathrm{d}x.$ [5]

$$\int 6\sin^2 2x\cos 2x\ \mathrm{d}x = \sin^3 2x + c_1 \text{ ----- M1}$$
$$\int 6\left(1-\cos^2 2x\right)\cos 2x\ \mathrm{d}x = \sin^3 2x + c_1 \text{ ----- M1}$$
$$\int 6\cos 2x - 6\cos^2 2x\cos 2x\ \mathrm{d}x = \sin^3 2x + c_1$$
$$\int 6\cos 2x - 6\cos^3 2x\ \mathrm{d}x = \sin^3 2x + c_1$$
$$\int 6\cos^3 2x\ \mathrm{d}x = \int 6\cos 2x\ \mathrm{d}x - \sin^3 2x + c_1$$
$$\int \cos^3 2x\ \mathrm{d}x = \frac{1}{6}\left[\int 6\cos 2x\ \mathrm{d}x - \sin^3 2x + c_1\right] \text{ ----- M1}$$
$$= \frac{1}{6}\left(\frac{6\sin 2x}{2}\right) - \frac{\sin^3 2x}{6} + c_2 \text{ ----- M1}$$
$$= \frac{1}{2}\sin 2x - \frac{1}{6}\sin^3 2x + c \text{ ----- A1}$$

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13 A particle is travelling in a straight line. It passes a fixed point A with a velocity of -7 m/s. Its acceleration, after passing A is given by $a = 8 - 2t$ for $0 \le t \le T$.
On reaching its maximum speed at time T seconds, the particle begins to decelerate at a constant rate of 1.5 m/s^2 until it comes to a rest at point B.

(a) Show that the particle first came to instantaneous rest when $t = 1$. [3]

$a = 8 - 2t$

$v = \int 8 - 2t \, dt$ – – – – – M1

$= 8t - t^2 + c$

When $v = -7, t = 0, c = -7$

$v = 8t - t^2 - 7$

$t^2 - 8t + 7 = 0$

$(t-1)(t-7) = 0$ – – – – – M1

$t = 1$ or $t = 7$.

Hence, the particle first came to instantaneous rest at $t = 1$. (shown) – – – – – A1

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(b) Find the maximum velocity attained by the particle and the value of T. [2]

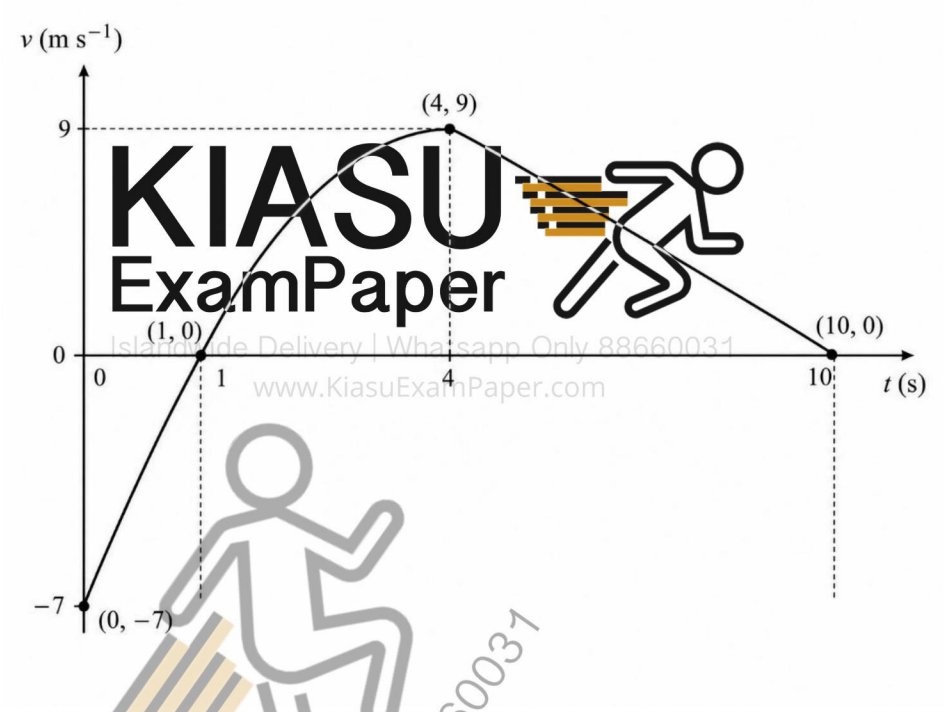
Maximum velocity attained when $a = 0$ – – – – – M1

$T = 4$

$v = 32 - 16 - 7$

$= 9$ m/s – – – – – A1

(c) Find the total distance travelled by the particle from A to B. [4]



Total distance travelled

$$= -\int_0^1 8t - t^2 - 7 \, dt + \int_1^4 8t - t^2 - 7 \, dt + \left(\frac{1}{2} \times 6 \times 9\right) \text{-----M1}$$

$$= -\left[\frac{8t^2}{2} - \frac{t^3}{3} - 7t\right]_0^1 + \left[\frac{8t^2}{2} - \frac{t^3}{3} - 7t\right]_1^4 + 27 \text{-----M1,B1 for 27}$$

$$= \frac{10}{3} + 18 + 27$$

$$= 48\frac{1}{3} \text{ m} \text{-----A1}$$

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END OF PAPER

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PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **31 August 2026**

2 hours 15 minutes

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Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

Total	**/90**

Checked by student: ..... Date: .....

This document consists of **25** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

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1 The simultaneous equations $y = x^2 + px - 6$ and $y = 2x + \frac{3}{2}$ have a common solution for which $x = 3$. Find the value of p, and hence solve the simultaneous equations. [5]

SOLUTIONS:

Given the common solution has $x = 3$,

$y = 2(3) + \frac{3}{2} = \frac{15}{2}$ ---------- [M1: find y when x =3]

Subst. into $y = x^2 + px - 6$:

$\frac{15}{2} = 3^2 + 3p - 6$

$p = \frac{3}{2}$ ---------- [A1]

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Hence

$x^2 + \frac{3}{2}x - 6 = 2x + \frac{3}{2}$ ---------- [M1: Substitution]

$2x^2 - x - 15 = 0$

$(2x + 5)(x - 3) = 0$

$x = 3$ or $x = -\frac{5}{2}$ ---------- [A1: Both x or y values]

Using $y = 2x + \frac{3}{2}$,

$x = 3,\ y = \frac{15}{2}$

$x = -\frac{5}{2},\ y = -\frac{7}{2}$ ---------- [A1: Both ordered pairs]

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2 (a) (i) In an architectural design, a support beam is modelled by the line

$$y = 2x + k$$

and a parabolic arch is modelled by

$$y = x^2 - 4x + 7 .$$

The beam is to touch, but not cut, the arch. Find the value of k. [3]

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SOLUTIONS:

$x^2 - 4x + 7 = 2x + k$ ---------- [M1: Form quadratic equation]

$x^2 - 6x + (7 - k) = 0$

For tangency, discriminant = 0.

$(-6)^2 - 4(1)(7 - k) = 0$ ---------- [M1: Use discriminant = 0]

$36 - 28 + 4k = 0$

$k = -2$ ---------- [A1]

(ii) The beam is then raised vertically by 3 units from this tangent position. Without drawing or referring to a graph, determine the number of points at which the beam intersects the arch. [2]

SOLUTIONS:

If raised vertically by 3 units, $k = 1$.

Discriminant = $(-6)^2 - 4(1)(6) = 12 > 0$ ---------- [M1: Finding value of discriminant]

Hence, there will be 2 points of intersection. ---------- [A1: State 2 intersections with valid reason]

(b) Represent the solution set of $(2x+1)^2 \geq 4x+8$ on a number line. [3]

SOLUTIONS:

$$(2x+1)^2 \geq 4x+8$$

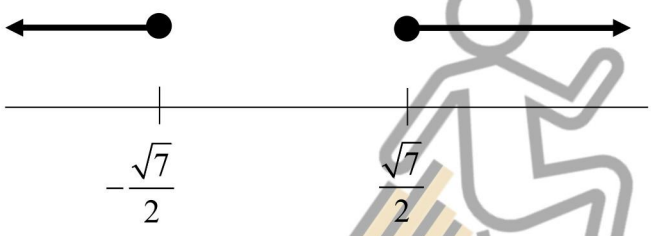
$$4x^2+4x+1 \geq 4x+8$$

$$4x^2-7 \geq 0 \quad \text{---------- [M1: Simplify to } \geq 0\text{]}$$

$$(2x-\sqrt{7})(2x+\sqrt{7}) \geq 0$$

$x \leq -\frac{\sqrt{7}}{2}$ or $x \geq \frac{\sqrt{7}}{2}$ ---------- [M1: Finding roots]

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---------- [A1: Correct number line]

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3 A cubic polynomial is given as $\mathrm{P}(x)=x^3+ax^2+bx+c$. The graph of $y=\mathrm{P}(x)$ cuts the y-axis at $(0,2)$. The remainder when $\mathrm{P}(x)$ is divided by $x+1$ is -3, and $x-2$ is a factor of $\mathrm{P}(x)$.

(a) Find the values of a, b and c. [4]

SOLUTIONS:

Since the graph cuts the y-axis at $(0,2)$,

$\mathrm{P}(0)=2 \quad \Rightarrow \quad c=2$ ---------- [B1: $c=2$]

Since $x-2$ is a factor,

$$\mathrm{P}(2)=0$$
$$8+4a+2b+2=0$$
$$2a+b=-5 \quad \text{--- (1)}$$

---------- [M1: Use of factor theorem]

The remainder when $\mathrm{P}(x)$ is divided by $x+1$ is -3, so

$$\mathrm{P}(-1)=-3$$
$$-1+a-b+2=-3$$
$$a-b=-4 \quad \text{--- (2)}$$

---------- [M1: Use of remainder theorem]

Solving (1) and (2), $a=-3,\ b=1,\ c=2$. ---------- [A1: Correct a, b and c]

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(b) Hence solve the equation $\mathrm{P}(x)=0$, leaving your answers in exact form. [3]

SOLUTIONS:

$\mathrm{P}(x)=x^3-3x^2+x+2$

Since $x-2$ is a factor, $\mathrm{P}(x)=(x-2)(x^2+dx-1)$.

Comparing coefficients of x,

$-2d-1=1$

$d=-1$

$\Rightarrow \quad \mathrm{P}(x)=(x-2)(x^2-x-1)$ ---------- [M1: Factorise using $x-2$]

Hence,

$(x-2)(x^2-x-1)=0$

$x-2=0 \quad$ or $\quad (x^2-x-1)=0$

$x=2 \quad$ or $\quad x=\dfrac{-(-1)\pm\sqrt{1^2-4(1)(-1)}}{2}$ ---------- [M1: Solve quadratic factor]

$=\dfrac{1+\sqrt{5}}{2}$

$x=2, \dfrac{1+\sqrt{5}}{2}, \dfrac{1-\sqrt{5}}{2}$ ---------- [A1: Exact roots]

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4 The amount of medicine remaining in a patient's bloodstream t hours after it is taken is M mg. It is modelled by the equation

$$\ln\left(\frac{M}{5}\right) = \ln 16 - \frac{t}{3}\ln 2 .$$

(a) Show that the model can be written in the form

$$M = 80e^{-\frac{t}{3}\ln 2} .$$ [2]

SOLUTIONS:

$$\ln\left(\frac{M}{5}\right) = \ln 16 - \frac{t}{3}\ln 2$$

$$\ln\left(\frac{M}{5}\right) = \ln 16 + \left(-\frac{t}{3}\ln 2\right)\ln e$$

$$\ln\left(\frac{M}{5}\right) = \ln 16 + \ln e^{-\frac{t}{3}\ln 2}$$

$$\ln\left(\frac{M}{5}\right) = \ln\left(16e^{-\frac{t}{3}\ln 2}\right)$$ ---------- [M1: Combine log terms]

$$\frac{M}{5} = 16e^{-\frac{t}{3}\ln 2}$$ ---------- [M1: Remove logarithm correctly]

$$M = 80e^{-\frac{t}{3}\ln 2}$$ (shown) ---------- [AG]

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(b) Find the time taken for the medicine to decrease to 20 mg. [3]

SOLUTIONS:

When $M = 20$,

$$80e^{-\frac{t}{3}\ln 2} = 20$$

$$e^{-\frac{t}{3}\ln 2} = \frac{1}{4} \quad \text{---------- [M1]}$$

$$\ln\left(e^{-\frac{t}{3}\ln 2}\right) = \ln\left(\frac{1}{4}\right)$$

$$-\frac{t}{3}\ln 2 = \ln\left(\frac{1}{4}\right) \quad \text{---------- [M1]}$$

$$t = \frac{-3\ln\left(\frac{1}{4}\right)}{\ln 2}$$

$$t = 6 \quad \text{---------- [A1]}$$

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Time taken is 6 hours.

(c) A second dose should be taken only when the amount of medicine remaining in the bloodstream has fallen to less than 15 mg.

A patient takes the medicine at 07 00. The patient asks whether the second dose can be taken at 14 20.

Use the model to determine whether the second dose can be taken at 14 20.
Give a reason for your answer. [3]

SOLUTIONS:

From 07 00 to 14 20, $t = \frac{22}{3}$.

$$M = 80e^{-\frac{22}{9}\ln 2} \quad \text{---------- [M1: Subst. } t = \frac{22}{3}\text{]}$$

$$= 14.7 \text{ mg} < 15 \quad \text{---------- [M1: Calculate } M \text{ and compare with 15 mg]}$$

Since the amount remaining has fallen below 15 mg,

the second dose **can** be taken at 14 20. ---------- [A1]

5 The graph of $y = a\cos(bx) + c$ has adjacent minimum points at $(0, -1)$ and $(4\pi, -1)$. It also has a maximum point at $(2\pi, 5)$.

(a) Find the values of a, b and c. [3]

SOLUTIONS:

Maximum value = 5 and minimum value = -1

$c = \dfrac{5+(-1)}{2} = 2$ ---------- [B1]

Amplitude = $\dfrac{5-(-1)}{2} = 3$ ---------- [B1]

Since $a < 0$, $a = -3$.

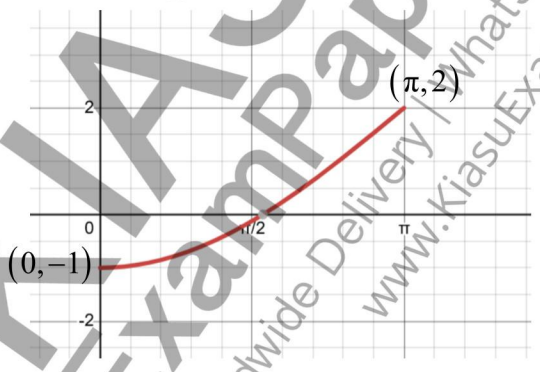
The period is the horizontal distance between adjacent minimum points, 4π.

$\dfrac{2\pi}{b} = 4\pi$

$b = \dfrac{1}{2}$ ---------- [B1]

(b) Hence, sketch the graph of $y = a\cos(bx) + c$ for $0 \le x \le \pi$. [2]

SOLUTIONS:



[B1: correct shape over smooth increasing quarter cosine curve]

[B1: correct endpoints]

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6 **(a)** Without using a calculator, show that $\tan\frac{\pi}{12} = 2-\sqrt{3}$. [2]

SOLUTIONS:

$$\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$$

$$= \frac{\tan\frac{\pi}{4}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{4}\tan\frac{\pi}{6}} \quad \text{---------- [M1]}$$

$$= \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{\left(\sqrt{3}-1\right)^2}{3-1} \quad \text{---------- [M1: Rationalise]}$$

$$= \frac{3-2\sqrt{3}+1}{2}$$

$$= 2-\sqrt{3} \quad \text{(shown)} \quad \text{---------- [AG]}$$

(b) Show that $\tan\left(\frac{\pi}{4}-\theta\right) = \frac{1-\tan\theta}{1+\tan\theta}$. [2]

SOLUTIONS:

$$\tan\left(\frac{\pi}{4}-\theta\right) = \frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta} \quad \text{---------- [M1: Use compound angle formula]}$$

$$= \frac{1-\tan\theta}{1+\tan\theta} = S \quad \text{(shown)} \quad \text{---------- [A1]}$$

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A rotating stage turns through an angle θ radians, where $0 \le \theta \le \frac{\pi}{4}$.

A sensor signal S is given by $S = \frac{1-\tan\theta}{1+\tan\theta}$. The sensor is aligned when $S \le 2-\sqrt{3}$.

(c) Using the results in **(a)** and **(b)**, find the range of values of θ for which the sensor is aligned. [3]

SOLUTIONS:

$$S \le 2-\sqrt{3}$$

$$S \le \tan\frac{\pi}{12}$$

$$\tan\left(\frac{\pi}{4}-\theta\right) \le \tan\frac{\pi}{12}$$ ---------- [M1]

$$\frac{\pi}{4}-\theta \le \frac{\pi}{12}$$

$$\theta \ge \frac{\pi}{6}$$ ---------- [M1]

Since $0 \le \theta \le \frac{\pi}{4}$,

range of values of θ: $\frac{\pi}{6} \le \theta \le \frac{\pi}{4}$ ---------- [B1]

(d) The rotating stage moves uniformly from $\theta = 0$ to $\theta = \frac{\pi}{4}$ in 24 seconds.

The locking mechanism is activated only if the sensor remains aligned continuously for more than 7.5 seconds.

Determine whether the locking mechanism is activated. Justify your answer. [2]

SOLUTIONS:

Aligned angular interval $= \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$

Total angular movement $= \frac{\pi}{4}$

Fraction of time aligned $= \frac{\frac{\pi}{12}}{\frac{\pi}{4}} = \frac{1}{3}$

Time aligned $= \frac{1}{3} \times 24 = 8$ seconds ---------- [M1: calculate aligned time 8 seconds]

Since 8 > 7.5, the locking mechanism is activated. ---------- [A1: Include all underlined parts]

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7 The points $A(-3,4)$, $B(5,0)$ and $C(1,-2)$ are joined to form triangle ABC. The perpendicular from C to AB meets AB at D. The point E lies on AB produced, such that $CE = CA$.

(a) Find the coordinates of point D. [4]

SOLUTIONS:

Gradient of $AB = \dfrac{0-4}{5-(-3)} = -\dfrac{1}{2}$ ---------- [M1]

Equation of AB:

$$y - 4 = -\frac{1}{2}(x+3)$$

$$x + 2y - 5 = 0 \quad \text{----- (1)}$$

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Equation of CD:

$$y + 2 = 2(x-1)$$

$$y = 2x - 4 \quad \text{----- (2)}$$ ---------- [M1]

Solving (1) and (2), $x = \dfrac{13}{5} = 2.6$ and $y = \dfrac{6}{5} = 1.2$. --------- [M1: Solve simultaneous eqns using elimination/substitution method]

D is $(2.6, 1.2)$. ---------- [A1]

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(b) Find the coordinates of point E. [3]

SOLUTIONS:

Equation of AB:

$$x+2y-5=0$$
$$x=5-2y$$

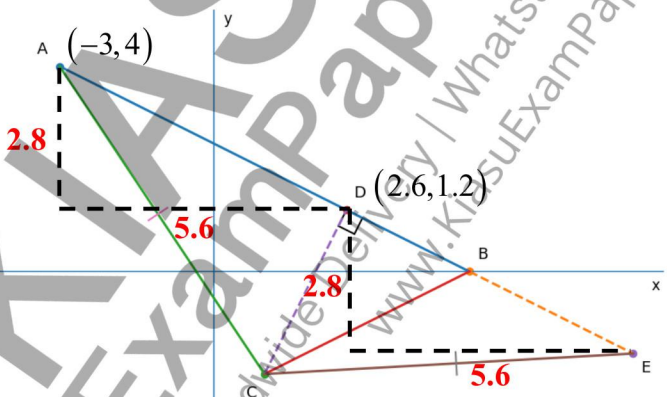
Since E lies on AB produced, let E be $(5-2y, y)$. ---------- [M1: Parameterise point E on AB]

Since $CE = CA$,

$$CE^2 = CA^2$$
$$(5-2y-1)^2+(y+2)^2=(1+3)^2+(-2-4)^2$$ ---------- [M1: Use $CE = CA$]
$$(4-2y)^2+(y+2)^2=16+36$$
$$16-16y+4y^2+y^2+4y+4=52$$
$$5y^2-12y+20=52$$
$$5y^2-12y-32=0$$
$$(5y+8)(y-4)=0$$
$$y=-\frac{8}{5}=-1.6 \quad \text{or} \quad y=4$$

Since E lies on AB produced, $y<0$, $y=-1.6$.
Point E is $(8.2, -1.6)$. ---------- [A1]

ALTERNATIVE SOLUTION:



Since $CA = CE$, $CD \perp AB$ and that E lies on AB produced, $AD = DE$.

----- [M1: Seen or implied]

$$x_E = 2.6+5.6=8.2$$
$$y_E = 1.2-2.8=-1.6$$

Point E is $(8.2, -1.6)$. ---- [A1, A1]

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(c) Find the area of triangle ACE. [2]

SOLUTIONS:

Area of triangle ACE

$$= \frac{1}{2}\begin{vmatrix} -3 & 1 & 8.2 & -3 \\ 4 & -2 & -1.6 & 4 \end{vmatrix}$$ ---------- [M1: Shoelace method o.e.]

$$= \frac{1}{2}(6 - 1.6 + 32.8 - 4 + 16.4 - 4.8)$$

$$= \frac{1}{2}(44.8)$$

$$= \frac{112}{5} = 22.4 \quad \text{sq. units}$$ ---------- [A1]

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8 The curve C has equation $y = x^3 + ax^2 + bx + 4$, where a and b are constants. It is given that C has a stationary point when $x = 1$. The tangent to C at $x = 2$ is parallel to the line $y = -3x + 7$.

(a) Find the values of a and b. [4]

SOLUTIONS:

$y = x^3 + ax^2 + bx + 4$

$\frac{dy}{dx} = 3x^2 + 2ax + b$ ---------- [M1]

Since C has a stationary point when $x = 1$,

$3 + 2a + b = 0$. ----- [M1: Use stationary condition at $x = 1$]

$2a + b = -3$. (1)

The tangent at $x = 2$ is parallel to

$$y = -3x + 7,$$

so the gradient is -3.

$12 + 4a + b = -3$ ----- [M1: Use gradient condition at $x = 2$]

$4a + b = -15$. (2)

Solving equations (1) and (2) simultaneously,

$\boxed{a = -6, b = 9}$ ---------- [A1]

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(b) Find the coordinates of all the stationary points of C, and determine their nature. [5]

SOLUTIONS:

$y = x^3 - 6x^2 + 9x + 4.$

$\frac{dy}{dx} = 3x^2 - 12x + 9$

$= 3(x^2 - 4x + 3)$

$= 3(x - 1)(x - 3).$

For stationary points,

$3(x - 1)(x - 3) = 0.$

$x = 1$ or $x = 3.$ ---------- [M1: Solve $\frac{dy}{dx} = 0$ to obtain values of x]

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So one stationary point is $(1,8)$.

The other stationary point is $(3,4)$.

$\frac{d^2y}{dx^2} = 6x - 12.$ ---------- [M1: Second derivative]

At $x = 1$, $\frac{d^2y}{dx^2} = 6(1) - 12 = -6 < 0,$ ---------- [M1: Sub point to 2nd derivative]

so $(1,8)$ is a maximum point. ---------- [A1: Point and nature]

At $x = 3$, $\frac{d^2y}{dx^2} = 6(3) - 12 = 6 > 0,$

so $(3,4)$ is a minimum point. ---------- [A1: Point and nature]

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9 The radius, r cm, of a circular chemical stain t seconds after it starts spreading is modelled by $r = 0.04t^2 + 0.6t$ and $0 \le t \le 15$.

(a) Find $\frac{\mathrm{d}A}{\mathrm{d}t}$ in terms of t, where A is the area of the stain. [3]

SOLUTIONS:

$r = 0.04t^2 + 0.6t$
$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$ ---------- [M1]
$\frac{dr}{dt} = 0.08t + 0.6$

Using the chain rule,

$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$
$= 2\pi r(0.08t + 0.6).$ ---------- [M1]

Substitute

$r = 0.04t^2 + 0.6t.$
$\frac{dA}{dt} = 2\pi(0.04t^2 + 0.6t)(0.08t + 0.6).$ ---------- [A1]

(b) Find the rate of increase of the area of the stain when $t = 10$. [2]

SOLUTIONS:

When $t = 10$,

$r = 0.04(10)^2 + 0.6(10)$
$= 4 + 6 = 10.$

$\frac{dr}{dt} = 0.08(10) + 0.6 = 1.4.$ ---------- [M1]

$\frac{dA}{dt} = 2\pi(10)(1.4)$
$= 28\pi = 88.0 \text{ cm}^2/\text{s}$ ---------- [A1]

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(c) For safety reasons, the laboratory has an automatic alert system. The system is activated if the area of the chemical stain is increasing at a rate greater than 100 cm^2/s.

This is because the cleaning team needs to respond immediately before the stain spreads beyond the containment mat.

Determine whether the alert system would already have been activated by the time $t = 14$. Justify your answer clearly. [3]

SOLUTIONS:

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When $t = 14$,

$r = 0.04(14)^2 + 0.6(14)$

$= 7.84 + 8.4$

$= 16.24$

$\frac{dr}{dt} = 0.08(14) + 0.6$ ---------- [M1]

$= 1.12 + 0.6$

$= 1.72.$

$\frac{dA}{dt} = 2\pi(16.24)(1.72)$

$= 55.8656\pi$

$\approx 175.5 \text{ cm}^2/\text{s}.$ ---------- [M1]

Since $175.5 > 100$, the area of the stain is increasing at a rate greater than 100 cm^2/s.
Therefore, the alert system would have been activated by the time $t = 14$.

---------- [A1: correct contextual conclusion]

10 A clinic uses an e-bike courier to deliver temperature-sensitive medicine to a care centre 12 km away.

The courier travels against a steady headwind. When riding at a speed of v km/h relative to the air, where $v > 5$, the actual ground speed is $v - 5$ km/h.

The total energy used for the journey, E watt-hours (Wh) consists of two parts.

- Motor energy: $\frac{v^2}{80}$ Wh for every kilometre travelled
- Cooling box energy: 112.5 Wh for every hour of the journey

(a) Show that $E = \frac{3v^2}{20} + \frac{1350}{v-5}$. [3]

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SOLUTIONS:

The actual ground speed is $v - 5$ km/h.

Time taken for the journey: $\frac{12}{v-5}$ hours ---------- [M1]

Motor energy used to overcome air resistance:

$12\left(\frac{v^2}{80}\right) = \frac{3v^2}{20}$ ---------- [M1]

Cooling box energy:

$112.5\left(\frac{12}{v-5}\right) = \frac{1350}{v-5}$.

Hence,

$E = \frac{3v^2}{20} + \frac{1350}{v-5}$ (shown) ---------- [A1]

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(b) Find the value of v for which E is stationary. [4]

SOLUTIONS:

$$E = \frac{3v^2}{20} + \frac{1350}{v-5}.$$

$$\frac{dE}{dv} = \frac{3v}{10} - \frac{1350}{(v-5)^2}$$ ---------- [M1]

For stationary value,

$$\frac{dE}{dv} = 0.$$

$$\frac{3v}{10} - \frac{1350}{(v-5)^2} = 0$$ ---------- [M1]

$$\frac{3v}{10} = \frac{1350}{(v-5)^2}.$$

$$3v(v-5)^2 = 13500$$

$$v(v-5)^2 = 4500.$$

$$v^3 - 10v^2 + 25v - 4500 = 0.$$

$$(v-20)(v^2 + 10v + 225) = 0.$$

For $v^2 + 10v + 225 = 0$,

Discriminant $= 10^2 - 4(225)$

$= -800 < 0$

(no real roots) ---------- [M1: Explain no real roots]

Therefore, $v = 20$ km/h ---------- [A1]

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(c) Determine with explanation whether this stationary value of E is a minimum. [3]

SOLUTIONS:

$\frac{d^2E}{dv^2} = \frac{3}{10} + \frac{2700}{(v-5)^3}$. ---------- [M1]

Since
$v = 20$,
$\frac{d^2E}{dv^2} = \frac{3}{10} + \frac{2700}{(20-5)^3} = 1.1 > 0.*$ ---------- [M1]

Therefore, the stationary value of E is a minimum. ---------- [A1: Include *]

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(d) For safety reasons, the courier is only allowed to choose a speed in the range

$$12 \le v \le 18$$

The battery can supply at most 155 Wh for this journey.

Determine the speed the courier should choose, and whether the battery limit is satisfied. Justify your answer. [2]

SOLUTIONS:

The stationary-energy speed is $v = 20$.

However, the allowed range is $12 \le v \le 18$.

Since $20 > 18$, the stationary-energy speed is not allowed.

From part (c), the minimum occurs at $v = 20$. Hence, for $12 \le v \le 18$, E decreases as v increases, so the least energy in the allowed range occurs when $v = 18$.

---------- [M1: Recognise that $v = 20$ is not allowed and choose $v = 18$ as the best allowed speed]

At $v = 18$,

$$E = \frac{3(18)^2}{20} + \frac{1350}{18-5}$$
$$= \frac{972}{20} + \frac{1350}{13}$$
$$= 152.446$$
$$\approx 152.4 \text{ Wh.}$$

Since $152.4 < 155$, the battery limit is satisfied.
Therefore, the courier should choose $v = 18$ km/h and the battery limit is satisfied.

---------- [A1: Calculate $E(18) \approx 152.4$ and give correct contextual conclusion]

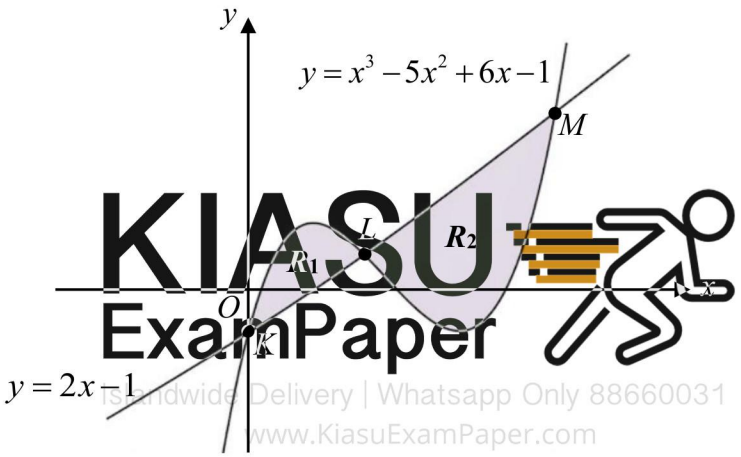
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11 The curve C has equation $y = x^3 - 5x^2 + 6x - 1$.

Line l has equation $y = 2x - 1$.

The curve C and the line l intersect at the points K, L and M, as shown in the diagram.

The shaded region consists of the two regions R_1 and R_2.



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(a) Find the coordinates of K, L and M. [4]

SOLUTIONS:

At the points of intersection,

$x^3 - 5x^2 + 6x - 1 = 2x - 1.$ ---------- [M1]

$x^3 - 5x^2 + 4x = 0$

$x(x^2 - 5x + 4) = 0$

$x(x - 1)(x - 4) = 0.$

$x = 0, x = 1, x = 4.$

$\boxed{K(0,-1), L(1,1), M(4,7)}$ ---------- [A1, A1, A1]

(b) Find the total shaded area. [6]

SOLUTIONS:

Area of R_1

$= \int_0^1 \left[\left(x^3 - 5x^2 + 6x - 1 \right) - \left(2x - 1 \right) \right] \mathrm{d}x$ ---------- [M1]

$= \int_0^1 \left(x^3 - 5x^2 + 4x \right) \mathrm{d}x$

$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + 2x^2 \right]_0^1$ ---------- [M1]

$= \frac{1}{4} - \frac{5}{3} + 2$

$= \frac{7}{12}$

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Area of R_2

$= \int_1^4 \left[\left(2x - 1 \right) - \left(x^3 - 5x^2 + 6x - 1 \right) \right] \mathrm{d}x$ ---------- [M1: $\int_1^4 [line - curve] \, \mathrm{d}x$]

$= \int_0^1 \left(-x^3 + 5x^2 - 4x \right) \mathrm{d}x$

$= \left[-\frac{x^4}{4} + \frac{5x^3}{3} - 2x^2 \right]_1^4$ ---------- [M1]

$= -\frac{256}{4} + \frac{320}{3} - 32 - \left(-\frac{1}{4} + \frac{5}{3} - 2 \right)$

$= \frac{45}{4}$

Total shaded area

$= \frac{7}{12} + \frac{45}{4}$ ---------- [M1]

$= \frac{71}{6}$ square units ---------- [A1]

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