NAME: ______________________________ (   )   CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2025

S4

ADDITIONAL MATHEMATICS   4049/01

Paper 1 Marking Scheme

READ THESE INSTRUCTIONS FIRST
Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

-

For Examiners' Use

Question	Marks	Question	Marks		
1		8			
2		9		Units	
3		10		Clarity / Logic	
4		11		Precision / Accuracy	
5		12		Total:	
6		13			
7		14			90
Parent's Name & Signature:					
Date:					

NOTE: *If the marks allocation is italicised, it has been modified from version 2 according to the suggestions given by the tester.*

1 **(a)** Find the range of values of p for which $y = x^2 - x - 2$ lies entirely above the line $y = p(x+2)$. [4]

	$x^2 - x - 2 = px + 2p$ $x^2 + (-1-p)x - 2 - 2p = 0$ $\Rightarrow$ discriminant < 0 $\Rightarrow (-1-p)^2 - 4(1)(-2-2p) < 0$ $\Rightarrow 1 + 2p + p^2 + 8 + 8p < 0$ $\Rightarrow p^2 + 10p + 9 < 0$ $\Rightarrow (p+9)(p+1) < 0$ $\Rightarrow -9 < p < -1$	M1 M1 for discriminant < 0 M1 A1

(b) Hence, deduce, without finding the discriminant, the number of intersection points when $p = 3$. [1]

	Line $y = p(x+2)$ does not intersect curve $y = (x+1)(x-2)$ when p is in the range $-9 < p < -1$. For $p = 3$, line intersects curve at 2 points.	B1

2 **(a)** By using substitution or otherwise, find the values of x for which $2^{2x-1} = 2^{x+3} - 24$, giving your answer where appropriate, to one decimal place. [5]

$2^{2x-1} = 2^{x+3} - 24$ $\frac{1}{2}\left(2^x\right)^2 = 2^3 \times 2^x - 24$ Let $u = 2^x$ $\frac{1}{2}(u)^2 = 2^3 \times u - 24$ $\frac{1}{2}u^2 = 8u - 24$ $u^2 = 16u - 48$ $u^2 - 16u + 48 = 0$ $(u-12)(u-4) = 0$ $u = 12$ or $u = 4$ $2^x = 12$ or $2^x = 4$ $\lg 2^x = \lg 12$ or $2^x = 2^2$ $x = \frac{\lg 12}{\lg 2}$ or $x = 2$ $x \approx 3.58496$ $x \approx 3.6$	M1 applying Laws of Indices to break them into product of two numbers M1 for getting quadratic equation M1 for using lg to solve. A2 for both answers

3 A closed circular cylinder has a volume of $\left(66\sqrt{2}+2\sqrt{3}\right)\pi\,\text{cm}^3$, a radius of $\left(\sqrt{2}+2\sqrt{3}\right)$cm and a height h cm. Express h in the form of $p\sqrt{2}+q\sqrt{3}$ where p and q are integers. [4]

Volume of cylinder = $\left(66\sqrt{2}+2\sqrt{3}\right)\pi$	
$\pi r^2 h=\left(66\sqrt{2}+2\sqrt{3}\right)\pi$	
$\left(\sqrt{2}+2\sqrt{3}\right)^2 h=\left(66\sqrt{2}+2\sqrt{3}\right)$	
$\left(2+4\sqrt{6}+12\right)h=\left(66\sqrt{2}+2\sqrt{3}\right)$	M1 – expansion of r^2
$h=\dfrac{66\sqrt{2}+2\sqrt{3}}{14+4\sqrt{6}}$	
$h=\dfrac{66\sqrt{2}+2\sqrt{3}}{14+4\sqrt{6}}\times\dfrac{14-4\sqrt{6}}{14-4\sqrt{6}}$	M1 – rationalisation
$h=\dfrac{924\sqrt{2}-264\sqrt{12}+28\sqrt{3}-8\sqrt{18}}{196-96}$	M1 – multiplication and simplification
$h=\dfrac{924\sqrt{2}-528\sqrt{3}+28\sqrt{3}-24\sqrt{2}}{100}$	
$h=\dfrac{900\sqrt{2}-500\sqrt{3}}{100}$	
$h=9\sqrt{2}-5\sqrt{3}$	A1 for answer

4 Express $\dfrac{2x^3-2x^2+7x-12}{x^3+4x}$ in partial fractions. [6]

$$\begin{array}{r} 2 \\ x^3+4x\overline{\left)2x^3-2x^2+7x-12\right.} \\ -\left(2x^3 \qquad +8x\right) \\ \hline -2x^2-x-12 \\ \hline \end{array}$$	**Marking Scheme modified from Version 2** B1 for long division working
$\therefore \dfrac{2x^3-2x^2+7x-12}{x^3+4x}=2+\dfrac{-2x^2-x-12}{x^3+4x}$	
$\dfrac{-2x^2-x-12}{x^3+4x}=\dfrac{A}{x}+\dfrac{Bx+C}{x^2+4}$	M1 for breaking into partial fractions
Multiply by $\left(x^3+4x\right)$ throughout, $-2x^2-x-12=A\left(x^2+4\right)+x\left(Bx+C\right)$	
Sub $x=0$, $-12=A(4)+0$ $A=-3$	*A1 for value of A*

Sub $x=1$,	
$-2-1-12=-3\left(1^2+4\right)+(1)(B+C)$	
$-15=-15+B+C$	
$B+C=0 \text{---------} (1)$	*M1 for forming simultaneous equations to solve for value of B and of C.*
Sub $x=-1$,	
$-2+1-12=-3\left(1^2+4\right)+(-1)(-B+C)$	
$-13=-15+B-C$	
$B-C=2 \text{---------} (2)$	
$(1)+(2): 2B=2$	
$B=1$	
Sub $B=1$ into (1):	
$1+C=0$	*A1 for value of B and of C.*
$C=-1$	
$\dfrac{2x^3-2x^2+7x-12}{x^3+4x}=2+\dfrac{x-1}{x^2+4}-\dfrac{3}{x}$	A1 for final answer
or	
$\begin{array}{r} 2 \\ x^3+4x\overline{\big)\,2x^3-2x^2+7x-12} \\ -\left(2x^3 \qquad +8x\right) \\ \hline -2x^2-x-12 \\ \hline \end{array}$	B1 for long division working
$\therefore \dfrac{2x^3-2x^2+7x-12}{x^3+4x}=2+\dfrac{-2x^2-x-12}{x^3+4x}$	M1 for breaking into partial fractions
$\dfrac{-2x^2-x-12}{x^3+4x}=\dfrac{A}{x}+\dfrac{Bx+C}{x^2+4}$	
Multiply by $\left(x^3+4x\right)$ throughout,	
$-2x^2-x-12=A\left(x^2+4\right)+x(Bx+C)$	
$=Ax^2+4A+Bx^2+Cx$	*M1 for expansion and grouping*
$=(A+B)x^2+Cx+4A$	
By comparing constants,	
$4A=-12$	
$A=-3$	*A1 for value of A*
By comparing coefficients of x,	
$C=-1$	

By comparing coefficients of x^2, $A+B=-2$ $-3+B=-2$ $B=1$ $\frac{2x^3-2x^2+7x-12}{x^3+4x}=2+\frac{x-1}{x^2+4}-\frac{3}{x}$	*A1 for value of B and of C.* A1 for final answer

5 Given the curve $y=kx^2+2kx-3$, where k is a constant,

(i) By expressing y in the form $k(x+b)^2-1$, determine the value of k and b. [3]

(i)	$y=kx^2+2kx-3$ $y=k(x^2+2x)-3$ $y=k(x^2+2x+1-1)-3$ $y=k(x+1)^2-k-3$ $b=1$ $-k-3=-1$ $k=-2$	**Marking Scheme modified from Version 2** M1 for $y=k(x+1)^2-k-3$ *A1 for value of b* *A1 for value of k*

(ii) Hence, show that the curve is always below the x-axis. [2]

(ii)	$y=-2(x+1)^2-1$ $(x+1)^2\geq 0$ $-2(x+1)^2\leq 0$ $-2(x+1)^2-1\leq -1<0$ Hence, the graph of y is always below the x-axis.	M1 A1
	or	
	Discriminant = $(-4)^2-4(-2)(-3)=-8<0$ Since discriminant < 0, the curve does not intersect the x-axis. Coefficient of x^2 is also negative, hence it is a quadratic curve with a maximum point. Hence, the curve is always above the x-axis.	M1 discriminant A1 for negative coefficient of x^2
	or	
	The turning point is $(-1,-1)$ and is a maximum point. Since the y-coordinate is less than zero, the curve is always above the x-axis.	M1 for maximum turning point. A1 for y-coor < 0

(iii) State the turning point of the curve and determine the nature of this point. [2]

(iii)	turning point $=(-1,-1)$ and is a maximum point	B1 B1

6 The binomial expansion of $(1+px)^n$ where $n > 0$, in ascending powers of x is

$$1-12x+28p^2x^2+qx^3+...$$

Find the value of p, of n and of q. [6]

$(1+px)^n$ $=1+\binom{n}{1}(px)+\binom{n}{2}(px)^2+\binom{n}{3}(px)^3+...$ $=1+npx+\frac{n(n-1)}{2}p^2x^2+\frac{n(n-1)(n-2)}{6}p^3x^3+...$ $=1-12x+28p^2x^2+qx^3+...$	**Marking Scheme modified from Version 2** *B1 for applying the formula for Binomial Coefficient*
By comparing coefficients of x^2, $\frac{n(n-1)}{2}=28$ $n^2-n=56$ $n^2-n-56=0$ $(n-8)(n+7)=0$ $n=8$ or $n=-7$ (N.A.)	*M1 for forming equation.* $\binom{n}{2}p^2=28$ *is not acceptable* A1 for both answers, must reject -7.
By comparing coefficients of x, $np=-12$ $p=\frac{-12}{8}$ $p=-1\frac{1}{2}$	*M1 for forming equation.* $\binom{n}{3}p=-12$ *is not acceptable* *A1 for value of p*
$q=\frac{8(7)(6)}{6}\left(-1\frac{1}{2}\right)^3$ $q=-189$	A1 for value of q

7 **(a)** The surface area of a solid cube is increasing at $0.2\,\text{cm}^2/\text{s}$. Find the rate of increase of the volume when the length of a side is 1cm. [4]

$A = 6x^2$ $\frac{dA}{dx} = 12x$ $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ $0.2 = (12x)\frac{dx}{dt}$ $\frac{dx}{dt} = \frac{1}{60x}$ $V = x^3$ $\frac{dV}{dx} = 3x^2$ $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$ $\frac{dV}{dt} = 3x^2\left(\frac{1}{60x}\right)$ $\frac{dV}{dt} = \frac{x}{20}$ Subst. $x = 1$, $\frac{dV}{dt} = \frac{1}{20}$ $\frac{dV}{dt} = 0.05\ \text{cm}^3/\text{s}$	M1 – form correct connect rates of change M1 – find $\frac{dx}{dt}$ M1 – form correct connect rates of change A1

(b) A curve has the equation $y = e^{1-8x}(\cos 2x)$, where $0 \le x \le \frac{\pi}{4}$. Find the gradient of the curve at the point where $y = 0$, leaving your answer in exact form. [4]

$y = e^{1-8x}(\cos 2x)$	
$\frac{dy}{dx} = e^{1-8x}(-2\sin 2x) + (\cos 2x)(-8)e^{1-8x}$	M2 – using product rule
$= -2e^{1-8x}(\sin 2x + 4\cos 2x)$	
When $y = 0$,	
$e^{1-8x}(\cos 2x) = 0$	
$e^{1-8x} = 0$ (NA) or $\cos 2x = 0$	
$2x = \frac{\pi}{2}, \frac{3\pi}{2}$	
$x = \frac{\pi}{4}, \frac{3\pi}{4}$ (NA)	M1 – finding x
$\frac{dy}{dx} = -2e^{1-8\left(\frac{\pi}{4}\right)}\left(\sin 2\left(\frac{\pi}{4}\right) + 2\cos 2\left(\frac{\pi}{4}\right)\right)$	
$\frac{dy}{dx} = -2e^{1-2\pi}$	A1

8 The equation of a curve is $y = -\frac{5}{8}x + \frac{6}{x}$, where $x > 0$. Given that the gradient of the normal at point P is 1, find

(i) the coordinates of P , [4]

$y=-\frac{5}{8}x+\frac{6}{x}$ $y=-\frac{5}{8}x+6x^{-1}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{5}{8}-6x^{-2}$ $-1=-\frac{5}{8}-\frac{6}{x^2}$ $-1+\frac{5}{8}=-\frac{6}{x^2}$ $-\frac{3}{8}=-\frac{6}{x^2}$ $3x^2=48$ $x^2=16$ $(x-4)(x+4)=0$ $x=4$ or $x=-4$ (N.A.) $y=-\frac{5}{8}(4)+\frac{6}{4}$ $y=-1$ P is $(4,\ -1)$	**Marking Scheme modified from Version 2** *B1 for differentiation* M1 – form equation from gradient of tangent A1 – find correct x A1 If student did not reject –4 in the previous part and find coordinates of P with –4, do not award final A1

(ii) the equation of the normal at the curve at P. [2]

$y-(-1)=1(x-4)$ $y=x-4-1$ $y=x-5$	M1 A1

9 **(a)** Solve the equation $\log_2(x+2)-\log_{\sqrt{2}}(x-1)=1$. [4]

$\log_2(x+2)-\log_{\sqrt{2}}(x-1)=1$	**Marking Scheme modified from Version 2**
$\log_2(x+2)-\dfrac{\log_2(x-1)}{\log_2\sqrt{2}}=1$	M1 for using Change-of-base Law
$\log_2(x+2)-\dfrac{\log_2(x-1)}{\log_2 2^{\frac{1}{2}}}=1$	
$\log_2(x+2)-\dfrac{\log_2(x-1)}{\frac{1}{2}\log_2 2}=1$	
$\log_2(x+2)-\dfrac{\log_2(x-1)}{\frac{1}{2}}=1$	
$\frac{1}{2}\log_2(x+2)-\log_2(x-1)=\frac{1}{2}$	
$\log_2(x+2)-2\log_2(x-1)=1$	
$\log_2\left(\dfrac{x+2}{x-1}\right)=1$	M1 for using Quotient Law
$\dfrac{x+2}{(x-1)^2}=2$	*M1 for removing log either by changing to index form or changing 1 to log₃3 and remove the log₃*
$x+2=2\left(x^2-2x+1\right)$	
$x+2=2x^2-4x+2$	
$2x^2-4x+2-x-2=0$	
$2x^2-5x=0$	
$x(2x-5)=0$	
$x=0$ or $2x-5=0$	
$x=0$ (N.A) or $x=2\frac{1}{2}$	A1 for both answers. Must reject 0.

(b) It is given that $\lg a - \lg b = \lg(a+b)$.

(i) Express a in terms of b. [2]

$\lg a - \lg b = \lg(a+b)$ $\lg \frac{a}{b} = \lg(a+b)$ $\frac{a}{b} = (a+b)$ $a = ab + b^2$ $a - ab = b^2$ $a = \frac{b^2}{1-b}$	M1 for using Quotient Law A1 for answer

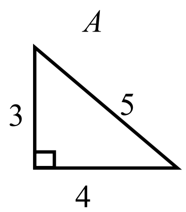
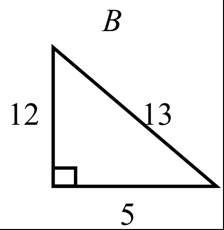
(ii) State the range of b. [1]

Since $a > 0$, $\therefore \frac{b^2}{1-b} > 0$ $\because b^2 > 0$, $\therefore 1 - b > 0$ $b < 1$ $\therefore 0 < b < 1$	B1

10 Given that $\sin A$ and $\cos B$ are in the same quadrant such that $\sin A = \frac{3}{5}$ and $\cos B = -\frac{5}{13}$.

Find, without using a calculator, the value of

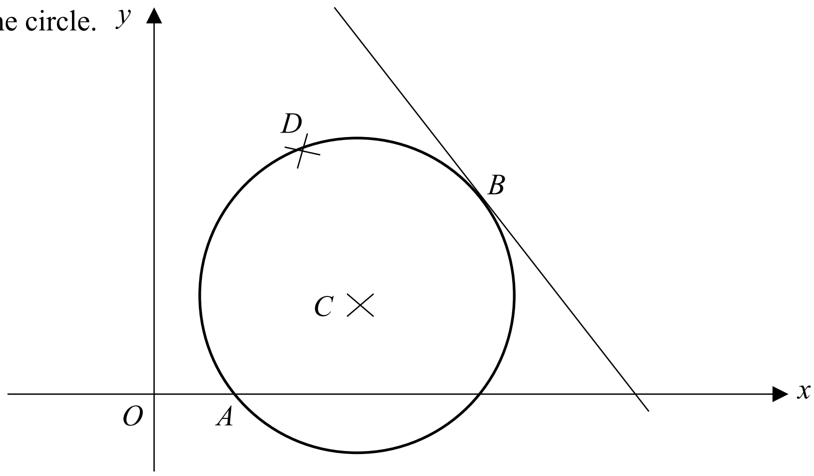
(i) $\cos(A-B)$, [2]

$\cos(A-B) = \cos A \cos B + \sin A \sin B$ $= \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$ $= \frac{56}{65}$ 	M1 A1

(ii) $\cos\dfrac{A}{2}$. [3]

	$\cos A = 2\cos^2\dfrac{A}{2} - 1$ $\cos A + 1 = 2\cos^2\dfrac{A}{2}$ $-\dfrac{4}{5} + 1 = 2\cos^2\dfrac{A}{2}$ $\dfrac{1}{5} = 2\cos^2\dfrac{A}{2}$ $\cos^2\dfrac{A}{2} = \dfrac{1}{10}$ $\cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1}{10}}$ $90° < A < 180°$ $45° < \dfrac{A}{2} < 90°$ $\cos\dfrac{A}{2} = \dfrac{\sqrt{10}}{10} \quad \left(-\dfrac{\sqrt{10}}{10}\text{(NA)}\right)$	M1 M1 A1 with rejection

11 Points A, $B(5,4)$ and D lie on a circle with centre C and AB is the diameter of the circle. The line $y = -x + 9$ is a tangent at point $B(5,4)$ on the circle.

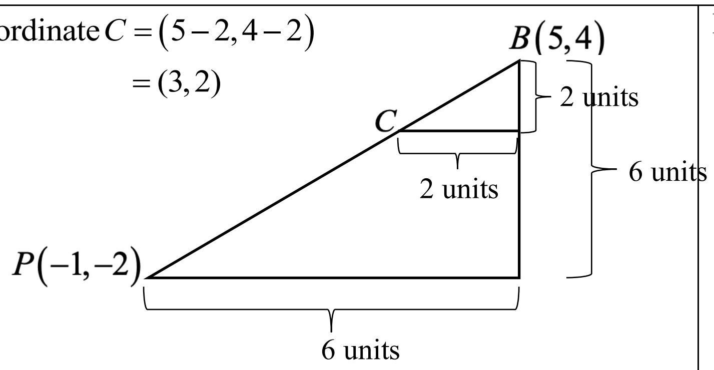


(a) Given that point P lies on BA extended and the x-coordinate of point P is -1, find the coordinates of P. [2]

Let the y-coordinate of P be a, $\dfrac{4-a}{5-(-1)} = 1$ $4 - a = 6$ $a = -2$ Coordinates of P is $(-1,-2)$	M1 – form equation A1

It is given that the distance *BP* is three times the distance *BC*.

(b) **(i)** Show that centre *C* is $(3,2)$. [1]

Coordinate $C=(5-2,4-2)$ $=(3,2)$ 	M1 – finding Coord *C*

(ii) Hence, find the equation of the circle. [2]

Distance $BC=\sqrt{(5-3)^2+(4-2)^2}$ $=\sqrt{8}$ units Equation of circle is $(x-3)^2+(y-2)^2=8$	M1 – finding distance *BC* A1

(c) Given that triangle *BCD* is right-angled at *C*, find the coordinates of *D*. [3]

Gradient of $CD=-1$ Equation of CD is $y-2=-1(x-3)$ $y=-x+5$ Subst. $y=-x+5$ into $(x-3)^2+(y-2)^2=8$, $(x-3)^2+(-x+5-2)^2=8$ $x^2-6x+9+x^2-6x+9=8$ $2x^2-12x+10=0$ $x^2-6x+5=0$ $(x-1)(x-5)=0$ $x=1$ or $x=5$(NA) Subst. $x=1$ into $y=-x+5$, $y=-1+5$ $y=4$ $D(1,4)$	M1 – find equation *CD* M1 – solving simultaneous equations A1

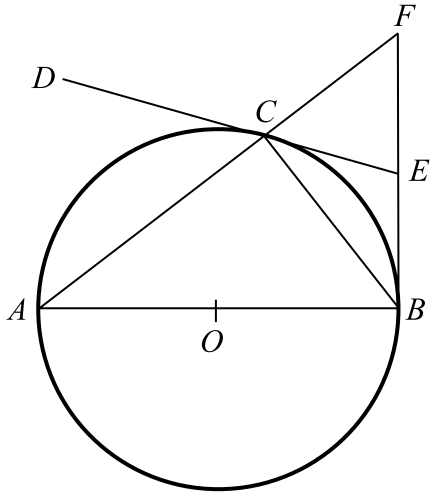
12 **(a)** Given that $y=\ln\left(\sin^2 x\right)$, show that $\dfrac{dy}{dx}=2\cot x$. [2]

$y=\ln\left(\sin^2 x\right)$ $=2\ln\left(\sin x\right)$ $\dfrac{dy}{dx}=2\left(\dfrac{1}{\sin x}\right)\left(\cos x\right)$ $=2\cot x$	M1: differentiation of ln M1: differentiation of sin x

(b) Hence show that the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\cot x-\cos^2 2x\right)dx=a+b\ln 2$, where a and b are constants.[4]

$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cot x-\cos^2 2x\,dx=\frac{1}{2}\left[2\ln\sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}-\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos 4x+1}{2}\,dx$	M1: for using (a) M1: for correct double angle formula
$=\frac{1}{2}\left[0-2\ln\frac{1}{\sqrt{2}}\right]-\frac{1}{2}\left[\frac{\sin 4x}{4}+x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	
$=\frac{1}{2}\ln 2-\frac{1}{2}\left[\frac{\pi}{2}-\frac{\pi}{4}\right]$	M1: definite integrals
$=\frac{1}{2}\ln 2-\frac{\pi}{8}$	
$=-\frac{\pi}{8}+\frac{1}{2}\ln 2$	A1

13 In the diagram, AB is a diameter of the circle with centre O. DE and BF are tangents to the circle at C and B respectively. DCE and BEF are straight lines. Prove that



(a) ΔABC and $\Delta\, BFC$ are similar, [3]

$\angle ACB = 90°$ (rt. $\angle$ in a semi-circle) $\angle FCB = 90°$ (adj $\angle$ on straight line) $\therefore \angle ACB = \angle BCF$ $\angle CBF = \angle CAB$ (tangent-chord theorem) Hence ΔABC and ΔBFC are similar.	M1 M1 M1

(b) Hence or otherwise, show that ΔCFE is an isosceles triangle. [4]

$\angle ABC = \angle DCA$ (tangent-chord theorem)	M1
$\angle FCE = \angle DCA$ (vertically opp. $\angle$s)	M1
$\angle ABC = \angle CFE$ (similar Δs)	M1
Since $\angle CFE = \angle FCE$, ΔCFE is isosceles (base $\angle$s are equal)	A1

14 The mass, m grams, of the decomposition of a radio-active substance is given by the formula $m = ae^{-bt}$, where a and b are constants, and t is the number of weeks after the initial measurement of the mass. Experimental values of t and m are given in the table below.

t	1	2	3	4	5
m	1.51	0.454	0.137	0.0224	0.0124

(i) What does the constant a refer to?[1]

The constant a refers to the **initial mass** of the radio-active substance. Or The constant a refers to the **mass** of the radio-active substance **when *t* = 0**.	B1

(ii) By drawing a suitable straight line graph using the data provided, estimate the value of a and of b.[7]

$m = ae^{-bt}$

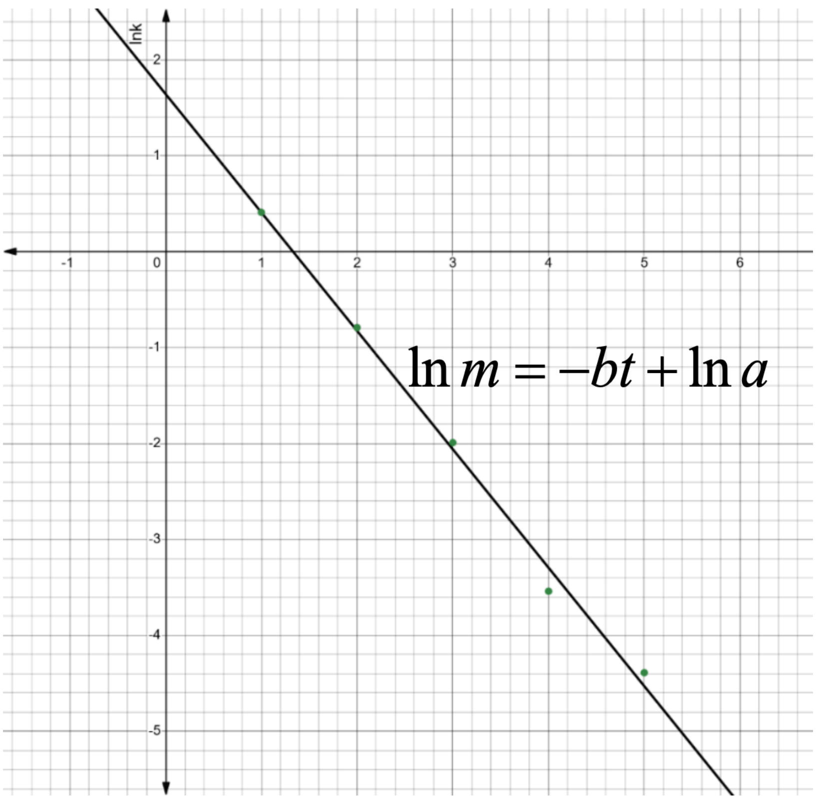
$\ln m = \ln\left(ae^{-bt}\right)$ M1

$\ln m = \ln a - bt$ A1

$\ln m = -bt + \ln a$

t	1	2	3	4	5
ln$m$	0.412	−0.790	−1.99	−4.00	−4.39

T1



G1: Plot all the points
G2: Best fit line
Penalise under Clarity if missing in labelling of axis / equation

$-b = \dfrac{-4-1}{4.7-0.5}$ $-b \approx -1.1904$ $b \approx 1.19$ $\ln a =$ Y-intercept $\ln a = 1.6$ $a \approx e^{1.6}$ $a \approx 4.9530$ $a \approx 4.95$	A2 (actual value of $a = 5 \pm 0.3$ and $b = 1.2 \pm 0.3$)

(iii) It is discovered that one of the readings of the mass, m, is incorrect. Using your graph, estimate the correct value for this incorrect reading. [2]

When $t = 4$, $\ln m = -3.15$ $m = e^{-3.15}$ $m \approx 0.042852$ $m \approx 0.0429$	M1 changing ln to exp A1