

Class	Register Number	Name
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南洋女子中學校

Nanyang Girls' High School

End-of-Year Examination 2023 Secondary 3

INTEGRATED MATHEMATICS 2

Monday 2 October 2023

2 hours
0845 – 1045

READ THESE INSTRUCTIONS FIRST

1. Write your name, register number and class on all the work you hand in.
2. Write in dark blue or black ink.
3. You may use an HB pencil for any diagrams or graphs.
4. Do not use staples, paper clips, glue or correction tape/ fluid.
5. Write your answers and working on the separate writing paper provided, unless otherwise stated.
6. Answer **all** questions.
7. Omission of essential working will result in loss of marks.
8. The use of an approved scientific calculator is expected, where appropriate.
9. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degree to one decimal place. For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .
10. At the end of the examination, fasten all your work securely together.
11. The number of marks is given in brackets [] at the end of each question or part question.

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NANYANG GIRLS' HIGH SCHOOL

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

- 1 Given that $6x^3 - 16x^2 + 10x - 4 = A(x - 1)^2(2x - 1) + B(x + 3)(x - 1) + C$ for all values of x , find the values of A , B and C . [4]
- 2 Express $\frac{10x+19}{(x-3)(2x+1)^2}$ in partial fractions. [4]
- 3 Simplify the expression $\frac{3(1+3\sqrt{5})}{3+\sqrt{5}}$ and leave your answer in the form $a + b\sqrt{5}$. [3]
- 4 The population, P , of a new town at the beginning of the year 2015 was 50000. The population increases such that after a period of t years, the new population can be modelled by $P = 50000e^{0.06t}$.
- (a) Find the population of the town at the beginning of 2023. [2]
- (b) How many years would it take for the population to reach 100 000? [3]
- (c) Is this model reliable in predicting the population after a long period of time? Explain your answer. [1]
- 5 The polynomial $2x^3 + ax^2 + bx - 8$, where a and b are constants, has a factor $(x - 2)$ but leaves a remainder of 9 when divided by $(x + 1)$.
- (a) Find the value of a and of b . [4]
- (b) Factorise $2x^3 + ax^2 + bx - 8$ completely. [2]
- (c) Hence, with the values of a and b found in **part (a)**, solve the equation $16y^3 + 4ay^2 = 8 - 2by$. [3]
- 6 (a) Solve the equation
- (i) $2^{2x} - 2^{x+1} = 2^x + 4$, [4]
- (ii) $3\log_x 3 = 8 - 4\log_3 x$. [5]
- (b) If $x = 1$ is a solution to the equation $2^{2x+p} - 2^{x+p} = 9(2^x) - 2$ where p is a constant, find the value of p . [2]

- 7 (a) Find the range of values of k for which the line $y = kx - 5$ intersects the curve $y = 2x^2 + 4x + 3$ at two distinct points. [4]
- (b) (i) Explain why the expression $2x^2 + 4x + 3$ is always positive for all real values of x . [2]
- (ii) Hence, solve the inequality $\frac{2x^2 + 4x + 3}{3x^2 - 7x - 6} < 0$. [3]
- 8 The function f is given by $f(x) = a \cos 2x + b$ for $0^\circ \leq x \leq 180^\circ$, where a and b are positive integers. $f(x)$ has an amplitude of 5 and a minimum value of -3 .
- (a) State the value of a and of b . [2]
- (b) State the period of $f(x)$. [1]
- (c) Sketch the graph of $y = f(x)$. [3]
- 9 (a) The graph of $y = \ln(x + 4)$ undergoes 2 successive transformations.
- I A scaling parallel to the y -axis with a factor of 2.
- II A translation parallel to the x -axis by 3 units.
- Find the equation of the resulting graph. [2]
- (b) Sketch the graph of $y = \ln(x + 4)$, showing clearly if any, the intercept(s) with the axes and the asymptote(s). [2]
- (c) Explain how you solve the equation $x = e^{x-1} - 4$ using your sketch in **part (b)**. Hence, determine the number of solutions to the equation. [4]
- 10 Given that $\sin \theta = m$ and θ is an acute angle, express, in terms of m ,
- (a) $\tan \theta$ [2]
- (b) $\cos(-\theta)$ [1]
- (c) $\operatorname{cosec} \theta$ [2]
- 11 Solve the equation
- (a) $4\cos^2 x + 4\sin x - 1 = 0$ for $0^\circ \leq x \leq 360^\circ$ [4]
- (b) $2\sin x - \tan x = 0$ for $0 \leq x \leq 2\pi$ [4]

- 12 (a) It is given that $\tan A = -\frac{12}{5}$ and $\tan B = -\frac{3}{4}$, where A and B are in different quadrants and $A < B$. Without using a calculator, find the value of $\cos(A + B)$. [4]
- (b) Given that $\frac{\sin(P + Q)}{\sin(P - Q)} = \frac{2}{3}$, evaluate $\frac{\tan Q}{\tan P}$. [3]

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2023 S3 IM2 EOY Marking Scheme

No.	
1	$A = 3$ $B = -1$ $C = -4$
2	$\frac{1}{x-3} - \frac{2}{2x+1} - \frac{4}{(2x+1)^2}$
3	$6\sqrt{5} - 9$
4(a)	80800 (3 s.f.)
4(b)	11.6 years
4(c)	As $t \rightarrow \infty$, $P \rightarrow \infty$ Thus, this model is not reliable in predicting the population after a long period of time. Or This model does not take into account deaths rate or falling birth rate, etc.
5 (a)	$a = 5$ $b = -14$
5(b)	$f(x) = (x-2)(2x+1)(x+4)$
5(c)	$y = 1, -\frac{1}{4}, -2$
6(a)(i)	$x = 2$
6(a)(ii)	$x = \sqrt{3}$ or $3\sqrt{3}$
6(b)	$p = 3$
7(a)	$k < -4$ or $k > 12$
7(b)(ii)	$-\frac{2}{3} < x < 3$
8(a)	$a = 5$ $b = 2$
8(b)	Period = 180°
8(c)	

9(a)	$y = 2 \ln(x + 1)$
9(b) (c)	
9(c)	Add $y = x - 1$ Number of solutions = 2
10 (a)	$\tan \theta = \frac{m}{\sqrt{1 - m^2}}$
10 (b)	$\cos(-\theta) = \sqrt{1 - m^2}$
10 (c)	$\operatorname{cosec} \theta = \frac{1}{m}$
11(a)	$x = 210^\circ, 330^\circ$
11(b)	$x = 0, \pi, 2\pi$ or $x = \frac{\pi}{3}, \frac{5\pi}{3}$
12(a)	$\frac{16}{65}$
12(b)	$-\frac{1}{5}$