SCHOOL OF THE ARTS SINGAPORE

Year 6 Mathematics: analysis and approaches
Higher level
Paper 2
Preliminary Examinations

Monday 28 August 2023

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your name and class on each answer sheet, and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper
- The maximum mark for this examination paper is [**110 marks**].

Section A		**Section B**	
Question	**Marks**	**Question**	**Marks**
1		10	
2		11	
3		12	
4			
5			
6			
7			
8			
9			

Name: ______________________

Class: ______________________

Index: ______________________

110

18 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows triangle ABC, with $\mathrm{AB} = 4.4$, $\mathrm{AC} = 5.8$, and $\mathrm{B\widehat{A}C} = 36°$.

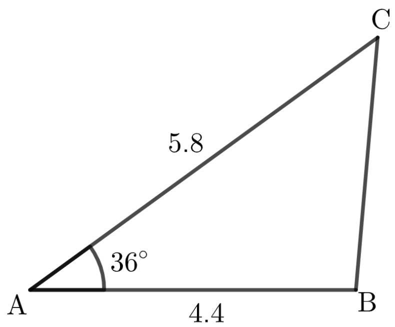

Diagram not to scale

(a) Find BC. [3]

(b) Find $\mathrm{A\widehat{C}B}$ [3]

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(Question 1 continued)

2. [Maximum mark: 6]

A company wishes to assess whether there is a linear relationship between the annual amount spent on advertising, $\$x$ thousand, and the annual profit, $\$y$ thousand for seven years. The following table shows the data collected.

Annual advertising amount ($\$x$ thousand)	24.0	25.9	30.0	27.0	32.4	37.0	33.0
Annual profit ($\$y$ thousand)	241	361	524	471	556	652	641

The regression line of y on x for this data can be written in the form $y = ax + b$.

(a) Find the value of a and the value of b. [2]

(b) Write down the value of the Pearson's product-moment correlation coefficient, r. [1]

(c) Use the equation of your regression line to estimate the annual profit, to the nearest thousand, during a year when $29 000 was spent on advertising. [2]

(d) A manager commented that this correlation coefficient shows that spending more money on advertising will cause annual profit to increase. Give a reason why this statement is not correct. [1]

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(Question 2 continued)

3. [Maximum mark: 6]

In an arithmetic sequence, the sum of the first 10 terms is 375 and the sum of the first 30 terms is 3525.

Find the 45th term of the sequence.

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Please do not write on this page.

Answers written on this page
will not be marked.

4. [Maximum mark: 7]

On Monday at noon, Kenny took an injection that combines two drugs, Anticold and Betacold. At time t hours, after the injection, the concentrations, measured in units, of Anticold and Betacold in Kenny's bloodstream can be modelled as

$$f(t) = 2\mathrm{e}^{-0.03t} \quad \text{and } g(t) = 4\mathrm{e}^{-0.08t} \text{ respectively.}$$

Each drug becomes ineffective when its concentration falls below 1 unit.

(a) Show that Betacold becomes ineffective before Anticold. [2]

Fifteen hours after the first injection, Kenny took a second injection.

(b) (i) Find the concentration of Betacold from the **first** injection in Kenny's bloodstream on Tuesday at noon.

(ii) Hence, find the total concentration of Betacold in Kenny's bloodstream on Tuesday at noon. [5]

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(Question 4 continued)

5. [Maximum mark: 6]

Consider the expansion of $(k+x)^2(3-2x)^7$, where $k \neq 0$. The coefficient of the term in x^8 is 448. Find the value of k.

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6 [Maximum mark: 5]

In an exam, there are three sections: A, B, and C. Each section contains 6 individual questions.

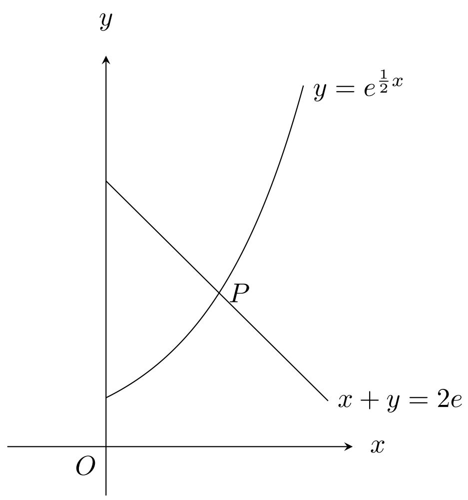
(a) A student is required to answer 8 questions. How many ways can they answer? [1]

A change in the exam requirements states that the student needs to answer a total of 8 questions with at least 2 questions from Section A, at least 1 question each from Section B and C, but no more than 3 questions from any section.

(b) How many ways can the student do this? [4]

7. [Maximum mark: 6]

Part of the graphs $y = e^{\frac{1}{2}x}$ and $x + y = 2e$ is shown below. The intersection of the graphs is given by the point P.



The enclosed region between the graphs and the y - axis is rotated 2π about the y - axis. Find the volume of the solid of revolution formed.

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(Question 7 continued)

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8 [Maximum mark: 7]

A continuous random variable $g(x)$ is defined as

$$g(x)=\begin{cases} kx\mathrm{e}^{\sin x} & 0 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k. [2]

Skewness of a distribution is defined as $\gamma = \dfrac{3(\text{mean} - \text{median})}{\text{standard deviation}}$.

where a distribution is symmetrical if $-0.2 \le \gamma \le 0.2$.

(b) Find whether the probability distribution of $g(x)$ is skewed. [5]

9. [Maximum mark: 6]

The equation $(\sec\gamma)x^2 + (2\sin\gamma)x - 24\cos\gamma\sin^2\gamma = 0$ has roots α and β where $\alpha < 0$, $\beta > 0$ and $0 \le \gamma \le \pi$.

Find the equation that has the roots $\frac{2\alpha}{\beta}$ and $\frac{\beta}{2\alpha}$.

Do **not** write solutions on this page.

Section B

Answer **all** questions on the answer sheets provided. Please start each question on a new page.

10. [Maximum mark: 17]

The daily revenue, X, of a minimart is normally distributed with a mean of $\$ 2040$ and standard deviation $\$ 210$.

(a) Find the probability that on a randomly chosen day, the daily revenue exceeds $\$ 2300$. [2]

(b) Find, on how many days out of 365 days in a year, the daily revenue exceeds $\$ 2300$ [2]

The owner of the minimart decides to boost the daily revenue by extending its operation hours. He has a new mean of $\$ 2250$ and a new standard deviation σ. The new probability of the daily revenue exceeding $\$ 2300$ has become 0.4.

(c) Find the new standard deviation σ. [5]

(d) After extending the operation hours, the owner calls a day as special day if the daily revenue exceeds $\$ 2300$.

(i) Find the probability that a special day has daily revenue of less than $\$ 2500$.

(ii) For 10 randomly chosen special days, find the probability that on fewer than 5 special days, the daily revenue is less than $\$ 2500$. [8]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

Consider the equation $e^{2x}y - y^2 = \sin x - x$ for $x \geq 0$ and $y > 0$.

(a) (i) Solve the equation for y when $x = 0$.

(ii) Show that $\dfrac{dy}{dx} = \dfrac{\cos x - 1 - 2e^{2x}y}{e^{2x} - 2y}$.

(iii) Find the value of $\dfrac{dy}{dx}$ when $x = 0$. [6]

(b) Use Euler's method, with a step length of 0.05, to find an approximate value for y when $x = 0.2$. Give your answer correct to six significant figures. [4]

(c) (i) Given that $\dfrac{d^2y}{dx^2} = 4$ and $\dfrac{d^3y}{dx^3} = 19$ when $x = 0$, find the first 4 terms of the Maclaurin series for y.

(ii) Use the Maclaurin series to find an approximate value for y when $x = 0.2$. Give your answer correct to six significant figures. [3]

(d) (i) Find the value for y when $x = 0.2$, correct to six significant figures.

(ii) Which method is the better approach to finding an approximate value of y when $x = 0.2$? Justify your answer. [4]

Do **not** write solutions on this page.

12. [Maximum mark: 21]

Consider the function $f(x)=\dfrac{x(x+1)}{x+2}$, $x \in \sim$, $x \neq -2$.

(a) (i) Find the stationary points of the graph $y = f(x)$.

(ii) Show that the function has an oblique asymptote $g(x)=x-1$.

(iii) Hence sketch the function, clearly showing any stationary point and asymptotes. [7]

A differential equation is given as $\dfrac{\mathrm{d}y}{\mathrm{d}x}+\dfrac{y}{f(x)}=\dfrac{x+1}{x^4+x^2}$.

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(b) Show that $\dfrac{1}{f(x)}$ can be written as $\dfrac{A}{x}+\dfrac{B}{x+1}$, where A and B are constants to be found. [2]

(c) (i) Show that the integrating factor can be given as $IF=\dfrac{x^2}{(x+1)}$.

(ii) Hence, find the general solution to the differential equation.

(iii) Using the values $x=1$ and $y=4$ find the unique solution $h(x)$. [10]

(d) Find the solution of x when $g(x)=h(x)$. [2]