

1. (a) Factorise the following expression completely. [2]

$$8x^3 - 27y^3 \quad (2x-3y)(4x^2+6xy+9y^2)$$

- (b) Express the following as a single fraction in its simplest form.

$$\frac{2}{x-3} - \frac{x+1}{x^2-4x+3} - \frac{x}{1-x} \quad (x+1)/(x-1) \quad [4]$$

2. Use the substitution $u = \sqrt{x-5}$ to solve the equation

$$3\sqrt{x-5} + \frac{8}{\sqrt{x-5}} = 14. \quad x=49/9 \text{ or } x=21 \quad [4]$$

3. Solve the equation $x + 3\sqrt{x+2} = 2$. $x=-1 \text{ or } 14(\text{rej}) \quad [4]$

4. (a) It is given that $(2-\sqrt{5})^2 - \frac{20}{3+\sqrt{5}} = p+q\sqrt{5}$, where p and q are integers.

Find the value of p and of q . $p=-6, q=1 \quad [4]$

- (b) Find the values of x and y which satisfy the equations

$$4^{x+y} \times 2^{-3y} = 1, \quad [4]$$

$$\frac{81^x}{3^{3x+2y}} = \frac{1}{27}. \quad x=1 \text{ and } y=2$$

- (c) Given that $y = x^2 + (w+6)x + \frac{3}{4}w + 18$, where w is an integer. Find the value(s) of w

such that x -axis is a tangent to the graph of y . $w=-12 \text{ or } w=3 \quad [4]$

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5. Solve the following equations.

(a) $\log_4(2-x) - 1 = \log_{16} x + \log_4\left(\frac{1}{4}\right)$ $x=1$ or $4(\text{rej})$ [5]

(b) $5^{2x-3} = 2e^{x+2}$ 3.39 [4]

6. A glass of liquid substance is left to cool in a room. Its temperature $T^\circ\text{C}$ after t minutes, is given by $T = 25 + T_0 e^{-at}$, where T_0 and a are constants. The initial temperature of the liquid substance is 85°C and it drops to 70°C after 10 minutes.

(i) Show that $T_0 = 60$ and $a = 0.0288$, correct to 3 significant figures. [3]

(ii) Find the temperature of the liquid substance an hour from the beginning, giving your answer to one decimal place. 35.7°C [2]

(iii) After a very long time, estimate the temperature of the liquid substance. Explain your answer. [2]

The temperature will be 25°C after a very long time, as e^{-at} will become very near to 0 and 25 is a constant.

7. The equation of a curve is $y = -x^2 - 6x + 3$. Find the range of values of the constant m for which the line $y = mx + 4$ intersects the curve at 2 distinct points. [4]

$$m < -8 \text{ or } m > -4$$

8. Given a cubic polynomial $f(x) = 10x^3 + x^2 - 5x + 1$,

(i) show that $2x - 1$ is a factor of $f(x)$. [1]

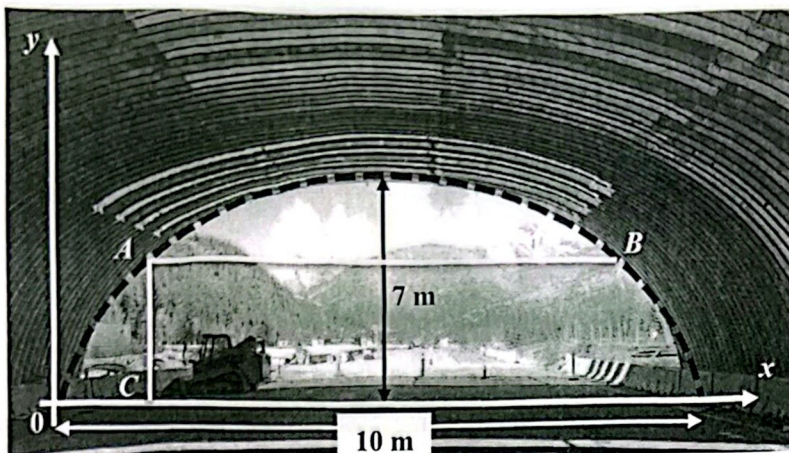
(ii) Hence, solve the equation $f(x) = 0$, leaving your answers in exact form. [5]

$$x = 1/2 \text{ or } x = (-3 \pm \sqrt{29})/10$$

9. A polynomial $P(x)$ leaves a remainder of 4 when divided by $x - 3$ and a remainder of -1 when divided by $x + 2$. Find the remainder when $P(x)$ is divided by $(x - 3)(x + 2)$. [4]

$$x + 1$$

10. An arched underpass has the shape of a parabola as shown below.



In the diagram, x m is the horizontal distance from left end of the arch on the ground and y m is the height of the arch above the ground. A one-way road passing under the arch is 10 m wide and the maximum height of the arch is 7 m.

The width of the arch at a point is defined as the horizontal distance from that point to the other side of the arch. One such width is illustrated in the figure above, AB .

The height of the arch at a point is defined as the vertical distance from that point to the level ground. One such height is illustrated in the figure above, AC .

- (i) Find a quadratic function in the form $y = a(x-h)^2 + k$ to represent the arch, where a , h and k are constants. $y = -7(x-5)^2/25 + 7$ [4]
- (ii) A point on the arch is 2.5 m vertically from the ground. What is the width of the arch at this point? 8.02 m [3]
- (iii) Find the height of the arch when its width is 4 m. 5.88 m [2]
- (iv) Decide whether it is possible for a truck that is 4 m wide and 6 m tall to navigate through the underpass. Explain your answer. [2]
It is not possible, as the height of the arch is 5.88m when its width is 4m, and 5.88m is less than 6m.

11. The graph of a cubic polynomial function $y = f(x)$ crosses the x -axis at 2 , $\sqrt{3}$ and $-\sqrt{3}$ and crosses the y -axis at 30. Find an expression for the cubic polynomial $f(x)$, express it in the general form, $f(x) = ax^3 + bx^2 + cx + d$, where a , b , c and d are integers. [4]

$$f(x) = 5x^3 - 10x^2 - 15x + 30$$

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12. Answer the whole of this question on a sheet of Graph Paper.

The table shows experimental values of two variables, x and y , which are connected by an equation of the form $e^y = ax^b$, where a and b are constants.

x	2	3	5	7	10
y	2.28	2.94	3.69	4.30	4.88

- (i) Plot y against $\ln x$ for the given data and draw a straight line graph. [3]
- (ii) Use your graph to estimate the value of a and of b . $a=3.21$ $b=1.61$ [4]
- (iii) By drawing a suitable straight line on the same axes, or otherwise, estimate the value of x when $e^y = x^5$. $x=1.40$ [2]

End of Paper