Hwa Chong Institution 华侨中学
Mathematics (SMTP)

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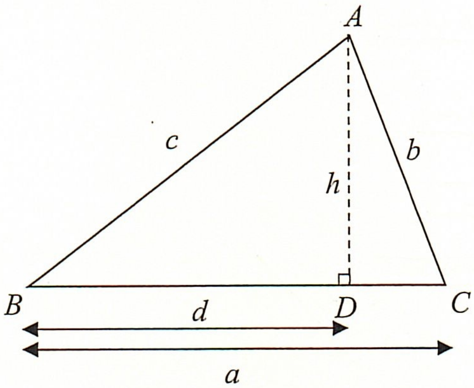
**MA304.3.X1 – Heron's Formula**

In this worksheet we will explore the Heron's Formula which is used to calculate the area of a triangle given only the lengths of the 3 sides.

S1. Suppose we have a triangle $\Delta ABC$ such that the lengths of the sides are given by $a$, $b$ and $c$ as shown below.




Let $D$ be the foot of perpendicular from $A$ to the line $BC$, such that $AD = h$ (height of the triangle with $BC$ as the base). Let $BD = d$ .

(a) By considering the two right-angle triangles $\Delta ABD$ and $\Delta ACD$, use Pythagoras' Theorem to express $d$ in terms of $a$, $b$ and $c$ only.

$(a-d)^2 + h^2 = b^2$

$a^2 - 2ad + d^2 + h^2 = b^2$ —— ①

$d^2 + h^2 = c^2$

$h^2 = c^2 - d^2$ —— ②

sub ② into ①

$\therefore a^2 - 2ad + c^2 = b^2$

$2ad = a^2 - b^2 + c^2$

$d = \dfrac{a^2 - b^2 + c^2}{2a}$

(b) Hence show that $h^2 = \dfrac{(a+c-b)(a+b+c)(b+c-a)(a+b-c)}{4a^2}$.

$$h^2 = c^2 - \frac{(b^2+c^2-a^2)}{4b^2}$$
$$= \frac{4b^2c^2 - (b^2+c^2-a^2)^2}{4b^2}$$
$$= \frac{(2bc)^2 - (b^2+c^2-a^2)^2}{4b^2}$$
$$= \frac{[(b+c)^2-a^2][(a^2-(b-c)^2]}{4b^2}$$
$$= \frac{(a+c-b)(a+b+c)(b+c-a)(a+b-c)}{4a^2} \text{ (shown)}$$

(c) By letting $s = \dfrac{a+b+c}{2}$, show that $A$, the area of the triangle $\Delta ABC$, is given by

$$A = \sqrt{s(s-a)(s-b)(s-c)}.$$

$$2s = a+b+c$$
$$h^2 = \frac{2s(2s-2b)(2s-2a)(2s-2c)}{4a^2}$$
$$= \frac{16s(s-b)(s-a)(s-c)}{4a^2}$$
$$h = \frac{2\sqrt{s(s-b)(s-a)(s-c)}}{a}$$
$$A = \frac{1}{2} \cdot a \cdot \frac{2\sqrt{s(s-b)(s-a)(s-c)}}{a}$$
$$= \sqrt{s(s-a)(s-b)(s-c)} \text{ (shown)}$$

S2. Find the area of the triangle whose sides are 6 cm, 10 cm and 12 cm.

Area of triangle $= \sqrt{14(14-6)(14-10)(14-12)}$ cm$^2$

$= 29.9$ cm$^2$ (3s.f.)

S3. Find a triangle (Heronian triangle) whose sides are of integer-valued lengths, and whose area is 36 cm$^2$. Is it the only one?